

STELLINGEN

BEHORENDE BIJ HET PROEFSCHRIFT

EFFECTS OF FADING AND INTERFERENCE IN NARROWBAND LAND-MOBILE NETWORKS

J.P.M.G. LINNARTZ

DELFT, 19 DECEMBER 1991

I

Aanwezigheid van 'Rayleigh fading' in een radiosysteem houdt in dat de natuur een Laplace-transformatie nabootst van prestatiekenmerken die direct samenhangen met het ontvangen signaalvermogen. (Hoofdstukken 3 en 8; Verhulst et al., "Slow frequency hopping multiple access for digital cellular radiotelephone", IEEE JSAC, Vol. 2 (1984), No. 4)

II

In enkele recente wetenschappelijke artikelen over radionetten met vrije-ruimtepropagatie wordt niet onderkend dat regelmatig (cellulair) frequentiehergebruik alleen mogelijk is doordat radiosignalen langs het aardoppervlak sterker gedempt worden dan in de vrije ruimte. (Hoofdstuk 8; G. Schilling, "Eindelijk weten we waarom het 's nachts donker is", *de Volkskrant*, 27 april 1991)

III

De cellulaire netstructuur met vaste frequentie-herhaalafstand is ontwikkeld voor circuitgeschakelde communicatie en is niet optimaal voor pakketgeschakeld berichtenverkeer. (Hoofdstuk 9)

IV

Bij prestatieanalyse van smalbandige mobiele netwerken is realistische modellering van de kanaalfading van groter belang dan nauwkeurige modellering van het detectieproces in de ontvanger. Desondanks wordt in recente publicaties grotere aandacht geschonken aan het detectieproces. (Hoofdstuk 7)

V

De Nederlandse politiek stoelt haar mediabeleid eenzijdig op het specifieke karakter van het medium televisie. De hiervan afgeleide regelgeving beperkt de radio-omroep dusdanig in zijn mogelijkheden dat er hierdoor nog meer luisteraars afhaken dan er al door de televisie-omroep in de avonduren worden weggetrokken. (J.P.M.G. Linnartz, "Radio in de schaduw van het succes van de televisie" *Informatie en informatiebeleid*, Vol. 8, No. 4, Winter 1990)

VI

Verkeersbeheersing van geïntegreerde telecommunicatienetten kan worden gestoeld op een keuze tussen kleine *vertraging* of kleine *kans op vermindering* van een bericht. Op overeenkomstige wijze kan de beheersing van het personenverkeer gebaat zijn bij een keuze tussen korte *reistijd* of meer *autonomie en privacy* van de mobilist. (J.P.M.G. Linnartz, "Verkeer, met snelheidsbegrenzers of met de lichtsnelheid, *Informatie en informatiebeleid*", Vol. 9, No. 3, Herfst 1991)

VII

Wetenschappelijk onderzoek in de telecommunicatie ondervindt beperkingen door het ontbreken van een kanaalcapaciteitstheorema waarbij, anders dan in de vermaarde wetten van Shannon, rekening wordt gehouden met synchronisatieproblemen, als zender en ontvanger niet bij voorbaat over dezelfde tijdsreferentie beschikken.

VIII

Leonardo da Vinci (1452-1519) liep in zijn aanwijzingen voor het tekenen van een boom, met name de dikte van zijn stam en takken, vooruit op de stroomwet van Kirchhoff (1824-1887) (E.H. Gombrich, *Art and illusion*, 1960; Leonardo da Vinci, *Diagram van de groei van bomen*, Bibliothèque de l'Institut de France, Parijs)

IX

Zowel de wetenschappelijk onderzoeker als de beeldend kunstenaar observeren en analyseren (natuurlijke of kunstmatige) systemen en formuleren hun eigen modelmatige wereld.

X

De neiging vooral progressieve en grensverleggende uitingen van kunst museaal te pousseren heeft een negatieve invloed op de volwassenwording van een cultuureigen, karakteristieke vorm van kunst.

XI

Veel grote efficiency-operaties leiden ertoe dat steeds meer werk wordt verzet door een steeds kleinere groep mensen, waarvan velen niet meer zijn opgewassen tegen de extra werkdruk. Dit heeft een averechts, destabiliserend effect op de efficiëntie waarmee de maatschappij omgaat met haar arbeidspotentieel.

XII

De gewoonte om krantekoppen opvallend, provocerend of aansprekend te formuleren leidt er regelmatig toe dat lezers niet meer beseffen dat een bepaald artikel over een voor hen interessant onderwerp gaat.

XIII

De door sommige pessimistische systeemontwerpers gehanteerde 'wet van behoud van hoeveelheid problemen' geldt hooguit als het systeem al optimaal ontworpen is.

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IN NARROWBAND LAND-MOBILE NETWORKS**

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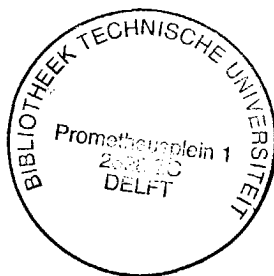
**INVLOED VAN SIGNAALFLUCTUATIES EN INTERFERENTIE
OP SMALBANDIGE LAND-MOBIELE NETWERKEN**

Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft
op gezag van de Rector Magnificus, prof. drs. P.A. Schenck,
in het openbaar te verdedigen ten overstaan van een commissie
aangewezen door het College van Dekanen
op 19 december 1991, te 14.00 uur

door

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geboren te Heerlen,
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Cover by Martin Linnartz, Bunde, The Netherlands, November 1991.

To my father and mother

PREFACE

Throughout history, man has shown a particular interest in communication. The demand for communication is closely related to mobility. Currently, a rapid growth of 'mobile communication' is seen, which is made feasible by recent technological developments. Mobile communication involves (radio-) communication in which at least one of the terminals can be in motion. Congestion of the radio frequencies allocated for mobile services requires highly efficient use of the electromagnetic spectrum.

This dissertation studies the performance of mobile radio systems with narrowband radio channels. It includes several results of research previously published by the author. In particular the effects of multipath and shadow 'fading' and of mutual 'interference' between mobile users is investigated. To this end, initially the relevant propagation mechanisms are addressed, focusing on statistical channel models. For the planning of real systems, the suitability of some propagation models using terrain data is empirically evaluated. The statistical channel description is used to compute the performance of mobile radio links. Expressions are given for the outage probability and for the average duration of fades, taking account of mutual interference between users. A simplified model for the bit error rate in digital transmission is proposed. Numerical results are given for mobile telephone networks.

To study the performance of packet-switched mobile networks, the statistics of randomly arriving data packets are described for a number of access protocols. Using the assessed properties of physical radio links, the capacity of mobile data networks is computed focussing on the spatial distribution of users over the service area. The network performance is calculated as a function of the distance between the mobile user and the fixed base station. In particular, the worst-case probability of successful access is computed.

Methods have been developed to find the effects of mutual interference between mobile terminals in a network of known dimensions. This allows selection of the most spectrum-efficient design that satisfies a prescribed network performance. Hence, the fundamental results presented or consolidated in this dissertation apply to spectrum-efficient spatial design (and planning) of infrastructures for mobile communication. The work forms a contribution to the joint research project of the three departments of Civil Engineering, Architecture and Electrical Engineering at Delft University of Technology, entitled "Ruimtelijk ontwerp van de toekomstige communicatie-infrastructuur" (Layout of the future communication infrastructure).

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CHAPTER 1

INTRODUCTION

1.1 HISTORICAL SETTING

The interest in communication appears to be a direct consequence of the human intellectual capabilities and our possibility to enhance knowledge and understanding by exchanging information. At the onset of the renaissance, a rational attitude dominated the European culture, illustrated in the statement by Francis Bacon (1561-1626): "The end of our foundation is the knowledge of causes and secret motions of things; and enlarging of the bound of Human Empire, to the effecting of all things possible" [1]. Bacon also stated: "Nam et ipsa scientia potestas est" (knowledge itself is power) [2]. In the same period, René Decartes (1596-1650) concluded "Cogito, ergo sum" [3]. The human knowledge praised in their theories was not necessarily to be

gathered through communication with other people, but speaking highly of knowledge obviously also stimulates communication and exchange of information.

At the end of the 19th century a number of philosophers studied the rapid technological and industrial innovations, and depicted man as a toolmaker [4],[5]. Tools and instruments were also developed to enrich the capabilities of communication. The invention of the printing press in the 15th century facilitated the spread of knowledge from the few to a wider public. Inventions and scientific discoveries, but also new personal ideas and viewpoints could be disseminated [6]-[8]. The printing press stimulated the changes in society in the 16th century and contributed to the end of feudalism [6]. Similarly, after the industrial revolutions that introduced such mechanical aids as the steam engine, *electrical* systems further refined the human use of tools and instruments, and also offered enhanced possibilities to communicate. This led to the development of communication infrastructures, exclusively intended for the exchange of information.

Only a few examples exist of non-electrical communication techniques for transfer of information via other infrastructures than those for physical transport: smoke signals, semaphores and signal flags as used in maritime operations are among them [9]. Early attempts to communicate visual signals by means of the semaphore, a pole with movable arms, were made in the 1830's in France. A similar experimental system was used by the Dutch during the 'ten days campaign' against the Belgian revolt in 1831/1832. In 1837, the House of Representatives passed a resolution requesting the Secretary of the Treasury to investigate the feasibility of setting up such a system in the United States [10]. The market interest in enhanced communication systems was also clearly illustrated by the fact that in 1860 the Pony Express started regular physical message services over land in the U.S. But at the same time, electronic systems for communication had started to develop. *Telecommunication* is defined by the International Telecommunication Union (ITU) as the transmission, emission or reception of any signs, signals or messages by electromagnetic systems [11]. The demonstration of (electrical) telegraphy by Joseph Henry and by Samuel F.B. Morse in 1832 followed shortly after the discovery of electromagnetism by Hans Christian

Ørsted and André-Marie Ampère early in the 1820's. In the 1840's, telegraph networks were built on the U.S. East Coast and in California. Rapid extension of their use followed; the first transatlantic cable was laid in 1858.

In 1864, James Clerk Maxwell postulated wireless propagation, which was verified and demonstrated by Heinrich Hertz in 1880 and 1887, respectively. Marconi and Popov started experiments with the radio-telegraph shortly thereafter, and Marconi patented a complete wireless system in 1897. In 1876, Alexander Graham Bell patented the telephone. The invention of the diode by Fleming in 1904 and the triode by Lee de Forest in 1906 made possible rapid development of long-distance (radio) telephony. The invention of the transistor by Bardeen, Brittain and Shockley, which later led to the development of integrated circuits, paved the way for miniaturisation of electronic systems. The continued advances in micro-electronic circuits have recently made the rapid development of mobile and personal communication systems feasible. Such systems would offer person-to-person communication, giving freedom of movement for the users and, if desired, eliminating the ineffective calls experienced with the fixed telephony service when the user is away from his or her terminal [43]. Moreover, new services, particularly those services employing mobile *data* communication become feasible [14],[34].

The effect of telecommunications on society at large may turn out to be of the same impact as the invention of the printing press [12],[16]. The introduction of new services, with due anticipation of their direct and indirect consequences, often requires conscious policies [17]. Nonetheless, user acceptance of new opportunities appears at least as important as political consensus. One of disciplines relevant to the understanding of the impact of the new developments, and thus also to formulating an information policy, whether liberal or more regulated, is the dynamic market theory [18]: Each innovation has a critical period during its introduction. Case studies [19] indicate that the simultaneous introduction of a new service *and* a new technology has often failed in the past, notwithstanding any impressive advantages the new service would offer. The above example of introduction of the electrical telegraph confirms that it was introduced as an improved technological solution to a service pioneered by

the semaphore and the pony service. As the concurrent experiments with semaphore communication show, the electrical telegraph was thus not a purely technology-push development: a clear interest (market-pull) in communication to bridge physical distances was present. The acceptance of these innovations came about as a novel and better technical implementation of a service previously offered with inferior techniques.

In mobile communication, new technological advances are developed in rapid pace, but experiments with new services and applications are performed also with known, mature, though sometimes inadequate technology. Examples of the latter range from the (not very successful) experiments with mobile data services using wireline telephone modems over mobile voice channels, to exchanging routine information (e.g. about traffic jams) in spoken form. New technologies such as packet-switching (store-and-forward) mobile data networks [20], automatic vehicle location to support fleet management [21], or datacasting of road traffic information to assist optimum choice of vehicle routes [15],[22] may be accepted by potential users once pilot systems have proved their usefulness.

This dissertation addresses some of the technical problems experienced in designing new systems for mobile (voice and data) communication, whereas the aspects of market demand and user acceptance are not treated in the main body of thesis. The aspect of mobility in relation to communication will be discussed in the next section.

1.2 COMMUNICATION AND MOBILITY

Evident in early history is the economic relation between mobility, trade and transport on the one hand, and exchange of information on the other hand. Shortly after the first coins were minted in Lydia in the seventh century B.C., professional currency exchange at the important commercial crossroads was made by the many monetary

sovereignties [23]. The location at crossroads ensured not only a continuous stream of clients but also information from all relevant parts of the ancient trading world.



Fig. 1.1:¹ *Historically, the infrastructure for communication coincided with the infrastructure for physical transport.*

The first news-gathering services were established by the Medici, the Fugger and the Welser families, who set up their own messenger service to assist cross-border commercial operations. Ships and gazettes reported about wars, crop failures, gold finds or natural catastrophes in foreign territories. In the 17th century, Louis Elsevier located the Copenhagen offices of his book printers and sellers company in the local Stock Exchange which ensured rapid communication with international partners. Here, the novel newspaper 'Amsterdam Corantos' was available, with the most up-to-date news items, exchange rates and commodity values, collected worldwide by crews of the Dutch merchant fleet. Hence, the fleet of ships transported not only goods,

¹ The cover and Figures 1.1 - 1.3 and 1.6 of this dissertation have been drawn by Martin Linnartz, father of the author. These pencil drawings are not included as a direct justification or visualisation of the scientific results reported in this dissertation: an artist's interpretation, visualised in these drawings fits the abstracting and delimiting character of an introductory chapter in a theoretical dissertation. By describing real-life systems by (idealised, mathematical) models and interpretations, scientific research has clear analogies with the work of an artist. Both the artist and the researcher observe and analyse (natural or artificial) systems and formulate their own (modelled) world. (see also Figs. 4.1-4.3)

including printed matter and maps, but also business information and newly discovered topographical data. Thus, shipping activities could *distribute* information (e.g. in printed form), and also *contribute* information [24]. The very combination of these two information services allowed the development of Holland into one of the most important international trading nodes during the 'Golden Age'². This historical example illustrates that information was originally communicated via the infrastructure for physical transport (see Fig. 1.1) and that information was merely transferred between locations of human activities.

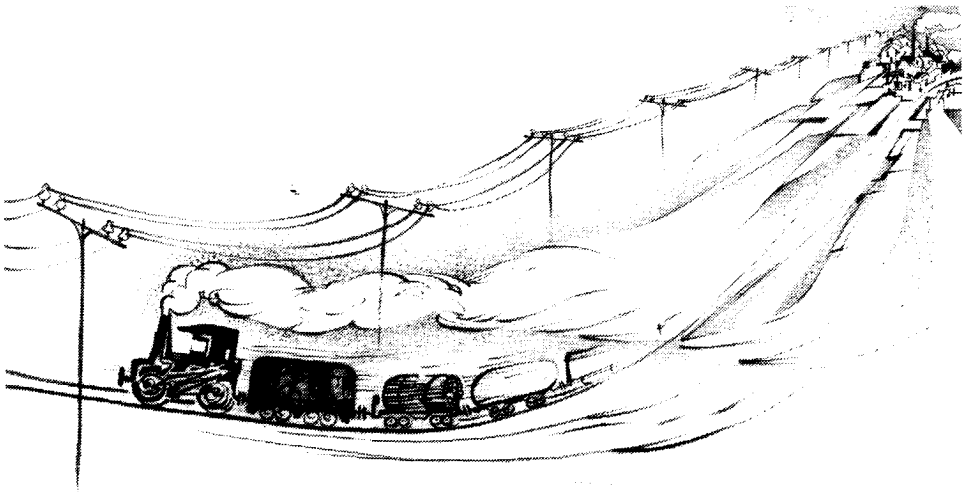


Fig. 1.2: *The invention of the telegraph caused separation of the infrastructures for communication and for physical transport.*

With the advent of *telecommunication*, the spatial relation between information and physical transport changed dramatically. In the early telegraph systems, the origination and destination of information traffic continued to be mainly determined by existing locations of human activities. But, although messages were still mostly sent from city to city, they no longer used media primarily intended for physical transport (Fig. 1.2). Introduction of the most basic telecommunication facilities appeared to sharply

²'De Gouden Eeuw', the (first part of the) 17th century, was a summit in the Dutch culture, sciences and trade.

improve the efficiency of logistics [13], [25]. At present, the improved techniques for the exchange of data allowed by computer communications (e.g., Electronic Data Interchange, EDI) are believed to significantly enhance the efficiency of logistic operations, or to remove market imperfections. The growth of facsimile is explained by the classical need for exchange of messages (between fixed locations) in written or printed form and the ubiquity of an electronic infrastructure, namely, the public circuit-switched telephone network. Still, facsimile messages are rarely sent to mobile users; origin and destination of the messages often coincide with origin, destination or major nodes in the logistic infrastructure.

Mobile communication, i.e., communication in which the origin of the message or its destination (or both) can be in motion, is one of the earliest applications of radio communication techniques. Radio further modified the spatial relation between the infrastructures for telecommunication and for physical transport. Not only are the media spatially separated; the origination and destination of messages generally no longer coincide with the origin or destination of physical transport. For instance, Marconi's wireless telegraph was used in ship-to-shore communications. The possibility of exchanging information with vessels during their motion from one point to another generalised the spatial relation between the intensity of information traffic and the infrastructure for physical transport into a more dynamic and area-oriented mode of operation.

As illustrated in Fig. 1.3, some recently proposed applications of telematics in traffic and transportation [15],[22],[26],[68] would also require support of intensive information streams along roads and highways. For instance, the increasing road congestion calls for improved management or pricing techniques which will require communication between vehicles and roadside base stations or between vehicles participating in road traffic. Satellite vehicle location systems are proposed to track transports of toxic goods [15]. Traveller information for optimum route selection requires broadcasting of traffic data [22].

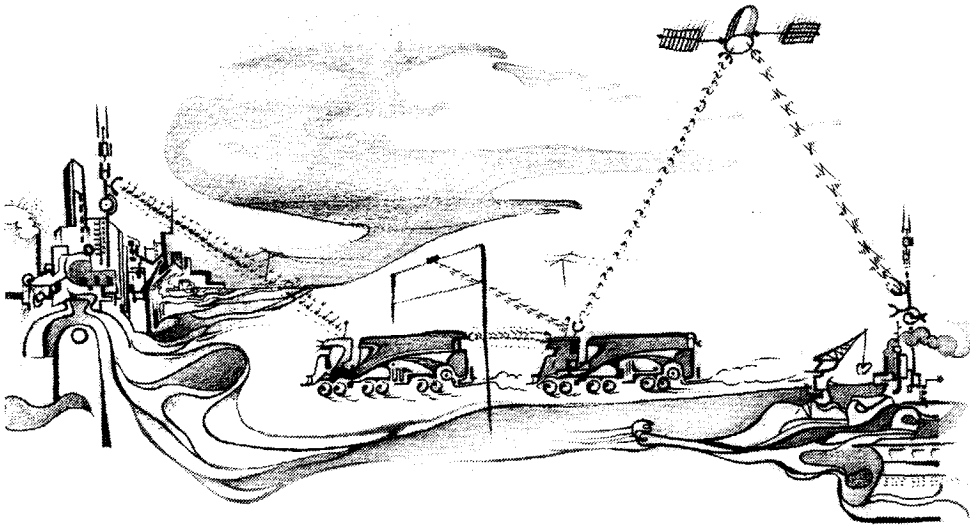


Fig. 1.3: *Telematic services in traffic and transport require a dense infrastructure for communication, which is closely related to the road infrastructure.*

The growth of the road traffic, and the increasing inconvenience and environmental damage caused by congestion, require better use of the infrastructure for physical transport [15]. This creates new demands for mobile communication, in addition to the present desire to extend conventional services (such as telephony) to mobile subscribers. Efficient use of the available radio spectrum and effective spatial management of the tele-traffic appear essential.

These observations illustrate the interaction of mobility and telecommunications throughout history. In dealing with mobile radio communication, this dissertation will particularly address issues of effective spatial use of the radio spectrum in relation to the specific properties of the mobile radio channel.

1.3 TRAFFIC MANAGEMENT IN TELECOMMUNICATION NETS

Where the intensity of the transport of information, people or goods grows beyond some extent, 'traffic' arises and mutual interaction between various individual contributions to this traffic becomes an issue. Traffic management is concerned with weighing the individual interests of the traffic participants and the settlement of potential mutual conflicts.

In recent policy notes on *road* traffic management (e.g. [26]), the word 'management' is used for a wide variety of measures taken to optimise the use of resources, including prior planning and design ('static management') and real-time control ('dynamic management'). In static management, systems and infrastructures are designed and planned to offer optimum performances within certain cost or environmental constraints. On the other hand, in dynamic management, the existing resources are used in such a way that the fluctuating traffic intensities are optimally supported.

In telecommunication networks, the issues of management (in the above wide sense) of intense collective streams of information have been studied for many decades and is still a relevant topic of research. Before discussing the management of telecommunication traffic in more detail, the layered organisation of the exchange of messages in telecommunications networks is described.

1.3.1 LAYERED ORGANISATION OF TELECOMMUNICATION NETS

The 'Open Systems Interconnection' (osi) reference model plays an important role in standardising protocols for exchange of data between complex information systems [27]. While originally intended to design computer protocols in a structured manner, the model recently proved helpful in formulating managerial and political issues

related to telecommunications and mass media, e.g., [17],[28]. In the following sections, the model is used to sketch the role of traffic management tools.

The OSI model consists of seven layers (1-7 in Table 1.1), each of which is specified to support reliable interconnection between two systems, **I** and **II**, irrespective of the technical implementation of lower layers. Each layer adds specific functions to lower layers to enrich the value of the services offered. Table 1.1 shows a division between the lower three and the upper four layers. The lower layers (1, 2 and 3) address 'transport' or 'bearer' services. The higher layers (4-7) add user-oriented functions necessary for services such as Electronic Data Interchange (EDI) or videotex. The combination of layers 1 to 7 offer 'value added' services.

Table 1.1: *(Extended) OSI reference model for exchange of information*

System		Layer	Tasks / Aspects
I	II		
(9)	(9)	Information delivery	Legitimate delivery to parties in economic and social processes
(8)	(8)	Information structure	Structuring and organization of information
7	7	Application layer	Interface to application software and user services
6	6	Presentation layer	Handling and translation of structured data
5	5	Session layer	Coordination and synchronization of the session
4	4	Transport layer	"End-to-end" transfer of data
3	3	Network layer	Routing and switching
2	2	Data link layer	Error control (coding)
1	1	Physical layer	Control of physical circuits
Physical media for interconnection (cables, radio links, optical fibre etc.)			

During the 1980's, the distinction between the three lower and the upper four layers was clearly drawn in legal and regulatory issues (e.g. in considering the extent of the PTT monopoly for transport services [28]). At present, the applications of information and computer systems become increasingly interwoven with telecommunication

networks [29]; this interaction is known as 'telematics'. For a more complete view of telematic services, one may therefore also have to consider the specific user applications which are located above layer 7. A corresponding extension of the OSI layers has been proposed by De Jong [30]: Social and juridical aspects of information transactions are addressed by the information structure layer (8) and the information delivery and access layer (9).

1.3.2 STRUCTURED TRAFFIC MANAGEMENT

In [66], Linnartz and Arnbak discussed a layered structure similar to the model in Table 1.1 to study traffic and transport in wide sense, i.e., including both telecommunications and physical transport. It was noted that in communication networks, and possibly also in infrastructures for physical traffic and transport, management of traffic can be organised in accordance with a layered structure.

1.3.2.1 Management above OSI layer 7:

In medieval Europe, the widespread illiteracy and the lack of appropriate means of communication gave a monopoly of information exchange (at the highest layer in Table 1.1) to the nobility and clergy. The effect of such a control on information flow can be illustrated by the refusal of the Greek Orthodox Church to place clocks in the church towers. This deprived the population of the enhanced time efficiency which the professional activities enjoyed in Roman Catholic Western Europe [4]. The effects of a liberal exchange of information became far more evident with the invention of the printing press in the 15th century and the ensuing developments in the 16th century [4],[6]-[8]. Printing provided a means of communicating huge amounts of information

to a much larger readership. During the reformation, the new possibilities offered by the novel technology were effectively exploited [7],[8]. Freedom of expression of thoughts, news gathering and the exchange of information are now accepted as fundamental human rights in a modern democratic society. Nevertheless, the widespread use of telecommunications and its impact on society have led to continuous political discussions throughout the past decades. The strict definition of aims of public broadcasting in The Netherlands and many other countries is still based on national cultural considerations, though a more liberal international attitude is being enforced, partly because the traditional protection of national broadcasting organisations is undermined by technical developments such as satellite broadcasting [17]. New technologies also inspire reconsideration of the longstanding exclusive monopolies granted to the national PTs on telecommunication services [28]. Liberalisation of the market for terminal equipment, and deregulation in certain transport services, such as mobile radio, have allowed faster changes in the variety of telematic services offered to private and business users.

The tendency towards more liberal regulations for (electronic) exchange of information does not imply a relief from the need for management of information streams at the uppermost layer. On the contrary, the more information becomes abundantly available, the higher the needs for management and structuring of the (authorised) information traffic [64]. For instance, requirements for information security, control of access to data bases, reduction of (distracting) information given to car drivers, avoidance of information overload in control rooms during calamities and selection of relevant documents in data base searches are currently receiving attention from researchers.

Maggi addressed the benefits of (electronic and other forms of) communication to remove uncertainties in business decisions in a formalistic way [31]. According to a recent consultant survey [32], 230.000 labour-hours are lost daily in The Netherlands because of ineffective answering or routing of telephone calls. This is expected to result in a yearly loss of 3 billion guilders. It may be asserted as a work hypothesis that, at the highest layers, the adverse effects of imperfect management of traffic is

usually not a consequence of any imperfect operation of (tele-) communication networks or services; these effects arise from inefficient organisation of activities at the layer of information organisation and information delivery. This is outside the scope of this dissertation.

1.3.2.2 Management at the upper OSI layers

Traffic management at layers 1-7 of the OSI reference model is more within the traditional domain of the telecommunication engineer. At the upper layers, traffic management is generally of a static nature, e.g. attempting to avoid the exchange of excessively redundant data or spreading the peaks of the traffic loads in time.

During the development of digital mobile radio systems, with tight restrictions on spectrum use, efficient voice coding received ample attention. The standard wireline analogue telephone circuit is recommended internationally to have a bandpass characteristic in the frequency range 300 to 3400 Hz with a signal-to-noise ratio of at least 34 dB [33]. To achieve this quality with standard digital PCM transmission, each voice channel requires a bit rate of 32 to 64 kbit/s. To conserve spectrum in digital mobile radio, source coding is applied to reduce the bit rate below 16 kbit/s [34]. The successes in voice coding during the last decade have provided the possibility to encode voice into, say, 10 kbit/s and still retain acceptable voice quality including easy recognition of the speaker. In the pan-European GSM network specifications, 9.6 kbit/s (or half this rate for special applications) is considered [34].

Another technique improving the efficiency of spectrum usage is to transmit digital coded messages rather than to use voice communication. To many groups of professional users of mobile communication, standardised data messages are of interest, and may even replace routine voice communication. Measurements of the characteristics of traffic in typical mobile radio channels show that many messages are short and stereotyped [35]. Most user categories require spoken message lengths of

about 15 seconds, except in taxi dispatch service (with shorter messages of about 8 seconds) and some services with longer messages. Short, stereotyped messages are well suited to be coded in a textual form. With mobile data communication, new services become possible such as vehicle printers or automatic repetition of the messages if the driver has been away from the vehicle. Encryption and processing become possible. If a positioning system is available in the vehicle, automatic remote location of the vehicle from its base are feasible [36]. From a spectrum-efficiency point of view, exchange of coded messages is many orders of magnitude more efficient than transmission of voice, particularly if names of streets etc. are to be transferred.

Tariff structures can be employed to manage telecommunications traffic at the transport layer. Higher tariffs during busy (business) hours and lower tariffs in the evening and at nighttime have proved a successful measure for spreading the peak load of the telephone net with substantial impact on the total volume of the busy-hour traffic, reducing the infrastructure investment required to meet peak traffic demands.

Despite their impact on the spectral efficiency of the network, these aspects of management at the higher OSI layers are not considered further within of this dissertation. However, they do have a number of implications for the design of the system at lower layers: Since highly efficient voice coding techniques became available, digital communication has become highly competitive to analogue voice communication in the attempts to conserve radio spectrum. Therefore, the discussion in the following chapters is not restricted to analogue voice communication. Digital voice communication, and some specific possibilities such as site diversity, is addressed specifically in a number of sections. Moreover, the advantages of computerised *data* messages compared to voice communication have been the motivation in devoting chapters 6 to 9 to packet-switching (store-and-forward) networks.

1.3.2.3 Management at the network layer

In telecommunication networks, static traffic management was first studied at the network layer, particularly by the Danish mathematician A.K. Erlang. In 1917, Erlang addressed the statistical behaviour of a large ensemble of telephone subscribers [37]. His studies offered the potential to effectively dimension the capacity of switches and trunks to the collective demand and to ensure the required grade of service to the individual subscribers. The laws of Erlang played a key role in design and planning at the network layer.

During the recent decades, studies similar to the telephony work by Erlang have been performed to investigate data traffic, e.g. [38]-[40],[65]. Prior to 1970, wire connections were the principal means for communication of data. Dial-up and leased telephone lines provided cost-effective and reasonably reliable means for interchange of data. It was recognised that this (circuit-switched) technology was inadequate for the needs of any computer communication network required to handle bursty traffic, i.e., traffic with a large peak-to-average ratio in the required data rate. Due to developments in computer hardware, around 1970 the cost of switching communication resources had dropped below the cost of the transmission resources (bandwidth) being switched [69]. Accordingly, the technology of packet-switched computer nets emerged as a cost-effective means of data exchange over shared wired channels.

In this dissertation, the discussion of the design of mobile communication networks is mainly limited to the steady-state performance of generic (voice and data) communication systems, given a certain (fixed) traffic intensity. In a number of cases the analyses can be applied to the planning of practical systems, for instance if terrain data is used to predict propagation losses. The effect of fluctuating traffic intensities and dynamic management of the communication resources *at the network layer* is outside the scope of the thesis. It offers an understanding of the 'static management' of mobile communication networks with multiple users.

1.3.2.4 Management at the physical layer

For many decades, multiplexed communication channels (trunks) between switches with bundled traffic from many subscribers, were expensive and thus limited in capacity. From the 1950's, national PRTs established microwave links to overcome the bandwidth and repeater-spacing limitations of coaxial trunk links. Nowadays, optical fibres offer capacity well in excess of what can be reached by any reasonable expansion of the microwave radio relay network, and this technology is likely to dominate in all new trunk systems, though back-up radio systems are maintained to ensure reliability.

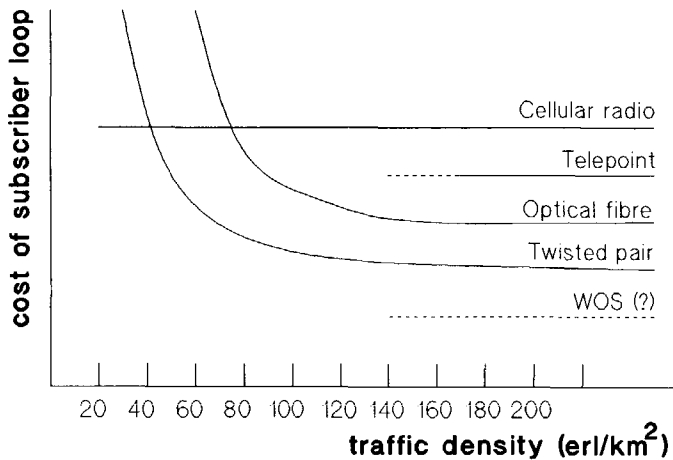


Fig. 1.4: *Relative cost of the subscriber loop versus the spatial traffic density [42].*

Radio communication is expected to have a much greater impact on future communication networks by providing local access for remote or mobile subscribers to the network. The density of telephone traffic covers many orders of magnitude [41]: less than 150 erlang/km² in villages and rural areas to perhaps 100,000 erlang/km² in office buildings. Optimum infrastructures for telecommunication networks may depend on the spatial distribution of the subscribers and the characteristics of the environment [42],[43]. Fig. 1.4 [42] relates the distribution of the telephone traffic intensity to an impression of the cost of the subscriber loop.

In *rural*, thinly populated *areas*, radio communication offers more economic subscriber loops than wired lines (probably including repeater/amplifiers) to every (remote) subscriber.

In *residential areas*, existing copper wires may remain more cost-effective than new radio access. However, new optical loops may eventually become competitive to installing or refurbishing the copper wire loop [44].

In *business centres*, cabled networks are often impractical because of frequent changes in the organisational structure and in the spatial distribution of activities which may lead to prohibitively frequent changes in the telecommunication infrastructure. Radio communication, in particular wireless office systems (wos) and tetherless access to PBXs, may offer much more flexible and economic solutions. The increasing mobility of staff and the desire for more effective exchange of information thus stimulates the development of wireless systems. However, the ensuing increasing demand for radio spectrum is likely to lead to severe shortages of frequency assignments in the near future [45]. Fig. 1.5 gives an estimate of the required frequency spectrum as a function of the user density [41].

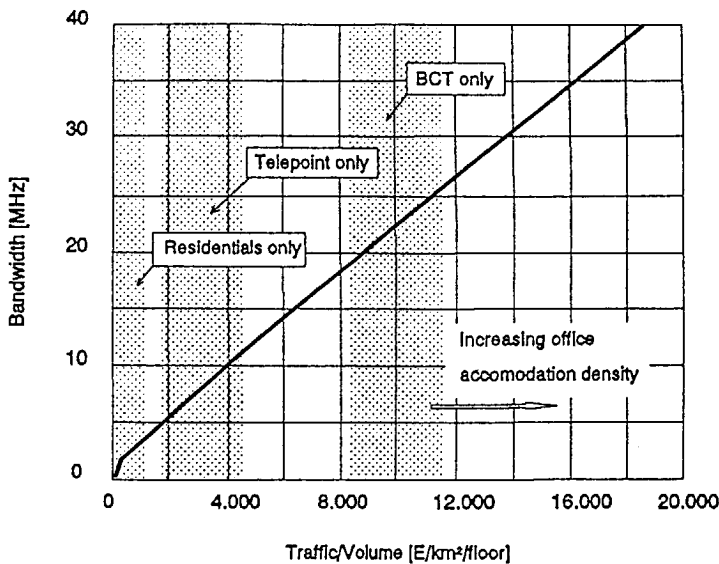


Fig. 1.5: Required radio spectrum in MHz, versus traffic density in $\text{erl/km}^2/\text{floor}$ [41]. (BCT: Business cordless telephone.)

According to various sources (summarised in [45] on p.65), personal communication services may require 40 to 230 MHz of spectrum, depending on the number of subscribers, on the technology employed and on the quantity of services. CCIR study group 8 recommends 60 MHz for low-power, personal tetherless access plus 170 MHz for vehicular radio.

Spectrum efficiency can be improved by using techniques for dynamic management at the lower OSI layers. In 'trunking' systems for voice communication with multiple closed user groups [47], voice channel frequencies are assigned only after a request for a call is received. (This is in contrast to the frequency assignment methods used hitherto, where a frequency assigned to one user group may not be used by other user groups even if the channel is idle). In packet-switched mobile data communications, common use of frequencies is also commonplace [38],[39]. Allowing random access of a large number of mobile terminals to a commonly used part of the radio spectrum requires management of the tele-traffic streams at the physical layer.

It can be concluded that, contrary to the common belief some years ago, radio communication has not been killed by optical communication, since radio nets both allow mobility and have a more attractive area-cost structure than fixed optical nets with their heavy installation costs.

From the above discussion of static and dynamic traffic management issues, it appears that many challenging technical problems are found in the management at the lower layers of digital radio networks, taking into account the physical characteristics of the shared radio channel. As will be illustrated by short literature surveys and reviews in the next chapters, many of the issues of design and planning of mobile networks are yet unsolved or require further investigation. Some of the aspects of static management will be addressed in the following. Dynamic spectrum management is implicitly assumed in the study of cellular telephone or random-access networks, where many users share common radio facilities according to the actual needs at a given moment. This dissertation does not particularly focus on aspects of dynamic behaviour of traffic intensities and the dynamic response of the system. Nonetheless,

in the case of a packet-switched (ALOHA) network, the stability is studied by considering the dynamic behaviour of (queueing of) packet traffic. This is included to verify the validity and accuracy of the stationary models and techniques employed here.

The next sections review the effects of mobility on communication systems and compare a number of radio techniques proposed and/or developed for mobile communications.

1.4 EFFECT OF MOBILITY ON COMMUNICATION SYSTEMS

User movements affect the communication system at many layers:

- At the *physical layer*, channel characteristics vary with the location of the user and because of mobility, vary in time. Because antennas mounted on vehicles have rather unfavourable positions and little 'clearance' (Fig. 1.6), the propagation characteristics in the mobile radio channel are notoriously poor. A mobile radio link is hindered by a number of propagation mechanisms, viz., multipath scattering from objects near the mobile antenna, shadowing by dominant obstacles and attenuation mechanisms on the propagation path between transmitter and receiver [48]. Multipath reception causes rapid fluctuations of the received signal power, whereas shadowing is experienced as a slow amplitude effect (see Fig. 1.7). Adaptive transmit power control can mitigate the adverse effects of individual shadowing to some extent [70]. Since most mobile networks are interference-limited rather than noise-limited, interfering signals are likely to produce substantially greater adverse collective impact if adaptive power control is used to overcome shadowing on their own transmission path. Mobile systems must live with signal fluctuations during transmission. Because of high demands on spectrum efficiency, extensive frequency re-use and mutual interference must be accepted. This means that the classical system analysis inspired by the stationary AWGN channel of conventional information theory may not be applicable.

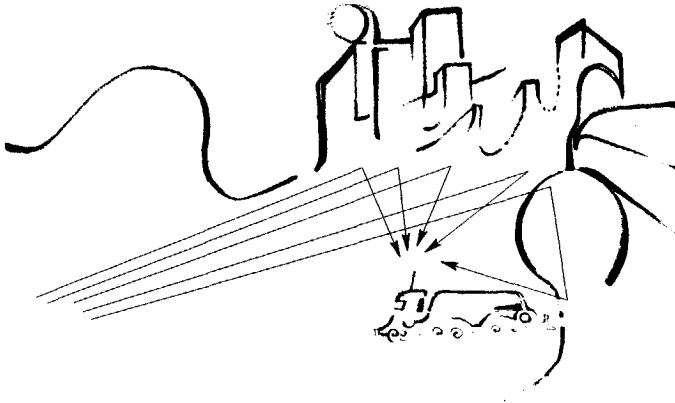


Fig. 1.6: *Multipath reception, caused by obstacles and terrain features in the vicinity of the mobile antenna.*

- At the *data link layer*, channel coding is to be chosen in accordance with the specific character of the fading radio channel. In contrast to AWGN channels with errors randomly distributed in time, burst of bit errors are experienced in the mobile channel. A typical mobile channel allows relatively reliable communication during certain periods, interrupted by other periods of particularly poor communication. The latter are called 'fades'.
- At the *network layer*, the routing of signals may change from time to time as the user moves through the service area of the network. This may require localisation of the terminal (or localisation of a specific user in more sophisticated 'personal' communication systems) to find optimum routing.
- At the *presentation layer*, improved source coding techniques have helped in achieving acceptable spectrum efficiency with digital systems, as discussed in section 1.3.2.2.
- At the *application layer* (and above), personal mobility and transportation of goods require specific communication facilities, as discussed in section 1.2.

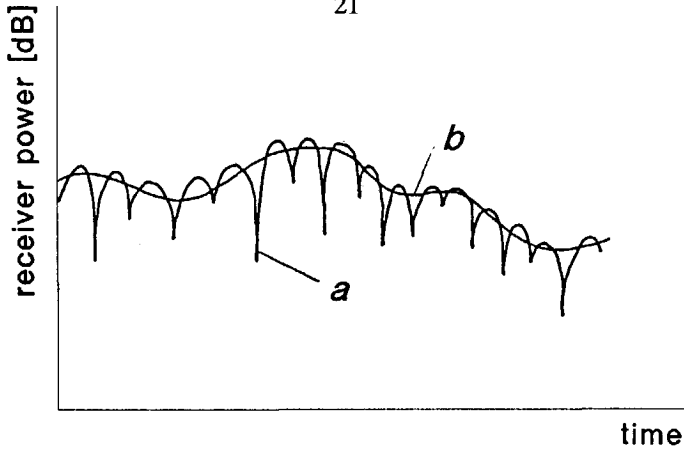


Fig. 1.7: *Instantaneous (a) and mean (b) received signal power (in dB) versus time in a typical mobile radio channel with multipath reception and shadowing.*

Despite this wide range of aspects of mobility, we shall generally focus on the effect that mobility has on channel behaviour (physical layer). It will be shown in later chapters that this indirectly also influences the system performance at higher layers. Above all, it is the effect of mobility on the channel that influences the spectrum efficiency of mobile networks: ample design margins to maintain reliable operation despite fluctuating signal strengths often lead to less efficient use of the spectrum than would be possible with non-fading channels.

1.5 SURVEY OF MOBILE RADIO NETWORKS

A number of different techniques are used to implement land-mobile radio systems. The channel characteristics addressed in this dissertation address narrowband VHF or UHF propagation over ranges larger than a few tens or hundreds of metres and less than, say, several tens of kilometres. Most systems using this type of channels are cellular systems [49]: frequency re-use is applied in a regular, systematic manner. The next subsection describes the cellular system concept and also briefly addresses the

differences with two other common techniques for mobile communication with vehicles, namely short-range communication and land-mobile satellite communication.

1.5.1 CELLULAR NETWORKS

1.5.1.1 Principles of cellular frequency re-use

In the cellular concept, frequencies allocated to the service are re-used in a regular pattern of areas covered by one base station, called 'cells'. In mobile-telephone nets these cells are usually hexagonal [49],[48]. In radio broadcasting, a similar concept has been developed based on rhombic cells [50],[51]. In Fig. 1.8 a typical example of a theoretical cellular structure for radio telephone networks is depicted.

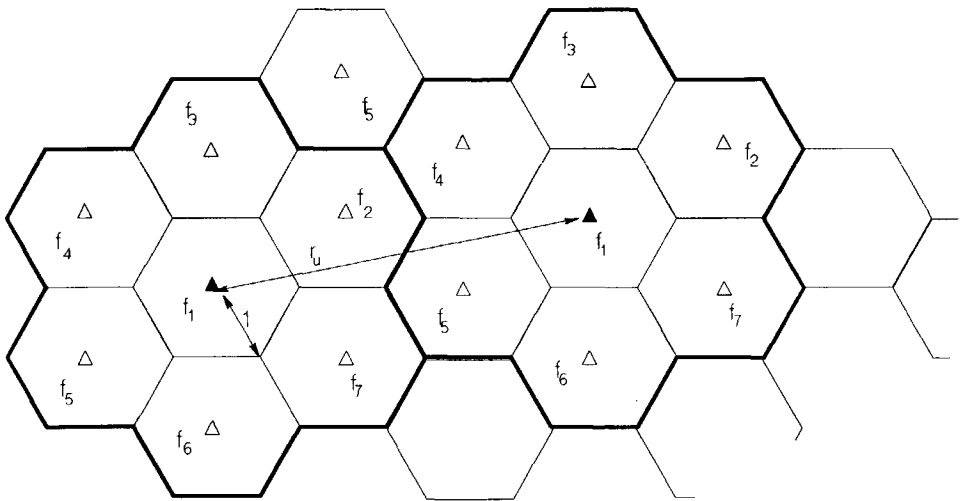


Fig. 1.8: Idealized cell structure according to a hexagonal frequency repetition pattern with clusters of $C=7$ cells.

To ensure that the mutual interference between users remains below a harmful level, adjacent cells use different frequencies. In fact, a set of C different cell frequencies

$f_1, \dots, f_C$ are used for each cluster of C adjacent cells. Clusters patterns and the corresponding frequencies are re-used in a regular pattern over the entire service area. Thus the total bandwidth for the system is C times the bandwidth occupied by a single cell. The closest distance between the centres of two cells using the same frequency (in different clusters) is determined by the choice of the cluster size C and the lay-out of the cell cluster. This distance is called the frequency 're-use' distance. The re-use distance r_u , normalised to the size of each hexagon, is

$$r_u = \sqrt{3C} \quad (1.1)$$

For hexagonal cells, i.e., with 'honeycomb' cell lay-outs commonly used in mobile radio, possible cluster sizes are $C=i^2+ij+j^2$, with integer i and j ($C=1, 3, 4, 7, 9, \dots$). In practical FM or digital cellular networks for public radio telephony, the cluster size is in the order of $C=7$ or 9 , though with special techniques, such as diversity reception, smaller re-use distances can be used [52].

When a mobile terminal moves outside the coverage area of a base station, the network management is assumed to take appropriate measures. A 'handover' to another base station is required to ensure sufficient quality of reception, including acceptable interference power levels. A mobile user experiences the worst average performance if the terminal is located at the boundary of two cells where the distances to base stations are maximum. The performance of the network is therefore often calculated for this case [53].

As seen from the above discussion, cellular engineering combines traffic assignment, interference management and spectrum conservation [54]. Cellular mobile radio differs from previous design strategies for mobile radio in two aspects: frequency re-use and cell splitting. Conventional radio systems were often designed with the objective to have each base station cover the largest possible area by using antennas mounted in high towers and using the maximum affordable power. Different frequencies were assigned to each base station. The lay-out of the base stations and their service areas did not change for the lifetime of the system. In contrast to this,

with cellular systems, the service area is divided into a large number of cells, e.g. as in Fig. 1.8, each with its own base station. The power radiated by the base stations is kept to a minimum, and the antennas are located just high enough above the ground to achieve the desired coverage. This enables non-adjacent cells to re-use the same set of frequencies.

As the demand for services increases, the number of cells may at some time become insufficient to provide the required grade of service. Cell splitting, i.e., subdividing of congested cells into smaller cells and re-using the frequencies in some efficient new pattern, can then be used to increase the traffic handled in a given area without the need for increasing the total bandwidth occupied by the entire system. In a practical environment with non-uniform distributions of the traffic and more significant propagation impairments in certain parts of the service area, optimal solutions lead to tailor-made structures with cells of different size. Often computer aids are invoked, using topographical databases [55].

1.5.1.2 The cellular radio service

During the 1980's, the more advanced national PTTs focused their marketing attention on providing the telephony service to mobile subscribers. The public radio telephone nets had been introduced during the seventies, based on the cellular concept for frequency assignment [34]. In The Netherlands, the first radio telephone net ATF-1 started operation in the 150 MHz band in 1981, followed by ATF-2 at 450 MHz in 1984 and ATF-3 at 900 MHz in 1989 [56]. Introduction of digital cellular telephone net GSM (ATF-4) in the 900 MHz band is planned for the beginning of the nineties. Despite its novelty and usefulness, it may be argued that the public cellular telephone nets are not a full-blown telematic service, but just an extension of the plain old telephone service (POTS) to mobile subscribers [57]. The circuit-switching protocols of the networks have been designed for voice communication lasting a few minutes, but the

digital channels in GSM provide a gross data rate of 16 kbit/s, which can be used for data communication with a bit rate up to 9.6 kbit/s.

Cox [46] concluded from observing the technological developments in the late eighties, that the technology of personal communication is progressing along two completely separate evolutionary paths: high-power *vehicular* cellular mobile radio technology and low-power pedestrian and stationary *cordless* telephone technology. The two techniques are essentially different. The vehicular cellular technology has been developed during the last 45 years, and new systems are still designed to meet the standards of the wireline telephony service. In contrast to this, the history of the handheld cordless telephone is, at most, 15 years and the system is designed with a focus on miniaturisation at the cost of some services. Particularly, restrictions imposed by the power consumption are relevant to handheld sets. Power-demanding digital processing operations, such as forward error-correction coding, speech coding and multipath channel equalisation are extensively employed in the proposed system for future vehicle telephone nets, but still appear impractical in low-powered handheld sets. Reduction of the power consumption during the stand-by mode is one of the reasons why the user of a handheld set can initiate a telephone call, but cannot receive calls unless a paging system is included.

Cellular voice networks are not limited to public telephony services. Closed trunking networks are installed in a number of countries. In 1991, the Dutch TRAXYS commenced operation in parts of the provinces Zuid-Holland, Noord-Holland and Limburg [58]. The advantages to be gained from trunking mobile voice communication for closed user groups are evident from the large quantity of traffic still handled on separate frequencies [47]. Combining this traffic is seen as a new development in mobile radio, although the concept is a simple extension of Erlang's classical results for wired telephone service to the management of radio channel usage.

Funkschau [59] sketched the situation in the Federal Republic of Germany in 1989, when trunking was not widely used. More than 100.000 mobile radio networks for

closed user groups supported traffic from more than 700.000 voice terminals in the business sector, plus 200.000 terminals in the public-services sector. Nine frequency bands, with a total of 550 channels were allocated. Companies in a city like Munich operated a total of 40.000 terminals in 4500 nets. This corresponds to an average of 100 to 150 users, usually belonging to four or five companies, per frequency.

Public packet-switched (store-and-forward) mobile networks for data communication are already in operation, for instance in the Nordic countries and in Canada, while in the U.S.A. a similar network is presently being developed. The Mobitex [20] communication protocol has been optimised for the exchange of data messages with a relatively short length, in the order of a few hundreds of bits or even less. A cellular frequency re-use pattern has been adopted.

1.5.2 SHORT-RANGE NETWORKS

Generally, the interest in short-range communication, i.e., communication over distances ranging from a few metres to a few hundred metres and mostly within line-of-sight, is growing rapidly. One reason is that the user capacity of cellular networks increases if cell sizes are reduced. This currently stimulates the development of micro- and pico-cellular networks, in which cell radii can be substantially smaller than a few hundred metres [60]. For these networks, the name 'personal' communication is often used: most cellular telephone nets are designed for use within cars and may be too bulky and power-consuming to be conveniently portable. With micro- and pico-cellular networks, the design is based on miniaturisation, at the cost of some services [46].

Further, short-range communication is of interest to special services that can be offered by using mobile communication in traffic and transport [15], e.g. control of road traffic flows to avoid congestion by dynamic route guidance or by road pricing.

Microwave line-of-sight or infra-red links, leaky feeders and communication via inductive loops may be considered for these links. In some applications, communication sessions occur at specific sites, but extended coverage areas, e.g. by cellular repetition, are not always required. Other services, e.g. intelligent cruise control, require exchange of information between vehicles, with distributed algorithms for management of the radio traffic.

In wireless office systems, communication also takes place at short range, mostly via indoor radio channels. While the problem of radio access in mobile radio is related to that in wireless office automation systems (wos) [61]-[62], in some respect they differ from each other. The traffic pattern in office automation is often very different from the occasional short standardised messages encountered in mobile radio, since file transfer or routine access to data bases, or data delivery to high-resolution video terminals may be supported. Hence, the required data rates may be much higher in wos than in mobile radio.

Design of systems with short-range communication links requires specific investigation of these propagation paths: the description of the short-range radio channel (including the micro-cellular channel) essentially differs from the radio channel with ranges longer than a few hundreds of metres [60]. Propagation is characterised by short delay spreads and possibly even a dominant line-of-sight component. Moreover, in the case of wos, the channel is almost stationary, whereas the mobile radio channel exhibits dynamic fading that can be fast compared to the duration of a data message. Although some of the analyses reported in this dissertation can, essentially, be modified or extended for short-range communication, general treatment is not attempted in this dissertation.

1.5.3 SATELLITE NETWORKS

Recently, mobile satellite communication has received increasing attention. The potential independence of a local earth-based infrastructure stimulates use of mobile satellite links. Particularly, international fleet management systems [21] employ satellite links for the exchange of messages because satellites can also be used for continent-wide paging and navigation on board the vehicle [36]. The absence of a pan-European earth-based infrastructure for packet data communication has been an important stimulus for satellite communication links between vehicles and their home bases. If fleet management systems would become more widely used in more local applications such as taxi dispatch or packet-delivery services, satellite communication systems may experience the disadvantage that local possibilities of frequency re-use are limited. In due course, communication may thus need to be transferred to ground-based cellular mobile networks. Once again, satellite technology would thus have proven its important ability to offer an early service only later becoming available in the standard fixed infrastructure. Earlier examples of this have been international satellite television programme distribution and video conferencing, now also possible by broadband optical cables, intercontinental digital leased lines and 'hot lines' for journalists and statesmen travelling in less developed regions. In this thesis, we consider the principles of a mature fully-fledged ground-based local radio infrastructure. Hence, innovative mobile satellite networks are outside our scope here, however interesting they may be as a trigger for new global services.

1.6 SCOPE OF THESIS AND SURVEY OF CONTENTS

In conclusion, it has been explained why this dissertation is confined to mobile communication in the VHF and UHF bands, over ranges in the order of a few hundreds metres to tens of kilometres. In this range most cellular systems operate, and the congestion of frequencies is most apparent. Since spectrum efficiency is of major

concern, mutual interference is considered a more important limitation to system performance than noise.

Only narrowband, i.e., frequency non-selective, radio channels are studied. Recently, also wideband transmission techniques, including spread-spectrum transmission of signals with a low user data rate, started to receive considerable attention from researchers throughout the world [71]. Wideband communication is made possible by developments in fast micro-electronic circuits, for instance for adaptive equalisers to combat selective fading. By the time the work reported in this dissertation was completed, definite results for the superiority of either narrowband or wideband concepts were not yet available. It is the author's impression that no general judgement can yet be made on this issue and that the optimum choice will depend upon many conditions. This dissertation is not intended to advocate the use of narrowband transmission techniques one-sidedly, but the results may assist in weighing narrowband and wideband design options.

The (osi) layered organisation of many communication networks has motivated the following overall structure of this dissertation: The physical medium for communication is described in chapter 2 in terms of propagation mechanisms. Based on these models for the physical channel, chapters 3 to 5 describe the performance of the radio link (physical layer). The chapters 6 to 9 are devoted to random multiple-access methods, focusing on the medium-access sublayer and taking account of the performance impairments at the physical layer. Most symbols and acronyms used in these chapters are listed in appendices.

The first part of *chapter 2* presents a review of some models for narrowband propagation, used in studies of UHF/VHF mobile radio networks. Rayleigh fading, shadowing and UHF groundwave propagation are discussed. These statistical models do not require specific terrain data, but are supplemented with results from rural propagation measurements in the VHF and UHF bands. The accuracy and validity of a number of relatively simple (both statistical and deterministic) propagation models are discussed.

Probabilities of signal 'outage' are derived in *chapter 3*, based on the fluctuations of received signal powers discussed in chapter 2. An outage of the wanted signal is assumed to occur if the signal-to-interference (C/I -) ratio drops below a certain threshold value. A Rayleigh-fading and shadowed signal is considered in the presence of multiple fading interference signals and additive white Gaussian noise. The results for signal outage are extended to 'site diversity', i.e., to the case of two cooperating receivers located at geographically distinct sites.

The time constants of fading in the mobile radio channel are considered in *chapter 4*. The threshold crossing rate, i.e., the number of times per second a certain C/I -ratio (called the 'threshold') is crossed, and the average non-fade duration are derived for a wanted Rayleigh-fading signal in the presence of multiple Rayleigh-fading interferers.

In *chapter 5*, an approximate model is proposed for the study of bit errors in a mobile radio channel. The effects of fading of the wanted and interfering signals are taken into account. This chapter is intended to offer a mathematically tractable model for investigation of random-access networks in later chapters.

Throughout chapters 2 to 5, the mobile radio channel and its effects on the reception of a wanted signal are discussed. This situation concerns particularly *cw* communication, such as telephony, where each receiver is expected to support 'wanted' signals from one particular transmitter. In contrast to this, random-access protocols are reviewed in *chapter 6*, in particular slotted ALOHA and Inhibit Sense Multiple Access (ISMA). At this stage the distinction between 'wanted' and 'interfering' signals becomes less obvious than in the chapters 3 to 5: in random-access networks, signals from multiple users compete to capture a common receiver; this receiver is successful if it locks on to (any) one of the signals and correctly decodes the data message [67]. In the case of bursty data communication, the behaviour of the communication protocols influences the amount of interference experienced. The performance of some medium-access schemes is evaluated, based on a generic description of the physical radio channel and of the data link layer. The well-known expression for steady-state throughput of data packets from an infinite population of

terminals, as eg. in [38], are reformulated for the probability of successful reception of an a-priori selected 'test' packet. Moreover, generalised probabilities of successful reception are considered: the specific effects of the mobile radio channel are added later, in chapters 7 to 9.

In *chapter 7*, various models for receiver capture, based on the previous discussion in chapter 3 to 5, are considered for a number of random-access protocols discussed in chapter 6. Models considered include: the probability of signal outage of the packet signal (cf. chapter 3), the influence of the packet duration (cf. chapter 4) and packet error rates for BPSK modulation (cf. chapter 5).

Chapter 8 focuses further on the capture model proposed in chapter 3: correct reception is assumed to occur if C/I -ratio is above a certain threshold. The performance of the ALOHA protocol is assessed for a number of propagation models, in terms of the total throughput, network stability and delay. It is observed that most of the retransmissions occur in areas with poor propagation to the central receiver. This has a disadvantageous effect on the traffic capacity and stability of the network.

Packet-switched networks with cellular frequency re-use are addressed in *chapter 9*. The spatial distribution of packet traffic generation in the service area is considered. The traffic to be offered to achieve a certain required throughput is studied, taking account of collisions with packets from the same cell, as well as interference between co-channel cells and noise. Since the results suggest that the frequency re-use distance can be very small, compared to cellular telephone networks, the case of contiguous frequency re-use, i.e., with the same frequency used everywhere in the entire service area, is also addressed. The common throughput of two receivers with data packets arriving from overlapping areas is computed.

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CHAPTER 2

VHF AND UHF PROPAGATION IN LAND-MOBILE COMMUNICATION

Land-mobile communication is burdened with particular propagation complications compared to the channel characteristics in radio systems with fixed and carefully positioned antennas. The antenna height at a mobile terminal is usually very small, typically less than a few metres. Hence, the antenna is expected to have very little 'clearance', so obstacles and reflecting surfaces in the vicinity of the antenna have a substantial influence on the characteristics of the propagation path. Moreover, the propagation characteristics change from place to place and, if the terminal moves, from time to time.

In generic system studies, the mobile radio channel is usually evaluated from 'statistical' propagation models: no specific terrain data is considered, and channel parameters are modelled as stochastic variables. The generally accepted statistical model for narrowband mobile propagation is summarised and reviewed in the first part of this chapter (section 2.1). Multipath fading, shadowing by local obstacles near the mobile antenna and (large-scale) attenuation mechanisms determined by the path profile in its entirety are discussed. Divergence of radio waves in free space, diffraction and reflection from the ground are the major large-scale mechanisms encountered in VHF and UHF land-mobile radio. For the statistical models employed in generic system analysis, the effect of the large-scale mechanisms is usually simplified by assuming an average attenuation, increasing with distance according to the 'attenuation power law'. This model will be used extensively in the system analyses in the next chapters.

The second part of this chapter (section 2.2) investigates deterministic propagation models, i.e., those requiring specific terrain data. More specifically, the assessment of a VHF/UHF propagation model appropriate for frequency assignment in rural areas is reported. Existing theoretical and empirical models for propagation phenomena, in particular diffraction and groundwave propagation, are verified with field measurements. Empirical models to account for the combined occurrence of multiple large-scale mechanisms are proposed. These results further verify the validity of the statistical models described in section 2.1.

2.1 REVIEW OF STATISTICAL PROPAGATION MODELS

Three mutually independent, multiplicative propagation phenomena can usually be distinguished: multipath fading, shadowing and 'large-scale' path loss [1],[2]. Multipath propagation leads to rapid fluctuations of the phase and amplitude of the signal if the vehicle moves over a distance in the order of a wave length or more. Multipath fading

thus has a 'small-scale' effect. Shadowing is a 'medium-scale' effect: field strength variations occur if the antenna is displaced over distances larger than a few tens or hundreds of metres. The 'large-scale' effects cause the received power to vary gradually due to signal attenuation determined by the geometry of the path profile in its entirety. This is in contrast to the local propagation mechanisms, which are determined by terrain features in the immediate vicinity of the antennas. The large-scale effects determine a power level averaged over an area of tens or hundreds of metres and therefore denoted as the 'area-mean' power $\bar{p}_i$. The index i is introduced to denote a signal from transmitter i .¹ Shadowing introduces additional fluctuations, so the received local-mean power $\bar{p}_i$ varies around the area-mean. The term 'local-mean' is used to denote the signal level averaged over a few tens of wave lengths, typically 40λ [2]. This ensures that the rapid fluctuations of the instantaneous received power due to multipath effects are largely removed. These propagation mechanisms will be described in the next sections.

2.1.1. LARGE-SCALE ATTENUATION

The characteristics of the path profile largely influence the behaviour of the radio link. In statistical models, the path loss is often expressed as a function of the distance between transmitter and receiver d , the corresponding antenna heights h_t and h_r , and the carrier frequency f_c . Although specific terrain data is not considered in statistical analyses, the average behaviour of the attenuation as a function of range is influenced by terrain parameters. The principal contributing large-scale mechanisms are free-space loss, groundwave propagation and diffraction.

¹ The index $i=0$ or j will be used to indicate a wanted (or reference) signal, whereas $i=1,2,\dots$ is mostly used to denote interference. In our studies of systems employing multiple receivers, the index A_i is used to denote signal powers at receiver A from transmitter i .

Free space loss:

In free space, the energy of radio waves diverges over an area proportional to the square of the propagation distance d . For isotropical antennas, the free space loss A_{fs} , expressed in dB, is

$$A_{fs} = 20 \log \left(\frac{4\pi d}{\lambda} \right), \quad (2.1)$$

with λ the wave length.

Groundwave loss:

Waves travelling over land interact with the earth's surface. For propagation over a plane earth, a general, analytical expression of the received signal has been derived by Norton [3] and simplified by Bullington [4],[5]. For isotropic antennas, the field strength is written as the complex sum of a direct line-of-sight wave, a wave reflected from the earth's surface and a surface wave [1], viz.,

$$E_t = E_{0t} \left(1 + R_c e^{j\Delta} + (1 - R_c) F(\cdot) e^{j\Delta} + \dots \right), \quad (2.2)$$

where R_c is the reflection coefficient, E_{0t} the theoretical field strength for propagation in free space, $F(\cdot)$ the complex surface wave attenuation, Δ the phase difference between the direct and reflected wave and j the imaginary number $\sqrt{-1}$. In UHF land-mobile communication, the propagation loss can be approximated by taking account of only the first two terms in (2.2), since $|F(\cdot)| \ll 1$ for the usual values of h_t and h_r . The phasor sum of the direct wave and the ground-reflected wave is known as the 'space wave' [6]. Its path loss is

$$A_G = -20 \log \left(\frac{\lambda}{4\pi d} \left| 1 + R_c e^{j\Delta} \right| \right). \quad (2.3)$$

If $d \gg 5h_t, h_r$, the phase difference Δ can be approximated [1]

$$\Delta \approx \frac{4\pi h_t h_r}{\lambda d} \quad (2.4)$$

and for most types of terrain the reflection coefficient approaches $R_c \rightarrow -1$ [1],[7]. The corresponding excess loss A_R , relative to free-space attenuation, is

$$A_R \approx -20 \log \left(2 \sin \left(\frac{2\pi h_r h_t}{\lambda d} \right) \right). \quad (2.5)$$

Except for communication at very short range, one may assume $d \gg 4h_r, h_t$. The total path loss can then be approximated as

$$A_K = A_{fs} + A_R \approx -20 \log \left(\frac{h_r h_t}{d^2} \right). \quad (2.6)$$

In this case, the field strength thus diminishes proportionally to the inverse of the fourth power of the distance. Experiments confirmed the fourth power law also for less perfect conditions of the surface [8]. At shorter range, the effect of groundwave attenuation is less pronounced, so the decrease with the signal strength is less rapid [9].

The excess loss A_R is called 'ground-reflection' loss. Nevertheless, the attenuation is not caused by superposition of the direct wave with a *specular* reflection in any clearly defined point on the path: It is the interaction of the radio wave with the earth's surface over the entire path which contributes to the second term. In land-mobile radio, a substantial part of the path profile may lie within the first Fresnel zone, so almost the entire path profile contributes to the existence of the ground-reflected wave. However, (2.5) and (2.6) can be derived from a 'two-ray model' [1] with a direct wave and a specularly reflected wave with reflection coefficient $R_c = -1$.

Diffraction loss:

If the direct line-of-sight between the terminals is obstructed, radio waves experience attenuation caused by diffraction. An exact solution is known for the diffraction loss caused by an idealised obstacle with the shape of a 'knife edge' located perpendicular

to the line-of-sight in free space. The geometry of the path is illustrated in Fig. 2.1. The diffraction parameter v is defined as

$$v \triangleq h_m \sqrt{\frac{2}{\lambda} \left(\frac{1}{d_t} + \frac{1}{d_r} \right)}, \quad (2.7)$$

where h_m is the height of the obstacle, and d_t and d_r are the terminal distances from the obstacle (Fig. 2.1).

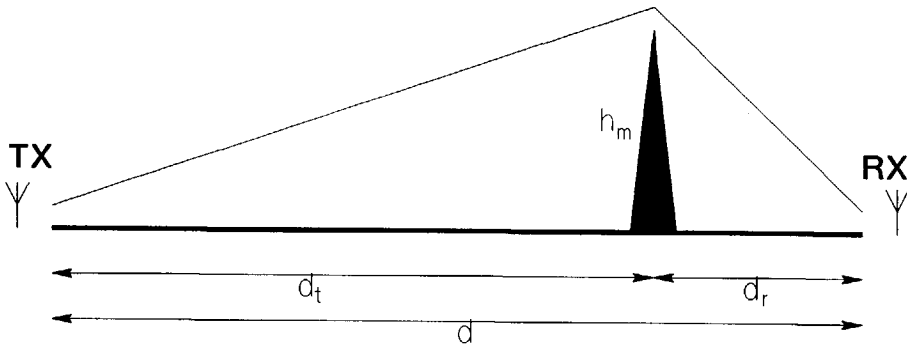


Fig. 2.1 Path geometry for a single knife edge in free space.

This definition implies that the obstacle h_m obstructs $\frac{1}{2}v^2$ Fresnel zones of the radio path. The diffraction loss A_d , expressed in dB, is closely approximated by [10]

$$A_d = \begin{cases} 6 + 9v - 1.27v^2 & 0 < v < 2.4 \\ 13 + 20 \log v & v > 2.4 \end{cases} \quad (2.8)$$

The total path loss is on the form of $A_K = A_{fs} + A_d$. The single knife-edge approximation of the path profile is not very realistic in VHF or UHF mobile radio communication; usually, the rest of the ground profile also influences the electromagnetic field at the receiver antenna. Approximation of a path profile by 'multiple knife edges' (MKE) is a frequently applied refinement. Techniques to estimate the loss over such a sequence of knife edges have been proposed by Bullington [11], Deygout [12] and Epstein and Peterson [13]. An analytical comparison of these

approximation techniques was reported in [14],[15]. Meeks [16],[17] found that measured path losses in forestal terrain at $f_c=110$ MHz are in good agreement with estimates proposed in [13]. The issue of the conversion from terrain profiles to an appropriate idealised multiple knife-edge model has been raised, but not uniquely resolved. In 1969, Okumura [18] presented a highly accurate propagation model for mobile communication in urban areas. The method takes detailed terrain data into account. Empirical curves for diffraction loss in [18] are claimed to be more accurate than the theoretical models, especially closely behind a large obstacle ($d_r < d$). The latter observation is also made by Preller and Koch [19]: closely behind an obstacle, losses are difficult to predict since reflected radio waves may enhance the received signal strength to higher levels than would occur due to diffraction only.

For typical path profiles encountered in practical mobile communication, the large-scale attenuation is caused by a combination of the above three mechanisms, interacting in a complicated manner. Theoretical discussion of the combined occurrence of reflection and diffraction loss is scarce. Bullington proposed an approximate technique to express the path loss over a knife edge located on a flat, plane earth [11], but results are approximate. It is stated that the influence of groundwave attenuation is reduced if substantial diffraction loss occurs over the path [20]. In some rare events, the path loss over a path with a sharp (knife-edge type) obstacle is less than in the absence of the obstacle [46]. Blomquist suggested the empirical estimate of the path loss [20]

$$A_K = A_S + \sqrt{A_d^2 + A_R^2}, \quad (2.9)$$

but most models, e.g. [1],[21], assume a simpler addition of losses

$$A_K = A_S + A_d + A_R, \quad (2.10)$$

where A_R is an empirically modified version of (2.5). Combined large-scale mechanisms will be further discussed in section 2.2, where detailed terrain profiles are considered, and empirical findings are reported.

If the detailed terrain profile is unknown, the total (area-mean) path attenuation A_K is usually written on the form of

$$A_K = 10\beta \log d + A_0 \quad (2.11)$$

with A_0 and β empirical constants. As seen from (2.1) and (2.6) theoretical values are $\beta=2$ in free space and $\beta=4$ in the case of propagation over a highly conducting plane earth. The statistical relation between the diffraction loss and the path length d depends strongly on the type of terrain and on the type of antenna positions considered. Egli [8] proposed the semi-empirical model

$$A_K = 40 \log d + 20 \log \left(\frac{f_c}{40 \text{ MHz}} \right) - 20 \log h_r h_t . \quad (2.12)$$

Evidently, this model is based on groundwave propagation, but an empirically determined frequency-dependent term ' $20 \log f_c / 40 \text{ MHz}$ ' is included. Other empirical values reported for the propagation exponent β in the UHF bands are 3 to 4 for distances in the range $3 \text{ km} < d < 50 \text{ km}$ [1],[2]. Anticipating upon field measurements to be reported in section 2.2 and 2.3.3, Fig. 2.2 presents measured average path loss for propagation at $f_c = 450 \text{ MHz}$ in the North German Plain. These curves would suggest that UHF propagation in forested areas is principally a diffraction effect in free space; the exponent was found to be $\beta \approx 2.6$, only somewhat higher than the theoretical value of $\beta=2$ for unobstructed propagation in free space. On the other hand, for relatively open areas, the plane-earth model is more appropriate ($\beta \approx 3.8$). The entire set of measurements showed a ' $32 \log d$ ' trend, i.e., ($\beta=3.2$), which is in agreement with a number of other experimental results, e.g. [1]. A more detailed description of the measurements is given in section 2.2, and in [22],[23].

In our analyses of multi-user networks from chapter 3 onwards, all transmitters will be assumed to have identical transmit power. For ease of notation, the normalised distance r_i and the normalised power $\bar{p}_i$ are defined such that

$$\bar{p}_i = r_i^{-\beta} , \quad (2.13)$$

where usually the distances r_i are normalised such that for the wanted signal $0 < r_0 < 1$.

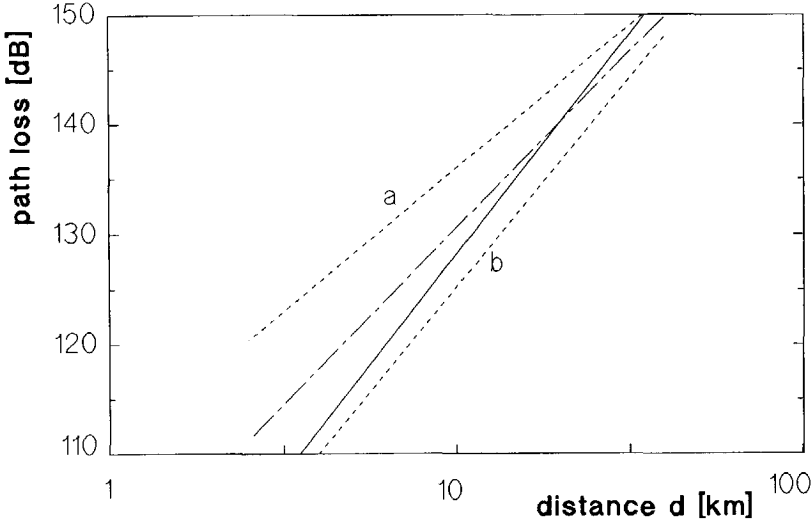


Fig. 2.2: (---) Aera-mean path loss measured in the North German Plain at $f_c=450$ MHz and antenna heights $h_r, h_t \approx 21$ m (see section 2.3.3). (- -,a) forestal paths, (- -,b) paths over open area, (—) model by Egli [8].

Throughout this thesis, mostly the path loss law $\beta=4$ is used in numerical examples, because this value is supported by the theoretical model for UHF-groundwave propagation, because this value appeared realistic in certain environments [8], and because a number of related papers, e.g. [51], also consider the case $\beta=4$. Moreover, $\beta=4$ produces some closed-form solutions for analyses of the random-access networks addressed in chapter 8 and 9. It is to be noted that in interference-limited networks, a fast decay of signal power (large β) gives a more favourable performance than a slow decay (small β). As a result, our curves for $\beta=4$ may be somewhat optimistic for areas with relatively irregular terrain where $\beta \approx 3$.

2.1.2 SHADOWING

Experiments reported by Egli [8] showed that the received local-mean power fluctuates with a 'log-normal' distribution about the area-mean power, with the area-mean power on the form of (2.12). By 'log-normal' is meant that the local-mean power expressed in logarithmic values, such as dB or neper, has a normal (i.e., Gaussian) distribution. The probability density function (pdf) of the local-mean power is thus on the form

$$f_{\bar{p}_j}(\bar{p}_j | \bar{\bar{p}}_j) = \frac{1}{\sqrt{2\pi} \sigma_s \bar{p}_j} \exp \left\{ -\frac{1}{2\sigma_s^2} \ln^2 \left(\frac{\bar{p}_j}{\bar{\bar{p}}_j} \right) \right\}, \quad (2.14)$$

where σ_s is the logarithmic standard deviation of the shadowing, expressed in natural units. The standard deviation s_s expressed in dB is found from $s_s = 4.34 \sigma_s$ [24],[25]. In (2.14), the area-mean power $\bar{p}_i$ corresponds to the *logarithmic* average μ_1 of the received local mean power, with μ_k defined as

$$\mu_k \triangleq \int_0^\infty [\ln(\bar{p}_j)]^k f_{\bar{p}_j}(\bar{p}_j | \bar{\bar{p}}_j) d\bar{p}_j, \quad (2.15)$$

so $\mu_1 = \ln(\bar{p}_i)$. The logarithmic variance is found from $\sigma_s^2 = \mu_2 - \mu_1^2$.

The *linear* average of the local-mean power is [24], [25]

$$E[\bar{p}_j] \triangleq \int_0^\infty \bar{p}_j f_{\bar{p}_j}(\bar{p}_j | \bar{\bar{p}}_j) d\bar{p}_j = \bar{\bar{p}}_j \exp \left\{ \frac{\sigma_s^2}{2} \right\}. \quad (2.16)$$

The linear average is thus not identical to the area-mean power, except in the limiting case of $\sigma_s = 0$.

For average terrain, Egli reported a logarithmic standard deviation of $s_s = 12$ dB [8]. Such large fluctuations are caused not only by local shadow attenuation by obstacles in the vicinity of the antenna, but also by large-scale effects leading to a coarse

estimate of the area-mean power. This log-normal fluctuation was called 'large-area shadowing' by Marsan, Hess and Gilbert in [26]; over semi-circular routes in Chicago, with fixed distance to the base station, s_s was found to range from 6.5 dB to 10.5 dB, with a median of 9.3 dB. Large-area shadowing thus reflects shadow fluctuations if the vehicle moves over many kilometres.

In contrast to this, in most papers on mobile propagation, only 'small-area shadowing' is considered: log-normal fluctuations of the local-mean power over a distance of tens or hundreds of metres are measured. Marsan et al. [26] reported a median s_s of 3.7 dB for small area shadowing. Preller and Koch [19] measured local-mean powers at 10 m intervals and studied shadowing over 500 m intervals. The maximum standard deviation experienced was about $s_s=7$ dB, but 50% of all experiments showed an s_s less than 4 dB.

Extending the distinction between large-area and small-area shadowing, the definition of shadowing is generalised here to cover any statistical fluctuation of the received local-mean power about a certain area-mean power, with the latter determined by (predictable) large-scale mechanisms. Multipath propagation is separated from shadow fluctuations by considering the local-mean powers. This definition implies that the standard deviation of the shadowing will depend on the geographical resolution of the estimate of the area-mean power. A propagation model which ignores specific terrain data produces about 12 dB of shadowing [8]. On the other hand, prediction methods using topographical data bases with unlimited resolution can, at least in theory, achieve an s_s of 0 dB. Thus, it can be argued that the standard deviation is a measure of the impreciseness (and hence the economy!) of the terrain description. It is a matter for network planners to consider how large s_s can be tolerated. A further discussion of this issue is deferred to section 3.2.1.

2.1.3 MULTIPATH FADING

In mobile communication with relatively low antenna positions, the received signal is usually a superposition of a large number of reflected waves. Wave interference therefore leads to fast fluctuations of the field strength if the antenna position is varied. Throughout this thesis, only narrowband channels are considered: the interarrival times of the reflected waves are small compared to the time scale of the variations in the modulating signal. In other words, in narrowband channels all reflections are assumed to add coherently (phasor addition).

We now consider an unmodulated carrier transmitted by terminal i . In a typical 'Rician-fading' channel, the received carrier is on the form of

$$v_i(t) = (C_i + \zeta_i) \cos \omega_c t + \xi_i \sin \omega_c t, \quad (2.17)$$

where C_i represents the direct (line-of-sight) component and ζ_i and ξ_i represent the in-phase component and quadrature component of the sum of the reflections. If the mobile terminal is in motion, ζ_i and ξ_i are functions of time, though this is not denoted explicitly. Rician fading occurs if the central limit theorem can be applied for the cumulations of in-phase components and quadrature components of each individual reflection. This occurs if the number of reflections is large and none of the reflections substantially dominates the joint reflected power. In this case, ζ_i and ξ_i are independently Gaussian distributed random variables with identical pdf's, on the form of $N(0, \bar{q}_i)$, i.e., with zero mean and variance equal to the local-mean reflected power $\bar{q}_i$. The received carrier $v_i(t)$ can also be expressed in terms of the amplitude ρ_i and the phase θ_i

$$v_i(t) = \rho_i \cos(\omega_c t + \theta_i), \quad (2.18)$$

with

$$\begin{aligned}\rho_i &= \sqrt{(C_i + \zeta_i)^2 + \xi_i^2} \\ \theta_i &= \arctan\left(\frac{C_i + \zeta_i}{\xi_i}\right).\end{aligned}\quad (2.19)$$

The instantaneous amplitude ρ_i has the Rician pdf [9]

$$f_{\rho_i}(\rho_i | \bar{q}_i, C_i) = \frac{\rho_i}{\bar{q}_i} \exp\left(-\frac{\rho_i^2 + C_i^2}{2\bar{q}_i}\right) I_0\left(\frac{\rho_i C_i}{\bar{q}_i}\right), \quad (2.20)$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind and zero order. The local-mean power is $\bar{p}_i = \bar{q}_i + \frac{1}{2}C_i^2$. The K -factor of the Rician distribution is defined as the ratio of the power $\frac{1}{2}C_i^2$ in the dominant component and the power $\bar{q}_i$ in the scattered component.

In the special case of 'Rayleigh fading' [1],[2],[27],[50], the power of the direct line-of-sight signal is small compared to the reflected signal power ($C_i \rightarrow 0$, $K \rightarrow 0$). In this case, the variance of ζ_i and ξ_i is equal to the local-mean power $\bar{p}_i$; the phase θ_i is uniformly distributed over $[0, 2\pi)$ and the instantaneous amplitude ρ_i has the Rayleigh pdf [50]

$$f_{\rho_i}(\rho_i | \bar{p}_i, C_i=0) = \frac{\rho_i}{\bar{p}_i} \exp\left(-\frac{\rho_i^2}{2\bar{p}_i}\right). \quad (2.21)$$

The corresponding total instantaneous power p_i ($p_i = \frac{1}{2}\rho_i^2 = \frac{1}{2}\zeta_i^2 + \frac{1}{2}\xi_i^2$) received from the i -th mobile terminal is exponentially distributed about the mean power, viz.,

$$\begin{aligned}f_{p_i}(p_i | \bar{p}_i) &= f_{\rho_i}(\rho_i | \bar{p}_i) \left| \frac{d\rho_i}{dp_i} \right| \\ &= \frac{1}{\bar{p}_i} \exp\left\{-\frac{p_i}{\bar{p}_i}\right\}.\end{aligned}\quad (2.22)$$

Measurements of mobile radio channels confirmed the Rayleigh distribution of the signal envelope in urban areas [1], while a dominant component C_i appeared present in open (rural) areas and in short-range communication, e.g. in micro-cells [9],[27].

2.1.3.1 Logarithmic moments of received signal power

If samples of the received power of a Rayleigh fading signal are statistically processed in dB values, one may define the k -th logarithmic moment m_k as

$$m_k \triangleq \int_0^{\infty} \left(10 \log \frac{p_i}{1 \text{ mW}} \right)^k \frac{1}{\bar{p}_i} \exp \left\{ -\frac{p_i}{\bar{p}_i} \right\} dp_i . \quad (2.23)$$

This can be solved for $k=1$ using known expressions for the Laplace transform of the logarithm (see (32.117) in [28])

$$m_1 = \frac{10}{\ln 10} \left[-\gamma + \ln \left(\frac{\bar{p}_i}{1 \text{ mW}} \right) \right] \approx 10 \log \left(\frac{\bar{p}_i}{1 \text{ mW}} \right) - 2.5 \text{ dB} , \quad (2.24)$$

where γ is Euler's constant ($\gamma \approx 0.577$) [29]. It follows that, for a Rayleigh fading signal, the *logarithmic* local-mean power m_1 will be some 2.5 dB below the *linear* power local-mean $\bar{p}_i$. From Laplace transform (32.119) in [30], the logarithmic variance is found to be

$$s_r^2 = \frac{100}{\ln^2 10} \left[\frac{\pi^2}{6} + \gamma + \gamma^2 \right] . \quad (2.25)$$

Thus a Rayleigh-fading signal shows a logarithmic standard deviation of about $s_r \approx 6$ dB.

2.3.1.2 Rate of fading

In mobile radio channels with high terminal speeds, Rayleigh fading causes the signal to fluctuate rapidly: If the mobile antenna moves a small distance ϵ , the k -th incident wave, arriving from the angle ϕ_k with respect to the instantaneous direction of motion, experiences a phase shift of

$$\Delta\theta_k = 2\pi \frac{\epsilon}{\lambda} \cos\phi_k. \quad (2.26)$$

It is often assumed that the angle ϕ_k is uniformly distributed within $[0, 2\pi)$ [1]. Hence, if ϵ is in the order of half a wave length λ or more, the phases of all incident waves become mutually uncorrelated, thus also the amplitude of the total received signal becomes uncorrelated with the amplitude at the point of departure. The normalised covariance $L(\epsilon)$ of the electric field strength for an antenna displacement ϵ is on the form of [1]

$$L(\epsilon) = J_0^2\left(2\pi \frac{\epsilon}{\lambda}\right). \quad (2.27)$$

with $J_0(\cdot)$ the zero-order Bessel function of the first kind [29]. It is seen in Fig. 2.3 that the signal remains almost entirely correlated for a small displacement, say $\epsilon < \lambda/8$, but becomes rapidly independent for larger displacements, say for $\epsilon > \lambda/2$.

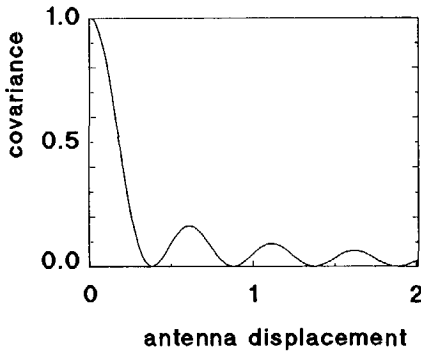


Fig. 2.3: Auto-covariance $L(\epsilon)$ of the electric field strength in a Rayleigh fading channel versus the normalised antenna displacement ϵ/λ (or $T_e f_m$) in horizontal direction.

The antenna displacement can also be expressed in the terminal velocity v and the time difference T_ϵ between the two samples ($\epsilon = vT_\epsilon$). So

$$\frac{\epsilon}{\lambda} = vT_\epsilon \frac{f_c}{c} = T_\epsilon f_m, \quad (2.28)$$

with f_m the maximum Doppler shift ($f_m = vf_c/c$). The rate of fading is further considered in chapter 4.

2.1.3.3 Coherence bandwidth

The foregoing properties of Rayleigh fading hold for unmodulated carrier signals as well as for narrowband signals. Earlier, the property 'narrowband' was defined in the time domain, considering interarrival times of reflections and the time scale of variations in the signal caused by modulation. Transformed into constraints in the frequency domain, this criterion is found to be satisfied if the transmission bandwidth does not substantially exceed the 'coherence' bandwidth B_c of the channel, which is the bandwidth over which the channel transfer function remains virtually constant, with [1, p.51]

$$B_c = \frac{1}{2\pi \Delta_t}, \quad (2.28)$$

where Δ_t is the delay time spread of the significant reflections.

For a digital signal, dispersion is experienced as frequency selective fading and intersymbol interference (ISI). No serious ISI is likely to occur if the symbol rate r_s is low, say if $r_s \ll (2\Delta_t)^{-1}$.

In mobile radio, delay spreads are mostly in the range from $\Delta_t \approx 0.1 \mu\text{s}$ to $10 \mu\text{s}$ [1]. A typical delay spread of $1/4 \mu\text{s}$ corresponds to a coherence bandwidth of $B_c \approx 640 \text{ kHz}$.

Measurements made in the U.S.A. [30],[31], indicated that delay spreads are usually less than $\Delta_t \approx 0.2 \mu\text{s}$ in open areas, about $\Delta_t \approx 0.5 \mu\text{s}$ in suburban areas, and about $\Delta_t \approx 3 \mu\text{s}$ in urban areas. Measurements in the Netherlands showed [32] that delay spreads are relatively large in European-style suburban areas, but rarely exceed $\Delta_t \approx 2 \mu\text{s}$. However, large distant buildings such as flats occasionally cause reflections with excess delays in the order of $25 \mu\text{s}$. In European densely and uniformly built (historical) centres, delay spreads may be somewhat less than in urban (business) centres in the U.S.A., although no experimental evidence in the literature has been found by the author.

2.1.4 COMBINED SHADOWING AND RAYLEIGH FADING

In the case of cellular VHF and UHF radio networks with propagation paths of $d=1$ to 60 km , the large-scale and small-scale mechanisms described in section 2.1.1 to 2.1.3 are generally assumed to be multiplicative, i.e., the narrowband propagation channel may be seen as a number of cascaded stochastic attenuators. Moreover, the attenuators (multipath fading, shadowing and path loss) are mostly assumed mutually independent.

For given area-mean power $\bar{p}_i$, the conditional pdf of the instantaneous power is [33],[34]

$$f_{p_i}(p_i|\bar{p}_i) = \int_0^{\infty} \frac{1}{\bar{p}_i} \exp\left\{-\frac{p_i}{\bar{p}_i}\right\} \frac{1}{\sqrt{2\pi} \sigma_{\bar{p}_i}} \exp\left\{-\frac{\ln^2 \frac{\bar{p}_i}{\bar{p}_i}}{2\sigma_s^2}\right\} d\bar{p}_i. \quad (2.29)$$

This is sometimes called a Suzuki distribution [33]. No analytical solution for this integral is known to the author.

In a number of network studies, the position of the terminal is a stochastic variable, too. If the pdf of the distance between the mobile terminal and the base station is denoted as $f_r(r)$, and if the area-mean is included on the form of (2.13), the unconditional pdf of the received power becomes, after combining shadowing,

$$f_{p_i}(p_i) = \int_0^{\infty} \int_0^{\infty} \frac{1}{\bar{p}_i} \exp\left(-\frac{p_i}{\bar{p}_i}\right) \frac{f(r)}{\sqrt{2\pi} \sigma_s \bar{p}_i} \exp\left\{-\frac{(\ln \bar{p}_i - \beta \ln r)^2}{2\sigma_s^2}\right\} dr d\bar{p}_i. \quad (2.30)$$

For some special cases of the $f_r(r)$, solutions for the integral over r can be found on closed form.

2.1.5 LAPLACE TRANSFORMS OF RECEIVED POWERS

To facilitate the analysis in the next chapters, the Laplace image of the signal power received in a mobile channel with combined Rayleigh fading and shadowing is now discussed, following our initial discussion in [35]. Laplace images of a pdf are particularly useful in the evaluation of the joint power received from multiple interfering signals. In general, the one-sided Laplace transform of a function $f(x)$ is defined as [29]

$$\mathcal{L}\{f(x), s\} \triangleq \int_0^{\infty} \exp(-sx) f(x) dx, \quad (2.31)$$

where s is the image variable. Moreover, Laplace images are closely related to 'moment-generating functions' and 'characteristic functions' probability theory, although mostly the kernel ' $\exp\{-j\omega x\}$ ' with $j=\sqrt{-1}$ is used for characteristic functions, rather than ' $\exp\{-sx\}$ ', as in (2.31).

The pdf (2.29) is now inserted for $f(x)$ in (2.31). The Laplace image of the power received from the i -th transmitting terminal, with area-mean power $\bar{p}_i$ is

$$\begin{aligned} \mathcal{L}\{f_{p_i}, s\} &= \int_0^{\infty} \int_0^{\infty} \frac{1}{\bar{p}_i} \exp\left\{-sx - \frac{x}{\bar{p}_i}\right\} \frac{1}{\sqrt{2\pi} \sigma_s \bar{p}_i} \exp\left\{-\frac{\ln^2 \frac{\bar{p}_i}{\bar{p}_i}}{2\sigma_s^2}\right\} d\bar{p}_i dx \\ &= \int_0^{\infty} \frac{1}{1+s\bar{p}_i} \frac{1}{\sqrt{2\pi} \sigma_s \bar{p}_i} \exp\left\{-\frac{\ln^2 \frac{\bar{p}_i}{\bar{p}_i}}{2\sigma_s^2}\right\} d\bar{p}_i, \end{aligned} \quad (2.32)$$

where the outer integral has been solved analytically. After substitution of

$$x \triangleq \frac{1}{\sqrt{2}\sigma_s} \ln\left(\frac{\bar{p}_i}{\bar{p}_i}\right), \quad (2.33)$$

the Laplace transform of a Suzuki pdf is found to be

$$\mathcal{L}\{f_{p_i}(p_i), s\} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2) dx}{1+s\bar{p}_i \exp(\sqrt{2}x\sigma_s)} \triangleq \phi_o(s\bar{p}_i). \quad (2.34)$$

where, for ease of notation, the function $\phi_o(s)$ was defined as

$$\phi_o(s) \triangleq \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2) dx}{1+s \exp(\sqrt{2}x\sigma_s)}. \quad (2.35)$$

For channels without shadowing ($\sigma_s=0$), this image becomes

$$\phi_0(s) = \frac{1}{1+s}. \quad (2.36)$$

This obviously corresponds to the Laplace image of the exponential pdf (2.22). Fig. 2.4 illustrates the image $\phi_o(s)$ for $\sigma_s=0, 1.36$ and 2.72 ($s_s=0, 6$ and 12 dB).

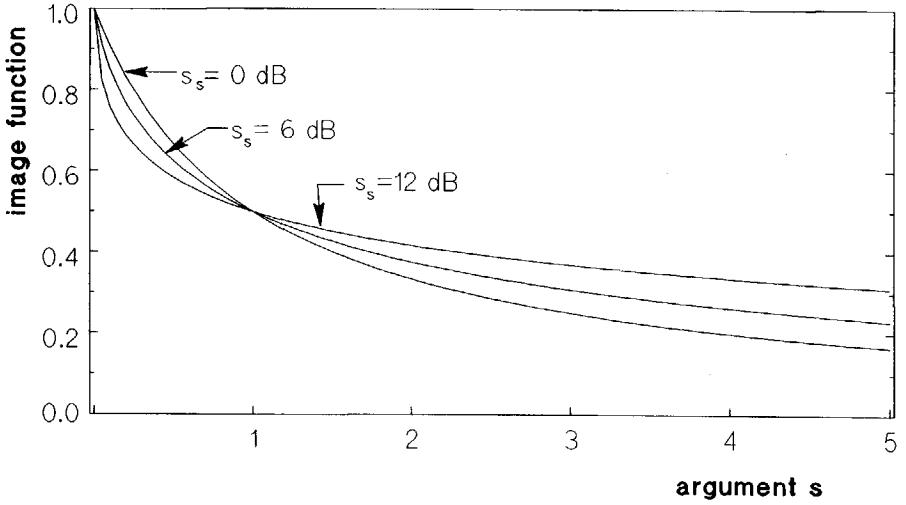


Fig. 2.4: Laplace image of a Suzuki pdf for 0, 6 and 12 dB of shadowing, versus its argument s .

It is noted that

$$\phi_o\left(\frac{1}{s}\right) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{s \exp(\sqrt{2}\sigma_s y)}{1 + s \exp(\sqrt{2}y\sigma_s)} e^{-y^2} dy = 1 - \phi_o(s), \quad (2.37)$$

where x has been substituted by $-y$. It follows that $\phi_o(1) = 1/2$ for any σ_s .

Assuming that the order of integration and differentiation may be interchanged, the m -th derivative of the image is

$$\begin{aligned} \frac{d^m \phi_o(s)}{ds^m} &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx e^{-x^2} \frac{d^m}{ds^m} \frac{1}{1 + s \exp(\sqrt{2}\sigma_s x)} \\ &= (-1)^m m! \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2 - m\sqrt{2}\sigma_s x)}{(1 + s \exp(\sqrt{2}\sigma_s x))^{m+1}} dx. \end{aligned} \quad (2.38)$$

Since the image $\phi_\sigma(s)$ also corresponds to the moment-generating function of the pdf of the instantaneous received power, the m -th order moment corresponds to the m -th derivative of the image in the point $s=0$. One finds $\phi_\sigma(0)=1$ and

$$\begin{aligned} \left. \frac{d^m \phi_\sigma(s)}{ds^m} \right|_{s=0} &= (-1)^m m! \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-x^2 + m\sqrt{2}\sigma_s x) dx \\ &= (-1)^m m! \exp\left\{\frac{1}{2}m^2\sigma_s^2\right\}, \end{aligned} \quad (2.39)$$

where the m -th linear moment of the pdf equals $(-1)^m$ times the m -th derivative (2.39). A full Taylor expansion near $s=0$, with

$$\phi_\sigma(s) = \sum_{m=0}^{\infty} (-1)^m s^m \exp\left\{\frac{1}{2}m^2\sigma_s^2\right\} \quad (2.40)$$

diverges, because the moments rapidly increase with m . Nonetheless, the first- or second-order terms proved useful in determining the approximate behaviour of $\phi_\sigma(\cdot)$.

Recently, immediately prior to the printing of this dissertation, further suggestions to evaluate the image function were offered to the author. A summary is reported in Appendix C, but further study is left as a recommendation. Numerical results in chapter 3 and 8 have been obtained with the Hermite polynomial method

$$\int_{-\infty}^{\infty} f(x) \exp(-x^2) dx = \sum_{l=1}^k w_l f(x_l) + R_k \quad (2.41)$$

The weight factors w_l at the sample points x_l for a k -point integration are given in table 25.10 of [29]. The remainder R_k equals

$$R_k = \frac{k! \sqrt{\pi}}{2^k (2k)!} f^{(2k)}(\xi) \quad (2.42)$$

for a certain $\xi \in (-\infty, \infty)$.

2.1.6 RECIPROCITY

Two-way communication requires facilities for 'inbound', i.e., mobile-to-fixed, as well as 'outbound', i.e., fixed-to-mobile communication. In circuit-switched mobile communication, such as cellular telephony, the inbound and outbound channel are also called the 'uplink' and 'downlink', respectively. The propagation aspects described above are valid for inbound and outbound channels. This is understood from the reciprocity theorem: If, in a radio communication link the role of the receive and transmit antenna are functionally interchanged, the instantaneous transfer characteristics of the radio channel remain unchanged. In mobile multi-user networks with fading channels, the reciprocity theorem does not imply that the inbound channel behaves identically as the outbound channel. Particular differences occur for a number of link aspects:

i) *man-made noise levels*

The antenna of the base station is usually mounted on an appropriate antenna mast, such that it does not suffer from attenuation caused by obstacles in its vicinity. The mobile antenna, on the other hand, is at most mounted a few metres above ground level. The man-made noise level, particularly automotive ignition noise, is likely to be substantially higher at the mobile antenna than at the base station antenna.

ii) *effect of antenna diversity and antenna directivity*

Multipath scatters mostly occur in the immediate vicinity of the mobile antenna. The base station receives more or less a TEM wave, whereas the mobile station receives a superposition of a set of reflected waves from random angles. Two antennas at the mobile terminal are likely to receive uncorrelated signal powers if their separation is more than a wave length (see (2.27)). At the base station site, however, all reflections arrive from almost identical directions. Therefore, diversity at the base station requires much larger separation of the antennae to ensure uncorrelated received signal powers at the two antennas [1]. For the same reason, antenna directivity has different effects at the mobile and the base station.

iii) *correlation of shadow fading of wanted signal and interfering signals*

In a cellular network, shadow fading of the wanted signal received by the mobile station is likely to be correlated with the shadow fading of the interference. In contrast to this, at the base station, shadow fading of the wanted signal presumably is mostly statistically independent from shadow fading of the interference. However, experimental results for correlation of shadow attenuation are scarce.

iv) *full-duplex channels*

In full-duplex operation, multipath fading of inbound and outbound channel, which operate at widely different frequencies, may be entirely uncorrelated.

v) *multiplexing and multiple access*

In a practical multi-user system with intermittent transmissions, inbound messages are sent via a *multiple-access* channel, whereas in outbound channel, signals destined for different users can be *multiplexed*. In the latter case, the receiver in a mobile station can maintain carrier and bit synchronisation to the continuous incoming bit stream from the base station, whereas the receiver in the base station has to acquire synchronisation for each user slot. Moreover, in packet-switched data networks, the inbound channel has to accept randomly occurring transmissions by the terminals in the service area. Random-access protocols are required to organise the data traffic flow in the inbound channel, and access conflicts ('contention') may occur.

In cellular networks with large traffic loads per base station, spread-spectrum modulation can be exploited in the downlink to combat multipath fading, whereas in the uplink, the signal powers from the various mobile subscribers may differ too much to effectively apply spread-spectrum multiple access [36].

vi) *industrial design*

From a practical point of view, the downlink and the uplink will be designed under entirely different (cost) constraints, such as power consumption [38], size, weight and other ergonomic aspects, energy radiated into the human body [37], and consumer cost aspects.

vii) *data traffic patterns*

In packet data networks applied for traffic and transportation, the characteristics of the data traffic flows are known to differ for the uplink and the downlink. For instance it was reported in [38] that (outbound) messages from a fleet management centre to the vehicles are more routine type, of a more uniform length and occur in a more regular pattern than messages in the opposite (inbound) direction.

2.1.7 TIME SCALES

Time variations of signals and system parameters play a key role in this study of mobile communication systems. This section summarises the various time scales T_i ($1 \leq i \leq 10$) considered in the observation of the system. Each time scale indicates an order of magnitude of the rate of variations caused by a certain phenomenon.

T_1 *Carrier frequency*

The frequency f_c of the carrier determines by far the fastest time scale $T_1 \approx 1/f_c$. The propagation models discussed here are appropriate in the frequency range of, say, $f_c = 150$ MHz to 2 GHz. So, the duration of a carrier cycle is in the order of $T_1 \approx 10^{-9}$ s. Only the interarrival time between the fictitious direct and ground-reflected wave in Eq. (2.2) are in the order of magnitude T_1 .

T_2 *Interarrival times between various obstacle-reflected waves*

If obstacles are present near the mobile antenna, reflected waves are received with interarrival times of, at most, a few microseconds. T_2 indicates the order of magnitude of the delay time spread Δ_t (see section 2.3.1.3).

T_3 *Bandwidth of the modulating signal*

A time scale T_3 is related to the bandwidth W of the modulating signal ($T_3 \approx 1/W$) or to the bit duration ($T_3 \approx T_b$). Only narrowband systems are considered, so modulation is

assumed to be slow as compared to the interarrival times of the various obstacle-reflected waves ($T_2 < T_3$).² The effect of delayed reflections ($T_3 < T_2$) can be described as intersymbol interference in the time domain or as frequency selective fading in the frequency domain [40].

T_4 *Propagation delays between transmitter and receiver*

The order of magnitude of the propagation time between transmitter and receiver is denoted as T_4 . In some systems, T_4 can be larger than the duration of a few bits. This delay may affect the performance in some full-duplex systems with feedback, for instance of feedback information on access conflicts between contending users in random-access networks. In addition, different distances of mobile users to the base station may require extension of time slots in slotted access systems.

T_5 *Duration of a data packet*

Blocks of L bits are considered ($T_5 \approx LT_3$, $L \gg 1$). In our analysis of slotted packet radio networks, the duration of data packets is assumed fixed. The duration of a data packet is often taken as the unit of time for analyses concentrating on protocol issues.

T_6 *Rate of Rayleigh fading*

The rate of fluctuations of the received signal amplitude determines the duration of intervals during which the channel performance is constant. Eq. (2.27) gives some impression of T_6 . The word 'instantaneous' is used to denote a value averaged over a period substantially shorter than T_6 . Gilbert [39] simplified the description of channel behaviour into two states: periods of unsatisfactory performance called 'fades' and periods of satisfactory performance called 'non-fade periods'. The event that the duration of a data block is short with respect to the average non-fade duration is called 'slow fading'. 'Fast fading' occurs when the packet duration is substantially longer than the average length of fade and non-fade intervals.

²Spread-spectrum communication, in which the transmission bandwidth B_T is intentionally enlarged ($B_T \gg W$), is not addressed in this dissertation. We assume narrowband channels with $B_T \approx W$.

T_7 *Rate of shadow fading*

Shadowing is much slower than multipath fading. With normal vehicle speeds, T_7 is in the order of a few seconds [26]. 'Local-mean' averages are taken over an observation interval T with $T_6 \ll T \ll T_7$.

T_8 *Waiting times between retransmissions*

A data block that cannot be decoded correctly by the receiver has to be retransmitted. The waiting time before a retransmission is performed is in the order of T_8 . As in all other studies of packet radio over mobile radio channels known to the author, retransmitted blocks will be assumed to experience uncorrelated Rayleigh fading ($T_6 \ll T_8$).

In most systems for data communication, messages lost because of excessive interference are retransmitted. Under certain conditions [41]-[43], this can lead to nonergodic (unstable) system behaviour: these retransmissions cause additional interference to other users which in its turn increases the number of retransmissions of messages in the system. Unless stated otherwise, stable systems are addressed. This corresponds to the assumption that T_8 is sufficiently large to ensure uncorrelated set of interfering signals during each (re-) transmission.

T_9 *Speed of motion through the service area (near/far fading)*

If the size of the service area is small, e.g. in micro-cellular nets, the position of the vehicle may change during a communication session. It is reasonable to assume $T_7 \ll T_9$. 'Area-mean' averages are taken over an observation interval $T_7 \ll T \ll T_9$.

T_{10} *Data traffic intensities*

Throughout this dissertation, the (Poisson) arrival rate of attempted transmissions is assumed constant.

In the analyses throughout this dissertation, the following relations are always assumed to be satisfied:

Because of the nature of radio communication	$T_1 < T_2$
Because of the nature of land-mobile communication	$T_2 < T_6 < T_7 < T_9$
Because of error-detection coding and terminal addresses	$T_3 < T_5$
Because of the assumption of narrowband systems	$T_2 < T_3$

2.2 ASSESSMENT OF DETERMINISTIC PROPAGATION MODELS

Statistical propagation models as discussed in the previous sections, are mainly used in generic system design. In contrast to this, deterministic models, considering specific terrain data, are used extensively in frequency management and during the planning of cell structures for mobile telephony networks. The effectiveness of the planning of radio links and selection of usable frequencies depends heavily on the availability of appropriate propagation models.

Narrowband propagation measurements in the VHF and UHF bands are analysed in this section. The investigation addresses radio propagation as encountered in point-to-point communication in rural areas, in particular in the ZODIAC network operated by the First Netherlands Army Corps in the North German Plain. The purpose of the investigation was to assess a model appropriate for the prediction of large-scale path losses. On the basis of this model, documented in [44], frequency management is conducted. Operational links cover 5 to 30 km over the slightly hilly terrain: the interdecile range of the terrain, defined as the difference Δh between the highest and the lowest points in the terrain after the extreme highest and lowest 10% of terrain heights have been removed, is in the range 20 to 100 m. The terrain is mainly agricultural, with scattered woodlands. Antenna heights range from 4 to 21 metres, which is often insufficient to exceed the tree-top level. Local propagation phenomena, both multipath reception and shadowing, introduce substantial fluctuations of the local field strength.

In frequency assignment for a complex radio relay network, evaluation of many path profiles is required in order to select the optimum location and frequencies for a relay node. This does not concern the operational paths only, because paths over which interference signals may travel have to be considered as well. Especially in transportable nets, e.g. for emergency aid or military applications, where relay nodes are deployed rapidly and may be relocated frequently, relatively simple algorithms are required for the prediction of propagation losses.

2.2.1 PROPAGATION FORECASTING USING TERRAIN DATA

Forecasting path loss from terrain data can be seen as a three-step process (see Fig. 2.5), namely interpretation and classification of the path profile (*step 1*), evaluation of the effects of relevant mechanisms (*step 2*), and appropriately combining the effects of each mechanism (*step 3*). The major part of the theoretical work on propagation concerns *step 2* only: given a set of path parameters, the theoretical losses due to isolated and idealised mechanisms can be calculated. Well-known examples, already discussed in section 2.1, are the diffraction loss over an obstacle with a known knife-edge geometry, and the loss caused by groundwave propagation over a smooth, plane and perfectly conducting earth, given antenna heights and the path distance. Nonetheless, the reliability of the prediction of a field strength depends on the first and third step. The *first step* is the interpretation of the terrain data by extracting parameters to be used in selected models. It is mainly in this step that many implementation decisions have to be made. Typical questions are "What is the effective height for diffraction of a hill covered by trees in winter?", "When does a terrain irregularity act as one single obstacle and when should it be treated as two (or more) separate hills?" and "What is the effective height of an antenna in an irregular, sloping and forested terrain?" [16],[17].

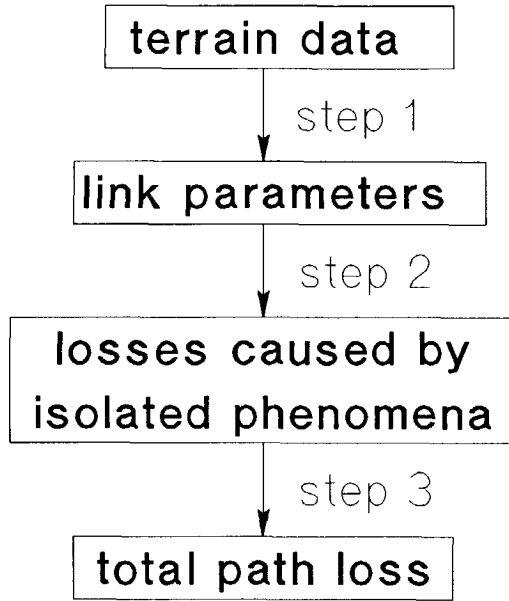


Fig. 2.5: *Forecasting propagation losses as a three-step process.*

Step 3 describes the transition from different theories, and sometimes conflicting approximate models, to a reliable estimate of the path loss. Experience and expertise are required to select and combine the relevant effects. The extent to which mechanisms are present and mutually interact is an important consideration in the selection of a model. Since ample literature is available on the second step, the experiments reported here are concentrating on the first and the third step. For the application considered here, it was concluded from the technical literature that diffraction over single or multiple obstacles and ground-reflection loss were the most relevant mechanisms of propagation. Moreover, fluctuations of the received power level were likely to occur because of local effects.

2.2.2 MEASUREMENT SET-UP

The propagation measurements were carried out in the period 1986 to 1988 by the Directie Materieel Koninklijke Landmacht (D.M.K.L./M.B.A.1) and by the Physics and Electronics Laboratory (F.E.L.-T.N.O.). Initially a set of stereotypical paths in The Netherlands was investigated, to isolate propagation mechanisms such as diffraction (Haarlerberg) and propagation over a plane earth (Flevopolders). The measurements in the North German Plain contained typical, mixed paths, as applied by the ZODIAC net. The transmitter was an Andret 740A signal generator with Ailtech 15100, 35512 and 5020 power amplifiers driven to a power level just below overload. Periodically, 4-second bursts of an unmodulated carrier were transmitted.

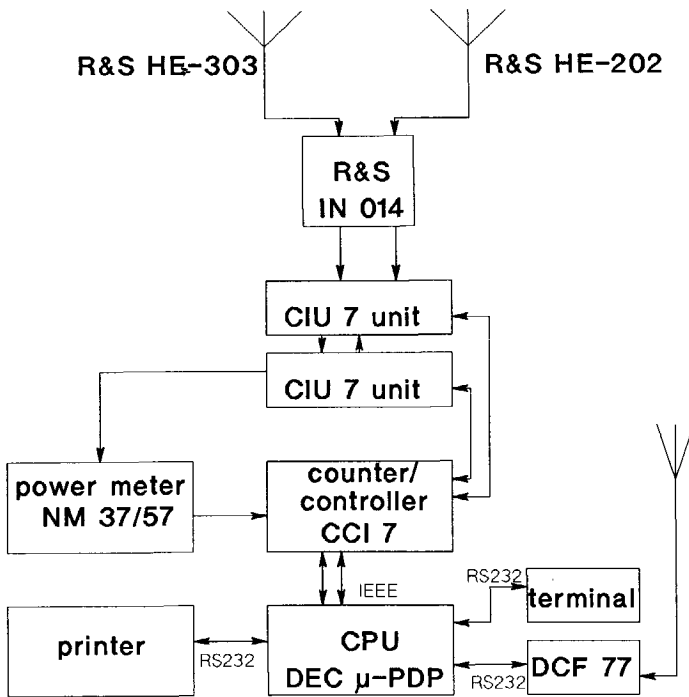


Fig. 2.6: Receiver system set-up for propagation measurements.

The transmit antenna was of a corner-type (Siemens s42044-Q16-A2), positioned at a mast of $h_t=21$ metres. The antenna gain was 15 dB. Antenna positions were chosen in accordance with common practice during military operations. This often resulted in

positions with relatively little antenna clearance and, probably, severe local multipath scattering. The receiving antenna was an active dipole (R&S HE-302 and HE-202) with horizontal polarisation. At the receiver sites, antenna heights were varied from $h_r=4$ to 22 metres. Due to the low antenna directivity and the lower antenna heights, local scattering was presumably more severe at the receiver site than at the transmitter site. An Ailtech NM37/57 CIU7/CCI7 receiver was used to measure the signal power level during each burst (see Fig. 2.6). To verify the reliability of these samples, the background noise level was measured between bursts. The equipment was controlled by an IBM PC at the transmitter site and by a micro PDP at the receiver end. Synchronisation of transmitter and receiver was obtained from the DCF77 time reference transmitter in Frankfurt. The solution of practical problems, such as frequency drift during the measurement of background noise, unbiased selection of the paths and regular calibration of all equipment, is reported in T.N.O. documents, e.g. in [22].

2.2.3 INITIAL, DEDICATED MEASUREMENTS IN THE NETHERLANDS

Measurements in The Netherlands were conducted to get an insight in the relevance of the individual propagation mechanisms. Measurements in the flat, plane and (in 1984 still) relatively bare Flevopolders closely agreed with theoretical propagation over a plane, perfectly conducting earth [45]. Diffraction over an isolated hill was studied based on measurements at seven locations on a path which crosses the Haarlerberg, which is located in a relatively smooth agricultural area near Zwolle and has a height of 55 metres relative to the surrounding area. The paths are characterised in Table 2.1 and Fig. 2.7. Measurements were taken at $f_c=240$ and 400 MHz. At location 1, the path loss closely agreed with the theoretical free-space loss. Observations during measurements in the North German Plain confirmed that, for short links, the measured path losses closely follow the free-space attenuation. Nulls caused by ground reflections have not been observed. Only for the longer paths,

i.e., beyond the turnover point $d=4\pi h_t h_r/\lambda$, the two-ray model (2.6) becomes appropriate unless significant obstacles cause diffraction losses.

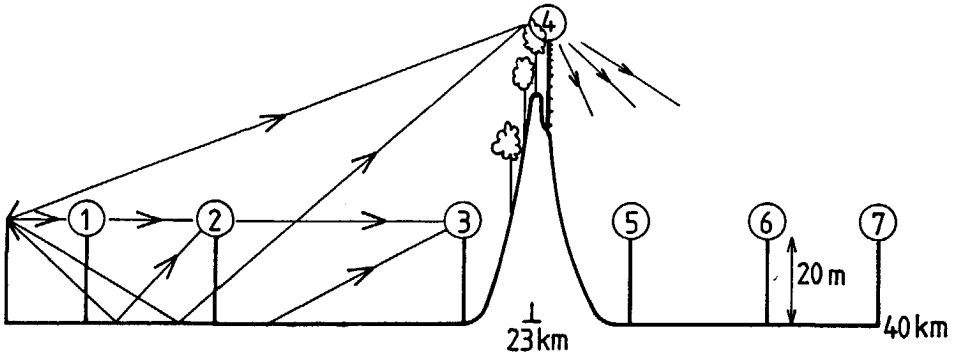


Fig. 2.7: Measurement locations near the Haarlerberg (1,...,7). Antenna height $h_r, h_t \approx 21$ m. Path length: (1) 3.4, (2) 9.4, (3) 21.2, (4) 23.9, (5) 28.3, (6) 34.0 and (7) 39.0 km.

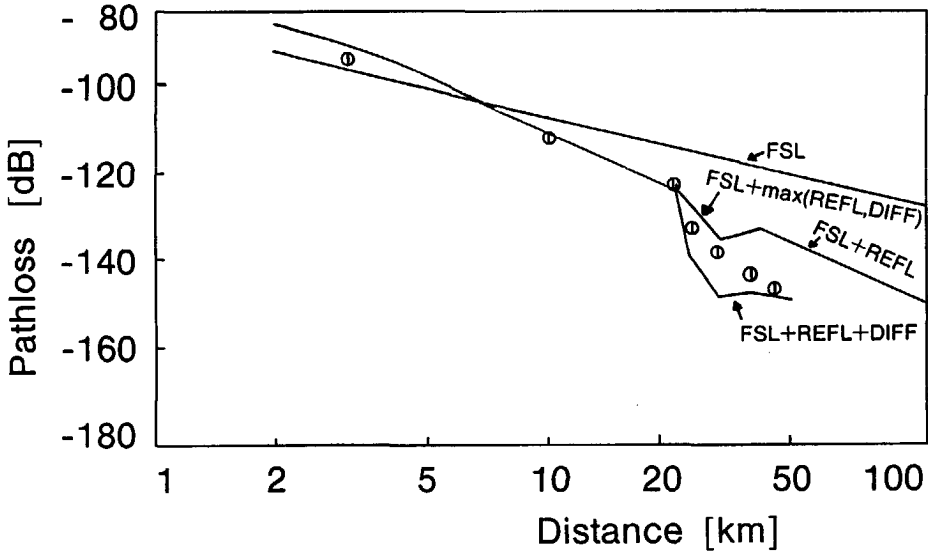


Fig. 2.8: Path loss in dB versus distance at 7 locations (○) and free-space loss (FSL), free-space loss plus the maximum of diffraction and ground-reflection loss ($FSL+MAX(DIFF, REFL)$) and free-space loss plus diffraction and ground-reflection loss ($FSL+DIFF+REFL$).

Table 2.1: *Behaviour of path loss at various locations*

<i>location</i>	<i>path loss</i>	<i>height gain</i>
1	$A_m \approx A_{fs}$	none
2,3	$A_m \approx A_{fs} + A_R$	6 dB/oct
4	$A_m \approx A_{fs} + A_d$	1.5 dB/m
5,6,7	$A_{fs} + A_R + A_d < A_m < A_{fs} + A_R + A_d$	6 dB/oct

The Haarlerberg measurements and the statistical results displayed in Fig 2.2 suggest that single knife-edge diffraction and ground reflection loss may not be independent, multiplicative mechanisms. Simple addition of logarithmic losses, as in the model $A = A_{fs} + A_d + A_R$, do not lead to an optimum model: In this case, interaction of both mechanisms is neglected. Moreover, the attenuation exponent β would exceed 4, which contradicts the measurements. On a path with dominant ground-reflection losses, isolated obstacles tends to temper reflection losses [46]. A more appropriate and accurate model for VHF / UHF propagation should change smoothly from a diffraction model (for paths mainly covered by vegetation) to a plane earth model (for paths over farmland). Antenna height gain tests [22] near the Haarlerberg further strengthened the impression that diffraction and reflection dominate different parts of the path.

2.2.4 STATISTICAL ANALYSIS OF MEASUREMENTS

2.2.4.1 Standard deviation of prediction error

Whenever a measurement of a path loss A_m is conducted, a discrepancy e with the loss A_K predicted by a certain model K is experienced. This discrepancy is caused by a number of effects. The statistical evaluation conducted here is based on the assumption that these effects are mainly multiplicative, i.e., the error introduced by

each effect is essentially an attenuation (or amplification) by a number of dBs. For A_m , A_K and e expressed in dB, the logarithmic standard deviation s_K of the error experienced when a propagation model A_K is verified, is defined as

$$s_K^2 \triangleq E[e - E[e]] = E[(A_m - A_K - E[A_m - A_K])^2] \quad (2.43)$$

where any systematical error $m_K \triangleq E[A_m - A_K]$ is removed from the variance. The variance s_K^2 is considered to occur due to four statistically independent effects: model deficiencies s_p , shadowing s_s , multipath scattering s_R and measurement errors s_m . So, if attenuation mechanisms are assumed multiplicative, the logarithmic variance s_K is given by

$$s_K^2 = s_m^2 + s_p^2 + s_s^2 + s_R^2 \quad (2.44)$$

Each term in (2.44) is discussed more in detail below:

Measurement errors

Using state-of-the-art equipment, measurement inaccuracies, though present, should not be of principal concern. Typical errors are of the order of $s_m \approx 1$ to 2 dB. Nonetheless, erratic measurements can greatly influence the results [22]. Faulty or loose connectors, damaged antennas, overloaded amplifiers etc. are not always spotted immediately. Such effects tend to introduce some outliers in the set of measurement data, rather than smoothly increasing the apparent s_K^2 by a small extra term. Daily calibration measurements [22] revealed that measurements over a few paths were unreliable; these have been removed. Similarly, samples taken with an insufficient carrier-to-noise ratio were discarded.

It was realised that the selection of paths to be used for measurements may not be limited to paths already used by radio links and known to give satisfactory communication. Hence, equipment had to be significantly more sensitive than operational radio relay sets, otherwise any resulting absence of paths with heavy loss would bias the population of measurements which would lead to an optimistic propagation model.

During the analysis, three out of 55 paths were suspected because losses largely disagreed with expected values. In the statistical evaluation these samples have been maintained, except in a number of special experiments (see e.g. sect. 2.3.5).

Local attenuation mechanisms

Shadowing and multipath reception introduce local fluctuations of the received power. As explained in sect. 2.1.2, shadowing is considered to be an effect of an area description using a finite resolution for the terrain data. The terrain data base contained the precise terrain heights at the antenna location, the maximum terrain heights in a 1kmx1km grid along the path profile, and the type of terrain features. Consequently, shadowing imposes an insurmountable limit in the achievable accuracy of forecasts ($s_K > s_s$), presumably in the order of 6 to 7 dB. Although the rough resolution limits the assessment to very basic models, the accuracy that can be achieved is still substantially better than 12 dB, which were expected if no terrain data would have been used at all [8].

Multipath fading is known to cause fluctuations of the received power for displacements over lengths in the order of a wavelength. In an environment with severe multipath reception, such as in urban mobile networks, multipath fluctuations are best removed by averaging over a number of samples. The averaging is to be performed over the received field strength in absolute values, rather than over dB values [22][47]. In the ZODIAC radio relay network, multipath scattering was initially not assumed to be a major cause of fluctuations. Therefore, the analysis was performed based on individual samples.

During the assessment of a model for large-scale propagation attenuation, the effect of shadowing and frequency-nonselective multipath scattering can be effectively reduced by taking a sufficiently large number of samples at each location and an appropriate averaging technique. Nonetheless, during the planning of the network, the local fluctuations of received power have to be accounted for. A relevant consideration is whether the network is to be operated with randomly located antennas, as in most mobile and personal communication networks, or with antennas

at carefully selected locations, as in point-to-point systems or (to some extent) in television broadcasting. Evidently, 'fade margins', deemed necessary in the planning procedures of a certain mobile network can be reduced if more sophisticated measures are taken to ensure favourable antenna positions, e.g. by diversity techniques. The influence of local propagation mechanisms on the efficiency of spectrum use will further discussed from chapter 3 onwards.

Model deficiencies

The remaining term s_p represents model deficiencies. The aim of the assessment of an appropriate model is to effectively reduce s_p to zero. The dearth of appropriate theoretical results for the combined occurrence of multiple propagation mechanisms is one of the main limitations on the achievable reduction of s_p . The next sections deal with empirical results of the effects of interacting propagation mechanisms.

It can be asserted that, given the limited resolution of the terrain data and the presence of multipath scattering, the best achievable accuracy s_K is in the order of 6 dB. This is in the same range as the accuracy reported for propagation models in mobile cellular networks. Only very complex models perform better, provided that antenna can be placed with sufficient clearance ($s_K \rightarrow 0$ dB). The model by Okumura [18] for frequency planning of cellular networks [21] is reported to achieve an accuracy of about 6 to 7 dB. The accuracy of most models, can be characterised by a standard deviation s_K of 8 to 10 dB [22].

2.2.4.2 Linear regression

The linear regression $R[e, X]$ between the residual forecast error e and a certain parameter X of the path profile is

$$R[e, X] = \frac{E[(A_m - A_k)X] - E[A_m - A_k]E[X]}{E[X^2] - E^2[X]} . \quad (2.45)$$

This measure proves an effective method to generate insight in the statistical dependence of model imperfections on typical path characteristics. Not only X but many other mechanisms influence the total path loss, so the regression is performed for samples with severe statistical noise. To underscore this, the 80% confidence interval of $R[e, X]$ was also computed. Regression has been performed with the following choices of the parameter X :

- $\log d/\text{km}$ the logarithm of path distance between transmitter and receiver.
- $\log f_c/\text{MHz}$ the logarithm of the carrier frequency
- v_e the effective diffraction parameter of the path, defined in eq. (2.7). The effective obstacle height h_m in (2.7) was computed according to the method by Bullington [11] (see sect. 2.3.1).

Using theoretical models to predict the combined effect of predicted a number of the path parameters, the forecast error e can also be correlated with the predicted losses due to these mechanisms. This provides insight in the accuracy by which each individual propagation mechanism is represented in the model A_K and thus assists in the validation of *step 3*. The models considered were:

- A_{mh} diffraction loss over the main hill on the path
- A_{Dg} theoretical multi-knife-edge diffraction loss according to the method by Deygout [12].
- A_{Bu} multi-knife-edge diffraction loss according to the method by Bullington, based on one equivalent obstacle [11].
- A_R theoretical (line-of-sight) reflection loss from a two-ray propagation model, above free-space attenuation. See Eq.(2.5), with $A_R=0$ dB if $d < 4h_t h_r/\lambda$.

If an empirically corrected model A_N is defined with

$$A_N \triangleq A_K + R[e,X] X, \quad (2.46)$$

the new standard deviation

$$\begin{aligned} s_N^2 &= E\{[A_m - A_k - R[e,X]X - E[A_m - A_k - R[e,X]X]]^2\} \\ &= s_K^2 - R^2[e,X] \text{var}[X] < s_K^2 \end{aligned} \quad (2.47)$$

will be obtained for that particular set of measurements. The value of s_N is reported in the following evaluation to give some impression of the effect of improved modelling of the parameter X on the model K . It is not the author's intention to claim that the empirical model A_N necessarily is better than the theoretical model A_K .

In the next sections, a propagation model will be refined step by step, considering additional propagation mechanisms, based on insight gained through regression with the above parameters. It is our aim to find a model that combines existing theoretical models (*step 3*) and still reasonably agrees with the measured data, rather than to establish an optimum statistical match of the newly proposed model with the data, ignoring generally accepted theoretical models. This, however, may imply that the final model can exhibit a residual correlation with the aforementioned parameters.

2.2.4.3. Number of samples

Theories from the combined occurrence of propagation mechanisms are scarce. As far as empirical results are reported, e.g. in [17], the validity in a different setting is not guaranteed in advance, so verification is required. Dedicated experiments, such as the Haarerberg experiment, can yield insight into selecting and combining the relevant mechanisms (*step 3*), if absence of local scattering and shadowing ensures the validity of this type of experiments considering a single path. However, usually a large set of samples is required to be able to draw statistical conclusions since the statistical noise

caused by local effects such as shadowing may well disguise the effect of mechanism under study. In the analysis, it was often found that

$$R[e, X] \sqrt{\text{var}[X]} < s_s, \quad (2.48)$$

i.e., the variation of the received power caused by variations of X had a smaller standard deviation than the shadowing and multipath reception. This necessitates the analysis of hundreds of measurements to achieve reasonable reliability of the regression. For instance, in the case of correlation with distance, it was found from the measurements that

$$10\beta \sqrt{\text{var} [\log d]} \approx 7\text{dB} . \quad (2.49)$$

i.e., the variation of the received power caused by variations of the distance d had a standard deviation of about 7 dB. In this case, fluctuations caused by shadowing and multipath fading are in the same order of magnitude or even larger. The results reproduced in Fig 2.2 were obtained with approximately 1300 measurements.

In the analysis of *step 1* (interpretation of the profile), the problem appears even more complicated than linear regression. The degrees of freedom in defining effective heights and other compensation parameters would appear virtually unlimited. Implementation decisions often have a discontinuous effect on path loss. Thus, small change in terrain profile, for example changing the modelled effective diffraction height of trees, can exert a step-wise influence on the predicted diffraction loss if the implemented algorithm switches from a single knife-edge to a double knife-edge interpretation. Further, conclusions are sensitive to outliers or exceptional events. Because of such difficulties, no general method can yet be recommended here from the experience gathered during the analysis. It does appear essential to take sufficient samples to smooth out discontinuities. In our measurements, 55 paths, 7 antenna heights and 4 UHF frequencies (i.e., a total of 1540 samples though about 200 samples were discarded because of insufficient C/N) did not fully achieve this. Experiments did reveal that the discontinuities sharply diminish with accuracy of the implemented

prediction model, despite the fact that a more detailed model generally has to take more decisions, each of which may contribute new discontinuities. For instance, the MKE diffraction model [12] consistently gave a significantly better defined optimum than SKE methods.

2.3 EMPIRICAL RESULTS FOR DETERMINISTIC PROPAGATION MODEL

In this section, empirical results are presented for a number of propagation models. Initially, a set of 34 paths was used; in the final stage of the project, measurements over a total of 55 paths became available.

2.3.1 MODELLING SINGLE KNIFE-EDGE (SKE) DIFFRACTION

The dominant hill in a radio path is defined as the point along the profile with the highest value v_m of the diffraction parameter v . Model 1 is defined as free-space loss plus the knife-edge diffraction loss caused by the dominant hill, so

$$A_1 = A_{fs} + A_{mh} . \quad (2.50)$$

The effective height of trees was taken 20 metres for coniferous woods and 10 metres for other woods. This model appeared to give optimistic estimates of the path loss, with $m_{\Delta E}[e]=20.6$ dB. The standard deviation s_l was found to be 11.7 dB. This method ignores the influence of any other part of the terrain than the dominant hill.

The increase of the path loss with distance is grossly underestimated and the frequency dependence is overestimated, as is seen from Table 2.2. Table 2.2-2.6 and 2.8-2.10 present the regression $R[e,X]$ with a number of relevant path parameters X , the 80% confidence interval of $R[e,X]$ and the standard deviation of an empirically

corrected model on the linear form of (2.46). Results in Table 2.2 - 2.6 and 2.9 - 2.10 were obtained from 34 paths.

Table 2.2: *Empirical corrections for model 1: diffraction over the dominant hill in free space*

$R[e,X]$	X	CONFIDENCE INTERVAL s_N	
-1.3	v_e	-0.55 .. 2.0	11.6 dB
+0.45	A_{Dg}	0.38 .. 0.52	11.2 dB
+0.01	A_{mh}	-0.14 .. 0.13	11.7 dB
+0.37	A_{Bu}	0.26 .. 0.47	11.5 dB
18.8	$\log d$	16.2 .. 21.4	11.1 dB
-6.7	$\log f_c$	- 8.9 .. - 4.5	11.6 dB

One of the propagation models suggested by Bullington includes a method where the effect of irregular terrain, including various hills obstructing the direct line-of-sight, is comprised into one effective obstacle [11]. The equivalent hill is found by drawing a fictitious line of sight from the transmitter antenna, touching its radio horizon and another line-of-sight from the receiver antenna, touching the radio horizon of the latter. The top of the equivalent hill is found at the point where both lines cross (see Fig. 2.9).

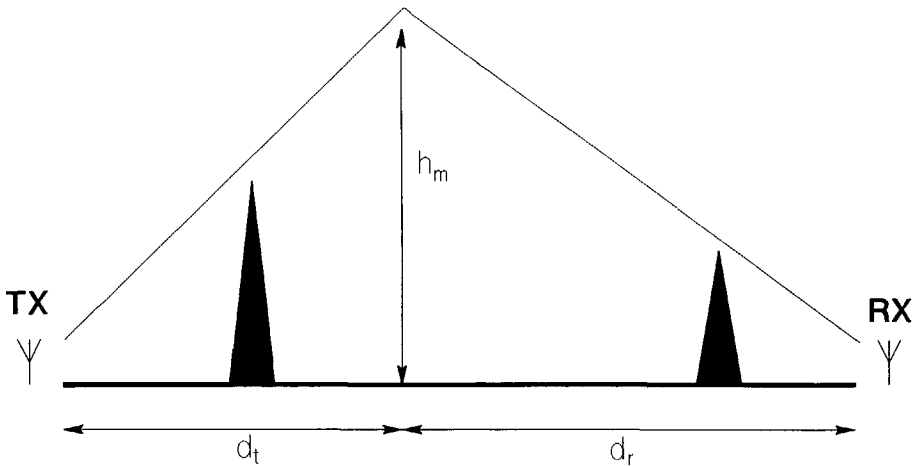


Fig. 2.9: *Construction of effective obstacle according to Bullington [11].*

This model particularly applies to obstacles in free space, i.e., on paths where the terrain, except for a few hills, does not significantly enter the first Fresnel zone. The propagation loss is then obtained as

$$A_2 = A_g + A_d(v_e) . \quad (2.51)$$

No ground-reflection loss is taken into account. Table 2.3 summarises the performance as assessed from the first set of measurements. The model accuracy is $m_2 \approx 19$ dB and $s_2 \approx 11.5$ dB.

Table 2.3 *Empirical corrections for model 2:
diffraction loss over the effective hill in free space*

$R[e,X]$	X	CONFIDENCE INTERVAL		s_N
-0.3	v_e	-1.0 ..	0.5	11.4 dB
+0.4	A_{Dg}	0.33 ..	0.47	11.1 dB
+0.13	A_{mh}	0.00 ..	0.26	11.5 dB
+0.19	A_{Bu}	0.08 ..	0.29	11.4 dB
18.1	$\log d$	15.5 ..	20.6	10.9 dB
-6.6	$\log f_c$	- 8.7 ..	- 4.4	11.3 dB

It is seen that the distance dependence is still grossly underestimated and that the frequency dependence is overestimated. With model 2, a problem of implementation was encountered for forested terrain, because the equivalent obstacle is often only determined by woods or isolated trees in the immediate vicinity of the antennas. In such events, all other terrain irregularities on the propagation path are ignored.

2.3.2 MODELLING MULTIPLE KNIFE-EDGE (MKE) DIFFRACTION

A significant improvement was found by applying multiple knife edges according to the method by Deygout [12]. The algorithm implemented takes account of up to three obstacles. First the dominant hill (v_{main}) is assessed. Secondary obstacles in the

(fictitious) line-of-sight between the transmitter and the dominant hill, with diffraction parameter v_1 , and between the dominant hill and the receiver, with diffraction parameter v_2 , are also taken into account. The propagation loss is then obtained as

$$A_3 = A_{fs} + A_d(v_{main}) + A_d(v_1) + A_d(v_2) . \quad (2.52)$$

The technique to obtain v_{main} , v_1 and v_2 is illustrated in Fig. 2.10.

Results are in Table 2.4. The systematic error of this model is $m_3 \approx 15.3$ dB and the standard deviation $s_3 \approx 10.7$ dB. This model thus appears almost one dB more accurate than the models considering diffraction losses over a single knife edge. The effective height of trees was empirically optimised and set to 25 metres for coniferous woods and 15 metres for other woods. This had a small effect on the accuracy achieved. No ground reflection loss is considered.

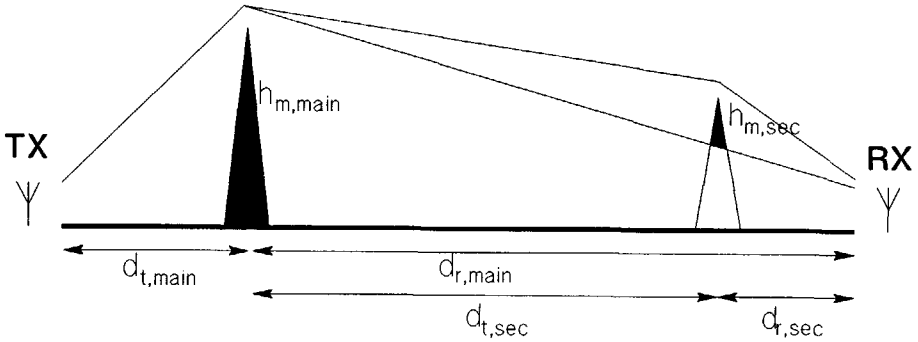


Fig. 2.10: Technique to obtain multiple knife-edge geometry according to Deygout [12].

Model 3 seems to overestimate diffraction losses. A possible explanation is the lack of proper account of groundwave propagation. Because of ground reflections, the error is anti-correlated with frequency and positively correlated with distance. Since reflection losses are not yet taken into account and since these losses may create substantial statistical noise, the improvement in accuracy compared to SKE methods was deemed to be relevant.

Table 2.4 *Empirical corrections for model 3:
Deygout triple knife-edge diffraction loss in free
space*

$R[e,X]$	X	CONFIDENCE INTERVAL s_N	
-2.04	v_e	-2.72 .. -1.37	10.6 dB
-0.17	A_{Dg}	-0.24 .. -0.11	10.6 dB
-0.35	A_{mh}	-0.48 .. -0.23	10.6 dB
-0.18	A_{Bu}	-0.28 .. -0.09	10.7 dB
15.0	$\log d$	12.6 .. 17.5	10.3 dB
-3.6	$\log f_c$	- 5.7 .. - 1.6	10.7 dB

2.3.3 MODELLING GROUND REFLECTION

The gross underestimate of the distance dependence of the path loss in model 1 to 3 can be explained by the ignorance of ground reflections. Height gain measurements confirmed the presence of the groundwave propagation mechanism: In the measurements in the North German Plain, the antenna height gain, as evaluated from measurements at $h_r = 4, 7, 10, 13, 16, 19$ and 21 metres, was assessed from the regression $R[A_{fs}, \log h_r]$. Measurements over 34 paths showed a '4.5 dB/oct' height gain, with an 80% confidence interval of 3.6 to 5.4 dB/oct. Perfect groundwave propagation would result in an antenna height gain of 6 dB per octave or, equivalently, '20 log h_r ' (see Eq. (2.7)). Possible explanations for the difference are that:

- i) effective antenna height when antenna sites are above the average ground level, e.g. on a hill, were not compensated;
- ii) the average height gain could be substantially influenced by one outlier, where an obstacle was close to the receiving antenna;
- iii) at relative short ranges ($d < 4h_r/\lambda$), the antenna height gain is not accurately approximated by a '6 dB/oct' trend since (2.6) becomes inappropriate if d is small;
- iv) multipath reception caused by local reflections, e.g. from trees, might have influenced the height gain.

The high gain did not always exactly follow a smooth 'dB/oct' trend: multipath scattering produced peaks and dips with a standard deviation of 2.5 dB for $f_c=250$ MHz, slowly increasing to 3.5 dB for $f_c=900$ MHz. Diffraction by nearby obstacles (shadowing) only influenced the height gain if obstructions were very close to the antenna. For instance, an exponential height gain of 1 or 2 dB *per metre* was measured closely beyond the Haarlerberg (Fig. 2.7, loc.4). Within small woods, an occasional 10 dB could be gained by placing the antenna between the trunks (at 7 to 10 m), rather than at the level of the highest leaves. Nonetheless, the general impression from the measurements was that ground reflections are the principal cause of the height gain.

For models including free-space loss and diffraction over multiple knife edges but ignoring ground reflections (such as model 3), the error e was found to contain a term of about ' $8 \log h_r$ '. This confirms that ground reflection and diffraction mechanisms interact.

Table 2.5 *Empirical corrections for the model 4, proposed by Egli [8]*

$R[e, X]$	X	CONFIDENCE INTERVAL		s_N
4.2	v_e	3.6 .. 4.8		11.3 dB
+0.87	A_{Dg}	0.80 .. 0.93		10.8 dB
-0.06	A_R	-0.13 .. 0.00		11.3 dB
-4.3	$\log d$	-7.0 .. -0.17		11.3 dB
-1.0	$\log f_c$	-3.2 .. 1.2		11.3 dB

The fourth model considers groundwave propagation and does not take any specific terrain profile into account. The model is based on theoretical results for propagation over a plane perfectly conducting earth. An empirical term ' $20 \log(f_c/40 \text{ MHz})$ ' was postulated [8]. The standard deviation is $s_e \approx 11.3$ dB for the set of 34 paths and 11.7 dB for the set of 55 paths, which closely agrees with $s_e \approx 12$ dB as reported in [8]. Interestingly, model 4 does not use any terrain data, but nonetheless is equally accurate as models 1 and 2. Model 4 was found to slightly overestimate path losses

($m_f \approx -2$ dB). Table 2.5 confirms that the empirical frequency correction is well within the confidence interval. The distribution of the error in Fig. 2.11. The ordinate is drawn such that any log-normal distribution of the error produces a straight line. The distribution somewhat departs from the log-normal distribution reported by Egli [8]. As Fig. 2.11 illustrates, in 10% of the measurements a field strength higher than 10 dB above the area-mean is encountered, whereas 10% of the locations produces an excess attenuation of 20 dB below the area-mean field strength. This asymmetry may have been caused by multipath scattering in a number of measurements. In fact, Rician fading would produce more pronounced down-fades than up-fades.

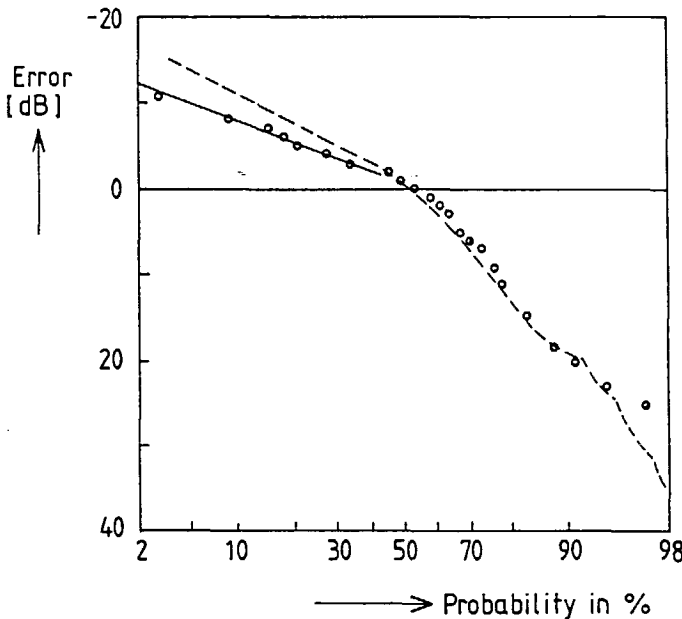


Fig. 2.11: *Distribution of the error (- -) of model 4, proposed by Egli, obtained from the unscreened set of measurements. Distribution of local-mean error, averaged per receiver location (o).*

After averaging over all measurements at one receiver location (seven heights, four frequencies), the 'local-mean' error was established and its distribution indicated by small circles (o) in Fig. 2.11. Following the definition in section 2.1.2, the local-mean error corresponds to large-area shadow attenuation.

A simple improvement of the model is made by distinguishing between paths over (plane) farmland and path covered by vegetation. The empirical vegetation parameter Ψ ($0 < \Psi < 1$) is introduced, defined as the portion of samples³ along the path that contains woods or at least a number of trees. For the paths considered, we found an average of 80% of all samples that contained woods of trees ($\mu_{\Psi} \Delta E[\Psi] \approx 0.8$). The standard deviation was $\sigma_{\Psi} \Delta E[(\Psi - \mu_{\Psi})^2] \approx 0.12$. Empirically the following regression was established:

$$A_s = A_{fs} + A_R + 20 \log \frac{f_c}{40 \text{ MHz}} \quad (2.53)$$

$$- 8 \log \frac{d}{20 \text{ km}} + 30(\Psi - 0.8) - 50(\Psi - 0.8) \log \frac{d}{20 \text{ km}} .$$

The standard deviation of this model was found to be $\sigma_s \approx 11.2$ dB, which indicates some improvement over the model proposed by Egli (model 4).

The mean μ_{Ψ} and standard deviation σ_{Ψ} suggest that a typical path over relatively open-area has $\Psi \approx \mu_{\Psi} - \sigma_{\Psi} \approx 0.68$, whereas a typical woodland path has $\Psi \approx \mu_{\Psi} + \sigma_{\Psi} \approx 0.92$. Comparison of the Ψ -value with the character of the path on a detailed map generally confirmed the usefulness of the parameter Ψ . Fig 2.2 is obtained by inserting $\Psi = 0.68$ and 0.92 into (2.53). For paths over woodland ($\Psi \approx 0.92$), $\beta \approx 2.6$, and for paths over open area ($\Psi \approx 0.62$), $\beta \approx 3.8$. This leads to the counter-intuitive conclusion that in open areas relatively shorter frequency re-use distances can be used, whereas in areas with more obstructions to propagation, longer re-use distance must be considered. An explanation is that in open area, interference power levels decay relatively rapidly because of ground reflections.

³Each terrain sample describes the highest point in the terrain and the highest terrain feature, e.g. trees, in a 1kmx1km square.

2.3.4 MODELLING COMBINED REFLECTION AND DIFFRACTION LOSSES

The simplest way to approximate the effect of combined diffraction and reflection losses, is to consider these mechanisms as independent and multiplicative (additive if dB values are considered). Model 6 takes account of Deygout triple knife edges [12] and groundwave reflection losses, with

$$A_6 = A_{fs} + A_R + A_d(v_{main}) + A_d(v_1) + A_d(v_2) \quad (2.54)$$

This model gave, with 34 paths, a systematic error of $m_6 \approx 0.8$ dB and a standard deviation of $s_6 \approx 10.6$ dB.

Table 2.6 *Empirical corrections for model 6*

$R[e, X] \sim X$		CONFIDENCE INTERVAL	s_N
-0.07	v_c	-0.69 .. 0.54	10.6 dB
-0.09	A_{Dg}	-0.16 .. 0.02	10.6 dB
-0.52	A_R	-0.58 .. -0.46	9.8 dB
-4.1	$\log d$	-6.6 .. -1.7	10.6 dB
+17.5	$\log f_c$	+15.6 .. +19.3	9.7 dB

It is evident from Table 2.6 that this model overestimates reflection losses. In particular, the frequency dependence of the reflection losses A_R in (2.54) impairs the accuracy. By adding a term ' $20 \log(f_c/40 \text{ MHz})$ ', thus

$$A_7 = A_{fs} + A_R + 20 \log \left(\frac{f_c}{40 \text{ MHz}} \right) + A_d(v_{main}) + A_d(v_1) + A_d(v_2) , \quad (2.55)$$

the standard deviation can be reduced to $s_7 \approx 9.8$ dB. Moreover, the regression of the error with most path parameters becomes relatively small.

It was noticed that the terrain features near the antennas had a substantial effect. This was studied by spitting the paths into two sets. One set contained paths with both antennas located in open area, the other file contained paths with at least one

antenna located in open area. For paths in the former set (wood-to-wood), the mean prediction error is $m_{\gamma} \approx 16$ dB, i.e., 16 dB too pessimistic. On the other hand, for paths in the latter set the mean error appeared $m_{\gamma} \approx 25$ dB, i.e., 25 dB too pessimistic. This observation stimulated the introduction of local shadow losses in the model to compensate for heavier losses for antenna's within woods.

2.3.5. MODELLING SHADOW AND SCATTER LOSSES

Scattering and shadowing in the vicinity of the antenna is an additional cause of propagation loss. Allsebrook and Parsons [48] draw attention to the 'clutter' factor, which depends on terrain features near the antenna. The extent to which scattering and shadowing occur depends on the antenna clearance, and therefore also on the antenna height. Dedicated experiments in the North German Plain indicated that in a relatively open area or for an antenna a number of metres above the tree tops, the influence of the antenna position is in the order of 2 to 3 dB. On the other hand, on wooded sites fluctuation is substantially stronger. Occasionally a 40 dB fluctuation was encountered within a few metres. According to [19], Rayleigh scattering may apply at UHF frequencies within woods; this could explain such large values. Practical circumstances often require that antenna clearance is much less than assumed in most theoretical propagation models for radio relay. In particular, models developed for civil radio relay were to be modified for rural communication or military point-to-point links with lower antenna positions. An antenna height of 21 metres is often insufficient to exceed tree top level in the North German Plain.

As discussed in section 2.1, shadow attenuation is taken into account statistically by assuming a log-normal distribution of the local-mean power. Evidently, local terrain features do not only affect the standard deviation but also the area-mean of this distribution. Damosso and Lingua [49] proposed empirical corrections for the area-mean power. At $f_c = 450$ MHz the excess attenuation A_L was found to be in the order of [49]

9 dB	in urban areas,
10 to 15 dB	in historical city centres,
0 dB	in sub-urban areas,
-8 dB	in open areas, and
-10 dB	on highways.

Figure 2.2-19 in reference [1] indicates that shadow attenuation sharply increases with carrier frequency. For the propagation paths in the North German Plain, an empirically acceptable correction for local effects was found on the form of

$$A_{loc} = \begin{cases} c_1 \log \frac{f_c}{40\text{MHz}} & \text{in cluttered area(woods)} \\ \frac{c_1}{2} \log \frac{f_c}{40\text{MHz}} & \text{at the fringe of a wood} \\ 0 \text{ dB} & \text{in open area} \end{cases} \quad (2.56)$$

Both at the receiver and the transmitter site a term on the form of (2,14) is considered. Model 8 considers

$$A_g = A_{fs} + A_d(v_{main}) + A_d(v_1) + A_d(v_2) + c_2 A_R + A_{loc,TX} + A_{loc,RX}, \quad (2.57)$$

where c_1 and c_2 have been jointly optimised.

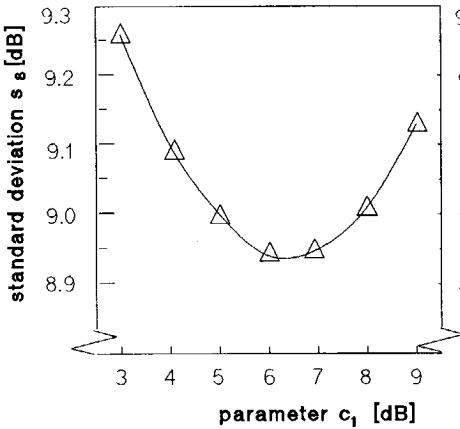


Fig. 2.12: Δ Standard deviation s_g versus empirical parameter c_1 in dB for shadow loss (52 paths)

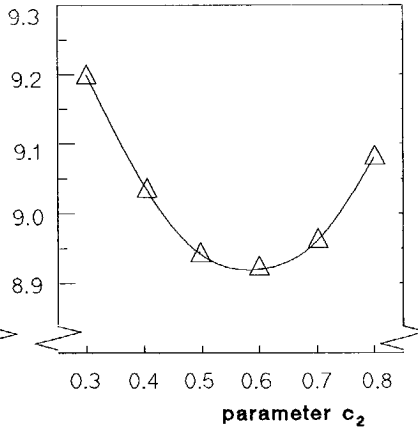


Fig. 2.13: Δ Standard deviation s_g versus empirical parameter c_2 for ground reflections. (52 paths)

The optimum values of c_1 and c_2 were found near $c_1 \approx 6 \dots 7$ dB and $c_2 \approx 0.6 \dots 0.7$.

Fig. 2.12 and Fig. 2.13 illustrate the effect the choice of c_1 and c_2 on the standard deviation, respectively. Fig. 2.12 and Fig. 2.13 have been obtained for a screened set of 52 paths, i.e. three outliers were removed. These figures show that the effect of the choice of c_1 and c_2 on the standard deviation is limited, i.e., in the order of 0.1 dB for a relatively wide range of values. The distribution of the error e of model 8 is presented in Fig. 2.14 for all samples ($\bullet$) and for path-average errors ($\times$) in the unscreened set of 55 paths.

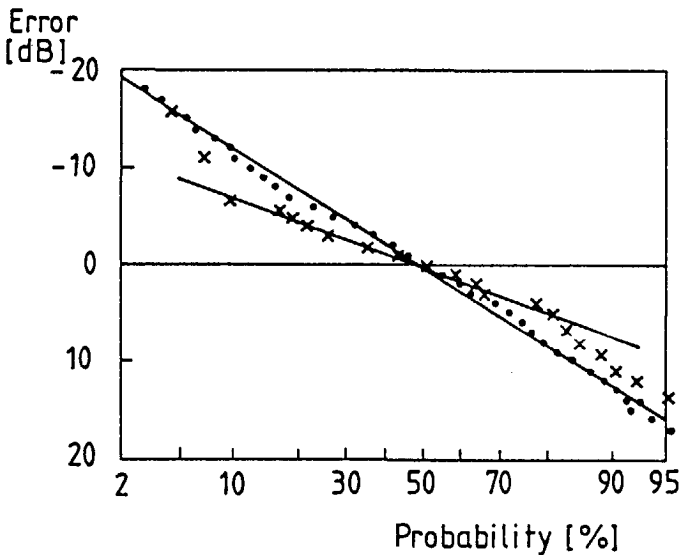


Fig. 2.14: *Distribution of the error ($\bullet$) of model 8 obtained from the unscreened set of measurements. Distribution of local-mean error, averaged per receiver location ($\times$).*

As illustrated by a straight line drawn in Fig. 2.14, a log-normal distribution with $s_y \approx 6$ dB is appropriate for the central part of the distribution. Because of outliers, the tails of the distribution shows some deviation from the log-normal distribution, which are seen in Fig. 2.14 as a departure from the straight line.

The standard deviation s_g depended on the particular set of paths and the screening of the data (see Table 2.7), while the mean error appeared less sensitive to the selection of paths ($m_g \approx 5.4$ dB). The standard deviation was about $s_g \approx 9$ dB for the set of all measurements taken at relatively high receiver antenna height ($h_r > 12$ m) and $s_g \approx 8$ dB for the screened set of 52 paths, which agrees with expected accuracies. For low antennas in a terrain with scattered woods, s_g remained above 9 dB for the screened set of 52 paths. This is explained by the fact that multipath reception may cause deviations in the order of 6 dB.

Table 2.7 *Standard deviation s_g for $c_1 = 6$ dB and $c_2 = 0.66$.*

NUMBER OF PATHS	s_g
33 (screened)	7.3 dB
34	8.3 dB
52 (screened)	9.0 dB
55	9.9 dB
52 (screened, $h_r > 12$ m)	8.0 dB
55 ($h_r > 12$ m)	9.0 dB

To verify the suitability of model 8 also under exceptional cases, an unscreened set of measurements over 55 paths have been taken into account to compile Table 2.8. No major shortcomings are observed, though the effect of path length appears to be slightly overestimated.

Table 2.8 *Empirical corrections for model 8 with reduced reflection loss and local shadowing (55 paths)*

$E[e, X]$	X	CONFIDENCE INTERVAL		s_N
0.77	v	0.35 ..	1.19	9.9 dB
-0.15	A_{Dg}	-0.20 ..	-0.10	9.8 dB
-0.11	A_R	-0.16 ..	-0.07	9.9 dB
-3.6	log d	-5.3 ..	-2.0	9.9 dB
0.7	log f_c	-0.8 ..	2.1	9.9 dB

2.3.6 EMPIRICAL MODEL BY BLOMQUIST

The model embodied in (2.57) is a linear combination of ground reflection losses and diffraction losses. Blomquist [20] suggested a non-linear function, viz.,

$$A_g = A_g + \sqrt{A_d^2 + A_R^2} . \quad (2.58)$$

This empirical formula models multiple knife edges in free space if the diffraction loss is relatively high ($A_D \gg A_R$) as is the case on many wooded paths. On the other hand, for relatively open area ($A_d \ll A_R$), groundwave propagation is recovered. As confirmed by other results, e.g. Fig. 2.2 or the Haarlerberg experiments, this is in agreement with experience.

Table 2.9 *Empirical corrections for model 9, by Blomquist [20] (34 paths)*

$R[e,X]$	X	CONFIDENCE INTERVAL s_N	
0.58	v_e	0.01 .. 1.15	9.7 dB
+0.10	A_{Dg}	0.03 .. 0.17	9.7 dB
-0.05	A_R	-0.10 .. 0.01	9.7 dB
5.2	$\log d$	2.9 .. 7.5	9.7 dB
7.1	$\log f_c$	5.2 .. 8.9	9.6 dB

With the given set of measurements, it appeared that (2.9) is more accurate than linearly adding the three loss terms as in (2.10), or model 6 described by Eq. (2.54). The model underestimates path losses by 8 dB and gave a standard deviation of $s_g \approx 9.8$ dB (for 34 paths). Correlation of the error with A_d and A_R (see Table 2.9) did not reveal significant deficiencies of the model. Distance dependence is underestimated by '5 $\log d$ ', while frequency dependence is underestimated by '7 $\log f_c$ '.

It was found that adding (2.26) for local shadow losses at the transmitter and receiver site, i.e.

$$A_{10} = A_f + \sqrt{A_D^2 + A_R^2} + A_{cl, TX} + A_{cl, RX} \quad (2.59)$$

reduces the standard deviation of the error by almost 1 dB ($s_{10} \approx 8.9$ dB, $m_{10} \approx -3.7$ dB with 34 paths). Table 2.10 shows that nonetheless some residual regression with A_R and A_{Dg} remains.

Table 2.10 *Empirical corrections for the model by Blomquist modified for local shadowing (34 paths)*

$R[e, X]$	X	CONFIDENCE INTERVAL	s_N
0.92	v_e	0.04 .. 1.43	8.8 dB
+0.10	A_{Dg}	0.04 .. 0.16	8.8 dB
+0.14	A_R	0.09 .. 0.19	8.8 dB
3.1	$\log d$	1.0 .. 5.2	8.9 dB
-3.5	$\log f_c$	-5.2 .. -1.8	8.8 dB

2.4 CONCLUSIONS

The propagation effects most relevant to VHF/UHF land-mobile radio have been discussed. A statistical model for propagation attenuation and for the received signal power has been formulated, for use in the next chapters in generic system design.

Field measurement of path losses in rural areas have been reported and used to evaluate the accuracy of a number of relatively simple propagation models. These idealised models take account of free-space loss, diffraction losses, ground-reflection losses and shadowing. Linear regression of the forecast error with relevant path parameters and with losses predicted by idealised models proved an effective method to investigate the performance and possible shortcomings of propagation models.

The groundwave model proposed by Egli in reference [8] gave the same accuracy as models considering diffraction loss over a single knife edge, i.e., the standard deviation is about 11 or 12 dB. Their accuracy appeared equivalent, despite the fact that the former model does not take into account any specific terrain data, whereas knife-edge models require evaluation of the path profile. Hence, the model by Egli is much faster in terms of required computer time. For more accurate results, it appeared necessary to perform multiple knife-edge computations and to carefully weigh the relative effects of groundwave and diffraction loss. The interaction of the effects of ground reflections and diffraction has been discussed by comparing models with field measurements. The empirical model proposed by Blomquist in reference [20] was verified. A linear addition of losses may also be applied for the application studied.

The measurements have also been used to verify the statistical models reviewed in the first part of this chapter. Because of ground reflections, the area-mean signal power decays faster in open area than in areas with dense vegetation. Thus larger separation of co-channel users in a wooded environment will be required to avoid harmful mutual interference. Moreover, because of shadowing and multipath scattering, received powers are likely to fluctuate more in a wooded area than in open area. This also necessitates the use of relatively large frequency re-use distances in wooded area.

The error of relatively accurate models appeared to have a log-normal distribution. This confirms the log-normal distribution of small-area shadowing. However, for the model by Egli, the distribution of forecast error showed some deviations from the log-normal distribution.

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CHAPTER 3

PROBABILITY OF SIGNAL OUTAGE

Cellular design of land-mobile radio networks meets the requirement to use the available spectrum in a highly efficient manner. Frequencies can be extensively re-used by keeping spatial separations between co-channel cells as small as possible compliant with the required system performance [1],[2]. However, determination of the minimum spacing between terminals transmitting with the same carrier frequency, i.e., the maximum acceptable interference power, is complicated by the fact that received signal powers fluctuate because of multipath fading and shadowing. Not only the statistical behaviour of any wanted signal power, but also that of all co-channel interferers determine the performance of mobile radio nets. The outage probability is a useful performance measure, expressing the probability that satisfactory

demodulation of the wanted signal fails because of excessive interference and noise power at the receiver input. In high-capacity cellular networks, it is interference, rather than noise, which imposes the main limits on the system performance. In this case, the outage probability is known as the 'co-channel interference probability' [2], i.e., the probability that the instantaneous C/I -ratio drops below a certain margin required by the receiver design.

This chapter gives an analytical determination of outage probabilities in multi-user mobile networks with narrowband radio channels. Section 3.1 formulates the model and reviews previous work on signal outage. The robustness of practical systems against co-channel interference is discussed and a criterion for signal outage based on the C/I -ratio is defined. Further, the statistical behaviour of the joint interference signal is discussed. In section 3.2, a novel and exact mathematical method is presented to express the co-channel interference probability. A noise-free system is considered and all co-channel cells are assumed to contain an active interferer. Results are found on the form of a multi-dimensional integral and techniques for its numerical quadrature are discussed. This basic evaluation is generalised in section 3.3 to include the effects of bursty interferers and noise impairments. Moreover, macro-diversity, i.e., reception of the wanted signal by cooperating receivers at multiple base station sites, is investigated. Relatively simple closed-form expressions are derived for the limiting case of large C/I -ratios. Numerical results derived from the general model are presented for the case of a cellular network with an idealised hexagonal repetition pattern.

3.1 MODEL FOR SIGNAL OUTAGES IN RADIO NETWORKS

In mobile telephony, the quality of service is often expressed in terms of the probability of outage experienced by subscribers near the boundary of the base station service area. Because of limited spectrum availability, radio networks become

increasingly limited by mutual interference between users. Therefore, the outage probability is usually determined in terms of the probability that the signal-to-joint-interference ratio drops below a minimum required ratio z . If noise is also significant, an outage occurs when the signal-to-noise-plus-interference ratio $C/(I+N)$ drops below the threshold z .

The receiver parameter z appears in many studies throughout technical literature. The parameter is called the "co-channel rejection ratio" [3], the "receiver threshold" [4],[22],[39], the "threshold ratio" [15], the "capture ratio" [6],[20] the "interference criterion" [5] or "protection ratio" [7],[8],[23].

The parameter 'protection ratio', however, is principally intended as a parameter in *system design* or *network planning* [8], describing the minimum required (average) C/I -ratio, rather than merely a *receiver* parameter. For instance, as indicated by the CCIR in Report 358-5, fade margins may be implicitly included in the required protection ratio to allow for the effects of multipath fading or terrain irregularities [8]. Since the effect of channel fading will be addressed explicitly in the next sections, the term 'protection ratio' is avoided here, and z will be considered as a pure receiver parameter.

Moreover, the protection ratio, as defined by the CCIR, is expressed in dB-values, while in our analyses, z will be expressed in absolute values.

The 'co-channel rejection ratio' is defined as the maximum tolerable interference power level with respect to the power of the wanted signal, thus the co-channel rejection ratio is z^{-1} . The terms 'threshold' and 'capture' were originally used in studies of the demodulation of FM broadcast transmissions (see chapter 7), whereas the analysis is not necessarily restricted to analogue exponential modulation. Nonetheless, in the light of the above observations, 'receiver threshold' is considered the most appropriate name for z . The choice of the value of z , in general, depends on the required quality of service and may therefore be somewhat arbitrary. The 'outage criterion' can, for instance, be a certain figure of merit subjectively determined by a representative panel of listeners [10], or a digital-word or bit error rate [11].

3.1.1 RECEIVER THRESHOLD

In general, the receiver threshold z depends both on the required output performance, i.e., at base-band, and on the type of interference. For linear modulation, such as AM and measured ssb, and any Gaussian interference, the well-known relation between the required signal-to-noise ratio at the detector output and z is linear, e.g. [9]. Typical receiver thresholds for ssb, a modulation technique used in some North American mobile telephone systems, are in the order of 20 dB [10]. With exponential modulation, which is more commonly used in mobile radio nets, the receiver threshold z can be reduced but cannot be substantially lower than 5 to 10 dB. This lower limit is imposed by a 'detection threshold' effect, which fundamentally limits the noise immunity of various types of non-linear modulation techniques [9],[12]. Gosling summarised typical receiver thresholds below which a voice baseband signal becomes almost unintelligible (see Table 3.1) [13].

Table 3.1 *Receiver threshold range for various types of analogue modulation [13]*

modulation technique and occupied bandwidth	receiver threshold (dB)	
	<i>minimum</i>	<i>maximum</i>
FM 25 kHz	3	8
FM 12.5 kHz	6	15
SSB 5 kHz:		
heavy companding	8	11
moderate companding	11	15
no companding	15	20

The CCIR indicates in Report 319 that $z \approx 80$ (about 19 dB) for FM voice with a small peak deviation of 4 to 5 kHz, thus with an occupied bandwidth $B_T \approx 12.5$ kHz, and $z \approx 7$ to 8 (8 to 9 dB) for FM voice with a wider peak deviation of 12 to 15 kHz ($B_T \approx 25$ to 30 kHz). A receiver threshold of 17 dB ($z \approx 50$) is used in the U.S. cellular radio network employing FM voice with a peak deviation of 12 kHz (and a channel spacing virtually identical to the occupied bandwidth of $B_T \approx 30$ kHz). For a C/I -ratio higher than 17 dB,

most subscribers experience the quality of voice reception as 'good' or 'excellent' [14]. An outage probability of 10% is tolerated. The receiver threshold below which the received voice becomes 'unintelligible' is much lower and may be in the order of 6 to 10 dB, as suggested in Table 3.1.

For digital modulation, the receiver threshold z may be defined as the instantaneous C/I -ratio at which the short-term average bit error probability P_b or the block error probability P_c exceeds a maximum tolerable value P_z , with P_z called the outage criterion. Chang studied the relation between P_z and z as a function of the type of interference experienced and presented outage probabilities in [11]. For interference with a Gaussian probability density, this relation is well known [9],[12]. For a single interferer with a peak-limited signal, such as a constant-envelope BPSK signal, the bit error rate is nearly a step function of the C/I -ratio [11]. Above a certain value z , error rates are negligible, whereas below this threshold, bit errors rapidly tend towards 50%. For the digital pan-European G.S.M. system using GMSK modulation, a receiver threshold of 9.5 dB is considered to describe the performance of practical receivers appropriately [15]. Oetting gives a review with comparison of various modulation techniques and their vulnerability to noise and cw interference in [16].

3.1.2 MODEL FOR JOINT INTERFERENCE SIGNAL

In order to determine the probability of outage, assessment of the statistical behaviour of the cumulative interference is indispensable in multi-user systems. Three cases of cumulation are distinguished, namely: The sum of multiple signals, each exposed to log-normal shadowing; the sum of multiple signals, each exposed to Rayleigh fading; and the sum of multiple signals, each exposed to combined shadowing and Rayleigh fading. In all three cases, the fading is assumed mutually independent for all interfering signals.

3.1.2.1 Multiple log-normal signals

In the past, two different methods have been used to estimate the probability distribution of the joint interference power accumulated from several log-normal signals. Fenton [17] and Schwartz and Yeh [18] both proposed to approximate the pdf of the joint interference power by a log-normal pdf, yet neither could determine it exactly. The method by Fenton assesses the logarithmic mean and variance of the joint interference signal directly as a function of the logarithmic means and variances of the individual interference signals. This method is most accurate for small standard deviations of the shadow-fading, say, for $s_i < 4$ dB. The technique proposed by Schwartz and Yeh is more accurate for s_i in the range 4 to 12 dB, which corresponds to the case of land-mobile radio in the VHF and UHF bands. Their method first assesses the logarithmic mean and variance of a joint signal produced by cumulation of two signals. Recurrence is then used in the event of more than two interfering signals. A disadvantage of the latter method is that numerical computations become time consuming. Table 3.2, compiled in [19], gives the logarithmic mean m_t in dB ($P_t = 10^{m_t/10}$) and standard deviation s_t of the joint interference power for various numbers of contributing signals determined by Schwartz and Yeh's method.

Table 3.2 Mean m_t and standard deviation s_t (in dB) of the joint power of n signals with uncorrelated shadowing, each with mean $m_i = 0$ dB and with identical standard deviation s_i [19].

n	$s_i = 0$ dB		$s_i = 6$ dB		$s_i = 12$ dB	
	m_t	s_t	m_t	s_t	m_t	s_t
1	0.00	0.00	0.00	6.00	0.00	12.00
2	3.00	0.00	4.58	4.58	7.45	9.58
3	4.50	0.00	6.90	3.93	11.20	8.40
4	6.00	0.00	8.43	3.54	13.62	7.66
5	7.00	0.00	9.57	3.26	15.37	7.13
6	7.50	0.00	10.48	3.04	16.74	6.74

Besides these methods, by Fenton [17] and Schwartz and Yeh [18], a number alternative (and often more simplified) techniques are used. For instance in VHF radio broadcasting, signals fluctuate with location and with time according to log-normal distributions. Techniques to compute the coverage of broadcast transmitters are in [41]-[43].

3.1.2.2 Multiple Rayleigh-fading signals

Cumulation of multiple Rayleigh-fading signals requires investigation of the nature of the signals contributing to the interference. The joint interference signal caused by n uncorrelated Rayleigh-fading narrowband signals with angular modulation is

$$v_i(t) = \sum_{i=1}^n \rho_i(t) \cos(\omega_{c_i} t + \theta_i(t) + \psi_i(t)) . \quad (3.1)$$

Similar to (2.18), ρ_i are the (random) amplitudes and θ_i the (random) phases of the n Rayleigh fading carriers. Angle modulation is represented by ψ_i . We now consider the signal behaviour during an observation interval T which is short compared to the rate of channel fading ($T \ll T_\phi$, see sect. 2.1.7). This implies that ρ_i and θ_i are virtually constant. Two extreme cases are distinguished in [20]:

i) Coherent (or phasor) cumulation: This occurs if, during the observation interval, the phase fluctuations caused by the modulating signals are sufficiently small, and the carrier frequencies of the n signals are exactly equal, thus if $\psi_i(t_0) \approx \psi_i(t_0 + T)$, $\theta_i(t_0) \approx \theta_i(t_0 + T)$ and $\omega_{c_i} = \omega_c$. The power during the interval T is found from

$$\begin{aligned}
 p_i &= \frac{1}{T} \int_{t_0}^{t_0+T} v_i^2(t) dt \\
 &= \frac{1}{2} \left(\sum_{i=1}^n \rho_i \cos(\theta_i(t_0) + \psi_i(t_0)) \right)^2 \\
 &\quad + \frac{1}{2} \left(\sum_{i=1}^n \rho_i \sin(\theta_i(t_0) + \psi_i(t_0)) \right)^2.
 \end{aligned} \tag{3.2}$$

The joint signal $v_i(t)$ behaves as a Rayleigh phasor, so the instantaneous power p_i is exponentially distributed. The local-mean power is equal to the sum of the local-mean powers of the individual signals. Coherent cumulation can occur only if phase modulation with a very small deviation is applied. Moreover, if the observation interval is taken short with respect to the rate of modulation, coherent cumulation is appropriate. The latter case occurs for instance in digital systems if the joint interference signal is studied during one bit interval or during the lock-in of a carrier-recovery loop in a synchronous detector.

ii) *Incoherent (or power) addition*: if the phases of each of the individual signals substantially fluctuate due to mutually independent modulation, the n signals add incoherently. The interference power experienced during the observation interval is the power sum of the individual signals. Thus,

$$\begin{aligned}
 p_i &= \frac{1}{T} \int_{t_0}^{t_0+T} v_i^2(t) dt \\
 &= \frac{1}{T} \int_{t_0}^{t_0+T} \left(\sum_{i=1}^n \rho_i \cos(\omega_{c_i} t + \theta_i(t) + \psi_i(t)) \right)^2 dt \\
 &= \frac{1}{2} \sum_{i=1}^n \rho_i^2 + \int_{t_0}^{t_0+T} \text{crossterms } dt = \sum_{i=1}^n p_i
 \end{aligned} \tag{3.3}$$

because incoherent cumulation ensures that the integrals over the crossterms vanish. Coherent and incoherent cumulation have previously been compared by Arnbak and

Van Blitterswijk in [20]. With coherent addition, the joint interference signal may exhibit deep fades, caused by mutual cancelling of phasors from the n signals. This cannot continue for a sustained period due to the phase variations caused by angle modulation of each signal or by slightly different carrier frequencies due to Doppler shifts and free-running oscillators. Hence coherent interference cumulation is seen to give optimistic results compared to incoherent cumulation for a given interference threshold z [20],[21].

With incoherent cumulation, the joint interference signal behaves as a band-limited Gaussian noise source if the number of components is sufficiently large. Moreover, any fade of one of the signals is likely to be hidden by the $n-1$ other interfering signals. Hence, the joint interference signal tends to exhibit less multipath fluctuations per unit of time than the signal from one individual interferer. The power sum of a number of Rayleigh fading signals has an approximate gamma distribution [20],[22]. This will be further elaborated upon in chapter 4.

3.1.2.3 Multiple signals with combined shadowing and Rayleigh fading

The early studies of signal outages and spectrum efficiency of cellular radio networks by Gosling [23] addressed channels with pure Rayleigh fading. As discussed in Sect. 2.1.2, neglecting shadowing is acceptable if the local mean powers of the signals are sufficiently known, for instance from computation of path losses from a high-resolution terrain database. Moreover, on the downlink, the individual shadow attenuation of different arriving signals may be highly correlated. In these cases, shadowing has a very limited effect on the fluctuations of the C/I -ratio. French [24] studied the effect of combined Rayleigh fading and shadowing in the special case of one single interfering signal. The case of multiple interfering signals was assessed by Cox using Monte Carlo simulation [25]. Later, a number of mathematical investigations of the effect of multiple interfering signals with Rayleigh fading and

shadowing were conducted [5],[8],[21],[26],[27],[28] though approximation techniques were used.

In the event of *coherent* cumulation of fading signals, the amplitude of the joint interference signal has a Rayleigh distribution. The local-mean power of the Rayleigh distributed envelope equals the sum of the power of the individual signals. If shadowing also occurs, the pdf of the local-mean power of the joint interference signal can be assessed by Fenton's [17] or Schwartz and Yeh's technique [18]. Muammar and Gupta [8] and Diakoku and Ohdate [26] extended the method by Fenton to evaluate cellular mobile radio systems with shadowing. The method by Schwartz and Yeh was extended in [27] and [28]. However, since the instantaneous power was assumed to be exponentially distributed about the local-mean power, these extensions were appropriate to coherent, rather than the more realistic incoherent cumulation of the instantaneous interference power.

For *incoherent* cumulation of multiple Rayleigh fading signals, the distribution of the instantaneous joint interference power depends on the local-mean power of each contributing signal. This will be discussed further in section 4.3. The joint interference signal cannot be characterised by the sum of the local-mean powers only, because the variance of the joint signal power depends on the number of contributing signals and on their relative powers [49]. This is in sharp contrast to the situation of coherent cumulation, and substantially complicates the analysis. To evaluate incoherent cumulation, Prasad and Arnbak [29] proposed to approximate the Suzuki pdf of the instantaneous power of each individual interfering signal (2.29) by a log-normal pdf with area-mean power $\bar{p}_{rs}$ and a logarithmic standard deviation s_{rs} . It was stated in [29] that the first and second-order *linear* moments of the approximate log-normal pdf and the exact Suzuki distribution match if one takes the logarithmic variance

$$\sigma_{rs}^2 = \sigma_s^2 + \ln 2 \quad (3.4)$$

and the logarithmic area-mean

$$\overline{\overline{P_{rs}}} = \frac{1}{2}\sqrt{2} \overline{\overline{P_0}} . \quad (3.5)$$

Here, the area-mean is the absolute value corresponding to the logarithmic area-mean power $\overline{\overline{P_{rs}}} = \exp\{m_{rs}\}$, with

$$m_{rs} = 10\log_{10} \frac{\overline{\overline{P_0}}}{1\text{mW}} - 5\log_{10} 2 . \quad (3.6)$$

This suggests that Rayleigh fading reduces the logarithmic mean of the signal by about 1.5 dB and gives a standard deviation of $\sigma = \sqrt{\ln 2}$ ($s = 4.34 \sqrt{\ln 2} \approx 3.6$ dB). For the wanted signal, the (exact) Suzuki distribution is considered in [21]. In [21],[28], Schwartz and Yeh's method is applied for incoherent cumulation of a number of signals with a log-normal distribution with mean (3.5) and variance (3.4).

In section 2.1.3.1 of this dissertation, the logarithmic mean and standard deviation of a Rayleigh fading signal have been derived. Since Rayleigh fading and shadowing cause statistically independent multiplicative fluctuations of the received power, attenuation in dB-values caused by shadowing and Rayleigh fading can be added. Thus the results in section 2.1.3.1 indicate that if *logarithmic* moments of the approximate log-normal distribution would be matched to the exact Suzuki distribution, the mean would be found to be 2.5 dB below the local-mean caused by shadowing, and a standard deviation of about 6 dB would be found appropriate to describe the effect of Rayleigh fading. Hence, if logarithmic mean and standard deviations are employed, the effect of Rayleigh fading appears larger than if linear moments are employed. This is explained by the fact that Rayleigh fading occasionally produces deep fades. Their effect is particularly noticed if dB values are studied, while the effect of deep fades on linear moments is much less. In [21], approximation is used only for interfering signals. Occasional deep fades of the interference may not be of great influence on the average harm caused by the interference. Moreover, as will be discussed in chapter 4, if multiple interference signals cumulate, deep fades of one interfering signal are likely to be hidden by the presence of other interfering signals. This suggests that the

approximation chosen by Prasad et al. [21],[29] is probably more appropriate than an approximation based on logarithmic moments.

Instead of these approximate techniques, an exact method of calculating the outage probability is now suggested. Since the principal part of the calculation can be performed analytically, this new technique is also faster in terms of computer time.

3.2 EXACT ANALYSIS OF CO-CHANNEL INTERFERENCE PROBABILITY

The complete statistical propagation model, i.e. including groundwave propagation, shadowing and Rayleigh fading, is now considered. The instantaneous power received at receiver A from the i -th mobile terminal is denoted as p_{A_i} . (This minor extension of the notation p_i will allow later extension of the analysis to networks with diversity reception). Similarly, the local-mean power and the area-mean power are denoted as $\bar{p}_{A_i}$ and $\bar{p}_{A_0}$, respectively. Throughout this analysis, the location of each mobile terminal and the corresponding area-mean power are assumed to be known, whereas the local-mean power and the instantaneous received power are considered to be stochastic variables. The distance between mobile terminal i and receiver A is denoted as a_i . The signal from terminal 0 is taken to be the 'wanted' signal. The event of successful reception at a certain instant of the wanted signal at base station A is denoted as A_0 . By transformation of stochastic variables, it was shown in [20] that

$$\begin{aligned} Pr(A_0) &\triangleq Pr\left(\frac{p_{A_0}}{p_i} > z\right) \\ &= \int_{z0^-}^{\infty} \int_{p_{A_0}}^{\infty} f_{p_{A_0}}(xy) f_{p_i}(x) x \, dx dy . \end{aligned} \quad (3.7)$$

Here, $f_{p_i}(\cdot)$ is the pdf of the instantaneous power p_i of the joint interference from n co-channel interfering signals. In a Rayleigh fading channel, the conditional pdf $f_{p_{A_0}}$ of

the instantaneous power of the wanted signal is the exponential distribution (2.22). Hence,

$$Pr(A_0|\bar{p}_{A_0}) = \int_{z=0}^{\infty} \int_{\bar{p}_{A_0}}^{\infty} \frac{1}{\bar{p}_{A_0}} \exp\left(-\frac{xy}{\bar{p}_{A_0}}\right) f_{p_i}(x) x dx dy . \quad (3.8)$$

After interchanging the order of integration, this reduces to

$$Pr(A_0|\bar{p}_{A_0}) = \int_{0^-}^{\infty} \exp\left(-\frac{xz}{\bar{p}_{A_0}}\right) f_{p_i}(x) dx . \quad (3.9)$$

The right-hand side of this equation equals the Laplace transform of the pdf of the joint interference power at the point $z/\bar{p}_{A_0}$ [39],[40], so

$$Pr(A_0|\bar{p}_{A_0}) = \mathcal{L}\left\{f_{p_i}, \frac{z}{\bar{p}_{A_0}}\right\} . \quad (3.10)$$

This observation will play a key role in our assessment of outage probabilities.

If the joint interference power p_i is due to incoherent accumulation of n independently fading signals, its pdf is the n -fold convolution of the pdf of the individual signal powers. Laplace transformation results in the multiplication of n factors, each containing a Laplace image of the pdf of the received power from an individual signal of the form (3.10), viz.,

$$Pr(A_0|n, \bar{p}_{A_0}) = \mathcal{L}\left\{f_{p_i}, \frac{z}{\bar{p}_{A_0}}\right\} = \mathcal{L}\left\{f_{p_{A_1}} \otimes \dots \otimes f_{p_{A_n}}, \frac{z}{\bar{p}_{A_0}}\right\} = \prod_{i=1}^n \mathcal{L}\left\{f_{p_{A_i}}, \frac{z}{\bar{p}_{A_0}}\right\} \quad (3.11)$$

where $\otimes$ denotes the convolution of two pdf's. The Laplace image of the Suzuki pdf of the instantaneous power received from the i -th interferer with area-mean power $\bar{p}_{A_i}$ was expressed in (2.34):

$$\mathcal{L}\left\{f_{p_{A_i}}, s\right\} = \Phi_{\sigma}\left(s\bar{p}_{A_i}\right) . \quad (3.12)$$

In the event that the i interferers transmits continuously, (3.12) can be inserted in (3.11). The effects of intermittent transmission to avoid unnecessary interference caused during speech pauses will be discussed in section 3.3 by appropriately adapting the expression for the Laplace image. Analogously, data networks with terminals randomly transmitting messages from unknown positions in the cell are addressed in chapter 7, 8 and 9.

By averaging (3.11) over the shadowing experienced by the desired signal, the probability becomes

$$\begin{aligned} Pr(A_0|n) &= \int_0^{\infty} f_{p_{A_0}}(\bar{p}_{A_0}|\bar{p}_{A_0}) \prod_{i=1}^n \mathcal{L}\left\{f_{p_i}, \frac{z}{\bar{p}_{A_0}}\right\} d\bar{p}_{A_0} \\ &= \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma_s \bar{p}_{A_0}} \exp\left\{-\frac{\ln^2 \frac{\bar{p}_{A_0}}{\bar{p}_{A_0}}}{2\sigma_s^2}\right\} \prod_{i=1}^n \Phi_{\sigma}\left(z \frac{\bar{p}_{A_i}}{\bar{p}_{A_0}}\right) d\bar{p}_{A_0} , \end{aligned} \quad (3.13)$$

provided that the area-mean powers of all signals are known. Using the logarithmic local-mean powers as integration variables, thus substituting

$$y_i \triangleq \frac{\sqrt{2}}{2\sigma_s} \ln \frac{\bar{p}_{A_i}}{\bar{p}_{A_0}} , \quad (3.14)$$

the probability is written in full as

$$Pr(A_0|n) = \int_{-\infty}^{\infty} dy_0 \frac{\exp\{-y_0^2\}}{\sqrt{\pi}} \cdot \left[\prod_{i=1}^n \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\bar{p}_0 \exp(\sqrt{2}y_0\sigma_s)}{\bar{p}_0 \exp(\sqrt{2}y_0\sigma_s) + z\bar{p}_i \exp(\sqrt{2}y_i\sigma_s)} \exp\{-y_i^2\} dy_i \right]. \quad (3.15)$$

This expression is the exact solution to the stated problem of the (area-mean) outage probability O in a cellular network with continuously transmitting interferers, since $O=1-Pr(A_0)$.

For channels without shadowing effects ($\sigma_s=0$, $\bar{p}_i=a_i^{-\beta}$), the probability (3.15) reduces to a simpler analytic expression previously given in [13],[24] etc., namely

$$Pr(A_0|n) = 1 - \prod_{i=1}^n \frac{a_i^{\beta}}{a_i^{\beta} + za_0^{\beta}}. \quad (3.16)$$

3.2.1 COMPUTATIONAL RESULTS

An idealised cellular network with a hexagonal cell layout ('honeycomb' structure), as discussed in chapter 1 considered. In a cellular network, signal outages are caused mainly by the interference from the active users in the n nearest co-channel cells. Typically, $n=6$ for idealised hexagonal cell lay-outs. Interference from more distant users in second- and higher-order co-channel cells can usually be neglected, provided that β sufficiently exceeds 2. Although an interfering user can be at any place inside his cell, it is assumed that all harmful interferers are at the re-use distance r_u ($a_i=r_u$ for $i=1,..,n$), where r_u is the distance between the centers of two co-channel cells. If $r_u > 1$, this approximation may be reasonable. As a worst case, it can be considered that interfering terminals are near the fringe of their cells where they are closest to the cell of the terminal transmitting the wanted signal. In this case, $a_i > r_u - 1$ for $i=1,..,n$. Here, as in most papers on outage probabilities, we adopt the former assumption ($a_i \approx r_u$),

with $P_i = r_u^{-\beta}$ for all first-order interferers. The affected user is considered to be at the cell boundary ($a_0=1$), which represents a worst case. UHF ground wave propagation is considered with $\beta=4$. For networks operating in irregular or urban terrain with $\beta \approx 3.2$, outage probabilities tend to be slightly larger.

To obtain numerical results in the presence of shadowing for $\sigma_s > 0$, the Hermite quadrature method has been applied, although the discussion in section 2.1.5 suggests that the technique for numerically evaluating (3.13) can presumably be refined. Fig. 3.1 presents the resulting outage probability, versus the normalised re-use distance r_u . Shadowing with an intensity of $s_s=6$ dB ($\sigma_s=1.38$) is assumed. The receiver threshold is taken 10 dB ($z=10$), which corresponds to the specifications of the digital G.S.M. system [15]. Moreover, analogue FM has a threshold of about 10 dB [9] (see also table 3.1).

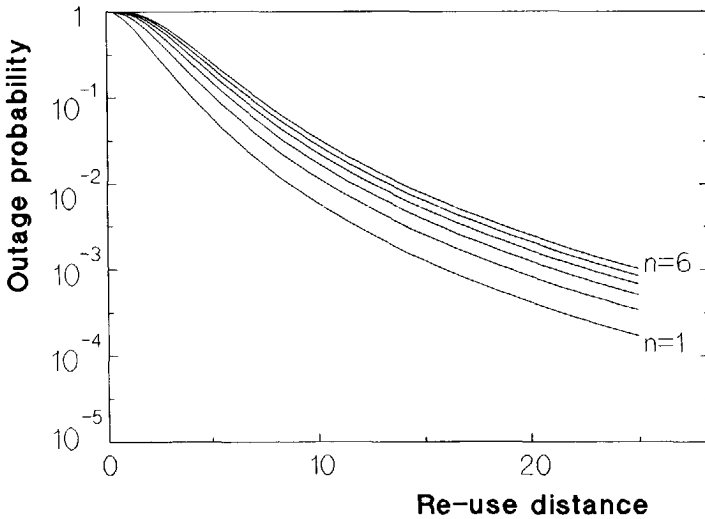


Fig. 3.1: Outage probability versus normalised re-use distance r_u for 1,2,...,6 co-channel interferers. Shadowing with $s_s=6$ dB, receiver threshold $z=10$ (10 dB). UHF groundwave propagation with $\beta=4$.

As a confirmation of the numerical technique, exact results can be found in closed form at particular points of the curve for a single interferer ($n=1$): After substituting $y \Delta y_0$ and $t \Delta \frac{1}{2} \sqrt{2} \cdot (y_I \cdot y_0)$, (3.15) becomes

$$\begin{aligned} Pr(A_0|n=1) &= \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\exp\{-2y^2 - 2\sqrt{2}ty - 2t^2\}}{1 + z r_u^{-\beta} \exp(2\sigma_s^2 t)} dy dt \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-t^2} dt}{1 + z r_u^{-\beta} \exp(2\sigma_s^2 t)} = \Phi_{\sqrt{2}\sigma_s}(z r_u^{-\beta}) \end{aligned}$$

Hence, for the special case $r_u = \beta / (z \exp\{2k\sigma_s^2\})$, the outage probability $O(r_u) \Delta 1 - Pr(A_0)$ can be found in closed form from expression (C.3) in Appendix C. For 6 dB of shadowing ($\sigma_s = 1.36$), $z = 10$ and $\beta = 4$, one finds the following outage probabilities:

re-use distance r_u	outage probability $O(r_u)$
$k=1: \quad r_u = \sqrt{\beta / (z \exp\{2\sigma_s^2\})} \approx 4.48$	$\frac{1}{2} \exp\{-\sigma_s^2\} \approx 0.07$
$k=2: \quad r_u = \sqrt{\beta / (z \exp\{4\sigma_s^2\})} \approx 11.3$	$\exp\{-3\sigma_s^2\} - \frac{1}{2} \exp\{-4\sigma_s^2\} \approx 3.8 \cdot 10^{-3}$

These results closely agree with the curve for $n=1$ in Fig. 3.1. The second example ($k=2$) also closely agrees with the approximate expressions for large C/I -ratios, which will be discussed further in section 3.4.5.

Fig. 3.2 shows the outage probability versus the standard deviation σ_s of the shadowing as calculated by our exact method (—) and by the approximate method by Prasad et al. (— —) in [21],[29]. The latter method produces slightly pessimistic outage probabilities, i.e., it overestimates outage probabilities by about 10% compared to our exact result (3.15). The accuracy of the method proposed in [21],[29] was found to improve with increasing σ_s , but appears relatively independent of the number of signals n .

The method proposed by Prasad and Arnbak [29], nevertheless, has the potential to combine shadowing and *Rician* multipath fading, which is relevant to micro-cellular networks [51]. In the case that $C_i > 0$, the modified mean (3.5) and variance (3.4) will be closer to the mean and variance caused by the shadowing. This propagation model, which is more appropriate to mobile radio outside city centres, can presumably not be investigated by the exact method presented here, which is based on the trick of interpreting the exponential power distribution of the Rayleigh-fading wanted signal in Eq.(3.9) as the kernel of a Laplace transform. This useful property is obviously lost with a Rician-fading wanted signal if $C_o > 0$. Integral kernels containing Bessel functions would occur with Rician fading, and do not have the simple interpretation in terms of Laplace functions exploited above.

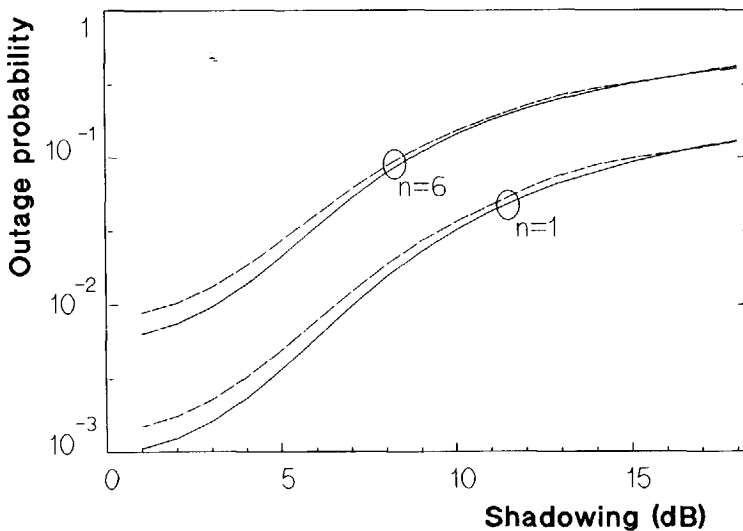


Fig. 3.2: Outage probability versus standard deviation of the shadowing. (—) exact results from Eq. (3.15), (---) approximate results. Re-use distance $r_u=10$. UHF groundwave propagation with $\beta=4$. Receiver threshold $z=10$ (10 dB).

Fig. 3.1 and Fig. 3.2 are given as examples of the use of Eq. (3.15); outage probabilities for other parameters can be computed without great computational effort. The parameters $s_s=6$ dB and $\beta=4$ were chosen in accordance with a number of other papers on this topic, e.g. [21],[24].

If low-resolution terrain data is used, large shadow fluctuations ($s_s \approx 12$ dB) require relatively large re-use distances to ensure reliable communication. Moreover, because of the large-area and small-area properties of shadowing, the resulting large-area-mean performance is likely to be non-uniformly distributed over the boundary of the cells. To ensure reliable communication on all parts of the cell boundary, the *large-area*-mean outage probabilities are to be kept significantly smaller than the minimum required *small-area*-mean outage probability P_c , required for intelligible reception of a voice signal. This leads to conservative network design: on most parts of the cell boundary the performance will be substantially better than prescribed by the criterion P_c .

On the other hand, for highly accurate terrain data, $s_s \rightarrow 0$ because all shadow fluctuations are included in the estimates of the area-mean powers $\bar{P}_i$. If small-area-mean outage probabilities are computed for each small area on the cell boundary, this allows the design of cellular structures customised for the particular propagation environment. If the cell layout is planned with small-area-mean outage probabilities not exceeding P_c , spectrum efficiency may be significantly improved over the case of theoretical network planning using no terrain data.

The use of terrain data to reduce s_s is particularly relevant since it is known from previous results, e.g. Fig. 2 in [21], that the re-use distance required for a certain outage probability differs tremendously for $s_s = 6$ dB and 12 dB. However, a general, objective comparison between theoretical and customised cell planning cannot easily be given because the spectrum efficiency highly depends on the creativity with which the practical cell layout is designed.

3.3 EXTENDED MODEL OF THE NETWORK

The analysis of section 3.2 is now generalised to take into account

- i) site diversity,
- ii) discontinuous voice transmission, and
- iii) a receiver noise floor.

First, the concept of diversity is introduced, and a general discussion of discontinuous voice transmission in cellular telephony networks is given. The limiting effect of a receiver noise floor is discussed. Based on this, the analysis in the previous section is extended. The case of two cooperating receivers at different sites is addressed with this generalised model. The technique for numerical evaluation is discussed.

Computational results are presented, both for the special case of channels without shadowing and for channels with shadowing. approximate expressions for high average carrier-to-interference ratios are presented.

3.3.1 THE CONCEPT OF DIVERSITY

Major improvements in the quality of reception can be achieved if one exploits the fact that Rayleigh fading of the signal received at two antennas, sufficiently spaced apart, is uncorrelated (see Eq. 2.27). The base station either selects the antenna that gives the best performance, or appropriately processes and combines the signals from both antennas, to enhance the quality of reception. This local technique is known as *antenna* (or *micro-*) diversity [30],[31],[32]. The use of diversity reception allows higher local interference power levels for a certain required system performance. Hence, higher spectrum efficiency can be achieved. Using *site* (or *macro-*) diversity [31]-[34], signals from more base stations are combined to improve the signal received from a mobile terminal. This is in contrast to most existing cellular telephony nets, where a formal handover to another base station than the primarily assigned one is required before it can participate in handling signals from a certain terminal. Macro-diversity

combats outages more effectively than micro-diversity, since not only Rayleigh fading but also shadowing and path loss can be uncorrelated at different receiver sites. In particular, site diversity is helpful for terminals near the boundary of a cell. If, in a conventional system, a terminal moves outside the range of one base station, a handover to an adjacent base station is performed. The optimum instant of handover is difficult to determine because the mobile radio channel is of a stochastic nature [35]. A terminal moving near the boundary of two adjacent cells may experience degraded channel performance if the handover instant is chosen at an unfortunate moment. Site diversity can offer a 'soft' handover for a terminal moving from one cell into an adjacent cell. Site diversity becomes of particular interest in future personal cordless telephone networks, since indoor wave propagation suffers from severe log-normal signal fluctuations caused by blocking or shadowing by walls [33].

The advantages of site diversity are obtained at the cost of increased complexity of the system hardware and of the access protocols. Signals arriving at different locations have to be compared and processed; this also requires an increased number of links in the fixed interconnection network. Moreover, if active terminals move over larger distances in a network with site diversity, determination of the involved *group* of tracking base stations is required. This is more complicated than a straightforward handover in conventional systems.

Analytical modelling of site diversity in land-mobile radio networks has received little attention in the past. Recently, the number of reported results has increased. Computer simulations of a cellular network with micro- and macro-diversity were reported in [32],[34]. In the following, it is assumed that each mobile terminal transmits a digitised voice channel in the form of frames (or segments) of bits with error detection coding. Whenever a base station receives an error-free voice segment, it is forwarded to the mobile switch centre (MSC) and immediately scheduled on the appropriate telephone circuit. If a segment is received correctly at more than one base station simultaneously, the MSC may ignore all but one copy of the segment. Thus, a form of dynamic routing of subsequent voice segments occurs at this stage, violating the concept of circuit switching in the strict sense. However, we will consider

a system which accepts an incidental loss of a voice segment due to an excessive burst of interference on the radio link (*loss system*). This is a major difference from packet radio networks where data packets lost on the radio channel are retransmitted (*delay system*), as e.g. studied in chapter 6 to 9. Communication between mobiles, base stations and the MSC is assumed instantaneous. As a result, no resequencing of voice segments can occur. We do not address the design of the wired trunk network and the supporting protocols. The selection of one of the receiver outputs at the MSC is relatively easy in a digital system with error detection coding: a voice segment will be dropped if the number of bit errors exceeds a certain maximum of correctable errors. For ease of analysis we do not evaluate exact block error rates, but shall assume a voice segment to be received correctly if its instantaneous power in at least one base station exceeds the joint interference power by at least a factor z . When the voice segment fails to capture any of the base stations, the user experiences a short outage of the telephone circuit. During the reception of each voice segment, the received power is assumed constant for all signals [22]. A lower word error rate could be achieved by combining the a posteriori information on detected bit sequences at the various receivers. This possibility is not addressed here.

This section derives generalised analytical results for outages in a telephony network with discontinuous voice transmission. The probability of loss of a voice segment is expressed for a mobile user on an end-to-end circuit basis. The analysis is limited to multiple access in the uplink, i.e., the inbound channel from mobile terminals to the fixed public switched telephone network (PSTN); multiplexing of signals in the downlink is not addressed here. The analysis includes interference and noise limitations.

3.3.2 DISCONTINUOUS VOICE TRANSMISSION

In the basic analysis, it was assumed that in all first-order co-channel cells interferers are active. This assumption is pessimistic for various reasons. First, if in a trunking

network all available frequencies are in use, the blocking probability for newly arriving calls is unity. This is evidently unacceptable to the terminal. Secondly, if mobile terminals in co-channel cells happen to be close to the base station, the transmit powers may be reduced to minimise the interference caused to other terminals. Thirdly, the transmitter may be switched off during speech pauses, which also reduces the mutual interference [45]. If one investigates the link quality experienced, an observation interval has to be chosen appropriately. If (small-area-mean) outage probabilities are observed during a period longer than a few seconds but substantially shorter than the duration of an average telephone call, the channel occupancy and the required transmit powers are likely to remain fixed, but the channel fading, including multipath fading and shadowing, and the on/off pattern of the voice are stochastic variables. We now address the worst-case situation that each relevant co-channel cell contains a terminal at maximum power, which is only switched off during speech pauses of the voice.

Each mobile terminal i ($i=0,1,\dots,N$ with N the size of the population of terminals) is assumed to transmit digitised voice segments (or 'packets') of fixed duration equal to the unit of time. The uplink channel is slotted; the time slots are perfectly synchronised throughout the entire network and known to all mobile and base stations. During a talkspurt of the i -th mobile terminal (event i_{ON}), voice segments are generated and transmitted in each successive time slot. No voice segments are generated during gaps between talkspurts; the transmitter is switched off during these periods. This technique is known as discontinuous voice transmission (DTX) or voice-excited transmission (VOX) [46]. The probability of activity of terminal i during a certain time slot is $Pr(i_{ON})$; the event of not transmitting is denoted by i_{OFF} , with $Pr(i_{OFF}) = 1 - Pr(i_{ON})$. The activity of terminals is assumed mutually independent. A typical value reported for speech activity is $Pr(i_{ON}) \approx 0.43$ for a coding strategy using a slow voice on-off detector [37][38]. For a fast coding strategy, detecting also short off-periods (mini-gaps) within a principal speech burst, the activity is $Pr(i_{ON}) \approx 0.36$ [37],[38].

If propagation delays can be neglected, the probability that a segment from terminal i causes interference to the wanted signal equals $Pr(i_{ON})$. Taking into account periods of inactivity, the pdf of the instantaneous interference power at the receiver is

$$f_{p_{A_i}}(x) = Pr(i_{OFF}) \delta(x) + Pr(i_{ON}) \int_0^{\bar{p}_{A_i}} \frac{1}{\bar{p}_{A_i}} \exp\left\{-\frac{x}{\bar{p}_{A_i}}\right\} \frac{1}{\sqrt{2\pi} \sigma \bar{p}_{A_i}} \exp\left\{-\frac{1}{2\sigma^2} \ln^2 \frac{\bar{p}_{A_i}}{\bar{p}_{A_i}}\right\} d\bar{p}_{A_i} \quad (3.17)$$

with the Laplace transform

$$\mathcal{L}\{f_{p_{A_i}}, s\} = Pr(i_{OFF}) + Pr(i_{ON}) \Phi_\sigma\left(s \bar{p}_{A_i}\right) \quad (3.18)$$

For ease of notation, an image function

$$\Phi(A, i) \triangleq Pr\left(p_{A_0} < z p_{A_i} \mid \bar{p}_{A_0}, \bar{p}_{A_i}, i_{ON}\right) = 1 - \Phi_\sigma\left(z \frac{\bar{p}_{A_i}}{\bar{p}_{A_0}}\right) \quad (3.19)$$

is introduced. This is a function of the *area-mean* power received from the interfering mobile i and the *local-mean* power received from the reference terminal 0, at base station A . It also depends on the standard deviation of the shadowing σ_s and on the receiver threshold z .

3.3.3 RECEIVER NOISE FLOOR

For a receiver with RF-bandwidth B_T , the total received noise power is $N_A = N_0 B_T$, if N_0 is the one-sided spectral power density of the AWGN at the receiver input. Band-pass Gaussian noise is known to have a Rayleigh-distributed envelope, so the momentary noise power p_n (observed during $T \ll T_3 \ll T_\phi$, see sect. 2.1.7) is exponentially distributed with pdf

$$f_{p_n}(x) = \frac{1}{N_A} \exp\left\{-\frac{x}{N_A}\right\}, \quad (3.20)$$

and Laplace image

$$\mathcal{L}\{f_{p_n}, s\} = \frac{1}{1+sN_A}. \quad (3.21)$$

The momentary noise power may be considered constant only for very short time intervals since the noise bandwidth is B_T . Signal impairments determined from (3.21) therefore also tend to report the effect of short noise spikes, rather than purely expressing harmful fading of the desired signal below the constant receiver noise floor. This is in contrast to the basic idea of an outage of the baseband signal during some period of time T , e.g. with T the duration of a voice packet. If we consider an observation time with $T \gg T_3$, the noise is wideband compared to the observation window. A more appropriate description is *constant* noise power, thus with (wideband) pdf

$$f_{p_n}(x) = \delta(x - N_A), \quad (3.22)$$

with Laplace image

$$\mathcal{L}\{f_{p_n}, s\} = \exp\{-sN_A\}. \quad (3.23)$$

To find the outage probability, $s = z/\bar{p}_{A0}$ is to be inserted (see 3.11). For high signal-to-noise ratios (large $\bar{p}_{A0}$), the behaviour of the image function for small values of s is relevant. The zero-th and first-order terms of the Taylor expansion of (3.23) and (3.21) are identical, viz,

$$\mathcal{L}\{f_{p_n}, s\} \approx 1 - sN_A + \dots \quad (3.24)$$

So, in practice for high C/N -ratios, the models (3.20) and (3.22) are not substantially different. This suggests that the exact behaviour of the pdf of the noise signal during the observation interval may only be of second-order relevance.

In the following, we use (3.23), corresponding to an observation interval that is large compared to the rate of change of the modulation of the wanted signal. This agrees with the assumption of incoherent cumulation of the signals from interfering terminals. The probability that the wanted signal with a known local-mean power is above the receiver noise floor at receiver A is

$$P_{NA} \triangleq \Pr(p_{A_0} > z N_A) = \exp\left\{-z \frac{N_A}{P_{A_0}}\right\}. \quad (3.25)$$

For large C/N , $p_{NA} \rightarrow 1 - (zN_A/P_{A_0})$.

The noise N_A is now included as an additional uncorrelated interference signal, i.e., P_{NA} is seen as an additional degrading factor in the product (3.11) describing the outage probability for a given local-mean power of the wanted signal. The above discussion addressed AWGN, band-limited by RF filters in the receiver. In mobile radio, man-made (burst) noise further threatens the quality of reception [44], [45]. It is the impression of the author, that by appropriately combining the wideband pdf (3.22) and the narrowband pdf (3.17), the effect of burst noise, i.e., noise with slower time constants than the modulating signal, can be addressed. Here, this is left as a recommendation for further study.

3.4 GENERALISED EXACT ANALYSIS OF OUTAGE PROBABILITY

Using (3.11), (3.17), (3.19) and (3.25), the long term probability of successful reception in the presence of noise, given the local-mean power of the wanted signal and the area-mean power of the interfering signals, becomes

$$Pr\left(A_0 \mid \bar{p}_{A_0}, \left\{\bar{p}_{A_i}\right\}_{i=1}^N\right) = P_{NA} \prod_{i=1}^N 1 - \phi(A, i) Pr(i_{ON}) . \quad (3.26)$$

The probability of correct reception, given the area-mean powers of all $N+1$ signals is found from averaging (3.26) over the range of local-mean power of the desired signal, i.e.,

$$Pr(A_0) = Pr\left(A_0 \mid \left\{\bar{p}_{A_i}\right\}_{i=0}^N\right) = \int_0^{\infty} f_{\bar{p}_{A_0}}(\bar{p}_{A_0} \mid \bar{p}_{A_0}) P_{NA} \left[\prod_{i=1}^N 1 - \phi(A, i) Pr(i_{ON}) \right] d\bar{p}_{A_0} . \quad (3.27)$$

In the following discussion, conditionality on area-mean powers will no longer be explicitly denoted. For $Pr(i_{ON})=1$ and $N_A=0$, (3.27) becomes identical to (3.15). It appears from (3.27) that the activity level $Pr(i_{ON})$ of interferer i and the weight $\phi(A, i)$ of the vulnerability of the wanted signal to this interference are multiplicative.

Eq. (3.27) gives the probability of loss of a wanted voice segment. An outage occurs if the user produces a talkspurt *and* the corresponding transmission is disturbed. If we assume that the system does not erroneously insert noise or crosstalk during speech pauses, the *time-average* outage probability is $Pr(0_{ON}) \cdot (1 - Pr(A_0))$. Transmitter identification codes can largely avoid disturbing signals in the baseband voice circuit when the desired carrier is switched off. Numerical results will be given for $Pr(A_0)$ and the corresponding outage probability $1 - Pr(A_0)$, thus conditional on the knowledge 0_{ON} that a wanted signal is transmitted.

3.4.1 A POSTERIORI PROBABILITY OF INTERFERENCE

We now explore the a posteriori information provided by the event A_0 about the activity of the interfering terminals. The probability $Pr(k_{ON} \mid A_0)$ that a terminal k has

transmitted an interfering voice segment, given that the wanted voice segment (of terminal 0) is received correctly by base station A , is found from Bayes rule

$$Pr(k_{ON}|A_0) = \frac{Pr(A_0|k_{ON})}{Pr(A_0)} Pr(k_{ON}), \quad (3.28)$$

where $Pr(A_0|k_{ON})$ is the probability that a wanted segment is received successfully, despite the knowledge that interferer k was active. Taking account of the noise floor, event k_{ON} , and possible activity of $N-1$ other interferers with probability $Pr(i_{ON})$,

$$Pr(A_0|k_{ON}, \bar{p}_{A_0}) = P_{NA} \{1 - \phi(A, k)\} \prod_{i=1, i \neq k}^N 1 - \phi(A, i) Pr(i_{ON}) \quad (3.29)$$

where we assumed the local-mean power of the wanted signal to be known. The conditional probability of activity of k can be expressed using (3.28), viz.,

$$\begin{aligned} Pr(k_{ON}|A_0, \bar{p}_{A_0}) &= \frac{P_{NA} \{1 - \phi(A, k)\} Pr(k_{ON}) \prod_{i=1, i \neq k}^N 1 - \phi(A, i) Pr(i_{ON})}{P_{NA} \prod_{i=1}^N 1 - \phi(A, i) Pr(i_{ON})} \\ &= \frac{1 - \phi(A, k)}{1 - \phi(A, k) Pr(k_{ON})} Pr(k_{ON}). \end{aligned} \quad (3.30)$$

Remarkably, the probability that k was active during a slot in which the wanted signal was received successfully at receiver A is defined uniquely by $Pr(k_{ON})$ and the image $\phi(A, k)$: only variables concerned with terminal 0 and k appear in (3.30). The a posteriori probability thus appears to be dependent neither on the noise level, nor on further interfering terminals.

For $Pr(k_{ON}) < 1$, the a posteriori probability $Pr(k_{ON}|A_0)$ is always less than the a priori probability $Pr(k_{ON})$. Successful reception of the wanted signal at receiver A gives the information that during that particular time slot the number of interfering signals is presumably relatively low.

Fig. 3.3 illustrates the effect of the position of the interfering mobile terminals k on the a posteriori probability of an interfering transmission from terminal k . The distance a_0 of the terminal transmitting the wanted signal to receiver A is normalised to unity ($p_{A_0}=a_0^{-\beta}=1$). Only normalised re-use distances up to $a_k=r_u=5$ (including the cases $C=1, 3, 4$ and 7) are shown, because relatively dense re-use patterns are possible if site diversity is used. For relatively severe shadowing, say $s_s>6$ dB, the a posteriori probability (3.30) significantly differs from the a priori probability $Pr(k_{ON})$, particularly for short re-use distances. For larger re-use distances ($r_u>5$), the differences become small for reasonable shadowing ($s_s<12$ dB).

Probability (3.30) is now used to express the probability of capture at receiver B , given the event A_0 .

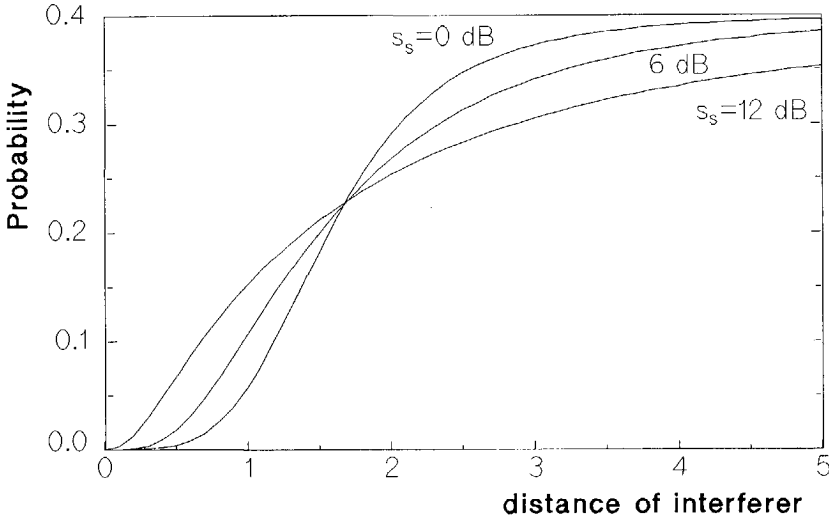


Fig. 3.3: A posteriori probability $Pr(k_{ON}|A_0)$ of activity of interferer k versus distance of the interferer a_k , given that the wanted signal from distance $a_0=1$ was received successfully. User activity $Pr(k_{ON})=0.4$. Receiver threshold $z=4$. Shadowing $s_s=0, 6$ and 12 dB.

3.4.2 SIGNAL OUTAGE WITH MACRO-DIVERSITY

Assume that the positions of the $N+1$ terminals are given. Terminal i is at a normalised distance a_i from base station A and at a normalised distance b_i from base station B (see Fig. 3.4). Both Rayleigh fading and shadowing are assumed to be uncorrelated at the two base stations. We now consider the probability that the wanted signal captures the receiver at B , given the *local-mean* powers of the wanted signal at receiver A and B , the *area-mean* powers of the interference at receivers A and B , and on the condition that the test signal is received correctly at receiver A (event A_0).

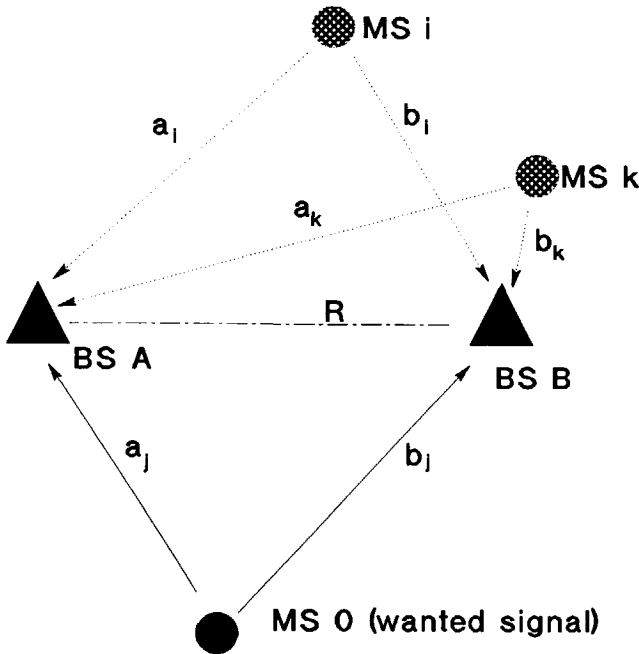


Fig. 3.4: Model for macro-diversity: Receivers are located in base stations A and B , the wanted signal arrives from mobile terminal 0 , interferers are i and k

Similar to (3.26), we write

$$Pr(B_0 | A_0, \bar{p}_{A_0}, \bar{p}_{B_0}) = P_{NB} \prod_{i=1}^N 1 - \phi(B, i) Pr(i_{ON} | A_0, \bar{p}_{A_0}), \quad (3.31)$$

where P_{NB} is the probability that the wanted signal is above the noise floor at receiver B . Inserting (3.30) gives

$$Pr(B_0 | A_0, \bar{p}_{A_0}, \bar{p}_{B_0}) = P_{NB} \prod_{i=1}^N 1 - \frac{1 - \phi(A, i)}{1 - \phi(A, i) Pr(i_{ON})} \phi(B, i) Pr(i_{ON}). \quad (3.32)$$

Since $1 - \phi(A, i)$ is smaller than $1 - \phi(A, i) \cdot Pr(i_{ON})$ if $Pr(i_{ON}) < 1$, the probability (3.32) is smaller than the unconditional probability (3.26) (if $P_{NA} = P_{NB}$). Hence, a voice segment which captures receiver A has an enhanced probability of capturing receiver B , too. This is explained by the fact that with relatively high a posteriori probability the interference level during that particular time slot is low. The probability that the wanted voice segment captures at least one of the two base stations, i.e., the event (A_0 or B_0 or both), equals

$$Pr(A_0 \vee B_0) = Pr(A_0) + Pr(B_0) - Pr(A_0) Pr(B_0 | A_0). \quad (3.33)$$

Conditional on the local-mean power of the wanted signal at receiver A and B , and by inserting (3.32) and (3.26), This probability becomes

$$\begin{aligned} Pr(A_0 \vee B_0 | \bar{p}_{A_0}, \bar{p}_{B_0}) &= P_{NA} \left[\prod_{i=1}^N 1 - \phi(A, i) Pr(i_{ON}) \right] \\ &+ P_{NB} \left[\prod_{i=1}^N 1 - \phi(B, i) Pr(i_{ON}) \right] \\ &- P_{NA} P_{NB} \left[\prod_{i=1}^N 1 - \psi(A, B, i) Pr(i_{ON}) \right], \end{aligned} \quad (3.34)$$

where we introduced the function

$$\psi(A, B, i) \triangleq \phi(A, i) + \phi(B, i) - \phi(A, i) \phi(B, i). \quad (3.35)$$

Assuming local-mean powers of the wanted signal at the two base stations to be mutually independent, averaging gives

$$Pr(A_0 \vee B_0) = \int_0^\infty \int_0^\infty Pr(A_0 \vee B_0 | \bar{p}_{A_0}, \bar{p}_{B_0}) f_{\bar{p}_{A_0}}(\bar{p}_{A_0}) f_{\bar{p}_{B_0}}(\bar{p}_{B_0}) d\bar{p}_{A_0} d\bar{p}_{B_0}, \quad (3.36)$$

where the conditional probability (3.34) and log-normal pdfs on the form of (2.29) are to be inserted. In a noise-free system with the activity level $Pr(i_{ON})=1$ for all terminals, that is, if continuous voice transmission is employed, the outage probability corresponding to (3.36) with (3.34) can be rewritten as

$$O \triangleq 1 - Pr(A_0 \vee B_0) = [1 - Pr(A_0)][1 - Pr(B_0)], \quad (3.37)$$

where each of the two outage probabilities for single base stations on the right-hand side of (3.37) has the form (3.27) with $Pr(i_{ON})$ equal to unity. This agrees with the fact that, for mutually independent shadowing and multipath fading on each signal path, outage probabilities at the two receivers may be multiplied to assess the overall probability of circuit outage.

3.4.3 COMPUTATIONAL RESULTS FOR RAYLEIGH-FADING CHANNELS WITHOUT SHADOWING

If shadowing is absent ($\sigma_s=0$), the image function (3.19) reduces to

$$\Phi(A, i) = \frac{\bar{z} \bar{p}_{A_i}}{\bar{z} \bar{p}_{A_i} + \bar{p}_{A_0}} = \frac{z a_0^\beta}{z a_0^\beta + a_i^\beta}. \quad (3.38)$$

For this case, results can be expressed in closed form. As an example, Fig. 3.5 gives a one-dimensional approximation of a situation frequently encountered in cellular networks. The mobile terminal transmitting the desired voice segments is located

equally far from two base stations, i.e., at the locus where a handover is most likely to occur in a conventional system without macro-diversity.

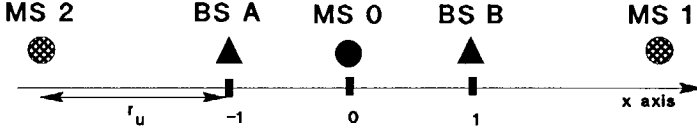


Fig. 3.5: Positions of base stations (BS) and mobile stations (MS) studied in section 3.4.3.

Interference is experienced from terminals located at the normalised re-use distance r_u from the receiving base stations. The normalised distances are

$$\begin{aligned} a_0 &= 1 & b_0 &= 1 \\ a_1 &= r_u + 2 & b_1 &= r_u \\ a_2 &= r_u & b_2 &= r_u + 2 \end{aligned} \quad (3.39)$$

The relevant outage probabilities are given as a function of the re-use distance for a receiver threshold of $z=10$ (10 dB). The propagation exponent is taken $\beta=4$.

Discontinuous voice transmission is considered with activity $Pr(i_{ON})=0.4$. Noise is not considered ($N_A=N_B=0$). In a conventional network without site diversity, the outage probability equals $O_1 = 1 - Pr(A_0)$ for reception at receiver A , with

$$Pr(A_0) = \left[1 - \frac{z}{z + r_u^\beta} Pr(2_{ON}) \right] \left[1 - \frac{z}{z + (r_u + 2)^\beta} Pr(1_{ON}) \right], \quad (3.40)$$

where we inserted (3.38) and (3.39) in (3.34). If a handover to receiver B has been performed, the terminal experiences an outage probability $O_2 = 1 - Pr(B_0)$. Since the positions of the receivers and the interferers are symmetric and since $Pr(1_{ON}) = Pr(2_{ON})$, the probabilities O_1 and O_2 are identical. For macro-diversity, the overall outage probability, denoted as O_3 , is found from

$$\begin{aligned}
 O_3 \Delta 1 - \Pr(A_0 \vee B_0) = 1 - 2 \left[1 - \frac{z}{z+r_u^\beta} \Pr(i_{ON}) \right] & \left[1 - \frac{z}{z+(r_u+2)^\beta} \Pr(i_{ON}) \right] \\
 + \left[1 - \left\{ \frac{z}{z+r_u^\beta} + \frac{z}{z+(r_u+2)^\beta} - \frac{z^2}{(z+r_u^\beta)(z+(r_u+2)^\beta)} \right\} \Pr(i_{ON}) \right]^2. & \quad (3.41)
 \end{aligned}$$

In purely Rayleigh-fading channels ($\sigma_s=0$), expression (3.34) also applies for micro-diversity at any one base station. The principle difference between micro and macro-diversity is the presence of fully correlated and uncorrelated shadowing, respectively. If shadowing is not present ($\sigma_s=0$), the expressions tend to become identical. The only remaining difference between the two systems is that with micro-diversity, path losses are identical to the two base stations for each terminal.

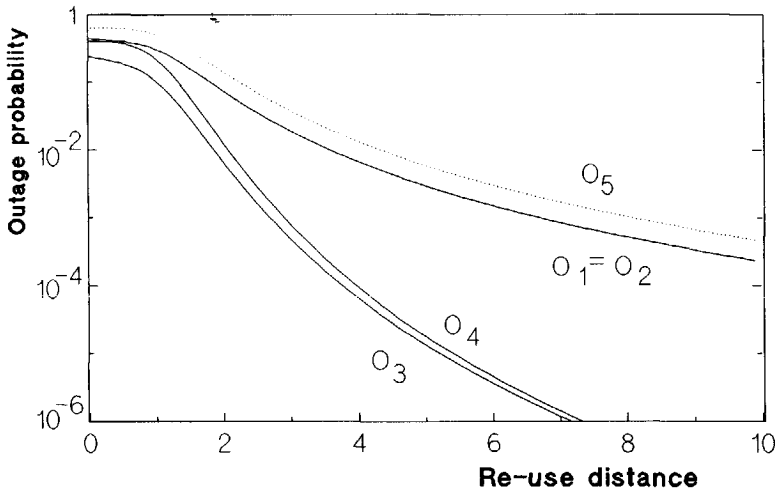


Fig. 3.6: Outage probability versus re-use distance. O_1, O_2 : one base station, O_3 : macro-diversity, O_4 : micro-diversity, O_5 : signal is not received at both receivers simultaneously. UHF groundwave propagation with $\beta=4$.

For micro-diversity, we consider the case that the receiver at base station B is moved to the site of base station A in Fig. 3.5. The local-mean interference powers are the same at receiver A and B for each signal and can be determined from the distances

$$\begin{aligned}
a_0 &= 1 & b_0 &= 1 \\
a_1 &= r_u + 2 & b_1 &= r_u + 2 \\
a_2 &= r_u & b_2 &= r_u
\end{aligned} \tag{3.42}$$

with (3.38). The probability of outage O_4 for micro-diversity, i.e., for two uncorrelated receivers at location A , is found analogous to (3.41), viz.,

$$\begin{aligned}
O_4 &= 1 - 2 \left[1 - \frac{z}{z + r_u^\beta} \Pr(2_{ON}) \right] \left[1 - \frac{z}{z + (r_u + 2)^\beta} \Pr(1_{ON}) \right] \\
&+ \left[1 - \left\{ \frac{2z}{z + r_u^\beta} - \frac{z^2}{(z + r_u^\beta)^2} \right\} \Pr(2_{ON}) \right] \left[1 - \left\{ \frac{2z}{z + (r_u + 2)^\beta} - \frac{z^2}{(z + (r_u + 2)^\beta)^2} \right\} \Pr(1_{ON}) \right].
\end{aligned} \tag{3.43}$$

For reasons of comparison, the probability of reception at the two base stations simultaneously (the event $A_0 \wedge B_0$) is also interesting. The corresponding probability $O_5 = 1 - \Pr(A_0 \wedge B_0)$ is equal to the last term on the right-hand side of (3.41).

It is noted that, for Rayleigh-fading channels without shadowing, macro-diversity gives a slight improvement over micro-diversity, which appears to be caused by different path losses to the two receivers. For small re-use distances, the mean interference power at the two base stations becomes substantially different for each interfering signal. The outage probability O_3 in the network with macro-diversity appears to be about 40% less than the outage probability O_4 with micro-diversity. On the other hand, for increasing re-use distances ($r_u \rightarrow \infty$), the interference situations experienced at A and B eventually become identical since $a_i \approx b_i \approx r_u$, so O_3 slowly approaches O_4 .

3.4.4 COMPUTATIONAL RESULTS FOR CHANNELS WITH SHADOWING AND RAYLEIGH FADING

Many proposals exist for the grid of receiver locations to support macro-diversity in cellular networks. Base stations with omni-directional or sector antennas can be

placed in the centre or in certain corners of each cell [34]. In this section we consider a wanted signal arriving from a point in the cell with the most adverse propagation conditions. In most concepts, this worst-case distance is also equal to the cell radius, which we normalise to unity ($a_0=b_0=1$). The distances from any interfering terminal to the two receivers A and B are assumed to be identical and equal to the re-use distance. Thus $a_i=b_i=r_u$ for all $i=1,2,..N$, so $p_{A_i}=p_{B_i}=r_u^{-\beta}$. Six equally interfering terminals are considered ($N=6$), corresponding to a network with omni-directional antennas.

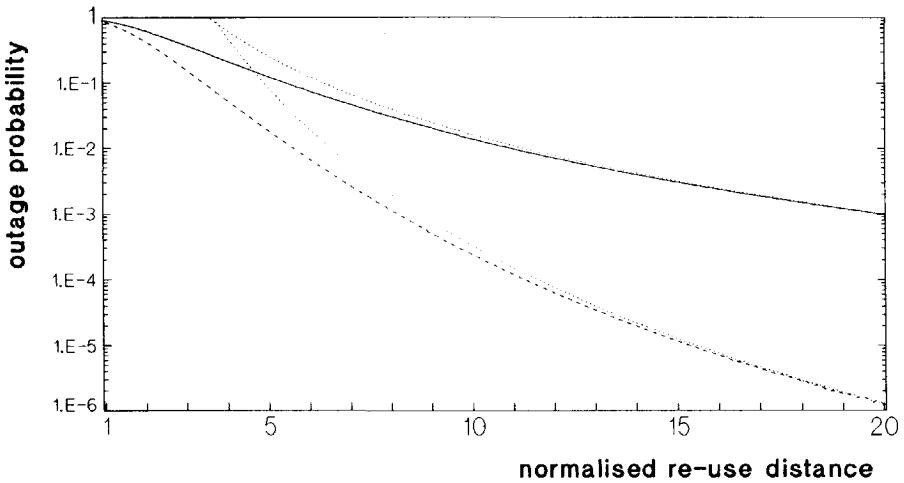


Fig. 3.7: Outage probability versus re-use distance. Shadowing of $s_s=6$ dB. Receiver threshold $z=10$ (10 dB). UHF path loss law $\beta=4$, 6 co-channel interferers, voice activity $Pr(i_{ON})=0.4$. (—) single receiver, (---) macro-diversity, (···) approximate expressions for large C/I-ratios.

Equation (3.34) will be used, implying that shadow fading of a signal from a terminal is assumed to be uncorrelated at receiver A and B . Fig. 3.7 and Fig. 3.8 give the outage probability for 2-branch macro-diversity for shadowing of $s_s=6$ and 12 dB, respectively. Results are compared with the case of no diversity described by (3.27). The curves have been computed from the Hermite polynomial method [40], using 20-point integration. For the outage probability with macro-diversity, a three-dimensional integration is required.

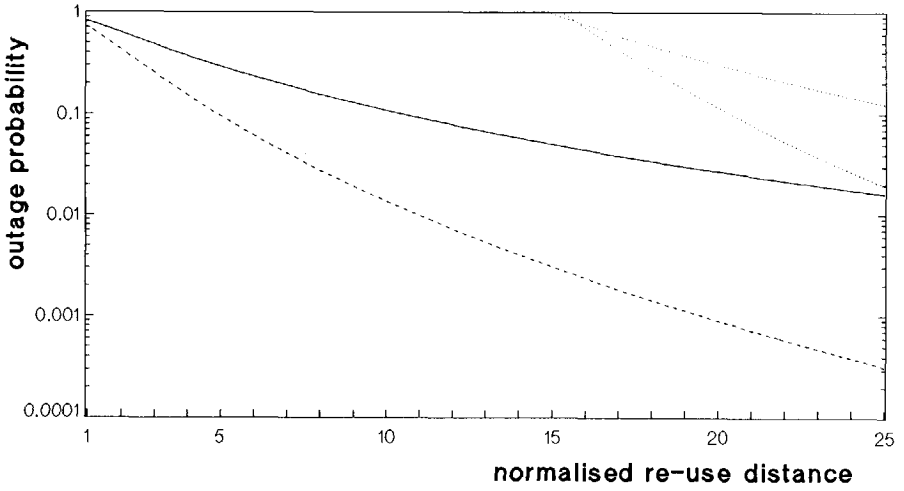


Fig. 3.8: Outage probability versus re-use distance. Shadowing of $s_s=12$ dB. Receiver threshold $z=10$ (10 dB). UHF path loss law $\beta=4$, 6 co-channel interferers, voice activity $\Pr(i_{ON})=0.4$. (—) single receiver, (— —) macro-diversity, (····) approximate expressions.

It is seen that macro-diversity substantially diminishes the outage probability, as compared to the case of one receiver: the curves suggest that, for large r_u , the outage probability with two-branch macro-diversity is almost equal to the square of the outage probability at a single receiver. This agrees with results depicted in Fig. 3.3, which show that the probabilities on the events A_θ and B_θ are not significantly correlated if large re-use distances are employed. However, to achieve a certain prescribed outage probability, e.g. 10^{-2} or 10^{-3} , macro-diversity is seen to allow relatively short re-use distances. Since $1-\Pr(A_\theta \vee B_\theta)$ departs from $\{1-\Pr(A_\theta)\} \cdot \{1-\Pr(B_\theta)\}$, correlation appears to be relevant.

3.4.5 LIMITING CASE FOR HIGH CARRIER-TO-INTERFERENCE RATIO

Although the formal expansion of $\phi_o(s)$ was seen to diverge (chapter 2), the second-order approximation of the image

$$\phi(A, i) \approx z \frac{\bar{p}_{A_i}}{\bar{p}_{A_0}} \exp\left\{\frac{1}{2}\sigma_s^2\right\} - z^2 \frac{\bar{p}_{A_i}^2}{\bar{p}_{A_0}^2} \exp\{2\sigma_s^2\} \quad (3.44)$$

is useful for relatively weak interference ($z\bar{p}_{A_i} < \bar{p}_{A_0}$). We use (3.44) to focus on the asymptotic behaviour for large C/I -ratios. If the noise is neglected ($p_{NA} = p_{NB} = 1$), after inserting this in (3.27), one finds

$$\begin{aligned} Pr(A_0) \approx \int_0^\infty d\bar{p}_{A_0} f_{\bar{p}_{A_0}}(\bar{p}_{A_0}) & \left[\prod_{i=1}^N 1 - z \frac{\bar{p}_{A_i}}{\bar{p}_{A_0}} \exp\left\{\frac{1}{2}\sigma_s^2\right\} Pr(i_{ON}) \right. \\ & \left. + z^2 \frac{\bar{p}_{A_i}^2}{\bar{p}_{A_0}^2} \exp\{2\sigma_s^2\} Pr(i_{ON}) \right]. \end{aligned} \quad (3.45)$$

Note that the approximation will not be valid for $\bar{p}_{A_0} \rightarrow 0$ and $\bar{p}_{A_i} \rightarrow \infty$, i.e., in the far tails of the log-normal distribution. Nonetheless, in the interval that provides the dominant contribution to the integral, the approximation seems reasonable. A formal proof of the accuracy of these approximations is not presented, but the approximate expression are compared with the exact results.

After substituting the integration variable according to (2.33), (3.45) becomes

$$\begin{aligned} Pr(A_0) \approx \int_{-\infty}^\infty dy \frac{\exp\{-y^2\}}{\sqrt{\pi}} \prod_{i=1}^N & \left[1 - z \frac{a_0^\beta}{a_i^\beta} Pr(i_{ON}) \exp\left\{-\sqrt{2}\sigma_s y + \frac{1}{2}\sigma_s^2\right\} \right. \\ & \left. + z^2 \frac{a_0^{2\beta}}{a_i^{2\beta}} Pr(i_{ON}) \exp\left\{-2\sqrt{2}\sigma_s y + 2\sigma_s^2\right\} \right]. \end{aligned} \quad (3.46)$$

Since the integrand contains a factor $\exp\{-y^2\}$, the effect of the approximations in the tails remains small. The product over N leads to 3^N terms. For $a_0 < a_i$, $(a_0/a_i)^{M\beta}$ rapidly vanishes with increasing M . Particularly for moderate values of shadowing, only low-order terms contribute significantly to the integral. Hence,

$$\begin{aligned}
 Pr(A_0) &\approx 1 - \sum_{i=1}^N z \frac{a_0^\beta}{a_i^\beta} \frac{Pr(i_{ON})}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-y^2 - \sqrt{2}\sigma_s y + \frac{1}{2}\sigma_s^2\right) dy + \\
 &\quad \sum_{i=1}^N \sum_{k=1}^N z^2 \frac{a_0^{2\beta}}{a_i^\beta a_k^\beta} Pr(i_{ON}) Pr(k_{ON}) \int_{-\infty}^{\infty} \exp\left\{-y^2 - 2\sqrt{2}\sigma_s y + 2\sigma_s^2\right\} dy \\
 &= 1 - z a_0^\beta \exp(\sigma_s^2) \sum_{i=1}^N a_i^{-\beta} Pr(i_{ON}) + \\
 &\quad z^2 a_0^{2\beta} \exp\{4\sigma_s^2\} \sum_{i=1}^N \sum_{k=1}^N a_i^{-\beta} a_k^{-\beta} Pr(i_{ON}) Pr(k_{ON}) .
 \end{aligned} \tag{3.47}$$

In the first-order term of (3.47), a factor $z \cdot \exp\{\sigma_s^2\}$ is multiplied by the sum of the area-mean powers of each interfering signal. This result is surprising since the method by Schwartz and Yeh indicates that, if multiple log-normal signals are added, the area-mean power of the joint signal tends to be significantly larger than the sum of the individual area-mean powers [28], i.e., simple addition of area-mean signal powers, as in the first-order term of (3.47), would not be a good approximation. But in the special case of very large C/I -ratios, (3.47) suggests that (i) N weak interfering signals cause the same outage probability as one signal with an N times stronger area-mean power, and (ii) Schwartz and Yeh's method may overestimate outage probabilities. Presumably, the method by Schwartz and Yeh focuses on finding a good approximation for the main body of the pdf, whereas in the case of large C/I -ratios, the tails of the pdf become increasingly important.

The mathematical appearance of this approximate outage probability would also suggest that the receiver threshold z can be multiplied by a shadow fade margin $\exp\{\sigma_s^2\}$ to obtain a 'protection ratio' suitable as a rule of the thumb for the planning of practical systems. As discussed in chapter 2, this shadow fade margin depends on the resolution of the terrain data used. Suppose that network planning with exact

estimates of the local-mean power ($\sigma_s=0$) requires a certain protection ratio determined by the threshold z . Then with modest shadowing ($s_s=6$ dB, $\sigma_s=1.36$) arising from less accurate terrain data, an extra protection margin of $\exp\{(1.36)^2\} \approx 6.36$ (or 8 dB) would be required to ensure the same (first-order) outage probability. Without terrain data at all, as in generic network design with $s_s=12$ dB ($\sigma_s=2.72$), the shadow fade margin should be even larger. However, it will be seen later (e.g. in Fig. 3.9) that (3.47) has lost its accuracy in this range, so a simple factor $\exp\{(2.72)^2\} \approx 1633$ (or 32 dB) would be unrealistically large. Nonetheless, one may conclude that, if in a practical system with a prescribed maximum acceptable outage probability, crude terrain data is used, this might be at the cost of the spectrum efficiency of the system.

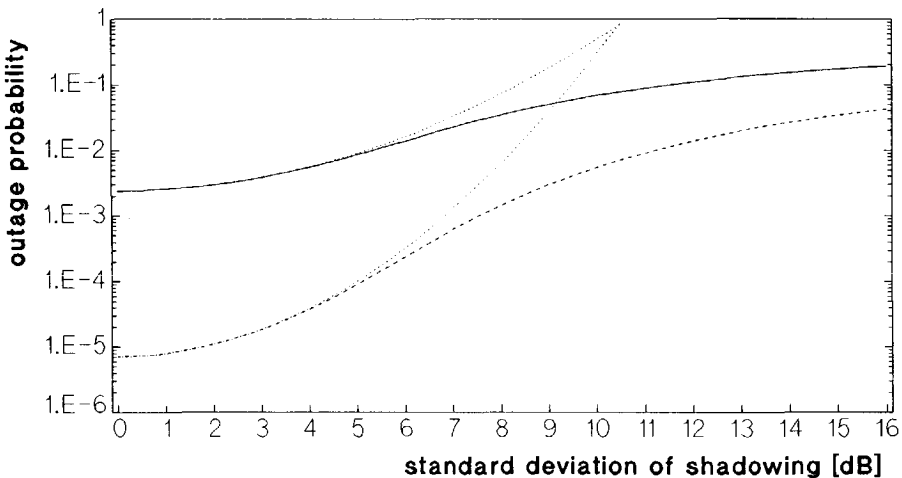


Fig. 3.9: Outage probability versus standard deviation of the shadowing. (—): single receiver, (---) macro-diversity, (···) Approximate expressions for large C/I -ratios. Re-use distance $r_u=10$. Receiver threshold $z=10$ (10 dB).

In Fig. 3.7 - Fig. 3.9, only the first-order approximation is shown. Second-order terms were required to allow investigation of the approximate expressions for site diversity, because all first-order terms in (3.34) cancel for two-branch site diversity. Most second-order terms also cancel; only the terms that arise from multiplication of first-

order terms in the third member of the right-hand side of (3.34) remain. Including these terms, the second-order approximate expression for the probability of successful reception at high C/I -ratios becomes

$$Pr(A_0 \vee B_0) \approx 1 - z^2 a_0^\beta b_0^\beta \exp(2\sigma_s^2) \left[\sum_{i=1}^N \sum_{\substack{k=1 \\ k \neq i}}^N a_i^{-\beta} b_k^{-\beta} Pr(i_{ON}) Pr(k_{ON}) + \sum_{i=1}^N a_i^{-\beta} b_i^{-\beta} Pr(i_{ON}) \right]. \quad (3.48)$$

In the special case where all interfering terminals are at the re-use distance ($a_i = b_i = r_u$) and their voice activity is identical ($Pr(i_{ON}) = P_{ON}$), (3.48) becomes

$$Pr(A_0 \vee B_0) \approx 1 - z^2 \left(\frac{a_0 b_0}{r_u^2} \right)^\beta \exp(2\sigma_s^2) (N(N-1)P_{ON}^2 + NP_{ON}). \quad (3.49)$$

Fig. 3.9 depicts the outage probability and the asymptotes as a function of the standard deviation of the shadowing. The accuracy of the asymptotes decreases with increasing shadowing (σ_s) or decreasing (r_u). Reasonably accurate results occur for $s_s < 6$ dB and $r_u > 10$. This suggests that the simple approximate expressions can be of interest if practical cell layouts are planned with the aid of a high resolution topographical database. On the other hand, when the standard deviation of the shadowing is as large as $s_s = 12$ dB, the approximate results appear to be inaccurate by more than an order of magnitude. For low C/I -ratios, the approximations tend to overestimate the outage probability, so they can thus be used in conservative designs.

3.5 CONCLUSIONS

The probability of a signal outage is a relevant measure of the performance of a mobile communication network with fading channels. In an interference-limited

network not only the fluctuations of the wanted signal, but also of the interference signals must be taken into account.

Assessment of outage probabilities began to be more developed during the eighties; a large number of technical papers appeared on this topic. By and large, these studies employed approximate techniques. An exact analytical technique has been presented in this chapter for the computation of outage probabilities in narrowband multi-user mobile radio channels, such as those for cellular telephony or trunking networks with closed user groups. The analysis includes the effects of both discontinuous voice transmission (DTX) and receiver noise. Radio wave propagation in a land-mobile radio channel has been modelled by taking account of uncorrelated Rayleigh fading and shadowing for each contributing signal path. Relevant parameters are the area-mean powers of the wanted signal and each interfering signal, the receiver threshold, the groundwave path loss exponent, the standard deviation of the shadow fading, and the voice activity of the interfering terminals. The analysis demonstrates that the standard deviation of the shadowing has a substantial effect on the outage probability. In the operational range of cellular networks, the outage probability tends to increase exponentially with increasing shadow fading.

If low-resolution terrain data is used for network planning, large shadow fluctuations ($s_s \approx 12$ dB) require large re-use distances to ensure reliable communication. Moreover, because of the large-area and small-area properties of shadowing, the resulting average performance is likely to be non-uniformly distributed over the boundary of the cells. On the other hand for highly accurate terrain data, all shadowing fluctuations are included in the estimates of the area-mean powers $\bar{P}_i$. This allows the design of cellular structures customised for the particular propagation environment, which is likely to yield improved spectrum efficiency.

The analysis was extended to two-branch site (or macro-) diversity in a cellular telephone network. Also, approximate expressions for large re-use distances and moderate shadow fading ($s_s < 6$ dB) have been derived. Although closed-form solutions only occurred in these asymptotical expressions, the method nonetheless leads to

expressions in a form lending themselves to numerical work. Thus, the method can be used to improve accuracy and speed of calculations of spectrum efficiency in planning of cellular radio systems. The method appears faster in numerical analysis than the approximate techniques first proposed by Schwartz and Yeh in reference [18] and later extended by Prasad and Arnbak in reference [21], but cannot readily be extended to Rician fading as occurring in non-urban areas or in micro-cellular networks.

In channels with shadow fading, site diversity gives substantial enhancement of the quality of the end-to-end circuit. In channels without shadowing, site diversity is still slightly more effective in combatting signal outage than micro-diversity, because the average powers of the interference signals differ due to the different losses for the two paths to the receivers. Numerical results have been given to assist in generic design of systems based on hexagonal cells with a uniform propagation model. The expressions for outage probability are on a form that would also allow more detailed computer-aided investigation of a 'real-life' cellular network, giving highly accurate estimates of the area-mean received powers, e.g., as can be obtained by using a topological data base.

A comparison of the effect of macro and micro-diversity for channels with shadowing has yet to be carried out. The special case of uncorrelated shadowing was considered. Correlation of the shadow attenuation reduces the differences in signal power received from each transmitter at the various sites; this is likely to reduce the effectiveness of macro-diversity (see e.g. [47],[48]). Investigation of site diversity on channels with correlated shadowing is therefore recommended. Expressions (e.g. Eq. (3.36)) appeared on a form that easily allows consideration of correlated shadow attenuation of the *wanted* signal. However, in cellular networks with macro-diversity, it is likely that, an interfering signal from a remote terminal is more correlated at the two receivers than the wanted signal. Correlation of interference seems more complicated to study with the Laplace technique reported in this chapter than correlated wanted signals.

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CHAPTER 4

THRESHOLD CROSSING RATE AND AVERAGE (NON-) FADE DURATION

The outage probability discussed in chapter 3 describes the percentage of time that the C/I -ratio on a mobile link is below a certain threshold. A further relevant performance measure is the duration of the outages. The effect of short but frequently occurring outages is entirely different from the effect of rare but lengthy outages. To understand the temporal behaviour of link outage better, this chapter studies the threshold crossing rate and the average (non-) fade duration in a Rayleigh-fading channel.

During a link fade in an *analogue* cellular network for voice communication, the user experiences not only an absence of the intended signal, but also a burst of noise or

cross-talk. Even if link outages are very short, they collectively degrade the system performance, although they may not be individually recognised. Generally only outages lasting longer than some tens of milliseconds are recognised as a drop-out of the telephony circuit. In *digital* cw communication, the fade duration determines the length of error bursts. If an error-correcting code with interleaving is employed to combat bursts of bit errors, the interleaving interval should be chosen substantially longer than the expected duration of a signal fade. However, in land-mobile radio, codes with short block lengths are most often used, rather than convolutional codes or codes with long block lengths and interleaving. A combination of forward error correction and error detection with retransmission is commonly applied. In voice communication, blocks are dropped if received with too many errors, whereas in computer communication retransmission of lost data is required. In both cases the probability that a block of a certain length is error-free or received with only a small number of bit errors, critically determines the performance of the system. Hence, an appropriate optimum block length ensures that the probability that a block fits inside a non-fade interval is sufficiently high. Shorter blocks suffer from relatively larger overheads, whereas longer blocks are more likely to experience one or more signal fades. It may be more efficient to retransmit an incidental lost packet than to employ a heavily redundant code on a channel with a low bit error probability most of the time.

The duration of the fades in a mobile radio channel has been discussed in several text books, such as [1],[2]. Most results for the level crossing rates and fade durations apply only for a single fading signal in the presence of stationary noise or a fixed receiver threshold. In this chapter, the classical theory developed by Rice for single-user channels [3],[4],[5] is extended for multi-user networks where the mutual interference between users limits the quality of the channel. The probability that the C/I -ratio drops below a certain threshold is determined, taking account of fluctuations of the wanted signal, but also of the joint interference signal. In section 4.1 and 4.2, respectively, the models for the wanted signal and for the joint interference signal are formulated. In section 4.3, the sum of multiple incoherent Rayleigh-fading narrowband signals is approximated as a joint interference signal with Nakagami fading. Outage

probabilities for a Rayleigh-fading signal in the presence of Nakagami-fading interference are derived in section 4.4. In section 4.5, expressions for the rate of crossing a specified C/I threshold are derived. The average fade and non-fade durations are found in section 4.6. It is shown in section 4.7 that the known expressions for the threshold crossing rate and the average non-fade duration for a single fading signal with respect to a stationary (non-fading) threshold level (e.g. determined by a receiver noise floor), occur as limiting cases of the results obtained in section 4.5 and 4.6. Section 4.8 discusses the distribution of the duration of non-fade periods. An approximate model is developed in section 4.9 to compute the probability that a signal outage lasts longer than a specified time. The probability that the C/I -ratio remains above the threshold for a prescribed duration (e.g. during reception of a data block) is studied in section 4.10. Conclusions are summarised in section 4.11.

4.1 SIGNAL MODEL

A sum of n uncorrelated Rayleigh-fading narrowband signals

$$v_i(t) = \sum_{i=1}^n \rho_i(t) \cos(\omega_{c_i} t + \theta_i(t) + \psi_i(t)) \quad (4.1)$$

interferes with a wanted signal $v_o(t)$ of the form

$$v_o(t) = \rho_o(t) \cos(\omega_{c_o} t + \theta_o(t) + \psi_o(t)) . \quad (4.2)$$

Analogous to (3.1), the carrier phases $\theta_i(t)$ and $\psi_i(t)$ describe any Doppler modulation with maximum frequency shift $f_m = \omega_m/2\pi$ arising from terminal mobility and angle modulation, respectively. This phase modulation and slightly different carrier frequencies are assumed to result in incoherent (power) addition of the signals. All $n+1$ signals present in the radio channel are subject to independent Rayleigh fading.

The amplitude ρ_i ($i=0, 1, \dots, n$) and its derivative $\dot{\rho}_i$ are statistically independent stochastic variables, with joint probability density function (pdf) [1],[5],[6]

$$f_{\rho_i, \dot{\rho}_i}(\rho_i, \dot{\rho}_i) = \frac{1}{\sqrt{2\pi} \sigma_{\rho_i}} \exp\left\{-\frac{\dot{\rho}_i^2}{2\sigma_{\rho_i}^2}\right\} \cdot \frac{\rho_i}{\bar{p}_i} \exp\left\{-\frac{\rho_i^2}{2\bar{p}_i}\right\}, \quad (4.3)$$

where $\bar{p}_i$ is the local-mean power of the i -th signal, and σ_{ρ_i} is the standard deviation of the time derivative of the amplitude, calculated from

$$\sigma_{\rho_i}^2 = \frac{1}{2} \omega_m^2 \bar{p}_i = 2\pi^2 \frac{v_i^2}{\lambda^2} \bar{p}_i \quad (4.4)$$

with v_i the velocity of terminal i and λ the wavelength. In contrast to the notation in chapter 2 and 3, μ and σ denote *linear* moments in this chapter.

Refined models for the pdf of a signal amplitude exposed to mobile fading have been suggested [7],[8],[14]. In [8],[14], the pdf of the amplitude ρ_i of a mobile signal was described by the Nakagami m -distribution [9], viz.,

$$f_{\rho_i}(\rho_i) = \frac{\rho_i^{2m-1}}{\Gamma(m) 2^{m-1} \bar{p}_i^m} \exp\left\{-\frac{\rho_i^2}{2\bar{p}_i}\right\}, \quad (4.5)$$

where $\Gamma(m)$ is the gamma function [10], with $\Gamma(m+1)=m!$. The corresponding instantaneous power p_i is gamma distributed, with pdf

$$f_{p_i}(p_i) = \frac{1}{\bar{p}_i \Gamma(m)} \left(\frac{p_i}{\bar{p}_i}\right)^{m-1} \exp\left\{-\frac{p_i}{\bar{p}_i}\right\}. \quad (4.6)$$

The parameter m is called the 'shape factor' of the Nakagami or the gamma distribution. In the special case $m=1$, Rayleigh fading (4.3) is recovered, with an exponentially distributed instantaneous power; for larger m , the fluctuations of the signal strength reduce compared to (4.3). The model behind (4.5) assumes multipath

scattering with relatively large delay-time spreads with different clusters of reflected waves. Within any one cluster, the phases of individual reflected waves are random, but the delay times are approximately equal for all waves ($T_2 \ll T_3$, see section 2.1.7). As a result the envelope of each cumulated cluster signal is Rayleigh-distributed. The average time delay is assumed to differ significantly between clusters. If the delay times also significantly exceed the bit time of a digital link ($T_2 \gg T_3$), the different clusters produce serious intersymbol interference, so the multipath self-interference then approximates the case of co-channel interference by multiple incoherent Rayleigh-fading signals.

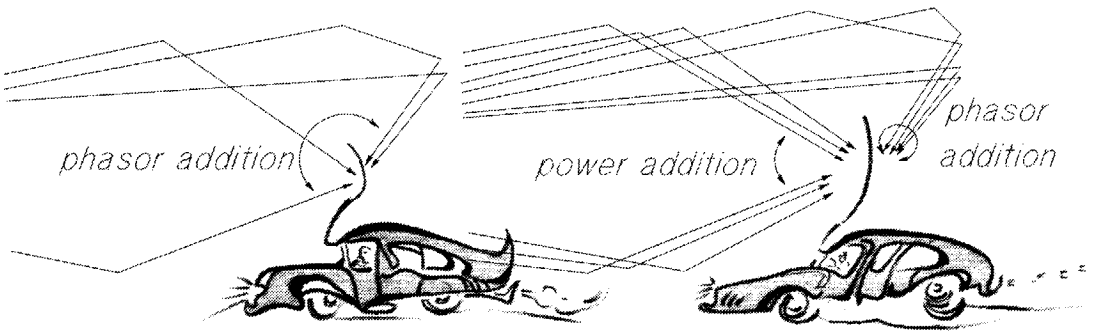


Fig. 4.1: Model of multipath wave interference for Rayleigh-fading (narrowband) channels.

Fig. 4.2: Model of multipath wave interference for Nakagami-fading (wideband) channels.

Fig. 4.1 and Fig. 4.2 illustrate the models of wave interference in Rayleigh and Nakagami multipath reception¹. Narrowband signals are addressed in the following. The corresponding model of Rayleigh fading for individual signals is retained, assuming delay time spreads to be small compared to the rate of modulation of the phase of signal ($T_2 \ll T_3$). The Nakagami model will be used to describe the interference cumulated from the multiple independently Rayleigh-fading sources.

¹See note with Fig. 1.1.

4.3 INTERFERENCE MODEL

Assume that the bit rate r_b of the angle-modulated data largely exceeds the Doppler modulation f_m ($T_3 \ll T_6$). So fluctuations in the phase $\theta_i(t)$ are relatively fast compared to amplitude fluctuations from mobility. Incoherence of the signals then allows power addition of the interference signals (see Fig. 4.3).

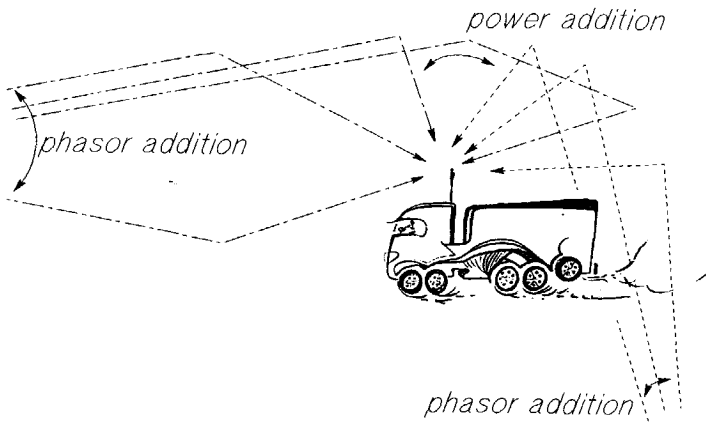


Fig. 4.3: Model for incoherent cumulation of multiple interfering (narrowband) Rayleigh-fading signals.

As in chapter 3, joint interference power pdf is found by convolving n pdf's of the individual interference powers, using the Laplace transformation of each pdf. The image function of the pdf of the instantaneous power of the i -th Rayleigh-fading signal with local-mean $\bar{p}_i$ is

$$\phi_i(s) \triangleq \mathbf{E}[\exp(-s p_i)] = \frac{1}{\bar{p}_i s + 1}, \quad (4.7)$$

with the expectation taken over the mobile fading of p_i . The Laplace image of the joint instantaneous interference power is found by multiplication, i.e.,

$$\Phi_t(s) = \prod_{i=1}^n \Phi_i(s). \quad (4.8)$$

For a small number of signals, say $n=1, 2$ or 3 , reverse transformation appears possible [10],[11]. For larger n , this approach becomes impractical. Another disadvantage is that the analytic form of the joint pdf abruptly changes if two (or more) local-mean powers happen to be equal. For instance, in the event of two interfering signals, the pdf of instantaneous interference power p_t is

$$f_{p_t}(p_t) = \begin{cases} \frac{1}{\bar{p}_1 - \bar{p}_2} \left[\exp\left\{-\frac{p_t}{\bar{p}_1}\right\} - \exp\left\{-\frac{p_t}{\bar{p}_2}\right\} \right] & \text{if } \bar{p}_1 \neq \bar{p}_2 \\ \frac{p_t}{\bar{p}_1} \exp\left\{-\frac{p_t}{\bar{p}_1}\right\} & \text{if } \bar{p}_1 = \bar{p}_2 \end{cases} \quad (4.9)$$

Intuitively, however, the experienced fading should not be highly sensitive to small mutual differences in the mean powers of the n components. Table 4.1 and Fig. 4.4 give the pdf (4.9) of the joint signal power if the local-mean power of the minor signal is $\infty, 10, 6, 3, 0.5$, and 0 dB below the local-mean power of the major interferer, in curve (a) - (f), respectively.

The probability density for low interference power p_t follows from the 'initial-value theorem' of the one-sided Laplace transform [11, p.20]

$$f_{p_t}(0) = \lim_{s \rightarrow \infty} s \Phi_t(s) = \lim_{s \rightarrow \infty} \frac{s}{\prod_{i=1}^n \bar{p}_i s + 1}, \quad (4.10)$$

which is zero for $n=2,3,\dots$, but equals $\bar{p}_1^{-1}$ for $n=1$. Thus, deep fades occur occasionally with one signal, but become very rare for larger n . An explanation is that with more signals, a deep fade of one signal is likely to be hidden by the signal power received from other signals.

4.3 APPROXIMATE MODEL FOR JOINT INTERFERENCE SIGNAL

Tractable analytical expressions for the pdf $f_{\rho,p}(\rho,\bar{\rho})$ for a joint interference signal cumulated from several fading signals with arbitrary local-mean powers, have not been found. In the following, we therefore approximate the pdf of the joint signal by a Nakagami-fading signal, with the same mean and variance as the exact pdf.

The k -th order moment μ_k of the pdf of the joint interference power, defined as

$$\mu_k \triangleq E[p_t^k] = \int_0^{\infty} p_t^k f_{p_t}(p_t) dp_t, \quad (4.11)$$

is found simply from the k -th derivative of the image function (4.8) with (4.7). The mean power of the joint interference power is identical to the sum of the mean powers of the individual signals, viz.,

$$\bar{p}_t = \mu_1 = -\phi'(0) = \sum_{i=1}^n \bar{p}_i. \quad (4.12)$$

Similarly, the variance is found from

$$\sigma^2 = \mu_2 - \mu_1^2 = \phi''(0) - \{\phi'(0)\}^2 = \sum_{i=1}^n \bar{p}_i^2. \quad (4.13)$$

If all n interference signals have equal mean power ($\bar{p}_i = \bar{p}$), the Laplace image reduces to

$$\phi_t(s) = \frac{1}{(\bar{p}s + 1)^n}. \quad (4.14)$$

Reverse transformation [10],[11] shows that the instantaneous power p_t of the cumulated interference is then gamma distributed, with mean $\mu_1 = n\bar{p}$ and variance $\sigma^2 = n\bar{p}^2$. The pdf of the amplitude of the joint interference ρ_t is the Nakagami m -distribution (4) with $m = n$. Consequently, in the special case of n interfering signals

with equal mean power, the Nakagami distribution accurately describes the pdf of the amplitude of the joint interference signal.

For the general case of interference caused by n signals with mutually different mean powers, the characteristic function of the joint interference power is approximated as

$$\phi_n(s) \approx \frac{1}{(\bar{p}s + 1)^m}. \quad (4.15)$$

This is equivalent to approximating the pdf (4.9) of the joint power by the Nakagami distribution (4.6). In this approximation, both the means and the standard deviations of the instantaneous signal power p_i in (4.9) and in (4.6) match each other if we take

$$\bar{p} = \frac{\sum_{k=1}^n \bar{p}_k^2}{\sum_{k=1}^n \bar{p}_k} \quad \text{and} \quad m = \frac{\left[\sum_{k=1}^n \bar{p}_k^2 \right]^2}{\sum_{k=1}^n \bar{p}_k^2} \quad (4.16)$$

and insert the local-mean power $\bar{p}_i = m\bar{p}$ for the joint interference power in the gamma distribution (4.6). Note that m determined from (4.16) is not necessarily an integer number, but $1 \leq m \leq n$.

As a example, the exact joint pdf of the instantaneous interference power p_i of two signals obtained by using the exact expression (4.9) is compared with the approximation (4.6) using (4.16), for several ratios of the local-mean powers in Fig. 4.4. The estimates for $\bar{p}$ and m are given in Table 4.1. It is asserted from Fig. 4.4 that the discrepancy is acceptable, even for the critical case $p_i \rightarrow 0$, especially if the interferers have similar mean powers.

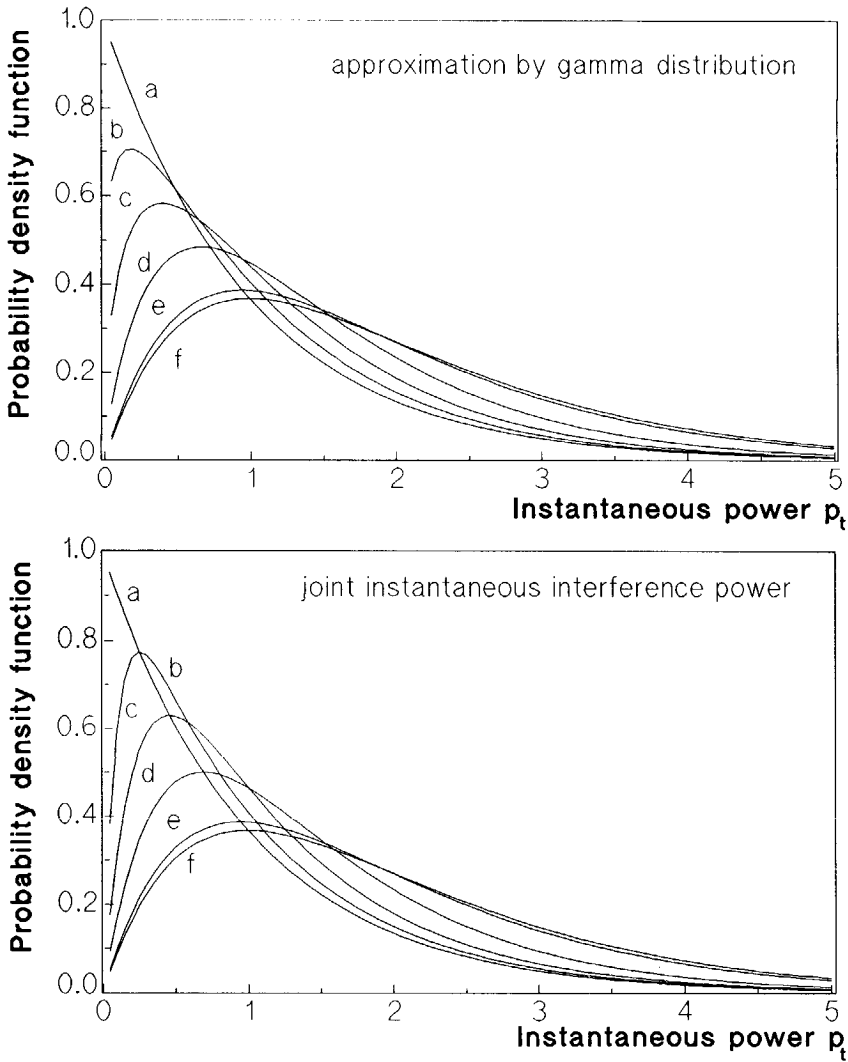


Fig. 4.4: Pdf of the instantaneous power of two signals with incoherent cumulation (exact, below) and an approximation by a gamma distribution, i.e., assuming a Nakagami-fading envelope (above). Curves (a, ..., f) are discussed in Table 4.1.

In the event of many interferers ($n \gg 1$), (4.5) leads to an unconventional definition of the signal amplitude. As known from the central limit theorem, the cumulated signal tends to bandlimited Gaussian noise. The definition of the *instantaneous* amplitude, i.e., averaged over a time scale T longer than the time constants of variations caused

by modulation but smaller than the time constants of the multipath fading ($T_3 \ll T \ll T_6$), can then only be related to the instantaneous power p_i ; No such phenomenon as an envelope can be defined. This is in contrast to the constant envelope of a single phase-modulated signal ($n=1$). At any moment with observation window $T_1 \ll T \ll T_3$, the joint interference power is exponentially distributed, as in Rayleigh fading, irrespective of n . The influence of n is recognised only after averaging the signal power over a suitable time T long enough to experience incoherent addition ($T \gg T_3$) but short enough to avoid fluctuations due to mobility ($T \ll T_6$). Remember that the influence of latter fluctuations on the cumulated signal decrease with increasing n : A temporary fade of any one of the n interfering signals is likely to be hidden by the interference received from other transmitters.

Table 4.1 Mean $\bar{p}$ and m -parameter of the joint signal for a major interferer plus a minor interferer with an local-mean power A dB below the major interferer

A	∞	10	6	3	0.5	0 dB
$\bar{p}$	1	0.92	0.85	0.83	0.95	1 $\bar{p}_1$
m	1	1.2	1.5	1.8	1.99	2
curve:	(a)	(b)	(c)	(d)	(e)	(f)

4.4 OUTAGE PROBABILITY

As in chapter 3, we assume that the wanted signal is correctly received as long as its instantaneous p_o power exceeds the joint interference power by at least a ratio z . Then the signal amplitude $\rho_o(t)$ exceeds the amplitude $\rho_i(t)$ of the joint interference by at least a ratio $\sqrt{z}$. As shown in chapter 3 for a Rayleigh-fading wanted signal $v_o(t)$, the probability that the power ratio C/I is above the threshold z is found from the Laplace image $\phi_i(.)$ of the distribution of joint power of n interferers, viz.

$$Pr(\rho_0 > \sqrt{z} \rho_i) = \Phi_i\left(\frac{z}{\bar{\rho}_0}\right) = \frac{1}{\left(\frac{\bar{\rho}}{\bar{\rho}_0} + 1\right)^n}, \quad (4.17)$$

where the latter equality holds for n interfering signals with identical mean powers. This simplification is adopted here in analogy with (4.15) to generate results that can be combined with results for the threshold crossing rate with Nakagami-fading interference.

4.5 THRESHOLD CROSSING RATE

The instantaneous amplitude $\rho_i(t)$ of the joint interference is found from the power sum

$$\rho_i^2(t) = \sum_{i=1}^n \rho_i^2(t). \quad (4.18)$$

The first derivative of $\rho_i(t)$ with respect to the time t gives

$$\dot{\rho}_i(t) = \frac{\sum_{i=1}^n \rho_i(t) \dot{\rho}_i(t)}{\rho_i(t)}. \quad (4.19)$$

According to (4.3), the derivatives of all individual amplitudes are zero-mean Gaussian distributed with variance (4.4). Because of the linear form of (4.19), the derivative $\dot{\rho}_i(t)$ of the amplitude of the cumulated signal, given all individual amplitudes $\{\rho_1, \dots, \rho_n\}$, has the Gaussian distribution $N(0, \sigma_{\rho_i}^2)$, i.e., one with zero mean and variance

$$\sigma_{\rho_t}^2 = \frac{\frac{1}{2} \sum_{i=1}^n \rho_i^2(t) \omega_m \bar{p}_i}{\rho_t^2(t)} . \quad (4.20)$$

Analytical results can be obtained if all interference signals can be assumed to experience uncorrelated fading and have equal mean power ($\bar{p}_i = \bar{p}$) and equal Doppler spectra ($\omega_{mi} = \omega_m$). The latter assumption also implies that all transmitters move with equal speed. Under these conditions, equation (4.20) simply reduces to

$$\sigma_{\rho_t}^2 = \frac{1}{2} \omega_m^2 \bar{p} . \quad (4.21)$$

Remarkably, this is independent of n , but the relative standard deviation σ_{ρ_t} decreases with $1/\sqrt{n}$, since the mean joint interference power $\bar{p}_t$ is equal to $n\bar{p}$.

In a interference-limited network, the C/I -ratio determines the quality of communication. For ease of notation, in this chapter we consider the ratio y between the *amplitudes* of wanted signal and the joint interference. Analogous to [6], we derive the joint pdf of this ratio and its derivative $\dot{y}$ by substituting

$$\rho_0 = \rho_t y \quad (4.22)$$

and

$$\dot{\rho}_0 = \dot{y} \rho_t + \dot{\rho}_t y . \quad (4.23)$$

The pdf of y and $\dot{y}$ results from the integration [6, eq.(9)]

$$f_{y,\dot{y}}(y,\dot{y}) = \int_0^\infty d\rho_t \int_0^\infty d\dot{\rho}_t \rho_t f_{\rho_0}(\rho_t y) f_{\dot{\rho}_0}(\dot{y} \rho_t + \dot{\rho}_t y) f_{\rho_t}(\rho_t) f_{\dot{\rho}_t}(\dot{\rho}_t) \quad (4.24)$$

where the 4-dimensional pdf can be separated because the instantaneous amplitude and its derivative are statistically independent for the both wanted and the interference signals. Inserting the pdfs (4.3), (4.5) and

$$f_{\dot{\rho}_t}(\dot{\rho}_t) = N\left(0, \frac{1}{2} \omega_m^2 \bar{p}\right) \quad (4.25)$$

in (4.24) results in

$$f_{y,y}(y,y) = \int_0^\infty \int_0^\infty d\rho_t d\dot{\rho}_t \frac{\rho_t^{2n+2} y \exp\{-a_1 \dot{\rho}_t^2 - a_2 \rho_t \dot{\rho}_t - a_3 \rho_t^2\}}{2\pi \sigma_{\rho_0} \sigma_{\rho_t} \bar{p} \bar{p}_0 (n-1)! 2^{n-1}}, \quad (4.26)$$

with

$$\begin{aligned} a_1 &\triangleq \frac{1}{2\sigma_{\rho_t}^2} + \frac{y^2}{2\sigma_{\rho_0}^2} \\ a_2 &\triangleq \frac{2yy}{2\sigma_{\rho_0}^2} \\ a_3 &\triangleq \frac{y^2}{2\sigma_{\rho_0}^2} + \frac{y^2}{2\bar{p}_0} + \frac{1}{2\bar{p}}. \end{aligned} \quad (4.27)$$

The integration over $\dot{\rho}_t$ of an exponent of a second-order polynomial can be performed (see Eq. (15.75) in [12])

$$f_{y,y}(y,y) = \int_0^\infty \sqrt{\frac{2}{\pi}} \frac{\rho_t^{2n+2} y}{\bar{p} \bar{p}_0 (n-1)! 2^n \sqrt{\sigma_{\rho_0}^2 + \sigma_{\rho_t}^2} y^2} \exp\left\{\frac{a_2^2 - 4a_1 a_3}{4a_1} \rho_t^2\right\} d\rho_t. \quad (4.28)$$

This integral over the amplitude ρ_t of the cumulate interference signal converges since

$$a_2^2 - 4a_1 a_3 = -\frac{1}{\sigma_{\rho_0}^2 \sigma_{\rho_t}^2} \left[y^2 + \frac{(y^2 \bar{p} + \bar{p}_0) \omega_m^2}{\bar{p} \bar{p}_0} \right] < 0 \quad (4.29)$$

and can be solved analytically using equation (15.77) in [12]. The joint pdf of the C/I-amplitude ratio and its derivative with respect to time is found to be on the form of

$$f_{yy}(y, \bar{y}) = \frac{1 * 3 * \dots * (2n+1) y (y^2 \bar{p} + \bar{p}_0)^{n+1} \omega_m^{2n+2}}{\bar{p}^n \bar{p}_0 (n-1)! 2^{2n+1} \left[y^2 + \frac{(y^2 \bar{p} + \bar{p}_0)^2 \omega_m^2}{2 \bar{p} \bar{p}_0} \right]^{n+\frac{3}{2}}} \quad (4.30)$$

This pdf can be used to determine the threshold crossing rate by applying the method used by Rice in [3]-[5]. The rate $N(z)$ at which the threshold z is crossed in the positive direction is found from

$$N(z) = \int_0^\infty \dot{y} f_{yy}(\sqrt{z}, \bar{y}) d\bar{y} \quad (4.31)$$

This integral is solved as

$$N(z) = \frac{\omega_m}{\sqrt{2\pi}} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\sqrt{n} \Gamma(n)} \frac{1}{\eta} \left(1 + \frac{1}{\eta^2 n}\right)^{-n} \quad (4.32)$$

where the fade margin η is defined as

$$\eta^2 \triangleq \frac{\bar{p}_0}{z \bar{p}_t} = \frac{\bar{p}_0}{z n \bar{p}} \quad (4.33)$$

The fade margin η is thus expressed as a ratio of amplitudes, rather than a ratio of powers. The threshold crossing rate is seen to be a function of the parameters ω_m , η and n only. Fig. 4.5 gives the threshold crossing rate as a function of fade margin η for various n .

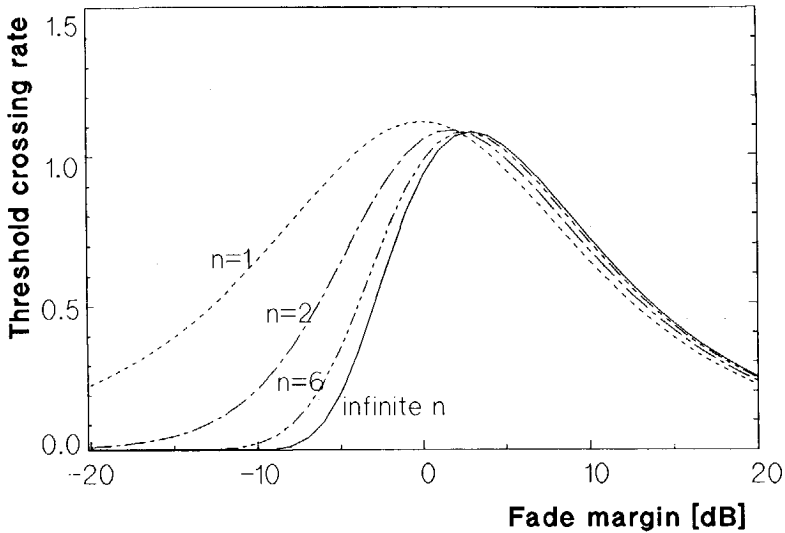


Fig. 4.5: The normalised threshold crossing rate $N(z)/f_m$ versus fade margin in dB for n interfering signals with equal mean power.

If the fade margin η is less than unity (0 dB), the signal has an average power $\bar{p}_0$ below the average minimum power $zn\bar{p}$ required for reliable reception. In this event, non-fade periods (with $p_0 > zp_i$) will be rare and short. It is noted that the threshold crossing rate heavily depends on the number of components. With many components ($n \gg 1$), the interference power is nearly constant. Crossing the threshold will be caused mainly by any temporary increase of the power of the wanted signal, which is rare; the corresponding threshold crossing rate increases sharply with η if η approaches unity (if $0.1 < \eta < 1$). On the other hand, for small n , the interference may exhibit fading, so a higher C/I can occasionally occur due to down-fading of the joint interference. The threshold crossing rate only smoothly increases with increasing η . As a control of the model, it is verified that for one interferer ($n=1$) the threshold crossing rate correctly is a symmetric function about $\eta=1$ (0 dB): if the mean powers of the wanted signal and the interferer are interchanged, the threshold crossing rate should remain unchanged (if $z=1$).

The case of conventional radio communication corresponds to relatively low joint interference power ($n\bar{p} < \bar{p}_0$) and a threshold z of a few dB, and hence a fade margin

η much higher than unity ($\eta \gg 1$). The threshold z will be crossed only when the wanted signal experiences a deep fade. Fading of the joint interference does not play a major role. Accordingly, the influence of the number of interferences is small [13].

This leads to the overall conclusion that, in the case of mobile telephony with its requirement of relatively high signal-to-interference during most of the time (high η), any number of interfering signals may well be approximated by its joint average power $\bar{p}_r \approx n\bar{p}$. In contention-limited mobile packet radio nets, however, severe interference is allowed to occur (low η) since we accept collisions of contending packets which can be retransmitted. In the latter case, down-fading of the interference signal can enhance the probability of correct packet reception.

4.6 AVERAGE (NON)-FADE DURATION

The probability of a signal outage ($C/I < z$) should be equal to the threshold crossing rate multiplied by the average duration of a fade. Similarly, the probability that $C/I > z$ equals the threshold crossing rate multiplied by the average non-fade duration.

The average non-fade duration is thus

$$\bar{\tau}_{NF}(z) = \frac{Pr(\rho_0 > \sqrt{z} \rho_t)}{N(z)} = \frac{Pr(y^2 > z)}{N(z)}. \quad (4.34)$$

Inserting (4.17) with (4.33), and (4.32) gives

$$\bar{\tau}_{NF}(z) = \eta \cdot \frac{\sqrt{2\pi}}{\omega_m} \cdot \frac{\sqrt{n} \Gamma(n)}{\Gamma\left(n + \frac{1}{2}\right)}. \quad (4.35)$$

This is a relatively simple expression since all factors in (4.17) cancel with factors in (4.32). The case $n=1$ is in agreement with [6]. In section 4.7, it will be shown that the limit for $n \rightarrow \infty$ is in agreement with [1]. Equation (4.35) is on the form of a product of a factor depending on the fade margin η , and a factor only depending on the number of interferers n . This is in contrast to the mathematical expression for the threshold crossing rate (4.32) or the outage probability (4.17), where both parameters are interacting.

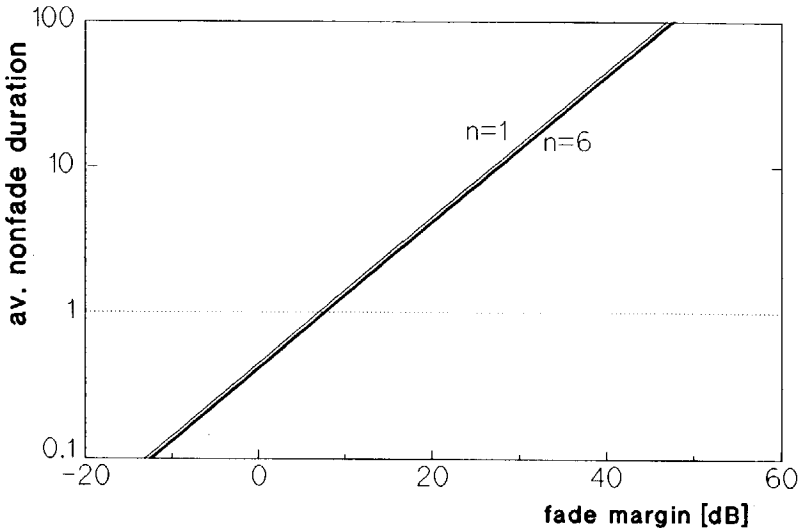


Fig. 4.6: Normalised average non-fade duration $\bar{\tau}_{NF} f_m$ versus the fade margin η for $n=1, 2, \dots, 6$ interfering signals.

Fig. 4.6 gives the average duration of a non-fade interval normalised to the maximum Doppler shift caused by vehicle mobility. It is seen that the influence of the number of interfering signals (or the m -parameter) is small, particularly if $n \geq 2$. This is understood from the asymptotic expansion (6.1.47) in [10]:

$$\chi \triangleq \frac{\sqrt{m} \Gamma(m)}{\Gamma\left(m + \frac{1}{2}\right)} \approx 1 + \frac{1}{8m} + \dots \quad (4.36)$$

The value of χ for $m=1$ is $\chi = \sqrt{2/\pi} \approx 1.13$ (Eq. 6.1.9 in [10]). The average non-fade interval decreases slightly with n . A possible explanation for this seemingly conflicting

result is that in the case of one interferer ($n=1$), the down-fades of the wanted signal may coincide with the fades in the interference and so, prolong the capture period. Significant up-fades of the interference, reducing the non-fade period, are not expected.

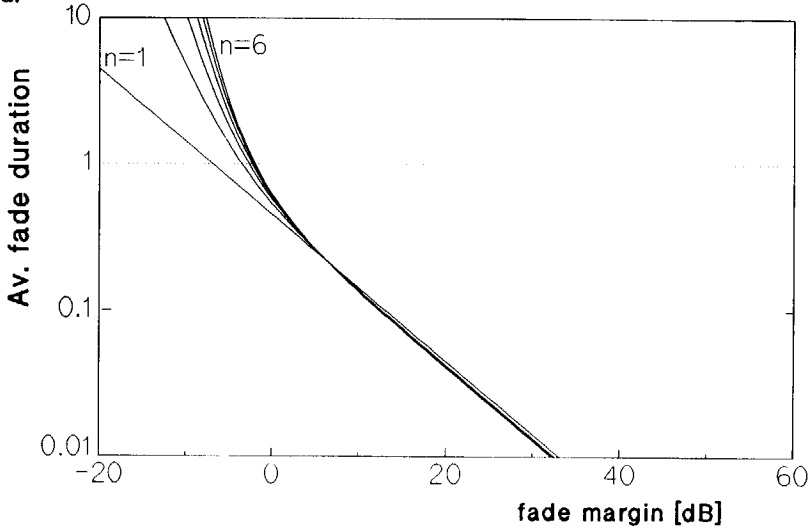


Fig. 4.7 Normalised average fade duration $\bar{\tau}_F f_m$ versus the fade margin η in dB for $n=1, 2, \dots, 6$ interfering signals.

For practical receivers, the threshold z , and hence the fade margin η , may well be a function of the type of interference, and thus of m . In practice, the non-fade duration might be more dependent on the character of the interference signal than suggested in Fig. 4.6.

The average *fade* duration is found from

$$\bar{\tau}_F(z) = \frac{Pr(\rho_0 < \sqrt{z} \rho_i)}{N(z)} = \frac{Pr(y^2 < z)}{N(z)}. \quad (4.37)$$

Inserting (4.17) with (4.33), and (4.32) gives

$$\bar{\tau}_F(z) = \eta \frac{\sqrt{2\pi}}{\omega_m} \left[\left(1 + \frac{1}{\eta^2 n} \right)^n - 1 \right] \frac{\sqrt{n} \Gamma(n)}{\Gamma\left(n + \frac{1}{2}\right)}. \quad (4.38)$$

For $\eta > 1$ (0 dB), the effect of the shape factor m (or n) of the interference is almost negligible (see Fig. 4.6, Fig. 4.7). However for $\eta < 1$, m has a substantial effect on the average fade duration: For small η and large n , the wanted signal is almost continuously in a fade ($\bar{\tau}_{F \rightarrow \infty}$), whereas for $n=1$, the fade of the wanted signal is occasionally interrupted by a fade of the interference signal, which causes $p_0 > z p_1$.

4.7 LIMIT FOR MANY INTERFERERS ($n \rightarrow \infty$)

Well-known results for the rate of crossing a fixed (non-fading) threshold can be verified from equations (4.32) and (4.35). To this end, the number of interferers is increased without limit, under the condition that the total power $n\bar{p}$ is constant. This inherently implies $\bar{p} \rightarrow 0$ and thus $\sigma_{\rho r} \rightarrow 0$. For large n the interference behaves like stationary (non-fading) Gaussian noise. Any temporary fade of the amplitude of one signal out of the n interferers is likely to be hidden by $n-1$ other transmissions. The results in [1] are verified by defining η as in (4.33), and investigating the limit $n \rightarrow \infty$ or $m \rightarrow \infty$.

Since $\Gamma(n+1/2)/(\sqrt{n}\Gamma(n)) \rightarrow 1$ for $n \rightarrow \infty$ and $(1+xn^{-1})^n \rightarrow e^x$ for $n \rightarrow \infty$, the limiting threshold crossing rate is

$$\lim_{n \rightarrow \infty} N(z) = \frac{\omega_m}{\sqrt{2\pi}} \frac{1}{\eta} \exp\left\{-\frac{1}{\eta^2}\right\}, \quad (4.39)$$

which is in agreement with Eq. 1.3-37 in [1]. Similarly, the average non-fade duration (4.35) and the average fade duration (4.36), (4.38) tend to the limits

$$\lim_{m \rightarrow \infty} \bar{\tau}_{NF}(z) = \lim_{m \rightarrow \infty} \eta \frac{\sqrt{2\pi m} \Gamma(m)}{\omega_m \Gamma\left(m + \frac{1}{2}\right)} = \eta \frac{\sqrt{2\pi}}{\omega_m} \quad (4.40)$$

and

$$\lim_{n \rightarrow \infty} \bar{\tau}_F(z) = \eta [\exp(\eta^2) - 1] \frac{\sqrt{2\pi}}{\omega_m}, \quad (4.41)$$

respectively. Eq. (4.41) agrees with Eq. 1.3-44 in [1], which provides a control of the consistency of the method presented in this chapter.

4.8 DISTRIBUTION OF (NON-) FADE DURATION

In [5], Rice presented a method to compute the pdf of the duration of fades and illustrated the method with an analysis of reception of a Rayleigh-fading signal with a Gaussian fading spectrum. A Gaussian spectrum applies for instance to the event of fading over ionospheric shortwave (HF) channels. In land-mobile radio communication at VHF or UHF, the Doppler spectrum of the multipath fading caused vehicle motion with constant speed, is certainly not a Gaussian spectrum. Spectral components at $f_c + f_m \cos \phi_i$ exist because waves are arriving from an angle ϕ_i (see section 2.3.1.2) with a certain (e.g. uniform) probability density. The spectrum is thus band-limited to $[f_c - f_m, f_c + f_m]$. If reflections arrive from random angles uniformly distributed over $[0, 2\pi)$, a high concentration of power occurs near the boundaries of the Doppler band [1], irrespective of the direction of motion of the mobile antenna with respect to the fixed base station antenna.

Analysis and experiments [15] showed that the distribution of the durations of deep fades (10 dB or more below the local-mean signal level) depends on the second and zeroth moments of the Doppler spectrum [1],[15] and is relatively unaffected by other characteristics of the spectrum. The distribution of shallower fades is influenced more by the exact shape of the Doppler spectrum. In this case, fade durations corresponding to the maximum Doppler frequency shift occur more frequently [15]-[17].

An exact analysis, taking account of typical mobile Doppler spectra, appears to be a complicated task, particularly if fading of the interfering signals is also to be considered. Simulation results have been reported for noise-limited systems, e.g. [15]-[17], thus ignoring the effect of fading interference. For interference-limited systems, with multiple fading interferers, no exact analysis of the fade duration pdf is known to the author.

In an urban environment, the different vehicle speeds significantly influence the (non-) fade duration. Van der Plas [18] showed that the duration of (non-) fade periods tends to be approximately exponentially distributed for vehicles with typical urban traffic speed statistics. It was noted, however, that a substantial percentage of the time an operating terminal may have zero speed. During these static periods no contributions to threshold crossings are made.

4.9 RATE OF DROP-OUTS

Short signal outages are experienced as 'clicks'. The clicks cumulatively degrade the circuit, but appear too rapidly to be individually distinguishable [19, p.20]. Only longer outages are experienced as temporary 'clipping' or a 'drop-out' of the circuit. The analysis of chapter 3 addressed the probability of outage, irrespective of whether this outage would be experienced as a click or as a drop-out. In this section, the probability that an outage exceeds a specified time T_0 is determined.

To this end, the significant time characteristics of speech are first reviewed. Human speech signals contain periods of activity and periods of silence [20]. (This fact was already exploited in section 3.2.2). A number of models have been proposed for voice activity, many of which were based on a Markov Chain. For example, Goodman et al.

[21] used a speech activity model to evaluate the performance of system with packetised voice, employing a random-access protocol (PRMA) to accommodate multiple users. A rough model of voice activity is obtained by considering principal talkspurts and principal gaps in the speech as a two-state Markov Chain. As a refinement, one can distinguish mini-spurts and mini-gaps in each principal talkspurt.

Table 4.2: Average duration of events in human speech [21]

<i>Condition</i>	<i>Av. duration</i>
principal talkspurt	1.00 s
principal gap	1.35 s
mini-spurt	0.275 s
mini-gap	0.050 s

A voice signal is seriously affected by an outage lasting for a significant part of the duration of a mini-spurt. Experiments on satellite links revealed that speech clips longer than $T_0=50$ ms at the beginning of a talkspurt (called a "freeze-out") cause perceptible mutilation of initial plosive, stop, fricative and nasal consonants [22]. For Time Assigned Speech Interpolation (TASI, DSI) on satellite links, freeze-outs are caused by the competition among signals from various subscribers due to statistical multiplexing of multiple voice signals. A typical requirement is that the probability of a clip longer than 50 ms at the beginning of a talkspurt should be less than 2%. In mobile radio, clips caused by RF-signal outages occur at random instants, thus not necessarily at the beginning of talkspurts as in TASI. Such random clips may be less serious than freeze-outs. Moreover, in digital voice communication with voice coding, a drop-out not exceeding a few frames may be overcome by extrapolating previous voice segments.

Typically, a voice waveform can be described as a stationary statistical process during intervals in the order of 20 to 40 ms. This interval is substantially shorter than the duration of a syllable. Consonant-vowel-consonant syllables were analysed and the following typical durations observed by Ricards [23]:

<i>Initial consonants:</i>	70-250 ms, average about 120 ms,
<i>Vowels:</i>	150-500 ms, average about 250 ms,
<i>Final consonants:</i>	100-350 ms, average about 190 ms.

These observations suggest that only outages longer than a few tens of milliseconds are distinguished as drop-outs.

In order to investigate the probability of such drop-outs, exponentially distributed fade durations are postulated, with memoryless threshold crossings. Thus, the distribution of the fade duration is taken

$$f_{\tau_F}(\tau_F) = \frac{1}{\tau_F} \exp\left\{-\frac{\tau_F}{\tau_F}\right\}. \quad (4.42)$$

The average number of dropouts per second, denoted as $N(z, T_0)$, is simply found as the number of threshold crossings per second multiplied by the probability that the fade lasts longer than T_0 , so

$$N(z, T_0) = N(z) \cdot \exp\left\{-\frac{T_0}{\tau_F}\right\}. \quad (4.43)$$

Inserting (4.32) and (4.36), (4.38) gives

$$N(z, T_0) = \frac{\omega_m \chi}{\sqrt{2\pi}} \frac{1}{\eta} \left(1 + \frac{1}{\eta^2}\right)^{-n} \exp\left\{\frac{-\omega_m T_0}{\eta \sqrt{2\pi}} \chi \left[\left(1 + \frac{1}{\eta^2}\right)^n - 1\right]^{-1}\right\}. \quad (4.44)$$

with χ defined in (4.36). Numerical results are in Fig. 4.8. for the drop-out rate, normalised to the maximum Doppler shift f_m , versus the fade margin η . A typical value of the maximum Doppler shift is $f_m \approx 60$ Hz, e.g. with a carrier frequency of $f_c = 900$ MHz and a vehicle speed of 72 km/h ($v = 20$ m/s). For $T_0 = 16$ ms, we find $T = 1$.

For very small fade margins ($\eta < 1$), the signal is almost continuously in a fade. Since the non-fade duration tends to zero if $\eta \rightarrow 0$, the drop-out rate approaches the

threshold crossing rate. For high fade margins, the drop-out rate is almost independent of n . Long drop-outs are relatively rare, particularly if the fade margin is higher than, say 10 dB. However, at lower vehicle speeds and relatively small fade margins, some signal outages will be experienced as drop-outs in human voice.

In digital transmission systems with protection against burst errors e.g. by error correcting coding, (4.44) is relevant to determine the probability of an error burst with excessive run length larger than the error correcting capability of the code.

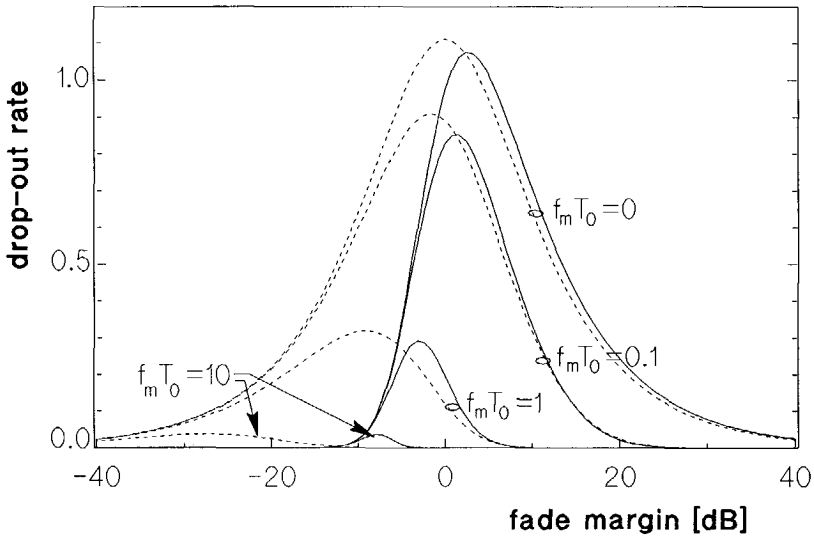


Fig. 4.8: Normalised rate of dropouts, $N(z, T_0)/f_m$, with duration longer than the normalised criterion $T_0 f_m = 0, 0.1, 1$ and 10 , versus the fade margin η in dB. (---) one interferer; (—) multiple interferers ($n=6$).

4.10 PROBABILITY OF FADE-FREE RECEPTION OF A DATA BLOCK

In digital communication using blocks of L bits to support a data rate r_b , the duration of each block is $T_s = L/r_b$. Successful reception of a data block is assumed to occur if

the received signal power exceeds the interference power for the entire duration of the block. The case of Rayleigh-fading signals with known local-mean powers is addressed. Exponentially distributed non-fade durations are postulated, with memoryless threshold crossings. Hence, in analogy with (4.42),

$$f_{\tau_{NF}}(\tau_{NF}) = \frac{1}{\tau_{NF}} \exp\left\{-\frac{\tau_{NF}}{\tau_{NF}}\right\}. \quad (4.45)$$

The probability that a non-fade period continues for a duration of at least T_s is found by integration of (4.45) from T_s to ∞ . The probability of successful reception (event A_0) becomes

$$\begin{aligned} Pr(A_0) &= Pr(c_0) Pr(\tau_{NF} > T_s) \\ &= Pr(c_0) \exp\left\{-\frac{T_s}{\tau_{NF}}\right\}, \end{aligned} \quad (4.46)$$

where c_0 denotes the event that the C/I -ratio is above the threshold. We now compare the case of interference caused by a single Rayleigh-fading signal with the event of cumulation of many weak interfering signals. The wanted signal experiences Rayleigh fading, with exponentially distributed received power. So, in the event of many interferers,

$$Pr(c_0|n \rightarrow \infty) = \exp\left\{-\frac{\bar{z}\bar{p}_i}{\bar{p}_0}\right\}. \quad (4.47)$$

For one Rayleigh-fading interferer, it was shown in chapter 3 (Eq. (3.16)) that

$$Pr(c_0|n=1) = \frac{\bar{p}_0}{\bar{p}_0 + z\bar{p}_1}. \quad (4.48)$$

The asymptotic behaviour for strong interference of (4.47) and (4.48) is seen to be essentially different. With many interferers, the probability c_0 decreases exponentially if the interference power is increased. This is in contrast to the case of one single

signal, where the probability is only inversely proportional to the interference power. In the event of $n=1$, the signal can be received correctly for the short duration of a deep fade of the interferer. However, we now show that this relatively slow degradation of $Pr(c_0)$ holds only for signal segments of infinitely short duration ($T_s=0$). The probability of correct reception of a segment of finite duration vanishes exponentially if the interference power is non-zero, even when the interference is caused by a single Rayleigh-fading signal.

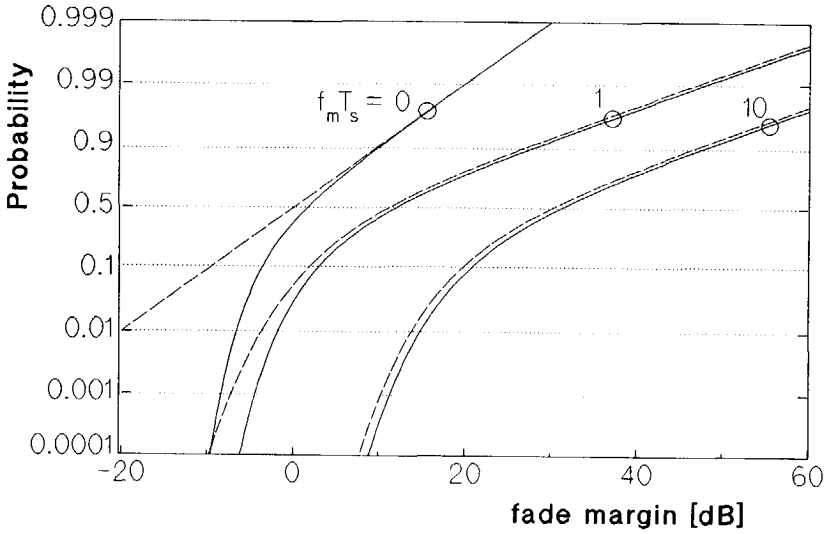


Fig. 4.9: Probability of reception of a data block without fade, as a function of the fade margin, (- -) for one interferer and (—) for many interferers. Normalised block duration $T_n = T_s f_m = 0, 1$ and 10 .

For a single interferer, the probability of successful reception is

$$Pr(A_0|n=1) = \frac{\eta^2}{\eta^2+1} \exp \left\{ -\frac{T_s \omega_m}{\sqrt{2\pi} \eta} \frac{\Gamma(1\frac{1}{2})}{\Gamma(1)} \right\} \quad (4.49)$$

where $\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$ and $\Gamma(1)=1$. In the event of interference composed of many weak signals, one obtains

$$Pr(A_0|n \rightarrow \infty) = \exp \left\{ -\frac{1}{\eta^2} - \frac{T_s \omega_m}{\sqrt{2\pi} \eta} \right\} \quad (4.50)$$

Fig. 4.9 illustrates the corresponding probabilities as a function of the fade margin η for various normalised packet durations T_n ($T_n = T_s f_m$).

The effective throughput, i.e., $Pr(A_0) \cdot L_u / (L_H + L_u)$ with L_u the number of user-data bits and L_H the number of overhead bits, is depicted in Fig. 4.10. The duration of the data block (or the data 'packet') is normalised to the Doppler modulation, according to $T_n = T_s f_m$. T_s , of course, depends on both the bit rate and the data format [24]-[28].

- In the High-Level Data Link Control (HDLC) protocol [24][27], a frame contains at least $L_H = 48$ overhead bits to accommodate a start flag, address and control bytes and CRC block error detection. At a bit rate of $r_b = 1200$ bit/s, this corresponds to $T_s = 0.04$ s ($T_n = 2.4$ if $f_m = 60$ Hz) and at $r_b = 16$ kbit/s to $T_s = 3$ ms ($T_n = 0.18$ if $f_m = 60$ Hz).

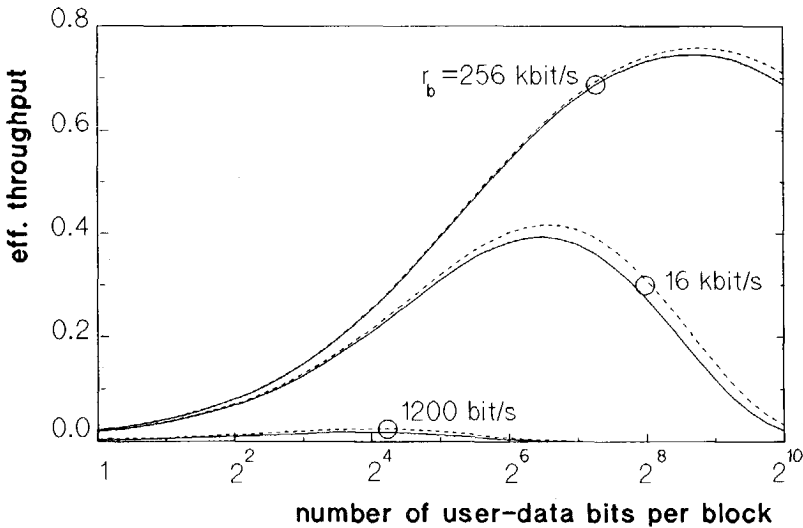


Fig. 4.10: Effective throughput versus the number of user-data bits L_u . Total block duration $T_s = (L_H + L_u) / r_b$, $L_H = 48$ header bits. Maximum Doppler shift $f_m = 60$ Hz. Bit rates $r_b = 1200, 16k$ and $256k$ bit/s. (—) many interferers, (---) one interferer. Fade margin $\eta = \sqrt{10}$ (10 dB).

- In the first generation of the 'Mobitex' public packet-switched mobile network [25], the bit rate is $r_b = 1200$ bit/s. For a data message of minimum duration ($T_s \approx 13.3$ ms, $L = 16$ bits), one finds $T_n = 0.8$. The maximum packet duration ($T_s = 3.315$ s for $L = 255$) corresponds to $T_n \approx 200$, though in this case the packet is split in smaller blocks and an selective repeat algorithm is used to enhance the throughput efficiency if blocks are lost in a signal fade.
- In systems with higher data rates, T_n is found to be substantially smaller. In the GSM-system, a time slot has duration $T_s \approx 0.577$ ms, so $T_n \approx 0.035$ if $f_m = 60$ Hz. Results in (4.49), (4.50) and in Fig. 4.10, have been derived for narrowband Rayleigh fading. For very high bit rates ($T_s < T_2$), however, reception may be impaired by intersymbol interference. In this case, the effect of increasing the bit rate is less favourable than depicted in Fig. 4.10 [29].

4.11 DISCUSSION AND CONCLUSIONS

In mobile radio, not only the probability of signal outage but also the duration of outages determine the quality of communication. The threshold crossing rate and the average non-fade duration have been studied analytically for the case of a Rayleigh-fading signal in the presence n interfering signals. The fading of the interfering signals has been assumed mutually independent, with identical mean powers and Doppler modulation widths. The mathematical analysis presented here is a good approximation of the outbound (base-to-mobile) channel in a cellular network. Since only the receiver is in motion and multipath scattering occurs in the vicinity of the receive antenna, the width of the Doppler spectrum is equal for the wanted signal and all interfering signals. For signals arriving from different sources, Rayleigh fading is (nearly) independent. Shadowing, caused by features in the vicinity of mobile, is heavily correlated and may thus be less relevant in an interference-limited design of the downlink. Further, interference is mainly caused by the (typically six) closest co-

channel base stations, which are all at about the re-use distance. Under such conditions, the average fade duration can be expressed analytically, as described here.

The expressions for the threshold crossing rate and the average non-fade duration have a more general applicator, whenever the joint interference signal can be approximated by a Nakagami-fading signal. In the inbound random-access channel of a packet-radio network, the assumption of equal mean power for all interferers is less realistic. For this case, it has been demonstrated that a Nakagami pdf closely approximates the pdf of the joint interference power. The results also suggested that mainly the rate of fading of the wanted signal will determine the average fade duration. A non-fade duration is most likely to be terminated by a downfade of the wanted signal, whereas an upfade of the joint interference signal is very unlikely. As far as the average non-fade duration is concerned, it has been shown that a further approximation may be reasonable. The exact distribution of the received power from the different fading interfering signals appeared less relevant. Non-fade durations may therefore be assessed by considering (multiple) interfering signals to be a noise-type signal with constant power.

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CHAPTER 5

AVERAGE BIT ERROR RATE IN CW COMMUNICATION

The performance of generic receivers in narrowband multi-user radio networks has been addressed in chapter 3 and 4, with outage probability used as the performance measure. In the specific case of digital communication, the probability of a bit error or a block error provides another relevant measure of the performance of a radio link. Most studies of the average bit error rate in a fading channel documented so far, e.g. in [1], consider only additive white Gaussian noise (AWGN) without interference. On the other hand, studies with co-channel interference mostly consider channels without fading, e.g. [2]. The combined effect of multipath fading and co-channel interference is rapidly becoming an area of active research, e.g. [3]-[5],[17]. Recently, Steele et al. [4]-[5] studied the BER for an MSK signal in mobile networks with Rayleigh fading and

multiple interfering signals. For our purpose, models as proposed in [5] appear prohibitively complicated. The motivation for this chapter was the need for a mathematically tractable, yet reasonably accurate expression to study the performance of digital communication systems. To this end, the bit error rate for channels with Rayleigh fading, noise and multiple interferers, and using standard receivers optimised for AWGN channels [1],[6], is considered with a number of simplifying assumptions, allowing us to compute the long-term average BER in micro and macro-cellular networks, and (in chapter 7) to calculate the throughput of random-access networks. Since the model proposed here is possibly too crude to compare the performance of slightly different digital modulation techniques, we confine ourselves to Binary Phase Shift Keying (BPSK).

This chapter is organised as follows: Section 5.1 briefly reviews standard BPSK receivers and associated models proposed in literature for co-channel interference in fading channels. Section 5.2 deals with the average BER for a desired BPSK signal with given (instantaneous) power, received from a mobile transmitter in the presence of additive white Gaussian noise and multiple interfering BPSK signals with Rayleigh fading. The local-mean BER is determined in section 5.3, and area-mean BERs in cellular networks are calculated in section 5.4 and 5.5 for micro- and macro-cellular networks, respectively. Approximate BER expressions are derived at high average signal-to-interference ratio and negligible noise level. Section 5.6 removes the (pessimistic) approximation that the bits of each interfering signal have precisely aligned bit timing. Conclusions are in section 5.7.

5.1 BRIEF REVIEW OF MODELS FOR BER IN MOBILE CHANNELS WITH CO-CHANNEL INTERFERENCE

Optimum receivers for BPSK are well known for the AWGN channel, and their bit error rates have been studied extensively [1],[6],[8]. An optimum receiver can be

implemented by correlating (e.g. by multiplying and integrating) the received signal with a locally generated perfect copy of the carrier. A block diagram of this receiver is in Fig. 5.1. Bandpass filters, present in any realistic receiver for practical reasons (e.g. to remove adjacent-channel interference) are not drawn in Fig. 5.1: From a theoretical point of view, they are obsolete [6], and imperfect bandpass filters may even degrade performance.

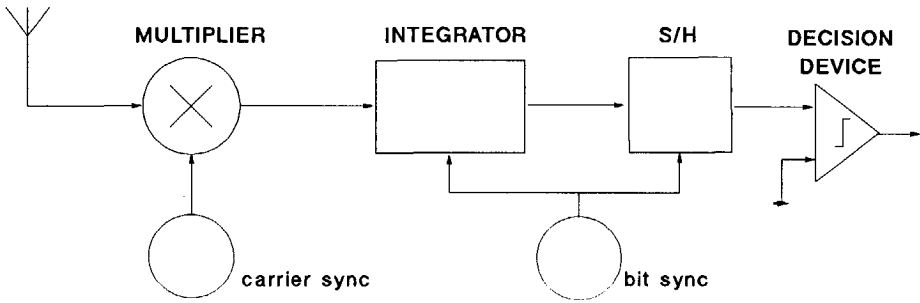


Fig. 5.1: Block diagram of a correlation receiver for coherent detection of BPSK signals

Ideally, the BER for coherent detection of BPSK in time-invariant AWGN channels is [1],[6]

$$P_b(e) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0}}. \quad (5.1)$$

where N_0 is the (one-sided) spectral density of the AWGN, E_b is the (constant) energy per bit ($E_b = p_0 T_b$) and erfc denotes the complementary error function [7]¹.

Extension of these results to analysis of the BER in realistic mobile fading channels with interference requires consideration of a number of possible issues, such as

- i) (man-made or thermal) additive noise,

¹The erfc -function is defined as $1 - \operatorname{erf}(\cdot)$, with the erf -function described in section 7.2. Expressions in terms of the Q -function [8] are obtained from $\operatorname{erfc}(k) = 2Q(\sqrt{2}k)$.

- ii) amplitude (or energy per bit) of the received signal,
- iii) fluctuations of the amplitude and phase of the wanted signal during detection of a bit (particularly if $T_3 > T_6$),
- iv) delay spread of the wanted signal, causing I.S.I. (particularly if $T_3 < T_2$),
- v) synchronisation impairments caused by fading (particularly phase fluctuations) of the wanted signal during previous bits,
- vi) parameters of the interfering signal(s), particularly
 - vi.1) carrier phase offsets,
 - vi.2) bit timing (phase) offsets,
 - vi.3) amplitudes of the interfering signals,
 - vi.4) carrier frequency and bit (clock) frequency offsets, and
 - vi.5) fading of the amplitude of interfering signals during detection of a bit,
- vii) synchronisation impairments of the receiver caused by the interfering signals,
- viii) delay spread of interfering signals,

Since only narrowband systems with frequency non-selective ('flat') fading channels are addressed here ($T_2 < T_3$), we ignore the effects *iv* and *viii*. The time constants of the fading [13] are assumed large compared to the bit duration ($T_3 < T_6$). This simplifies *iii* and *vi.5*. Assuming receivers perfectly synchronising to the wanted signal, the impairments *v* and *vii* are neglected.

A number of models and expressions have been proposed for the BER in the presence of co-channel interference, for instance:

1) Sheikh, Yao and Wu [9] assumed bit error to occur if the power of the wanted signal fails to exceed the power of the strongest interfering signal, thus

$$P_b(e) = Pr \left(p_0 < \max_{i(i=1,2,\dots,n)} p_i \right). \quad (5.2)$$

The analysis closely resembles the analysis of outage probabilities.

2) Zhang, Pahlavan and Ganesh [10] approximated co-channel interference as AWGN, resulting in the bit error probability

$$P_b(e|p_0, p_t) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0 + p_t T_b}} . \quad (5.3)$$

Here, the variance of the Gaussian noise is increased with the *instantaneous* power of the joint interference signal.

3) Habbab, Kavehrad and Sundberg [11] and a later paper by Zhang and Pahlavan [14] also assumed that co-channel interference behaves as AWGN, but now with error probability

$$P_b(e|p_0, \bar{p}_t) = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{N_0 + \bar{p}_t T_b}} . \quad (5.4)$$

In contrast to (5.3), the variance of the noise is increased with the *local-mean* power of the joint interference signal. It was argued by Linnartz, Goossen and Hekmat in [12] that the latter assumption can be more realistic than the former one.

4) Wong and Steele [5] studied the BER for MSK modulation. The effect of co-channel interference on the decision variable, as extracted by a standard receiver, was taken into account for independent bit timing for all (independent) transmitters (*vi.1* and *vi.2*). These results were subsequently averaged over the pdf of the received signal amplitudes caused by fading (*ii* and *vi.3*).

The approaches in 1-3 considerably simplify the analysis. In section 5.2 it will be shown that model 3 can be recovered by simplifying model 4. The effect of one of the simplifications is studied in section 5.5.

5.2 INSTANTANEOUS BER FOR BPSK SIGNALS WITH RAYLEIGH-FADING CO-CHANNEL INTERFERENCE

First, precisely overlapping bit periods are assumed for all interfering signals, i.e., none of the interfering carriers makes a phase reversal during the integration over the bit duration of the wanted signal. Interfering signals are assumed to have exactly the same carrier and bit clock frequency as the wanted signal: phase fluctuations, e.g. caused by frequency drifts or Doppler shifts, are considered negligible during each bit duration. In fact at $f_c = 900$ MHz, random Doppler shifts are usually in the order of $f_m = 50$ Hz, whereas usual bit rates are at least 1200 bit/s ($T_b < T_d$). The momentary derivative of the phase $\dot{\theta}_i$ may fall outside the Doppler spectrum, but the probability that $|\dot{\theta}_i| > f_m$ is small and this event usually coincides with deep fades of the (interference) signal [13]. Hence, the effect of multipath reception on phase fluctuation during one bit interval is presumably negligible. The joint receiver input signal $v(t)$ is on the form

$$v(t) = \rho_0 \kappa_0 \cos(\omega_c t + \theta_0) + \sum_{i=1}^n \rho_i \kappa_i \cos(\omega_c t + \theta_i) + n(t), \quad (5.5)$$

where κ_i ($\kappa_i = \pm 1$) represents the antipodal binary phase modulation of the i -th carrier and $n(t)$ is an additive white Gaussian noise process. The received energy per desired bit is $E_b = \rho_0 T_b = \frac{1}{2} \rho_0^2 T_b$. In the detector, $v(t)$ is multiplied by a locally generated cosine ($2\cos\omega_c t$) and integrated over the entire bit duration T_b . The decision variable ν for synchronous bit extraction from the desired BPSK signal (having index 0) in the presence of n interferers with random (but constant) phase relative to the local oscillator is thus

$$\begin{aligned} \nu &= \frac{2}{T_b} \int_{kT_b}^{(k+1)T_b} v(t) \cos(\omega_c t) dt \\ &= \rho_0 \kappa_0 \cos(\theta_0) + \sum_{i=1}^n \zeta_i \kappa_i + n_I, \end{aligned} \quad (5.6)$$

with n_I the (in-phase) sample of the Gaussian noise and $\kappa_i \rho_i$ is the sample of the i -th interfering signal. As shown in Appendix A, the variance of this noise sample is

$$E[n_I^2] = \frac{N_0}{T_b} . \quad (5.7)$$

In a Rayleigh-fading channel, the in-phase components ζ_i ($\zeta_i = \rho_i \cos \theta_i$) of the n interferers are all independent Gaussian variables (see section 2.1.3). Phase reversals caused by modulation of κ_i do not affect the Gaussian distribution of the n interference samples, provided that the interfering bits are exactly aligned to the bits of the wanted signal. The variance of the i -th interference sample is

$$E[\zeta_i^2] = \bar{p}_i . \quad (5.8)$$

Thus, AWGN as well as the Rayleigh-fading interference have a Gaussian pdf, but the correlation between the samples during successive bit intervals differs substantially. For AWGN, successive samples are uncorrelated, whereas for a slow-fading BPSK signal successive samples have (almost) identical amplitude ζ_i . The Gaussian-distributed in-phase samples $\{\zeta_i\}$ originating from the interference signals plus the Gaussian noise sample produce a new Gaussian variable. The corresponding conditional bit error probability for a receiver locked to the wanted signal with BPSK is thus, for the event that $\kappa_0 = -1$,

$$\begin{aligned} P_b(e|\theta_0=0, \bar{p}_0, \bar{p}, \kappa_0=-1) &= Pr\left(\sum_{i=1}^n \zeta_i \kappa_i + n_I > \rho_0\right) \\ &= \int_{\rho_0}^{\infty} \frac{1}{\sqrt{2\pi\left(\bar{p}_i + \frac{N_0}{T_b}\right)}} \exp\left\{-\frac{\frac{1}{2}\rho^2 T_b}{N_0 + \bar{p}_i T_b}\right\} d\rho \\ &= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{\rho_0 T_b}{\bar{p}_i T_b + N_0}}}^{\infty} \exp\{-t^2\} dt . \end{aligned} \quad (5.9)$$

Because of symmetry, this BER also holds if the wanted signal carries a "1", thus if $\kappa_0=1$. Thus, writing the last integral in (5.9) as the complementary error function,

$$P_b(\epsilon|\theta_0=0, p_0, \bar{p}_i) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{p_0 T_b}{K_i \bar{p}_i T_b + K_n N_0}} \right), \quad (5.10)$$

with the constants $K_i=1$ and $K_n=1$, respectively. This expression confirms model 3 in section 5.1.

If, analogously, one assumes interference with exact bit alignment in the cases of Quadrature Phase Shift Keying (QPSK) or Binary Frequency Shift Keying (BFSK) signals, one finds $K_i=2$ and $K_n=1$, or $K_i=1$ and $K_n=2$, respectively [15],[16]. It is well known that in classical time-invariant AWGN channels, antipodal modulation (such as BPSK) is 3 dB more effective than orthogonal modulation (such as FSK) to combat AWGN. This is confirmed by the above values of K_n . Interestingly, the results suggest that BPSK and BFSK are *equally* resistant to the co-channel interference in narrowband Rayleigh-fading channels and 3dB *better* than QPSK, when standard correlation receivers are employed. Of course, a more detailed model is required to verify whether this conclusion holds for the more realistic assumption of mutually independent transmitters, i.e., with random timing and clock offsets.

The expression (5.10) is now used to compute the average BER in cellular networks.

5.3 LOCAL-MEAN BER IN MICRO AND MACRO-CELLULAR NETWORK

For a Rician fading wanted signal, the local-mean BER is found by integration of (5.10) over the Rician distribution (2.20) of the signal amplitude:

$$P_b(e|\theta_0=0, \bar{q}_0, \bar{p}_t) = \int_0^\infty \frac{\rho_0}{\bar{q}_0} \exp\left\{-\frac{\rho_0^2 + C_0^2}{2\bar{q}_0}\right\} I_0\left(\frac{\rho_0 C_0}{\bar{q}_0}\right) \operatorname{erfc}\sqrt{\frac{\frac{1}{2}\rho_0^2 T_b}{\bar{p}_t T_b + N_0}} d\rho_0. \quad (5.11)$$

Numerical results are given in Fig. 5.2. The local-mean C/I -ratio is defined as $\bar{p}_0/\bar{p}_t = (\bar{q}_0 + \frac{1}{2}C_0^2)/\bar{p}_t$. Various values of the Rician K -factor ($K = C_0^2/2\bar{q}_0$) are given.

At high C/I -ratios, bit errors mainly occur if the wanted signal happens to be in a deep fade, thus for small ρ_0 . The zeroth-order modified Bessel function $I_0(x)$ is always larger than one for real and positive arguments x , and $I_0(x) \approx 1 + \frac{1}{2}x^2 + \dots$ for small x (Eq. 9.6.12 in [7]). The behaviour during fades (small ρ_0) mainly determines the BER at large C/I -ratios. Thus, the integral (5.11) can be overbounded by

$$\begin{aligned} P_b(e|\theta_0=0, \bar{p}_0, \bar{p}_t) &\geq e^{-K} \int_0^\infty e^{-x} \operatorname{erfc}\sqrt{\frac{\bar{p}_0 T_b x}{(1+K)(\bar{p}_t T_b + N_0)}} dx \\ &= \frac{1}{2} - \frac{1}{2} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + (K+1)(\bar{p}_t T_b + N_0)}} \\ &\rightarrow (K+1) e^{-K} \frac{\bar{p}_t}{4\bar{p}_0} \quad \text{for } \bar{p}_0 \gg \bar{p}_t \gg \frac{N_0}{T_b}. \end{aligned} \quad (5.12)$$

Fig. 5.2 shows that (5.12) asymptotically approaches the exact expression for large C/I -ratios.

In the limiting case of Rayleigh fading, the instantaneous power p_0 of the test signal is exponentially distributed ($C_0=0, \bar{p}_0=\bar{q}_0$). This gives the local-mean BER

$$P_b(e|\theta_0=0, \bar{p}_0, \bar{p}_t) = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_t T_b + N_0}} \quad (5.13)$$

for any C/I -ratio. The special case of a noise-limited channel without co-channel interference (AWGN channel; $p_i=0$) agrees with [1].

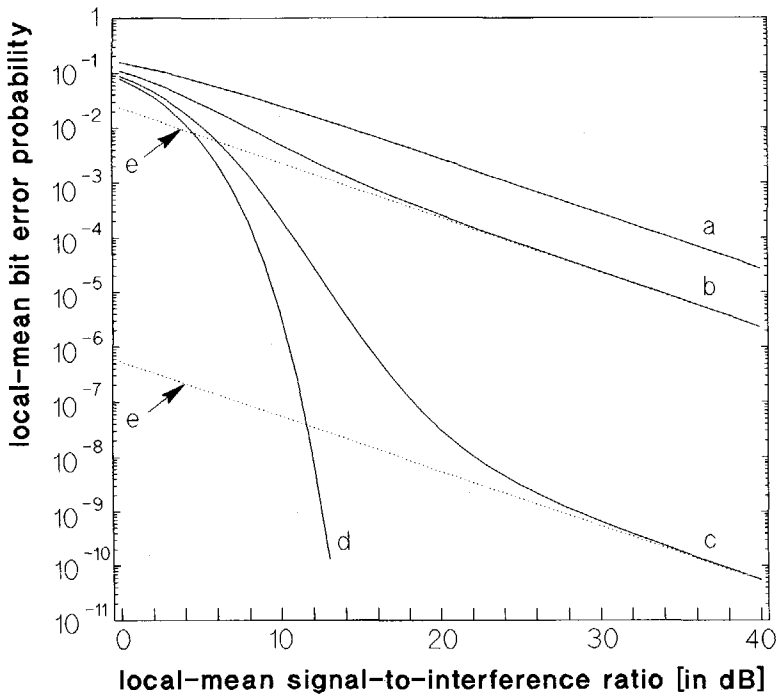


Fig. 5.2: Local-mean BER versus the local-mean signal-to-joint-interference ratio. Rician parameter: (a) $K=0$ (Rayleigh fading), (b) $K=4$ (6 dB), (c) $K=16$ (12 dB) and (d) $K \rightarrow \infty$ (no fading). ($\cdots$, e) approximate expression for large C/I -ratios.

As seen from Fig. 5.2, the Rician K factor highly affects the local-mean BER. Even if the scattered component is relatively weak ($K > 16$ (12 dB)), Rician fading substantially increases the BER above to the ideal (non-fading) case with $K \rightarrow \infty$.

5.4 AREA-MEAN BER IN MICRO-CELLULAR NETWORK

In a micro-cellular network, the terminal transmitting the wanted signal is often within line-of-sight of the base station. The wanted signal is then not likely to experience shadowing, but interference signals may propagate over obstructed paths.

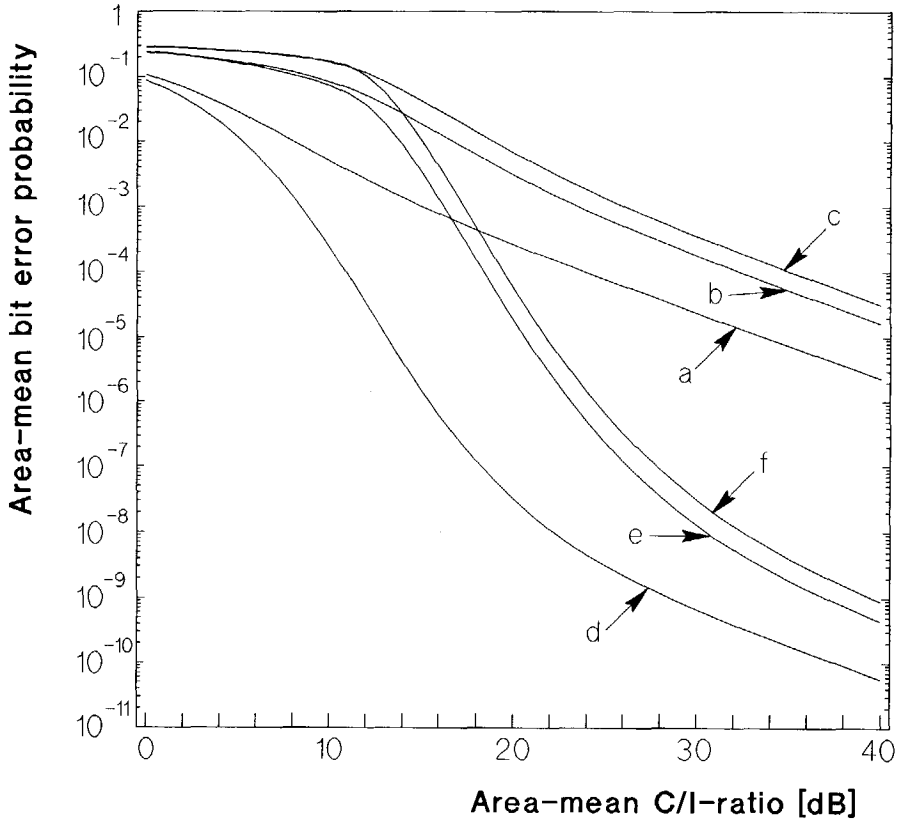


Fig. 5.3: Area-mean BER in a micro-cellular network with six interferers, versus area-mean signal-to-interference ratio P_0/P_i . Wanted signal: (a,b,c) $K=4$ (6 dB), (d,e,f) $K=16$ (12 dB). Rayleigh-fading interfering signal: (a,d) without shadowing, (b,e) shadowing with $s_s = 6$ dB ($P_i = 11.1P_0$ and $s_i = 3.04$ dB), and (c,f) shadowing with $s_s = 12$ dB ($P_i = 43.7P_0$ and $s_i = 6.74$ dB).

As discussed in section 3.1, the power sum of a number of n log-normally distributed signals has been studied, but no closed-form solution for the pdf of the local-mean power $\bar{p}_l$ of the joint interference signal has yet been found. To a good approximation, the local-mean joint interference power also has a log-normal distribution. The area-mean power and the standard deviation obtained with the method proposed by Schwartz and Yeh are given in table 3.1. The conditional bit error probability (5.11) is averaged over the log-normal distribution (2.14) of the local-mean joint interference power $\bar{p}_l$.

In Fig. 5.3, the area-mean C/I -ratio is defined as $(\frac{1}{2}C_0^2 + q_0)/\bar{p}_l$, which is in contrast to the use of *joint* interference powers in Fig. 5.2. The definition adopted here allows the reader to use Fig. 5.3 without first having to apply Schwartz and Yeh's method.

It can be shown (see [15] and the method in section 5.5.1) that, for high C/I -ratios, the BER tends to

$$P_b(e|\bar{p}_0, \bar{p}_l) \rightarrow \frac{\bar{p}_l}{4\bar{p}_0} (K+1) \exp\left\{-K + \frac{1}{2}\sigma_l^2\right\}. \quad (5.14)$$

5.5 AREA-MEAN BER IN MACRO-CELLULAR NETWORK

In macro-cellular networks, the wanted and interfering signals experience Rayleigh fading and shadowing. The local-mean BER is given by Eq. (5.13). To compute the area-mean BER, this expression must be averaged over the log-normal distribution of the local-mean powers of the wanted and the joint interference signal. The integration variables $\bar{p}_0$ and $\bar{p}_l$ are substituted by

$$x^2 \Delta \frac{1}{2\sigma_x^2} \ln^2 \left(\frac{\bar{p}_0}{\bar{p}_0} \right) \quad (5.15)$$

and

$$y^2 \triangleq \frac{1}{2\sigma_i^2} \ln^2 \left(\frac{\bar{p}_i}{\underline{p}_i} \right), \quad (5.16)$$

respectively. This gives

$$P_b = \frac{1}{2} - \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy \exp(-x^2 - y^2) \sqrt{\frac{\bar{p}_0 T_b \exp(\sqrt{2} x \sigma_s)}{\bar{p}_0 T_b \exp(\sqrt{2} x \sigma_s) + \bar{p}_i T_b \exp(\sqrt{2} y \sigma_i) + N_0}}. \quad (5.17)$$

No closed-form solution has been found for the integrals in (5.17). To obtain numerical results, the Hermite polynomial method [7] is used twice, resulting in the bit error probability being approximated by a double sum. Fig. 5.4 presents the average bit error rate for channels without Gaussian noise ($N_0=0$). The area-mean C/I -ratio is defined as $\bar{p}_0/\bar{p}_i$, which corresponds to r_u^β and $(3C)^{1/2\beta}$.

Shadowing of the desired signal is taken $\sigma_s=1.36$ and 2.72 ($s_s=6$ dB and 12 dB). Six co-channel interfering signal with mutually independent shadowing with identical standard deviation ($\sigma_i=\sigma_s$ for $i=1,2,..6$) are assumed. As seen from Table 3.1, the joint interference signal has as a logarithmic standard deviation of $s_i=3.04$ dB and 6.74 dB for $s_s= 6$ dB and 12 dB, respectively. For 12 dB of shadowing ($\sigma_s=2.72$), the area-mean BER remains unacceptably high even for area-mean C/I -ratios of 40 dB ($\bar{p}_0/\bar{p}_i=10^4$) which corresponds to $\bar{p}_0/\bar{p}_i \approx 22.9$ (14 dB).

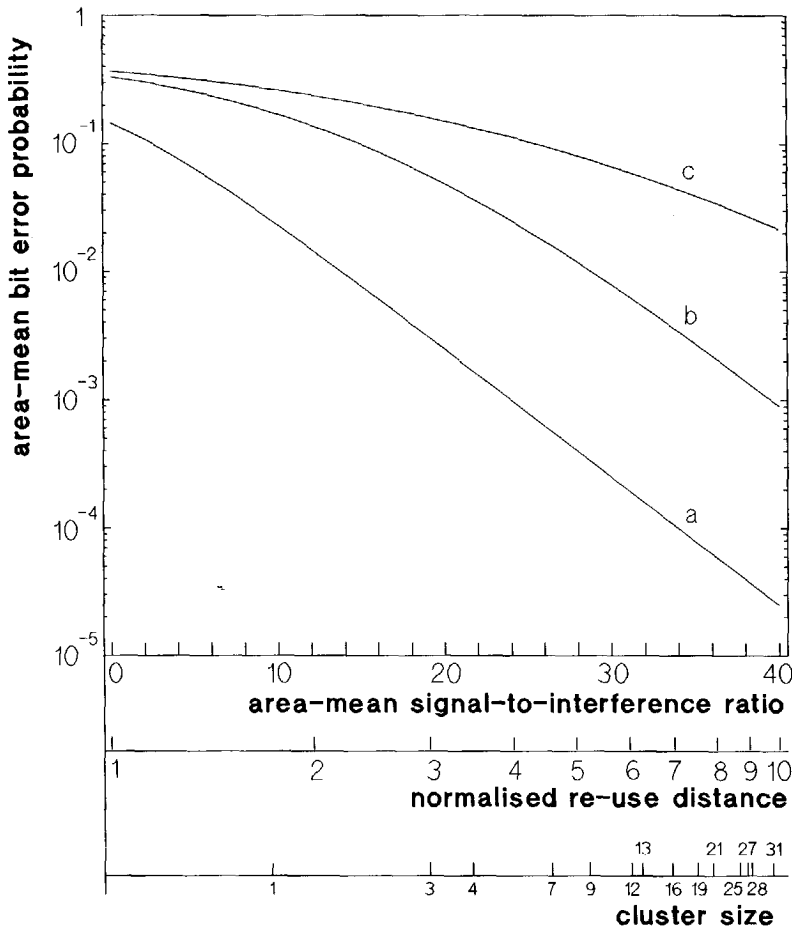


Fig. 5.4 Area-mean BER for BPSK in macro-cellular network with six interfering signals versus area-mean C/I-ratio per interferer and corresponding re-use distance and cluster size, (a) without shadowing, (b) shadowing with $s_s = 6$ dB ($\bar{p}_t = 11.1 \bar{p}_b$, $s_t = 3.04$ dB), and (c) shadowing with $s_s = 12$ dB ($\bar{p}_t = 43.7 \bar{p}_b$, $s_t = 6.74$ dB). UHF groundwave propagation with $\beta = 4$.

5.5.1 APPROXIMATE EXPRESSIONS FOR LARGE C/I-RATIOS

In the event of negligible noise and high signal-to-interference ratios ($\bar{p}_0 \gg \bar{p}_t \gg N_0$), (5.17) can be approximated by

$$P_b \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy \exp(-x^2 - y^2) \left[1 - \sqrt{1 - \frac{\bar{p}_t \exp(\sqrt{2} y \sigma_s)}{\bar{p}_0 \exp(\sqrt{2} x \sigma_s)}} \right]. \quad (5.18)$$

After a first-order expansion of the square root, one finds

$$P_b \approx \frac{\bar{p}_t}{4\pi \bar{p}_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp(-x^2 - y^2 + \sqrt{2} y \sigma_s - \sqrt{2} x \sigma_s) dx dy, \quad (5.19)$$

which is solved as

$$P_b \approx \frac{\bar{p}_t}{4\bar{p}_0} \exp\left\{\frac{\sigma_s^2 + \sigma_t^2}{2}\right\}, \quad (5.20)$$

where $\bar{p}_t$ depends on the standard deviation σ_t (see table 3.2). It is concluded that shadowing substantially affects the area-mean BER. For instance, with $s_i = 12$ dB for the wanted signal and each interfering signal ($s_i = 6.74$ dB and $\bar{p}_t = 43.7\bar{p}_i$, see Table 3.2), the area-mean BER is approximately $1.1 \cdot 10^3$ times larger than the BER in channels without shadowing ($s_s = 0$ dB, $s_i = 0$ dB and $\bar{p}_t = 6\bar{p}_i$). If the shadowing of the wanted signal and the shadowing of the joint of interference signal are correlated, the BER presumably is less influenced.

5.6 EFFECT OF NON-SYNCHRONOUS INTERFERENCE

In the previous sections, the receiver and interference model were highly simplified to obtain mathematically tractable expressions for the BER. This required relatively strong assumptions and approximations, which were not qualified. In the author's impression, the assumption of perfect bit alignment of all signals is particularly unrealistic in

practical distributed communication networks: signals arriving from independent transmitters will certainly not contain exactly synchronised bit streams.

In this section, it is still assumed that the bit duration T_b is the same for all transmitters, but the starting time of the bits may now differ from terminal to terminal. The receiver is assumed to be in perfect bit synchronisation with the wanted signal, and hence no longer with the interfering signals. This is illustrated in Fig. 5.5.

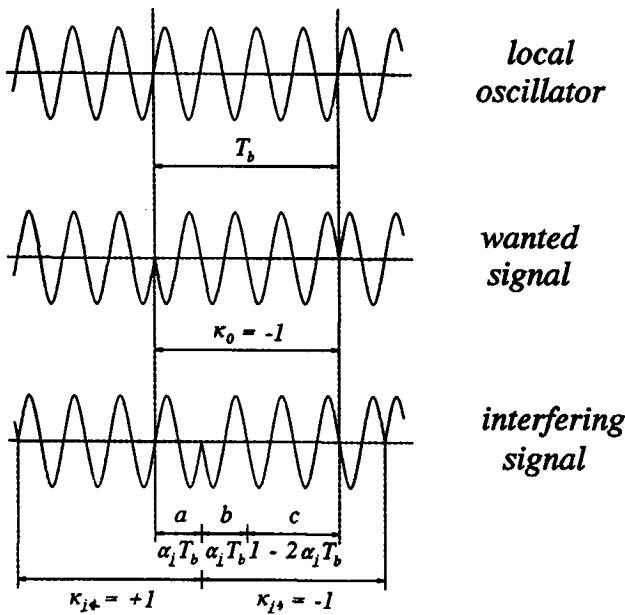


Fig. 5.5: Wanted BPSK signal and the i -th interfering BPSK signal with bit synchronisation offset α_i . Interference introduced during period (a) cancels interference introduced during period (b).

The bit synchronisation offset is denoted as α_i ($0 < \alpha_i < 1$), i.e., if a carrier reversal occurs it happens at the instant $(k + \alpha_i)T_b$. We denote the bit value of the i -th interfering signal before and after the phase reversal as κ_{i-} and κ_{i+} , respectively. Thus the interfering signal has the form $\kappa_{i-}\rho_i\cos(\omega_c t + \theta_i)$ during the interval

$\langle (k-1+\alpha_i)T_b, (k+\alpha_i)T_b \rangle$ and the form $\kappa_{i\rightarrow} \rho_i \cos(\omega_c t + \theta_i)$ during the interval $\langle (k+\alpha_i)T_b, (k+1+\alpha_i)T_b \rangle$.

The statistical decision variable becomes

$$v = \rho_0 \kappa_0 \cos(\theta_0) + \sum_{i=1}^n \zeta_i \{ \kappa_{i\leftarrow} \alpha_i + \kappa_{i\rightarrow} (1 - \alpha_i) \} + n_I. \quad (5.21)$$

If the interfering transmitters are independent, the bit-timing offset α_i is likely to be uniformly distributed. Because of symmetry of the case $0 < \alpha_i < 1/2$ with the case $1/2 < \alpha_i < 1$, it can be assumed that

$$f_{\alpha_i}(\alpha_i) = \begin{cases} 2 & 0 < \alpha_i < \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases} \quad (5.22)$$

If a phase reversal of the i -th signal occurs at the instant $k + \alpha_i$, thus if $\kappa_{i\leftarrow} = -\kappa_{i\rightarrow}$, the signal in the period $\langle kT_b, (k + \alpha_i)T_b \rangle$ cancels the phase-reversed signal during the following interval $\langle (k + \alpha_i)T_b, (k + 2\alpha_i)T_b \rangle$. In this case, the variance of the interference sample is effectively reduced to $p_i(1 - 2\alpha_i)^2$. For a fixed value of α_i , the sample from an interfering signal with a phase reversal is Gaussian distributed. However, if α_i is treated as a random variable, the interference sample in (5.9) becomes non-Gaussian.

5.6.1 SINGLE INTERFERING SIGNAL

If "0"s and "1"s are equiprobable with independent probability of occurrence, the probability of a phase reversal is one half. Thus, if the interference consists of only a single signal ($n=1$) with a bit synchronisation offset of $\alpha_i T_b$, the bit error probability is

$$P_b(e|\theta_0=0, \bar{p}_0, \bar{p}_1, \alpha_1) = \frac{1}{2} - \frac{1}{4} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}} - \frac{1}{4} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b (1 - 2\alpha_1)^2 + N_0}} \quad (5.23)$$

For $\alpha_1=0$, i.e., in the case of perfect bit synchronisation, (5.23) reduces to (5.10). Integrating (5.23) over the uniform pdf (5.22) of α_1 gives

$$P_b(e|\bar{p}_0, \bar{p}_1) = \frac{1}{2} - \frac{1}{4} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}} - I, \quad (5.24)$$

where I is the integral

$$I = \frac{1}{4} \sqrt{\frac{\bar{p}_0}{\bar{p}_1}} \int_0^{\frac{1}{2}} \left(\sqrt{\alpha_1^2 - \alpha_1 + \frac{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}{4\bar{p}_1 T_b}} \right)^{-1} d\alpha_1 \quad (5.25)$$

Since

$$\frac{d}{dx} \ln(x+a + \sqrt{x^2 + 2ax + b}) = \frac{1}{\sqrt{x^2 + 2ax + b}} \quad (5.26)$$

one finds

$$I = \frac{1}{4} \sqrt{\frac{\bar{p}_0}{\bar{p}_1}} \ln \left(x - \frac{1}{2} + \sqrt{x^2 - x + \frac{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}{4\bar{p}_1 T_b}} \right) \bigg|_{x=0}^{\frac{1}{2}} \quad (5.27)$$

Hence, the BER becomes, after substituting (5.27) in (5.24),

$$P_b = \frac{1}{2} - \frac{1}{4} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}} + \frac{1}{4} \sqrt{\frac{\bar{p}_0}{\bar{p}_1}} \ln \left(\frac{\sqrt{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0} - \sqrt{\bar{p}_1 T_b}}{\sqrt{\bar{p}_0 T_b + N_0}} \right) \quad (5.28)$$

Numerical results are shown as curve *b* of Fig. 5.6.

5.6.2 MANY INTERFERING SIGNALS

If the interference is caused by the sum of many weak signals, the joint interference term in (5.21) becomes a Gaussian distributed random variable. Since ζ_i and α_i are independent stochastic variables, the variance of a sample of an interfering signal that makes a phase reversal is

$$\begin{aligned} E[\zeta_i^2 (1 - 2\alpha_i)^2] &= E[\zeta_i^2] E[(1 - 2\alpha_i)^2] \\ &= \bar{p}_i \cdot 2 \int_0^{\frac{1}{2}} (1 - 4\alpha_i + 4\alpha_i^2) d\alpha_i \\ &= \bar{p}_i \cdot \frac{1}{3} . \end{aligned} \quad (5.29)$$

Alternatively, if no phase reversal occurs, the variance is found from (5.8). If a phase reversal occurs with probability one half, and if the bit alignment offset is random, the experienced total interference sample has the variance $\frac{1}{2}(1 + \frac{1}{3})\bar{p}_i$. This corresponds to a reduction of 1.8 dB of the effective interference power. Thus, for random bit offsets, and sufficiently many interfering signals, the average BER for coherent detection of a wanted BPSK signal becomes

$$P_b(\epsilon | \theta_0 = 0, \bar{p}_0, \bar{p}_i, n \rightarrow \infty) = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \frac{2}{3} \bar{p}_i T_b + N_0}} \quad (5.30)$$

which is depicted as curve *c* in Fig. 5.4.

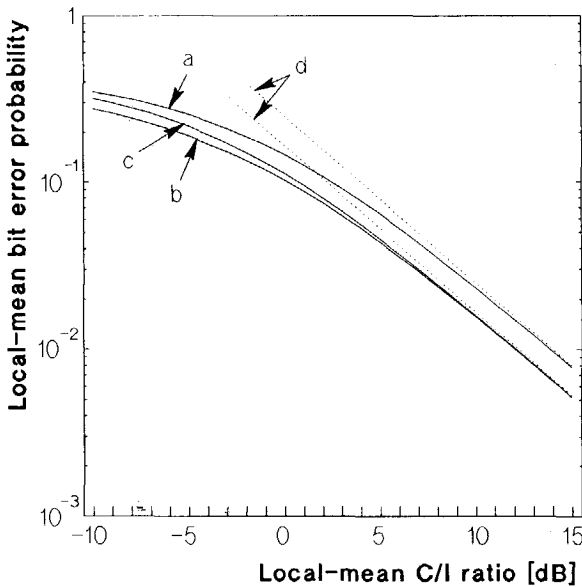


Fig. 5.6: *Local-mean BER of BPSK in a Rayleigh-fading channel versus local-mean signal-to-joint-interference ratio, for $n=1$ and ∞ interferers, with (a) perfect bit alignment and (b,c) random bit alignment for one (a,b) and many (a,c) interfering signals. (···, d) corresponding approximate expressions.*

As seen from Fig. 5.6, the latter result for $n \rightarrow \infty$ closely approximates the BER for one single interferer ($n=1$). A small effect is noticeable for local-mean carrier-to-interference ratios below 0 dB ($C/I < 1$). However, the behaviour for $C/I > 1$ (0 dB) is of more practical interest: in this range (5.28) and (5.30) produce almost identical results.

5.7 CONCLUSIONS

The average bit error rate is a relevant measure of the performance of a digital mobile network with continuous wave transmission. The local- and area-mean BERS have been approximated for the case of a wanted signal in the presence of multiple fading interfering signals. The results were used to calculate the local- and area-mean BER in micro- and macro-cellular networks.

The analysis showed that the specular-to-scatter (Rician) ratio K of multipath fading and the standard deviation σ_s of any shadowing on the channel has a very substantial effect on the average BER: It appeared that the rapid decrease of the BER with increasing power of the wanted signal, which occurs in time-invariant channels, is not experienced in mobile channels with fading. Even for high signal-to-interference-plus-noise ratios, the average BER remains relatively high. Comparing results for micro- and macro-cellular networks, it is seen that, within the usual parameter ranges, say, less than 12 dB of shadowing ($0 < \sigma_s < 2.72$) and Rician fading with $0 < K < 1000$ (30 dB), shadowing affects the BER less than the Rician factor. One may conclude that in micro-cellular networks with the terminal transmitting the wanted signal within line-of-sight, BERS are substantially smaller than in macro-cellular networks. This is intuitively explained by the fact that for small K , as in Rayleigh-fading (macro-cellular) channels, deep fades occur frequently, which largely contribute to the BER.

The effect of interference has been studied for bit intervals that are exactly overlapping and, more realistically, randomly aligned with the bit transitions of the wanted signal. It was seen that the assumption of exactly aligned bits overestimates the BER somewhat. Nonetheless, for conservative system and network design, this may be an acceptable approximation.

Perfect carrier synchronisation has been assumed. Particular in networks with burst transmissions, such as in TDMA inbound channels for mobile telephony or in packet radio networks, imperfect receiver synchronisation may severely limit the

performance. This aspect has not been addressed here and is recommended for further study.

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CHAPTER 6

RANDOM MULTIPLE ACCESS TO MOBILE RADIO CHANNELS

Chapters 3, 4 and 5 addressed the case of mobile communication from a known transmitter (sending the 'wanted' signal) to the receiver, in the presence of 'unwanted' interference and AWGN. Terminals were assumed to transmit a continuous stream of information. In this chapter, data networks with a large number of participating terminals are considered. Each terminal is considered to generate messages in the form of data packets, which are to be transmitted to a common receiver.

Transmissions are of a burst type: the average data rate being low, all communication is concentrated in occasional peaks of high data rates. As discussed in chapter 1, burst traffic is commonly encountered in mobile information systems, such as paging and messaging networks, traffic information and route guidance systems, and fleet

management systems. In such networks, the instantaneous flow of data from mobile terminals has to be organised and controlled dynamically. This is solved by (random) multiple-access schemes. The issue of random access of short messages is not restricted to networks for data communications. In complex systems for circuit-switched *voice* communication, a common inbound (mobile-to-fixed) random-access signalling channel is employed to handle call requests and specific signalling messages from mobile subscribers.

Scarcity of the radio spectrum requires efficient use of the available frequencies and has led to the development of a wide variety of multiple-access schemes, e.g. [1]-[4]: the same radio channel is used by a number of subscribers, each of which only uses the channel for a limited amount of time. In Time Division Multiple Access (TDMA), the time interval reserved for each user is assigned according to a periodic scheme. A disadvantage of such fixed assignments is that a user must await his time slot before being allowed to transmit a packet. The delay may become unacceptably large if the number of subscribers is large, especially slots assigned to non-active users remain idle. In contrast to this, in dynamic access, resource reservation is flexible (demand assignment) or not made in advance at all (random access), so subscribers compete for access to the channel [4],[5]. The obvious disadvantage of random access is that two (or more) competing terminals may simultaneously attempt to transmit a data packet over the radio channel. Such a contention or access conflict is called a collision. To avoid collisions, various dynamic access control methods are possible, at the expense of signalling capacity and control complexity.

This chapter reviews a number of simple random-access protocols, namely slotted ALOHA [6],[7] Carrier Sense Multiple Access (CSMA) [8],[9], and Inhibit Sense Multiple Access (ISMA) [9],[10]. A method for the evaluation of their performance is presented, which is an extension of techniques reported earlier, e.g. by Kleinrock and Togabi [8], to address the performance experienced by individual terminals rather than the average performance for all participating terminals. Random-access techniques commonly used in wired local areas networks (LANs), were initially used in mobile radio networks. However, it soon appeared that the performance of many random-

access schemes substantially differs for wired and radio channels, e.g. [11], being highly dependent on the physical characteristics of the channel. The characteristics of the mobile channel are not included explicitly in this chapter: the physical radio link is modelled simply by stating the generic probability of successful reception of a data packet, given the presence of a certain number of contending packets. Detailed investigation of the effect of the channel will be deferred to chapter 7, 8 and 9.

The organisation of this chapter is as follows: first, in section 6.1, some random-access schemes are briefly reviewed. A first step in our analytical evaluation is the formulation of the model for Poisson arrivals of data packets in section 6.2. Section 6.3 discusses how the physical radio channel, can be included in the consideration of random-access protocols. Performance measures from random access channels are discussed in section 6.4. Combining the various models and definitions from sections 6.2 - 6.4, section 6.5 goes on to determine the throughput and the near-far effect in slotted ALOHA. Similar results for ISMA and CSMA are determined in section 6.6. Section 6.7 discusses the throughput of an ISMA channel when also taking account of the increase of the propagation delay with increasing separation between the mobile terminal and the base station. Section 6.8 contains the conclusions of this chapter.

6.1 REVIEW OF SOME RANDOM ACCESS PROTOCOLS

6.1.1 (SLOTTED) ALOHA

The earliest contention technique, known as ALOHA, was developed for a packet-radio network carrying data from remote terminals to a host computer at the University of Hawaii. The decision in 1969 to implement the ALOHA protocol was based on network simplicity [12]: many users send their 'bursty' packet traffic at high speed on a single (common) radio channel without mutual regulation, thus accepting the risk of access conflicts. Whenever a data packet is generated, this is transmitted on the random-

access channel, without considering a priori whether or not it will interfere with other transmissions. If more than one packet is transmitted at the same moment, a collision occurs so transmitted data packets may not correctly reach the destination. In the central receiver the correctness of a received packet is determined. If the packet is deemed correct, i.e., if it contains a correctable number of bit errors, an acknowledgement is broadcast on a separate feedback channel. Since such (outbound) feedback transmissions do not experience competition from other transmitters, this feedback channel is likely to be substantially more reliable than the inbound collision channel. If a mobile terminal does not receive an acknowledgement of its transmission, it will therefore assume that its packet has been lost in a collision, and will retransmit the packet after waiting a random time.

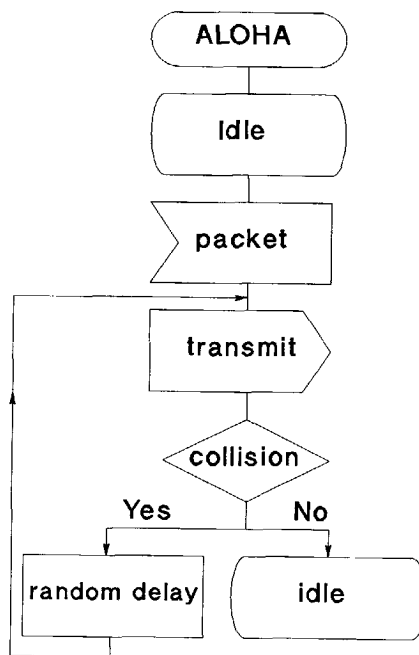


Fig. 6.1: Flow diagram for the activities of each mobile terminal in ALOHA random access [13].

The ALOHA access technique is illustrated in Fig. 6.1 [13], using symbols proposed in the Specification and Description Language (SDL) of the CCITT [14].

An improvement can be made by dividing time into slots; terminals may only start a packet transmission at the *beginning* of a time slot. If the durations of all packets equal the slot time, contending packets either overlap completely or not at all. The event that a packet is lost due to *partially* overlapping interference is avoided. This has been shown to effectively double the throughput of the ALOHA channel [7].

6.1.2 PROTOCOLS EXPLOITING FEEDBACK INFORMATION

With increasing traffic loads, collisions occur more frequently in the ALOHA channel. This reduces the throughput and may eventually lead to instability if retransmitted packets continue to collide [15], [16], [24], [25], [27]. A number of other protocols have been proposed to mitigate this problem. Collision resolution algorithms (CRAS) improve the stability of the channel and reduce the delay caused by retransmission waiting times by employing the feedback channel more effectively [15], [16]. For instance, the central station can report whether the inbound channel was idle, a collision occurred, or a successful packet reception happened. A more efficient retransmission strategy can be followed given knowledge of this feedback information. Examples of CRAS are the splitting (or tree) algorithm [14] and the stack algorithm [16]. Dynamic frame length ALOHA is another technique to ensure the stability of an ALOHA network [28].

Busy-Channel Multiple Access (BCMA) is the class of multiple-access schemes in which no new packet transmissions are allowed when the inbound channel is busy. Various strategies have been proposed to acquire this information about the channel state [8]-[10], [17], [18], [31]-[33], [35]-[36].

6.1.3 NONPERSISTENT CARRIER SENSE MULTIPLE ACCESS

Carrier Sense Multiple Access (CSMA) is one of the best-known examples of BCMA. CSMA employs feedback, in the standard form of the positive acknowledgements of correct reception. Additionally, no new packet transmission is initiated by (mobile) terminals if they sense that the inbound channel is already busy [8]. Fig. 6.2 gives a flow diagram of 'nonpersistent' CSMA: Nonpersistent means that if the channel is sensed busy, the terminal attempts a new transmission only after waiting a random time. Thus with nonpersistent CSMA, a transmission attempt does not necessarily lead to a transmission on the channel.

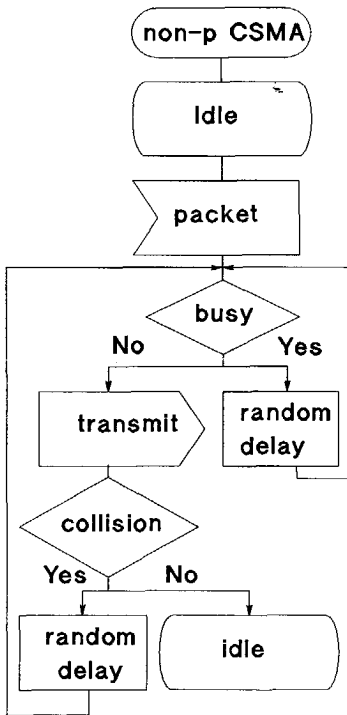


Fig. 6.2: Flow diagram of the activities of each mobile terminal in nonpersistent CSMA [13].

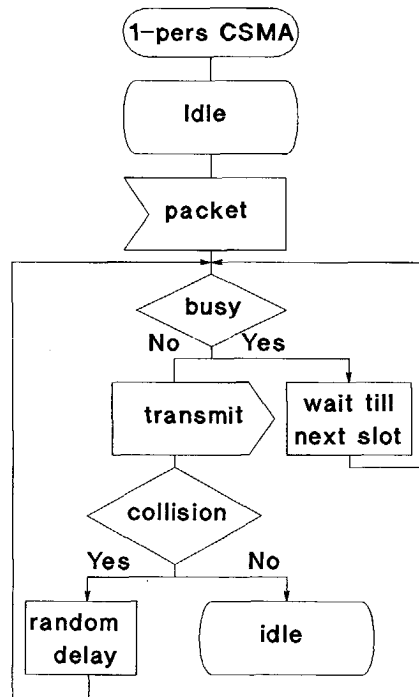


Fig. 6.3: Flow diagram of the activities of each mobile terminal in 1-persistent CSMA [13].

6.1.4 1-PERSISTENT CARRIER SENSE MULTIPLE ACCESS

If, in 1-persistent CSMA, a mobile senses the channel busy, it waits only until the channel becomes idle, thus it *persistent* on sensing, and then immediately transmits the packet. The terminal activities are depicted in Fig. 6.3.

6.1.5 SLOTTED p -PERSISTENT CARRIER SENSE MULTIPLE ACCESS

It is the impression of the author that a number of different interpretations of p -persistent CSMA are used in the analyses reported in technical literature, e.g. [8], [36]. This section describes slotted p -persistent CSMA as described in [8]. Time is divided into (mini-) slots with a duration that is a fraction of the packet duration. Any packet transmission occupies a fixed, integer number of successive slots. A terminal with a packet to be transmitted starts transmission with probability p if the channel is sensed idle, and so waits with probability $1-p$ until the next slot. If, on the other hand the channel is busy, the terminal waits until the next slot and again senses the channel. This is repeated until either a destructive collision or a successful transmission of any packet occurs, in which cases the packet is rescheduled. Fig. 6.4 [13] illustrates the terminal activities.

6.1.6 UNSLOTTED p -PERSISTENT CARRIER SENSE MULTIPLE ACCESS

p -Persistent CSMA was originally defined for slotted channels [8]. Here, a modification of the protocol is made to allow evaluation of unslotted networks (see Fig. 6.5):

If a packet arrives at an instant when the channel is sensed idle, the packet is transmitted immediately. If, on the other hand, the channel is busy at the instant of

arrival of the packet in the terminal buffer, the terminal performs a binary random experiment: With probability p the terminal transmits the packet as soon as the channel becomes idle. Such an attempt is considered successful unless the packet is destroyed in a collision. Alternatively, with probability $1-p$, the packet is rescheduled. These unsuccessful attempts are considered to contribute to the 'attempted packet traffic G_i' ' (see section 6.2) even though no transmission occurred. A next attempt is performed after waiting a random time.

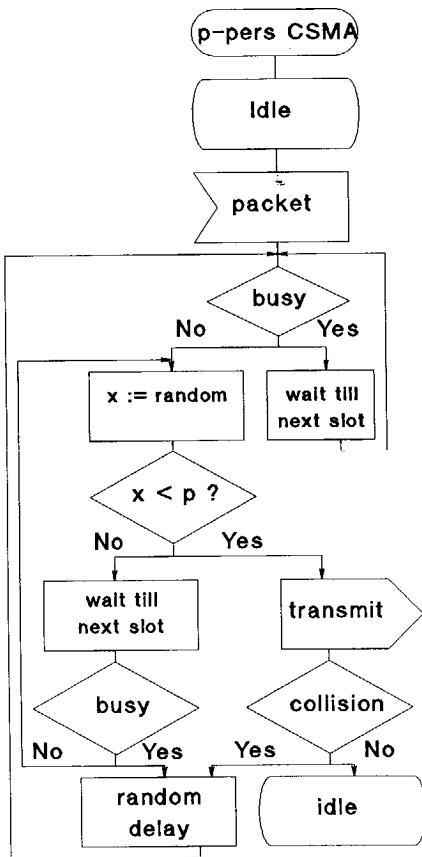


Fig. 6.4: Flow diagram for the activities of each mobile terminal in slotted p -persistent CSMA [13].

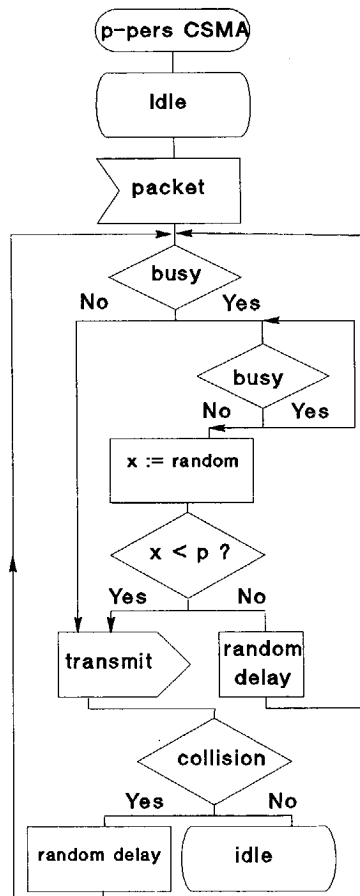


Fig. 6.5: Flow diagram for the activities of each mobile terminal in unslotted p -persistent CSMA.

Intuitively, as compared to unslotted nonpersistent CSMA, unslotted p -persistent CSMA is expected to have lower delays because waiting terminals may start transmitting with some appropriately chosen probability p immediately after the channel becomes idle. Particularly in channels with capture, this may reduce the delay. Compared to the slotted p -persistent access scheme, unslotted p -persistent has the advantage that slot timing is not required, which is advantageous if substantial (random) propagation delays occur. Moreover, if packets arrive when the channel happens to be idle, the packet is transmitted immediately with unslotted p -persistent CSMA. Another major difference occurs if an initially busy channel becomes idle: In unslotted p -persistent CSMA the terminals performs one binary experiment. If the outcome prohibits transmission, the packet is rescheduled, whereas in slotted p -persistent CSMA the terminals tries immediately again, thus in the next mini slot.

According to the above description, (unslotted) nonpersistent CSMA becomes the limiting case of unslotted p -persistent CSMA, for $p \rightarrow 0$. This observation is not valid for the slotted p -persistent CSMA.

6.1.7 INHIBIT SENSE MULTIPLE ACCESS

Successful application of CSMA requires that all mobile terminals can receive each other's signals on the inbound frequency, e.g. as on a local-area network (LAN). However, in radio nets, especially with fading channels, a mobile terminal might not be able to sense a transmission by all other terminals. This complication, known as the hidden-terminal problem [9], is avoided in Inhibit Sense Multiple Access (ISMA), where the base station transmits a busy signal on an outbound channel (in order to forbid all other mobile terminals to transmit) as soon as an inbound packet is being received [10]. A disadvantage of ISMA is the necessity of a real-time (i.e., non-slotted, non-fading) feedback channel. A mobile terminal may also still not sense the presence of the 'busy' tone, because of noise corruption or fading of the feedback signal [17]. This

problem may be circumvented by broadcasting an (active) *idle* tone, rather than an (active) busy tone, or reduced by transmitting the 'busy reports' with error-control coding.

Even if signalling messages on the feedback channel were always received correctly by all mobile terminals, collisions would nonetheless occur in ISMA for two reasons: *i*) New packet transmissions may start during the transmission delay of the inhibit signal, and *ii*) packets from two or more persistent terminals, awaiting the channel to become idle, may collide immediately after the termination of the previous packet transmission.

6.2 PACKET TRAFFIC MODEL

All data packets are assumed of uniform duration, equal to the unit of time and, in the event of slotted ALOHA, equal to the duration of a time slot. The average number of attempted transmissions contributed by all users in the entire network, expressed in packets per unit of time (ppt), is denoted by G_p , known as the 'attempted' or 'offered' traffic. The number of participating terminals in the network is N . If the i -th terminal ($i=1,...,N$) attempts to transmit a data packet in a time slot with probability $Pr(i_{ON})$, the total attempted traffic is

$$G_t = \sum_{i=1}^N Pr(i_{ON}) \quad (6.1)$$

Most of the investigations reported in this and the following chapters address an infinite population ($N \rightarrow \infty$), albeit with bounded total offered traffic G_p . Accordingly, the probability of an attempt per terminal is assumed to approach zero ($Pr(i_{ON}) \rightarrow 0$). The number of arrivals in a certain interval is assumed to be Poisson distributed. This

assumption may be reasonable for the packet traffic generated in some typical mobile networks, such as the call-request channel of a cellular telephone network.

The (Poisson) arrival rate λ is expressed in the average number of arrivals during a unit of time. So, for packets of unit duration, $G_i = \lambda$. The probability of n packet arrivals in a time interval T is [19],[20]

$$P_n(n) = \frac{(\lambda T)^n}{n!} \exp(-\lambda T) . \quad (6.2)$$

A generalised derivation of (6.2) is contained in appendix B. The distribution of the number of arrivals in any time interval $(t, t+T)$ is independent of starting time t . It can be shown that (6.2) also expresses the probability of n contenders to a given test packet already known to be present.

The probability of no arrivals during period of duration τ_0 is

$$P_n(0) = \exp\{-\lambda \tau_0\} = \int_{\tau_0}^{\infty} f_{\tau}(\tau) d\tau \quad (6.3)$$

where $f_{\tau}(\tau)$ is the pdf of the duration between two arrivals. Thus, interarrival times have the negative exponential distribution

$$f_{\tau}(\tau) = \lambda \exp\{-\lambda \tau\} \quad (6.4)$$

with mean λ^{-1} .

Two extensions of this basic model are now introduced. A Poisson arrival rate of data packets depending on transmit distance to the central base station and on time is considered:

Spatial distribution: In general, packet offerings are not uniformly distributed over the service area of the network. For instance, remote terminals with weak signals may have to perform more retransmissions than nearby terminals. In the case of an infinite population of terminals, the attempted traffic may be considered to be continuously spread over the area. Abramson [11] defined $G(r)$ as the average number of attempted packet transmissions per unit of time and per unit of area, offered by terminals at a distance r from the central receiver. Assuming no angular variations of traffic, the total traffic per unit of time is found from the polar integration

$$G_t = \int_0^{\infty} 2\pi r G(r) dr . \quad (6.5)$$

In a number of investigations, r is considered as a stochastic variable, distributed with the probability density function $f_r(r)$. So, for a randomly selected packet one may write

$$f_r(r) = \frac{1}{G_t} 2\pi r G(r) . \quad (6.6)$$

Non-stationarity: It is generally assumed that the arrival rate λ is constant with time. This assumption will be relaxed in section 6.8, where the time-dependent arrival rate $\lambda(r,t)$ is introduced, resulting in a non-stationary Poisson process. As shown in appendix A this leads to

$$P_n(n) = \frac{(\bar{\lambda}T)^n}{n!} \exp\{-\bar{\lambda}T\} , \quad (6.7)$$

where $\bar{\lambda}$ is the average arrival rate during T , i.e.

$$\bar{\lambda} \triangleq \frac{1}{T} \int_{space} \int_{timeT} \lambda(r,t) 2\pi r dt dr . \quad (6.8)$$

Appendix B focuses on the integration over *time*, since the integration over *space* was previously considered in a number of papers, e.g. [11],[21].

6.3 IMPACT OF THE RADIO CHANNEL

Poisson packet arrivals are the foundation for a simple model for the probability of a collision with n interfering packets. Considering these collisions, classical papers such as [8] determine the throughput based on the two assumptions that, firstly, a data packet is always received correctly in the absence of collisions and, secondly, all packets involved in a collision are lost. In mobile radio nets, both assumptions should be reconsidered: channel imperfections may cause loss of a data packet even if no interference from other terminals occurs [22], and in the event of a collision, the strongest contending radio signal may capture the receiver, see e.g. [7],[21],[23]. Compared to remote terminals, nearby terminals therefore experience a higher probability of success in transmitting a packet; propagation fluctuations may also contribute to the capture effect (see chapter 7).

In order to determine the probability of successful transmission in terms of the behaviour of the physical layer, the following quantities are defined

$q_n(r)$ is the probability of successful reception of a test packet transmitted from a distance r from the central receiver, given the presence of interference from n other terminals with unknown positions.

q_n is the probability of successful reception, given that the arbitrary test packet is transmitted in the presence of n interfering signals, averaged over all expected positions of the mobile terminals, including the one transmitting the test packet. So, using (6.6),

$$q_n = \frac{1}{G} \int_0^{\infty} 2\pi r q_n(r) G(r) dr . \quad (6.9)$$

C_{n+1} is the expectation value of the number of successful packets in a collisions with $n+1$ competing packets, with $C_{n+1} = (n+1)q_n$. If receiver capture is mutually exclusive for each of the $n+1$ packets, i.e., if successful reception of one packet ensures that a packet from another terminal cannot be correctly received, C_{n+1}

also expresses the probability that one out of $n+1$ simultaneously transmitted packets is received correctly.

These probabilities will be determined taking account of the mobile channel characteristics from chapter 7 onwards. In some equations, the more explicit notation $q_n(r_j)$ and $Q(r_j)$ is used, index j denoting the 'test' packet.

6.4 MEASURES OF STEADY-STATE PERFORMANCE

Considering the arrival process of (interfering) packets offered to the channel and the probability of packet success, given a certain number of contenders, the efficiency of a certain random-access scheme will now be evaluated. A number of measures have been proposed to express the performance of the network. In this section, stationary behaviour of the network is assumed.

The evaluation of the 'total channel throughput' and the individual terminal 'success probability' is performed here for steady-state behaviour: Attempts to transmit a packet are considered to be a stationary Poisson process. If an attempt is unsuccessful, for instance if the packet is lost in a collision, the packet is retransmitted after a (long) random waiting time. Similarly, if a nonpersistent terminal is inhibited from transmission, a new attempt is scheduled only after a (long) random waiting time. The assumption of Poisson arrivals implies that in the event of a collision, such random retransmission waiting times are required to be sufficiently long to assure that the retransmitted packets experience interference that is uncorrelated with the interference experienced during previous attempts. This steady-state analysis simplifies the dynamic behaviour of the terminals in response to collisions. In practice, random waiting times should be kept as short as possible. However, if retransmissions are performed too soon, the steady-state assumption is inappropriate. In such cases, the random-access channel can become unstable. This will be addressed in section 8.6.

6.4.1 NEAR-FAR EFFECT

Because of propagation attenuation, the probability of successful access may depend on the distance r between the mobile terminal and the receiver. This leads to unfairness between terminals in the system. To evaluate this near-far effect, $Q(r)$ is defined as the unconditional probability of successful reception of a test packet generated for transmission from a terminal at a distance r , taking into account the probability of its permission to transmit and the statistics of the number of interfering packets n . So

$$Q(r) = \sum_{n=0}^N P_n(n) q_n(r) . \quad (6.10)$$

As will be described later, the probability $P_n(n)$ of n contending packets is on the form of (6.2) for ALOHA networks. For CSMA, ISMA, and presumably also for CRA, $P_n(n)$ is not necessarily a Poisson distribution.

For an individual subscriber, the (time-average) success rate $Q(r)$ can differ widely from the ensemble average Q , where

$$Q_t = \frac{1}{G_t} \int_0^{\infty} 2\pi r G(r) Q(r) dr. \quad (6.11)$$

Analogous to the offered traffic per unit of area, the throughput per unit of area $S(r)$ may be considered, with $S(r) = Q(r)G(r)$.

6.4.2 THROUGHPUT

From a point of view of spectrum efficiency, the total throughput S_t is an important measure of the overall performance of a random-access network. It is defined as the average number of packets per unit of time that is received correctly at the base station:

$$S_t = \int_0^{\infty} 2\pi r Q(r) G(r) dr . \quad (6.12)$$

The ensemble-average probability Q_t of a successful attempt is found as $Q_t = S_t / G_t$.

6.4.3 QUEUEING DELAY

The packet delay can be considered to consist of a 'terminal queueing delay' D_q , before the packet arrives at the first position in the terminal buffer, and an 'access delay' D_a , caused by retransmission waiting times if the packet is lost in a collision [38]. Models based on a Markov chain offer the possibility to evaluate the backlog, packet delay and also the stability of the network. In this section we address queueing delays within the terminal buffer, whereas a discussion of the access delay is deferred to section 8.6.

In [30], a model for packet queueing in a terminal buffer was addressed, assuming the probability of successful transmission $Q(r)$ to be stationary; thus the network and the channel traffic were assumed in equilibrium. A Markov chain was used to model the stochastic behaviour of the number of packets in the buffer of a particular terminal.

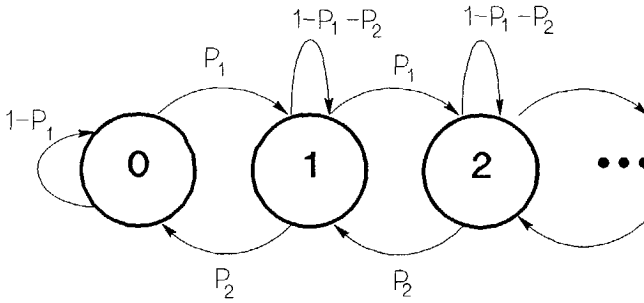


Fig. 6.6: Markov chain model for packet queueing in a terminal buffer.

P_o is the probability that a new packet is generated and enters the queue. Here the probability of arrival of new packets is assumed independent of the number of packets in the terminal buffer. A packet in the first position of the queue leaves the system with probability $P_r Q(r)$, where P_r is the probability of permission to transmit the packet in the first position of the queue. Hence, for a queue with non-zero length, the total number of queued packets in the terminal increases with probability $P_1 = P_o(1 - P_r Q(r))$, and decreases with probability $P_2 = (1 - P_o)P_r Q(r)$. The delay experienced before this packet reaches the first position in the buffer is [30]

$$D_q(r) = \frac{P_o + P_r^2 Q(r)^2}{P_r Q(r) - P_o}. \quad (6.13)$$

Hence, the queueing delay D_q in the transmitter buffer can be expressed merely in terms of P_o , P_r and $Q(r)$. The rest of the dissertation will focus on assessing the distant dependent probability of successful transmission $Q(r)$; the queueing delay is not explicitly expressed during the analyses. It is seen that stability of the queue in the terminal buffer is ensured as long as $P_o < Q(r) P_r$.

6.5 NEAR-FAR EFFECT AND THROUGHPUT OF SLOTTED ALOHA

In slotted ALOHA, a (mobile) terminal transmits a packet arriving at a random instant t during the immediately following time slot, regardless of other transmissions in the same time slot (see section 6.1). The number of simultaneous transmissions in an arbitrary time slot is thus found by studying the arrival process at all terminals during the previous time slot. For Poisson arrival processes, the probability that the test packet experiences interference from n other (contending) signals in the same time slot is also Poisson distributed, according to (6.2). The probability $Q(r)$ of a successful transmission is

$$Q(r) = \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} q_n(r). \quad (6.14)$$

Using (6.12) and (6.9), the total throughput becomes

$$S_t = G_t e^{-G_t} \sum_{n=0}^{\infty} \frac{G_t^n}{n!} q_n. \quad (6.15)$$

The classical collision channel without receiver capture [4], [8], [11], [12], [20] is recovered by inserting

$$\begin{aligned} q_0(r) &= 1 \\ q_n(r) &= 0 \quad \text{for } n=1,2,\dots, \end{aligned} \quad (6.16)$$

for all $r \geq 0$. Hence

$$S_t = G_t \exp(-G_t). \quad (6.17)$$

For low traffic loads ($G_t \ll 1$ ppt), any packet is likely to be received successfully ($S_t \rightarrow G_t$). For high traffic loads ($G_t \gg 1$ ppt), however, collisions severely reduce the throughput ($S_t \ll G_t$). The maximum $S_t \approx 0.368$ ppt is found for $G_t = 1$ ppt, though in a practical system this would lead to instability [16], [28].

6.6 NEAR-FAR EFFECT AND THROUGHPUT OF INHIBIT SENSE MULTIPLE ACCESS

The Inhibit Sense Multiple Access (ISMA) access technique was discussed in section 6.1. Typical examples of packet transmissions on the common radio channel are shown in Fig. 6.7 and Fig. 6.8 for unslotted nonpersistent and unslotted p -persistent ISMA, respectively. We now establish a terminology and a notation for mathematical analysis. The results described in the next sections also apply to CSMA, provided that each terminal can perfectly sense transmissions by any other terminal.

In ISMA, the radio system is supplemented by an outbound signal with the status of the channel: either busy or idle. An inbound packet arriving at the idle receiver will be called the 'initiating packet'. When the base station receives an inbound initiating packet, a busy signal is immediately broadcast to all mobiles to inhibit them from transmission. In a real system, broadcasting the busy signal occurs after a short processing delay d_1 . This delay is normalised to the unity duration of each data packet, and it is reasonable to consider only the case $d_1 < 1$. After completion of all $n+1$ contending transmissions, the base station starts transmitting an idle signal after a delay d_2 . In ISMA, delays may occur for instance if a narrowband feedback channel prohibits rapid transitions.

In CSMA, these delays are mainly included to account for the time a mobile terminal takes to switch from reception to transmission mode (power-up), after sensing the radio channel for carriers from other active terminals [31].

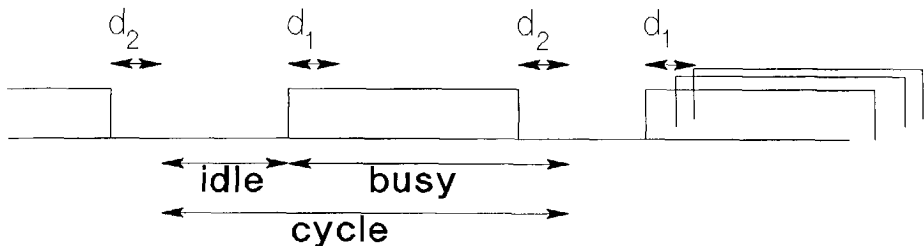


Fig. 6.7: Typical event of packet transmissions in non persistent ISMA.

A 'busy period' is defined as a period during which the base station broadcasts a busy signal plus the preceding signalling delay d_1 . This corresponds to the time interval between the first arrival of a packet until the moment that the channel becomes idle. Moreover, immediately after the channel becomes idle, persistent terminals that sense a busy signal may start to transmit. In such cases the busy period has a duration longer than two (or more) units of time. Hence, the duration of the busy period depends on the signalling delay d_1 and d_2 , and on the persistency p in sensing the feedback channel [8]. The average duration of the busy period is denoted as B .

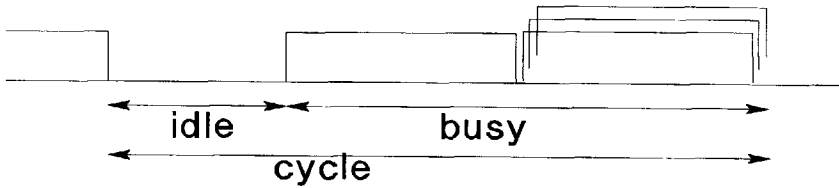


Fig. 6.8: Typical event of packet transmissions in unslotted p -persistent ISMA.

For memoryless Poisson arrivals, the duration of the 'idle period', i.e., the time interval between at the release of the channel and the first packet arrival, is exponentially distributed with mean $I = G_i^{-1}$ (see Eq. (6.4)).

An idle period plus the following busy period is called a cycle, with average duration $A = I + B$.

6.6.1 NONPERSISTENT ISMA

For nonpersistent CSMA and ISMA, rescheduling always occurs if the channel is busy at the instant of sensing. So, if a packet arrives at a nonpersistent terminal when the base station transmits a busy signal (denoted by event H_B), the attempt is considered to have failed. This occurs with probability

$$Pr(H_B) = \frac{B - d_1}{I + B} \quad (6.18)$$

If the feedback channel is imperfect, a transmission may erroneously be started [17], but this case is not considered here. The packet is rescheduled for later transmission.

With probability $I/(I+B)$ a test packet, transmitted from a distance r , starts at an instant when the channel is idle, i.e., the test packet becomes an initiating packet. This event is denoted as H_I . A collision can occur if one or more other terminals start transmitting during the time delay d_I of the inhibit signal. The conditional probability of n transmissions overlapping with the initiating test packet is

$$R_n(n | H_I) = \frac{(d_I G_I)^n}{n!} \exp(-d_I G_I). \quad (6.19)$$

Alternatively, the test packet itself starts during a period of duration d_I when the channel is busy because of a transmission by another initiating terminal, but seems idle since the inhibit signal is not yet being broadcast. This event, denoted as H_d occurs with probability $d_I/(B+I)$. The test packet thus experiences interference from the initiating packet, but possibly also from other arriving packets. The additional $n-1$ contending signals occur with a Poisson arrival rate during the interval d_I . Hence, the conditional probability of n interferers is found from

$$R_n(n | H_d) = \frac{(d_I G_I)^{n-1}}{(n-1)!} \exp(-d_I G_I), \quad (6.20)$$

where $n=1, 2, \dots$. Taking account of the above three possible events H_B , H_I and H_d , the unconditional probability of successful transmission $Q(r)$ is

$$\begin{aligned} Q(r) &= P(H_I)Q(r | H_I) + P(H_d)Q(r | H_d) \\ &= \frac{I}{B+I} \sum_{n=0}^{\infty} R_n(n | H_I) q_n(r | H_I) + \frac{d_I}{B+I} \sum_{n=1}^{\infty} R_n(n | H_d) q_n(r | H_d). \end{aligned} \quad (6.21)$$

The probability of capture $q_n(r)$ depends, among other things, on the probability to acquire receiver synchronization, which, in general, depends on the channel status (H_d or H_i) at the arrival of the packet. In some cases, only initiating packets (arriving at an idle receiver) can be handled. Because of necessary resynchronisation of the receiver with respect to carrier, bit and packet timing in an unslotted network, and re-initialisation of the communication protocol at the data-link level, packets arriving at a busy receiver may be not be accepted. In this case, $q_n(r | H_d) = 0$. The busy period was shown in [8] to be of average duration

$$B = 1 + d_1 + d_2 - \frac{1}{G_i} [1 - \exp(-d_1 G_i)]. \quad (6.22)$$

If for a certain channel the conditional capture probabilities $q_n(r | H)$ are known, $Q(r)$ can be computed from (6.19)-(6.22) and $I = G_i^{-1}$.

Previously [32],[33], the total throughput S_i was derived by extending the analysis in [8], i.e., by considering Poisson arrivals of packets at the receiver in the base station without addressing any particular 'test' packet. This approach produced

$$S_i = \frac{1}{B+I} \exp(-d_1 G_i) \sum_{n=0}^{\infty} \frac{d_1^n G_i^n}{n!} C_{n+1}. \quad (6.23)$$

For channels without capture ($C_i = 1$ and zero otherwise), results in [8] are recovered from (6.23). Also, for instantaneous inhibit signalling ($d_1, d_2 \rightarrow 0$), collisions will not occur in nonpersistent ISMA, and Eq. (6.23) reduces to $S_i \rightarrow G_i (1 + G_i)^{-1}$, in agreement with [8].

As a control of the method presented here, the total channel throughput S_i is expressed by integration of $Q(r)$ in (6.21), viz.,

$$\begin{aligned}
S_i &= \int_0^{\infty} \frac{2\pi r G(r)}{B+I} \exp(-d_1 G_i) \sum_{n=0}^{\infty} \left[\frac{d_1^n G_i^{n-1}}{n!} q_n(r|H_i) + \frac{n d_1^n G_i^{n-1}}{n!} q_n(r|H_d) \right] dr \\
&= \frac{G_i e^{-d_1 G_i}}{B+I} \sum_{n=0}^N \frac{d_1^n G_i^n}{n!} \int_0^{\infty} \frac{2\pi r G(r)}{G_i} [q_n(r|H_i) + n q_n(r|H_d)] dr
\end{aligned} \tag{6.24}$$

Hence we recover (6.23), provided that $C_{n+1} = q_n|_{H_i} + n q_n|_{H_d}$. This appears correct since, in any cycle, the initiating packet has capture probability $q_n(r|H_i)$, each of the n other packets has capture probability $q_n(r|H_d)$.

The derivation of $Q(r)$ reported here appeared necessary despite the availability of results in [8] because the distribution of the number of signals competing with the test packet was not known previously. From the above expression it appears that $P_n(n)$ tends to a Poisson distribution with mean $d_1 G_i$ provided that $I \gg B$, thus if G_i is relatively small. On the other hand, at high offered traffic loads (large G_i , $I < B$), the number of interferers to a test packet tends to one plus a Poisson-distributed random number with mean $d_1 G_i$. The explanation is that, for small G_i the test packet usually is an initiating packet, whereas for large G_i the test packet is only seldom an initiating packet. Thus, in contrast to the case for slotted ALOHA, the distribution of the number of interfering signals in ISMA is *not* Poisson, except in the limiting case $G_i \rightarrow 0$.

6.6.2 UNSLOTTED p -PERSISTENT ISMA

We now consider unslotted p -persistent ISMA without signalling delay ($d_1 = d_2 = 0$). A busy period can consist of a number of packet transmissions in succession because any terminal may start transmitting as soon as the previous transmission by another terminal is finished. If a packet arrives during an idle period (event H_i), the probability of correct reception of this first packet is $q_0(r)$. During the transmission of this packet, a random number of k terminals sense the channel busy with probability

$P_n(k)$. When the channel goes idle, each of the k terminals starts transmitting with probability p . For a test packet arriving during a busy period (event H_B), the probability of n interfering packets is thus

$$P_n(n|p) = \sum_{k=n}^{\infty} \binom{k}{n} P_n(k) (1-p)^{k-n} p^n = \frac{(pG_p)^n}{n!} \exp(-pG_p). \quad (6.25)$$

In particular, the probability that the busy period is terminated, i.e., none of the k terminals starts transmitting, is

$$P_n(0|p) = \exp(-pG_p). \quad (6.26)$$

The probability $P_m(m)$ of transmissions by other persistent terminals, during m units of time, concatenated to the first packet transmission, is

$$P_m(m) = \exp(-pG_p) [1 - \exp(-pG_p)]^m. \quad (6.27)$$

On the average, a busy period thus has the total duration

$$B \triangleq E_m[1+m] = 1 + \exp(-pG_p) \sum_{m=0}^{\infty} m [1 - \exp(-pG_p)]^m = \exp(+pG_p), \quad (6.28)$$

where E_m denotes the expectation over m . The probability of a successful transmission $Q(r)$ is

$$\begin{aligned} Q(r) &= P(H_I) Q(r|H_I) + P(H_B) Q(r|H_B) \\ &= \frac{I}{B+I} q_0(r) + \frac{B}{B+I} \sum_{n=0}^{\infty} p P_n(n|p) q_n(r). \end{aligned} \quad (6.29)$$

Using $I = G_I^{-1}$, (6.25), (6.28) and (6.29) becomes

$$Q(r) = \frac{q_0(r) + p G_I \sum_{n=0}^{\infty} \frac{(pG_p)^n}{n!} q_n(r)}{1 + G_I \exp(pG_p)}. \quad (6.30)$$

After integration, the total channel throughput S_t is obtained from

$$S_t = G_t \frac{q_0 + \sum_{i=1}^{\infty} \frac{(pG_t)^i}{i!} C_i}{1 + G_t \exp(pG_t)}. \quad (6.31)$$

In the special case $p=0$, we recover the result for unslotted nonpersistent ISMA without signalling delay ($d=0$). The classical case of unslotted 1-persistent ISMA on wired channels is recovered by inserting $p=1$, $q_0=1$ and $q_n=0$ for $n=2,3,\dots$ in (6.31), yielding

$$S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)}. \quad (6.32)$$

This agrees with expressions derived for Poisson arrivals at a receiver without capture [8].

6.7 NEAR-FAR EFFECT AND THROUGHPUT OF NONPERSISTENT ISMA IN CHANNELS WITH PROPAGATION DELAYS

In nonpersistent ISMA, the central base station continuously broadcasts the channel status on the outbound channel. Because of the finite speed of radio waves, this feedback signal is received by a mobile terminal only after a certain delay. Hence, remote terminals have a retarded view of the channel status, whereas nearby terminals have more up-to-date information. The previous sections dealt with a propagation delay that is identical for all participating terminals. The impact of propagation delays increasing with propagation distance was investigated in [34] for wired LAN networks. This section deals with radio data networks, assuming delays that consist of a fixed part, called the 'processing delay', and a distant-dependent part, called the 'propagation delay'. Again, the processing delay is caused by the necessary

operations performed in the central station, for instance to recognise an incoming packet signal or by transmitter power-up times in the mobile terminals. Propagation delays may be negligible compared to the processing delays in a typical cellular mobile data network employing a low bit rate, such as 1200 bit/s. At present, however, new mobile information systems are designed for increasingly high data rates. In a high-capacity mobile data net with a service area of 30 km, packets of 200 bits and a bit rate of 200 kbit/s, thus with a packet duration of 1 ms, the maximum roundtrip delay is 20% of the duration of a packet, so propagation delays may seriously affect the system performance.

Propagation delays, unlike processing delays, lead to unfairness in the probability of performing a successful packet transmission, since remote terminals have more retarded information of the receiver status than nearby terminals. It will be shown (in section 6.7.6) that with reasonably low packet traffic loads, propagation delays cause the probability of successful access to reduce approximately linearly with the distance to the common receiver. This further increases the near-far effect that occurs if weak radio signals from remote terminals have an increased probability of being lost in excessive noise or interference, as addressed in the previous sections.

The analysis of channels with propagation delays is complicated by the fact that packet arrivals at the receiver do not comply with an on-off switched Poisson process, as was considered so far: As the leading edge of the busy signal propagates outwards, more and more terminals will be inhibited from potential transmissions so the packet arrival rate at the base station will decrease gradually, rather than step-wise. Similarly, when the channel is released by the transition from a busy signal to an idle signal, the resulting arrival rate of packets will increase only gradually. A perfect feedback channel is considered, so each mobile terminal is assumed to receive the signalling messages without errors. Fading, noise or interference corrupting the signalling are neglected. Also, the time resolution is assumed to be perfect. In practice this would require a feedback channel of infinite bandwidth. For feedback channels with finite bandwidth, the transitions between busy and idle states contribute to the (fixed) processing delay. Imperfections of the feedback channel are addressed in [17].

6.7.1 THE 'WAKE'- ISMA CHANNEL TRAFFIC MODEL

Extending the 'switch' model considered in section 6.6, the following additional time intervals are defined for a terminal at a distance r from the central station:

The 'idle period', with duration $i(r)$, is the interval of time during which the following two conditions are satisfied simultaneously: the terminal receives an idle signal *and* a packet transmitted by the terminal will indeed arrive at an idle receiver. Because of propagation delays, the duration $i(r)$ of the idle period is a function of the location of the terminal. The 'vulnerable period', with duration $v(r)$, is the interval of time during which the terminal receives an idle signal, while a packet transmitted in this interval is bound to arrive at an already busy receiver. The 'inhibited period', with duration h , is the period during which a busy signal is received. A subsequent vulnerable and inhibited period was simply called a 'busy period' in section 6.6. The total duration of a 'cycle', i.e., the period from beginning of a inhibited period until the beginning of the next inhibited period is $a = h + i(r) + v(r)$. Evidently, a and h cannot depend on the distance r . Since packet arrivals are random, a , h , $i(r)$ and $v(r)$ are stochastic variables. Corresponding expectation values are denoted in capitals, thus as A , H , $I(r)$ and $V(r)$, respectively.

In the analysis of channels without propagation delays, these periods can be drawn on a (one-dimensional) time axis (see [8] or Fig. 6.7 and 6.8). To consider the effect of propagation delays and the resulting signalling wake, the distance to the base station is taken as the second dimension in the 'time-space' diagram given in Fig. 6.9.

A packet transmitted in the idle period arrives at an idle receiver in the central station. During a certain cycle, the first (and only) packet arriving at the idle receiver is called the 'initiating' packet. After a processing delay of normalised duration d_1 , the central station starts transmitting a busy signal. This busy status is maintained for the entire duration of the packet reception. If a number of overlapping packets is received, the busy signal is also broadcast. After the reception of last of the colliding signals terminates, the idle signal is transmitted with a delay d_2 . The processing delays d_1 and d_2 are parameters of the central station and are considered to be fixed.

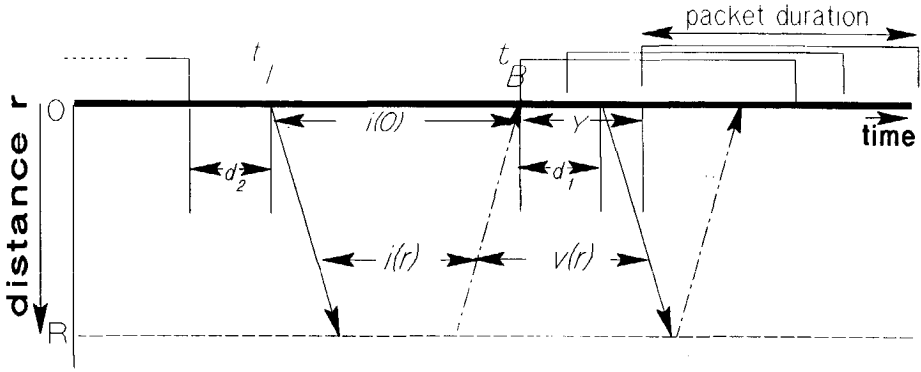


Fig. 6.9: Time-space relation for inbound (---) and outbound (—) signals travelling over the radio channel.

We define the instant t_I as the instant when the base station starts broadcasting an idle signal. This signal arrives at a distance r from the base station at the instant $t_I + r/u$, with u the speed of light, normalised to the packet duration and the radius of the service area. A terminal only has permission to transmit if it receives an idle signal. Thus, the first instant when a packet from a distance r can arrive at the base station is $t_I + 2r/u$. The instant of arrival of the initiating packet at the central station is denoted as t_B , where $t_B = t_I + i(0)$. This event implies that, for a terminal at distance r , the idle period ended at $t_B - r/u$, and the vulnerable period was entered. Taking account of the possibility that the initiating packet arrives before the idle signal has reached all terminals ($i(0) < t_p$), the idle period at a distance r is found as the interval

$$\left\langle t_I + \frac{r}{u}, \max\left(t_I + \frac{r}{u}, t_I + i(0) - \frac{r}{u}\right) \right\rangle.$$

Thus, $i(r) = \max(0, i(0) - 2r/u)$.

After reception of the initiating packet, the central station broadcasts the busy signal with a (normalised) delay d_1 , thus at the instant $t_I + i(0) + d_1$. The vulnerable period at a distance r is found as the time interval

$$\left\langle \max\left(t_I + \frac{r}{u}, t_I + i(0) - \frac{r}{u}\right), t_I + i(0) + d_1 + \frac{r}{u} \right\rangle,$$

with duration $v(r)=d_1 + \min(2r/u, i(0))$. Colliding packets may arrive until t_B+d_1+2r/u . Termination of the inhibited period occurs $1+d_2$ units of time after the arrival of the last packet in the cycle considered. If only the initiating packet is present, this occurs at t_B+1+d_2+r/u . However, in general, the duration of the inhibited period depends on the (stochastic) arrival times of interfering packets. The inhibited period nonetheless always ends before $t_B+d_1+t_p+1+d_2+r/u$.

6.7.2 PACKET ARRIVAL RATE IN CHANNELS WITH PROPAGATION DELAYS

A terminal has permission to transmit only if it receives an idle signal. Only after a duration of $t_p=2R/u$, with R the normalised cell radius ($R=1$) and t_p the (normalised) maximum round-trip delay, packets can arrive from any location in the circular area. We assume $d_1+t_p < 1$. Between t_I and t_I+t_p , the arrival rate steadily grows from 0 to G_t . Taking account of the time-space volume from which data packets can arrive, the arrival rate is found as $\lambda(t)=\lambda_1(t-t_I)$, with

$$\lambda_f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}ut \int_0^{t/t_p} 2\pi r G(r) dr = G_t \frac{t^2}{t_p^2} & 0 < t < t_p \\ G_t & t > t_p \end{cases} \quad (6.33)$$

Here it has been assumed that the central station does not start again transmitting a busy signal during the interval $\langle t_I, t_I+t \rangle$. The arrival rate $\lambda(t)$ is depicted in Fig. 6.10. Using (6.7), the probability that no packet arrives at the central receiver during the period $\langle t_I, t_I+x \rangle$ is

$$P_{N_{(t_I, t_I+x)}}(n=0) = \exp \left\{ - \int_0^x \lambda_f(t) dt \right\}. \quad (6.34)$$

The cumulative distribution function (cdf) $F_{i(0)}(x)$ of the lapse of idle time $i(0)$ at the central station is

$$F_{i(0)}(x) \triangleq \Pr\{i(0) < x\} = 1 - P_{N_{i(t), x}}(n=0) = 1 - \exp\left\{-\int_0^x \lambda_i(t) dt\right\}. \quad (6.35)$$

It is seen that, in contrast to the conventional case with *stationary* Poisson arrivals ($\lambda(t)$ constant), the interarrival time is not exponentially distributed.

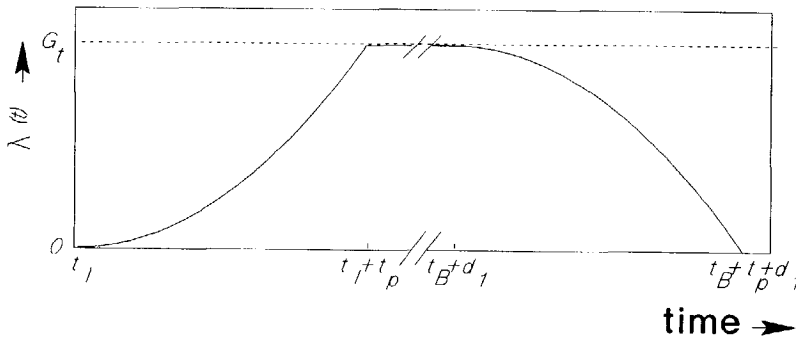


Fig. 6.10: Rate of arrival $\lambda(t)$ versus time after the start of an idle period (t_I). $t_B + d_1$: instant when base station starts transmitting the busy signal.

6.7.3 DURATION OF THE IDLE PERIOD

The average duration $I(r)$ of an idle period is obtained from

$$\begin{aligned}
I(r) &\triangleq E\left[\max\left(0, i(0) - \frac{2r}{u}\right)\right] = \int_{\frac{2r}{u}}^{\infty} \left(x - \frac{2r}{u}\right) \frac{dF_{i(0)}(x)}{dx} dx \\
&= \int_{\frac{2r}{u}}^{t_p} \left(x - \frac{2r}{u}\right) \lambda_f(x) \exp\left\{-\int_0^x \lambda_f(t) dt\right\} dx \\
&\quad + \int_{t_p}^{\infty} \left(x - \frac{2r}{u}\right) \lambda_f(x) \exp\left\{-\int_0^{t_p} \lambda_f(t) dt - \int_{t_p}^x \lambda_f(t) dt\right\} dx.
\end{aligned} \tag{6.36}$$

The first term is evaluated from integration by parts, with

$$-\int_{\frac{2r}{u}}^{t_p} \left(x - \frac{2r}{u}\right) d\exp\left\{-G_t \int_0^x \frac{t^2}{t_p^2} dt\right\} = -\left(t_p - \frac{2r}{u}\right) \exp\left\{-G_t \frac{t_p^2}{3}\right\} + \int_{\frac{2r}{u}}^{t_p} \exp\left\{-G_t \frac{x^3}{3t_p^2}\right\} dx \tag{6.37}$$

and the second term becomes

$$\int_{t_p}^{\infty} \left(x - \frac{2r}{u}\right) G_t \exp\left\{-G_t \left(\frac{t^3}{3} + x - t_p\right)\right\} dx = \left(t_p - \frac{2r}{u} + \frac{1}{G_t}\right) \exp\left\{-\frac{1}{3} G_t t_p^3\right\}. \tag{6.38}$$

So, one finds

$$\begin{aligned}
I(r) &= \int_{\frac{2r}{u}}^{t_p} \exp\left\{-G_t \frac{x^3}{3t_p^2}\right\} dx + \frac{1}{G_t} \exp\left\{-\frac{1}{3} G_t t_p^3\right\} \\
&= \frac{1}{3} \sqrt[3]{\frac{3t_p^2}{G_t}} \left[\gamma\left(\frac{1}{3}, \frac{1}{3} G_t t_p^3\right) - \gamma\left(\frac{1}{3}, \frac{2}{3} G_t \frac{r^3}{R^2 u}\right) \right] + \frac{1}{G_t} \exp\left\{-\frac{1}{3} G_t t_p^3\right\},
\end{aligned} \tag{6.39}$$

where the incomplete gamma function is defined as [37]

$$\gamma(\alpha, x) \triangleq \int_0^x e^{-t} t^{\alpha-1} dt. \tag{6.40}$$

As a verification of the analysis, it is confirmed that for channels without propagation delay ($t_p=0$), the average duration of the idle period (6.39) correctly tends to $I(r)=G_t^{-1}$. Fig. 6.11 gives $I(r)$ as a function of r , and the approximation $\max(0, I(0)-2r/u)$ for small G_t .

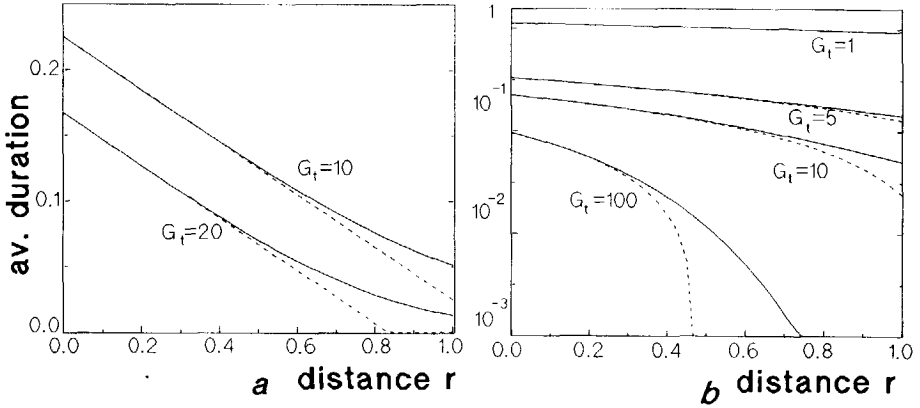


Fig. 6.11: (—) Average duration of the idle period $I(r)$ in packet times on a linear axis (a, left) and on a logarithmic axis (b, right), versus distance r , and (---) linear approximation by $\max(0, I(0)-2r/u)$. Roundtrip delay $t_p=0.2$.

Fig. 6.12 presents $I(r)$ versus G_t for various r and the bounds $G_t^{-1} \leq I(0) \leq G_t^{-1} + t_p$ and $G_t^{-1} - t_p \leq I(1) \leq G_t^{-1}$. It is seen that for a terminal at a certain distance, the average duration of the idle period rapidly reduces to zero if the attempted traffic load G_t exceeds a certain intensity. If an interfering packet always destroys the initiating packet (no capture), the probability $Q(r_0)$ of a successful transmission of a packet from a distance r_0 is found from

$$Q(r_0) = \frac{I(r_0)}{A} \Pr\{\text{no interfering packets}\} \quad (6.41)$$

where A is the average duration of a complete cycle. In (6.41), $I(r_0)$ is the only factor depending on r_0 . Apart from a multiplicative factor independent of r_0 , the near-far unfairness is thus determined by the distance-dependence of the idle period. Compared to the investigation of $I(r)$, the determination of A , which includes assessment of the average duration of the vulnerable period and the inhibited period,

is relatively complicated. This is caused by the necessity to take into account the presence of the busy signal corresponding to the foregoing cycle if $i(0)$ happens to be shorter than the maximum round-trip delay t_p . Therefore we initially simplify the analysis by considering an approximate model, appropriate in particular to light traffic loads.

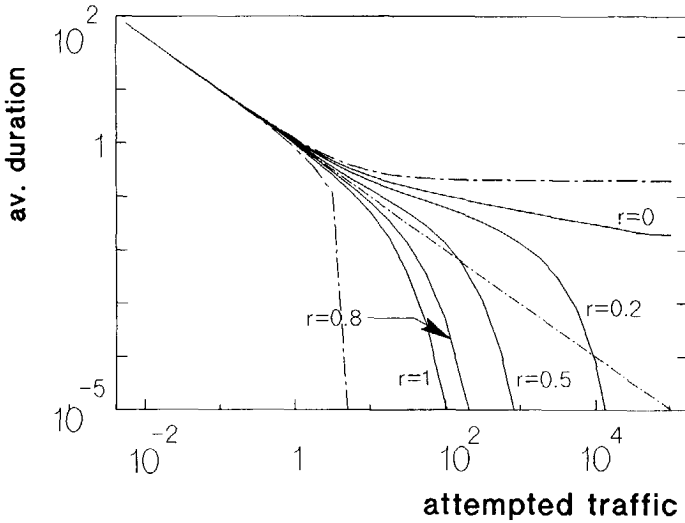


Fig. 6.12: (—) Average duration $I(r)$ of the idle period as a function of attempted traffic load G_t for various distances. Maximum roundtrip delay $t_p=0.2$. (·—·) upperbound $I(r) < G_t^{-1} + t_p$ and lowerbound $I(r) > G_t^{-1} - t_p$ and (···) Average duration in channels without propagation delays (G_t^{-1}).

6.7.6 APPROXIMATE MODEL FOR LIGHT TRAFFIC LOADS

Initially, we assume that $i(0)$ is always larger than the round-trip delay t_p , which is reasonable if $F_{i(0)}(t_p) = 1 - \exp\{-1/3G_t t_p\} \approx 1/3G_t t_p$ is small. If this condition is satisfied, each idle period is likely to have non-zero duration $i(r) = i(0) \cdot 2^{-r/u} > 0$ for any location in the service area. In other words, this assumption ensures that a propagating busy

signal corresponding to a previous cycle does not affect the durations of the next vulnerable and inhibited periods. In this case, the duration of the vulnerable period loses its stochastic nature and becomes identical to $v(r) = d_1 + 2r/u$ in each cycle.

We now address the duration of the inhibited period by initially considering the arrival rate of interfering packets. After arrival of the initiating packet at the instant t_B , the central station starts broadcasting a busy signal at the instant $t_B + d_1$. This signal reaches a terminal at distance r at the instant $t_B + d_1 + r/u$. Taking account of the time interval in which transmissions can occur, packets from the area $0 < r < R_0$ and destined to collide, continue to arrive at the central station with the rate $\lambda(t) = \lambda_B(t - t_B - d_1)$, where

$$\lambda_B(t) = \begin{cases} G_t \frac{R_0^2}{R^2} & t < 0 \\ \int_{\frac{1}{2}u}^{R_0} 2\pi r G(r) dr = G_t \left[\frac{R_0^2}{R^2} - \frac{t^2}{t_p^2} \right] & 0 < t < 2R_0/u \\ 0 & t > 2R_0/u \end{cases} \quad (6.42)$$

We insert $R_0 = R = 1$ to take account of all arriving packets, including those from the most remote terminals. R_0 is introduced here, because later we will consider systems with receiver capture, in which weak interfering packets ($r > R_0$) do not cause any harmful interference.

The arrival rate of interfering packets decreases smoothly (see Fig. 6.10) and becomes zero only at the instant $t_B + d_1 + t_p$. The time lapse y between the arrival of the initiating packet and the last interfering packet in the vulnerable period has the cdf

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \Pr\{\text{no arrivals in } (t_B + y, t_B + d_1 + t_p)\} & y > 0 \end{cases}$$

or

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \exp\left\{-\int_{-d_1+y}^{t_p} \lambda_B(\tau) d\tau\right\} & 0 < y < d_1+t_p \\ 1 & y > d_1+t_p \end{cases} \quad (6.43)$$

Inserting (6.42) gives

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ \exp\left\{-\frac{2}{3}t_p G_t - d_1 G_t + y G_t\right\} & 0 < y < d_1 \\ \exp\left\{-G_t[t_p+d_1-y] + \frac{1}{3}G_t\left[t_p - \frac{(y-d_1)^3}{t_p^2}\right]\right\} & d_1 < y < d_1+t_p \\ 1 & y > d_1+t_p \end{cases} \quad (6.44)$$

At $y=0$, this cdf exhibits a step of a size equal to the probability that no interfering packet is present. The average duration of the inhibited period is $H=E[y]+1-d_1+d_2$, or

$$\begin{aligned} H &= y F_Y(y) \Big|_0^{d_1+t_p} - \int_0^{d_1+t_p} F_Y(y) dy + 1 - d_1 + d_2 \\ &= t_p - \int_0^{d_1} \exp\left\{\left(-\frac{2}{3}t_p - d_1 + y\right)G_t\right\} dy - \int_{d_1}^{d_1+t_p} \exp\left\{-G_t\left[\frac{2}{3}t_p + d_1 - y + \frac{(y-d_1)^3}{t_p^2}\right]\right\} dy + 1 + d_2 \\ &= t_p + e^{-\frac{2}{3}t_p G_t} \left[-\frac{1}{G_t} (1 - e^{-d_1 G_t}) - \int_0^{t_p} \exp\left\{+x G_t - \frac{x^3}{t_p^2} G_t\right\} dx \right] + 1 + d_2. \end{aligned} \quad (6.45)$$

Limiting cases are $H \rightarrow 1-d_1+d_2$ for $G_t \rightarrow 0$ and $H \rightarrow 1+t_p+d_2$ for $G_t \rightarrow \infty$, though (6.42)-(6.45) may become inaccurate for large traffic loads because of the assumptions discussed at the beginning of the section. For channels without propagation delays ($t_p \rightarrow 0$), $H \rightarrow (1 - \exp\{-d_1 G_t\}) G_t^{-1} + 1 + d_2$, which agrees with [8]. Fig. 6.13 portrays (6.45) and an exact solution [35] which will be discussed later.

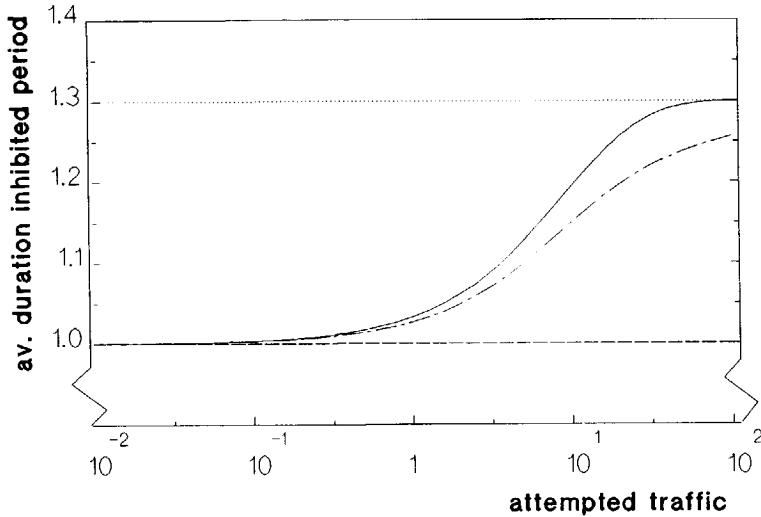


Fig. 6.13: Average duration H of the inhibited period in packet times versus attempted traffic load G_p , according to (—) approximate and (— · —) exact method. $t_p=0.2$, $d_1=d_2=0.1$.
 Bounds: (— · —) $H > 1 - d_1 + d_2 = 1$ and (···) $H < 1 + d_2 + t_p = 1.3$.

The overall probability of a successful access $Q(r)$ is found from the probability $I(r)/A$ that the attempt occurs in the idle period and the probability that no harmful interfering packet is transmitted in the vulnerable period. In a realistic radio channel, an initiating packet may be received correctly despite the presence of weak interfering packets. The capture effect will be studied in more detail for slotted ALOHA and ISMA with fixed (distance-independent) propagation delays in chapter 7. Nonetheless, to gain some initial insight in the combined (near-far) effects of delay and attenuation, we consider a basic capture model: the vulnerability-circle model, presented in 1977 by Abramson [11] for ALOHA channels, can conveniently be introduced here: An initiating packet from distance r_0 is assumed to be received correctly if none of any interfering packets arrives from a distance closer than $c_z r_0$, with c_z a system constant. Further, we assume that no non-initiating packets can be received correctly because the receiver is already busy receiving and synchronised to another (i.e., to the initiating) packet at the time of arrival. The probability of no packets harmfully

interfering to the initiating packet is found from integrating the arrival rate in (6.42) and taking $R_0 = \min(R, c_z r_0)$. So

$$Pr(\text{capture}|r_0) = \exp\left\{-\left(\frac{\min(R, c_z r_0)}{R}\right)^2 \left[d_1 + \frac{2}{3}t_p\right] G_t\right\}. \quad (6.46)$$

The probability of a successful transmission (6.41) now becomes

$$\begin{aligned} Q(r_0) &= \frac{I(r_0)}{A} Pr\{\text{capture}|r_0\} \\ &= I(r_0) \frac{\exp\left\{-G_t \left(\frac{\min(R, c_z r_0)}{R}\right)^2 \left(d_1 + \frac{2}{3}t_p\right)\right\}}{I(0) + d_1 + H}. \end{aligned} \quad (6.47)$$

Here, (6.39) and (6.45) are to be inserted. Fig. 6.14 and Fig. 6.15 present numerical results of $Q(r)$ for $G_t = 2$ and 10 ppt, respectively.

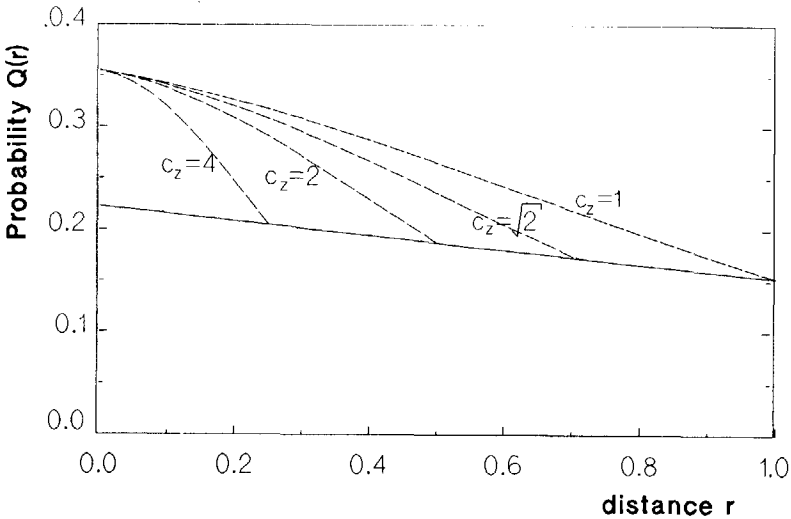


Fig. 6.14: The probability of successful transmission $Q(r)$ versus distance r for $t_p=0.2$ and $d_1=d_2=0.1$ (- -) for various vulnerability parameters c_z , and (—) without capture. The total channel traffic load G_t is 2 ppt.

The total channel throughput S_t expressed in the average number of successfully received packets per unit of time, is obtained as

$$S_t = \int_0^R G(r) Q(r) 2\pi r dr \quad (6.48)$$

and illustrated in Fig. 6.16.

For CSMA, it was previously suggested [8] to approximate the effect of variable propagation delays by considering all terminals to be at an identical (worst-case) distance to the base station. This corresponds to inserting the modified delays d_1' and d_2' , with $d_1' = d_1 + R/u$ and $d_2' = d_2 + R/u$, in formulas such as (6.22) and (6.23). For ISMA, as addressed in this section, three approximate methods are compared in Fig. 6.16 for the special case $d_1 = 0.1$, $d_2 = 0.1$ and $t_p = 0.2$: Method *a* adds the *one-way* propagation delay to d_1 and d_2 ($d_1' = 0.2$, $d_2' = 0.2$), whereas the method *b* adds the *round-trip* delay to d_1 and d_2 ($d_1' = 0.3$, $d_2' = 0.3$). Method *c* adds the round-trip delay to d_1 but does not account for propagation delays in d_2 ($d_1' = 0.3$, $d_2' = 0.1$).

Although method *a* might offer worst-case results for CSMA [8], for ISMA it is optimistic, i.e., it overestimates the throughput. On the other hand, method *b* yields relatively conservative estimates. It is seen that the approximation *c* gives about 5% lower results than the exact analysis, and may offer a sufficiently tight bound for practical system design, particularly in the practical range $G_t < 1$ ppt.

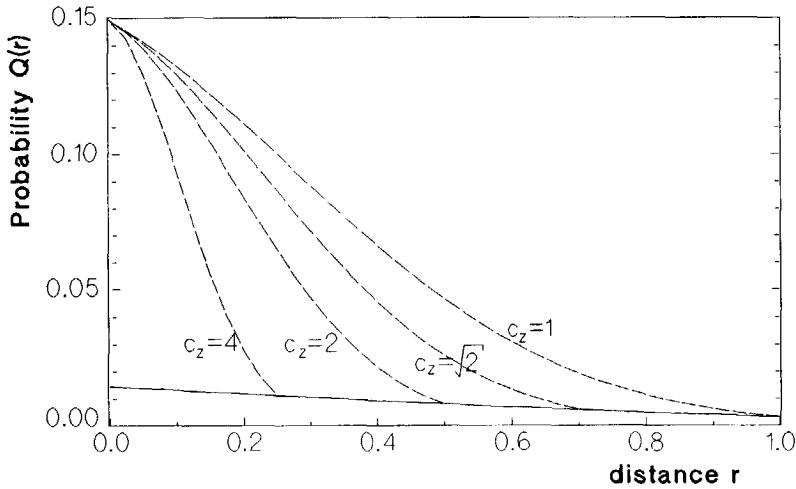


Fig. 6.15: The probability of successful transmission $Q(r)$ versus distance r for $t_p=0.2$ and $d_1=d_2=0.1$ (---) for various vulnerability parameters c_z , and (—) without capture. The total channel traffic load G_t is 10 ppt.

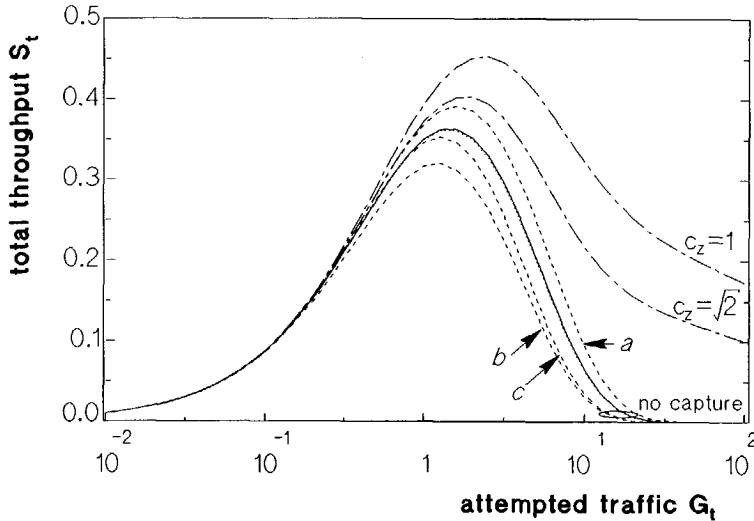


Fig. 6.16: Throughput S_t versus attempted traffic G_t in ppt. Normalised delays: $d_1=0.1$, $d_2=0.1$, $t_p=0.2$.
No capture: (—) exact method and (---) approximate method proposed in sect. 6.7.6; (---, a,b,c) approximation by constant, distance-independent delays, see text. (—) channels with variable delay and with capture.

This conclusion is explained intuitively by the fact that method *c* takes account of the effect that roundtrip delays substantially widen the vulnerable period, and thus increase the expected number of interfering signals. Method *a* overestimates throughput mainly because it tends to underestimate the number of interfering signals. As seen from (6.46), for a uniform distribution of the attempted traffic, the probability of no packets interfering with an initiating packet is $\exp\{-d_1^{-2/3} t_p\}$, thus d_1 is effectively prolonged by 133% of the one-way propagation delay.

The effect of receiver capture ($c_z=1$ and $\sqrt{2}$) is seen to be significant, particularly for high offered traffic loads ($G_t > 1$ ppt). This shall be addressed in more detail in chapters 7 to 9.

6.7.7 COMPARISON WITH EXACT ANALYSIS

The 'wake' diagram in Fig. 6.17 illustrates the effect of the event $i(0) < t_p$ on the vulnerable period. This event also affects the average duration of the inhibited period and the probability of no (harmful) interference. Given the duration of the idle period $i(0)$, the duration of the vulnerable period is determined by $v(r) = d_1 + \min(t_p, i(0))$. The probability of no harmfully interfering packets can be computed by integration over the appropriate time-space volume, which is a tedious exercise. Moreover, the duration of the inhibited period is a stochastic variable, dependent, among other things, on the duration of the foregoing vulnerable period, thus indirectly on the duration of the foregoing idle period.

The unconditional average duration of the inhibited period is obtained from

$$H = \int_0^{\infty} H(x) f_{i(0)}(x) dx \quad (6.49)$$

where $H(x)$ is the average duration of the inhibited period for a given $i(0)=x$. Evaluation of (6.49) also becomes tiresome because an analytic expression of the integrand only exists for sub-intervals of the domain of the integration variable, and various segments of the time-space volume are to be treated separately in the integration. This has been endeavoured by Venema in his graduation work [35].

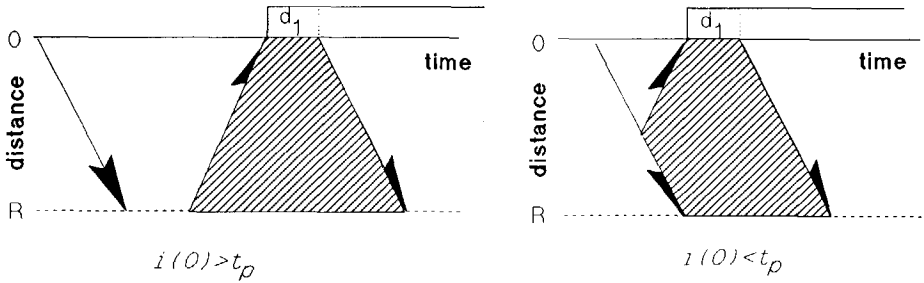


Fig. 6.17: Time-space ('wake') diagram and vulnerable period (///) for the events $i(0) < t_p$ and $i(0) > t_p$.

Numerical results are given by (—) in Fig. 6.13 and by (---) in Fig. 6.16 to discuss the accuracy of the approximate model presented here. It is seen in Fig. 6.13 that the duration of H is overestimated by (6.45). This leads to an underestimation of $Q(r)$ and S_r . Moreover, if $i(0) < t_p$, the time-space volume from which interfering packets may arrive is smaller than assumed in (6.46). Hence, the number of interfering signals is overestimated. This also leads to an underestimation of the channel performance. Nonetheless, even the combined effect of these approximations turns out to be very small, and is barely distinguishable in Fig. 6.16 (compare (· · ·) dotted and (—) solid line).

6.8 CONCLUSIONS

This chapter discussed protocols for random multiple access and analytical models to study their performance in mobile channels. The channel performance experienced by individual mobile terminals may differ widely from the mean performance averaged over all terminals. Two types of near-far effects occur in a mobile random access network:

- i)* effects because weaker signals are received from remote terminals and
- ii)* effects caused by propagation delays.

Ad *i)* Calculation of the effect of path losses required extension of the known techniques to compute the total throughput of ALOHA, CSMA and ISMA in wired channels. Here, the probability of capture and total channel throughput have only been expressed in terms of the probability that a test signal survives a collision with n contenders. As will be shown in chapter 7, the latter probability can be obtained from the models for channel fading and receiver capture developed in chapter 3 to 5.

Ad *ii)* The performance of nonpersistent ISMA was also studied for radio networks with substantial propagation delays. The principal contribution of variable propagation delays to the near-far unfairness is caused by the 'wake'-effect of the idle period decreasing almost linearly with the distance between terminal and the central station. The unfairness caused by propagation delays increases the unfairness caused by the fact that weak signals from remote terminals experience a higher probability of being lost in a collision. It was seen that for reasonable traffic loads, say $G_t < 5$ ppt, propagation delays cause less unfairness than propagation attenuation. At high traffic loads in nonpersistent ISMA networks, delays are expected to become the major cause of unfair access probabilities for remote terminals: beyond a certain offered traffic load, terminals at a certain distance experience an highly limited probability to successfully transmit a data packet.

Exact analysis of the ISMA with propagation delays appeared a tedious task if cycles can overlap. Fortunately, an approximate technique, taking exact account of the duration of the idle period, but ignoring the effect that overlapping cycles have on the duration of the vulnerable period and the busy period, proved to give relatively accurate results. It was seen that propagation delays principally affect the delay in the transition of the feedback information from idle to busy.

The approximation by means of fixed (distance-independent) propagation delays was seen to lead to somewhat pessimistic results for the throughput.

The present development of public networks for packet switched mobile data communication and wireless office automation systems tends to focus on wireless communication links over shorter and shorter ranges. This seems to divert interest from the propagation delays in the network. Note, however, that these propagation delays are to be considered in relation to the packet duration, which decreases inversely proportional with the chosen bit rate.

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CHAPTER 7

MODELS FOR RECEIVER CAPTURE IN MOBILE RANDOM-ACCESS NETWORKS

As seen in chapter 6, the performance of random-access protocols strongly depends on the probability of a successful transmission of an individual data packet over the common radio channel. The probability that a selected 'test' packet experiences a collision with n contending packets was presented for various protocols. The probability that the test packet is received successfully despite its n contenders depends, among other things, on the characteristics of the radio channel such as the rate of fading, the type of modulation, the receiver design, and the distribution of the terminals over the service area. To find the probability of successful reception of a packet, this chapter summarises and compares a number of models for receiver capture.

The organisation of this chapter is as follows: In section 7.1, the capture effect is discussed. Moreover, capture is compared with the event of 'receiver success' as studied in coding theory. Section 7.2 presents the quasi-uniform spatial distribution of the attempted packet traffic over the service area. This special case allows relatively convenient analysis of the network behaviour for most of the capture models considered. Each of the sections 7.3 - 7.7 discusses a particular capture model in more detail and presents numerical results for the probability of capture and the channel throughput. Section 7.8 presents the probability of successful access according to the models discussed in sections 7.3 - 7.7. The suitability of the capture ratio model is addressed in more detail in section 7.9.

7.1 RECEIVER CAPTURE

The name 'capture effect' was originally used to describe the effect of the threshold in non-linear modulation and demodulation of a continuous signal. Wozencraft and Jacobs [1] discuss the capture effect and the receiver threshold in a generalised way: Fig. 7.1 illustrates non-linear modulation of an analogue (continuous) signal $x(t)$, with $(0 < x(t) < 1)$ on a two-dimensional carrier $\{\phi_1, \phi_2\}$. The resulting signal $x_c(t)$ is on the form¹

$$x_c(t) = F_1(x(t)) \phi_1(t) + F_2(x(t)) \phi_2(t) \quad (7.1)$$

¹ In the case of FM-radio, e.g. in VHF radio broadcasting, the orthogonal waveforms are on the form of

$$\begin{aligned} F_1 &= \cos \left(2\pi f_\Delta \int_0^t x(\lambda) d\lambda \right) & \phi_1(t) &= \cos \omega_c t \\ F_2 &= \sin \left(2\pi f_\Delta \int_0^t x(\lambda) d\lambda \right) & \phi_2(t) &= \sin \omega_c t \end{aligned}$$

where f_Δ is the frequency deviation. The signal locus is formed by multiple rotations over the unit circle.

where $F_1(\cdot)$ and $F_2(\cdot)$ are continuous, bounded, non-linear functions of the message $x(t)$. The locus of the signal $x(t)$, i.e., the position of the signal $x_c(t)$ in the two-dimensional $\{\phi_1, \phi_2\}$ -plane as a function of $x(t)$, is depicted in Fig. 7.1. If

$$\left(\frac{dF_1}{dx}\right)^2 + \left(\frac{dF_2}{dx}\right)^2 \gg 1 \quad (7.2)$$

the post-detection signal-to-noise ratio can be greatly enhanced as compared to baseband transmission or linear modulation, provided that the received pre-detection signal-to-noise ratio is above the 'threshold'.

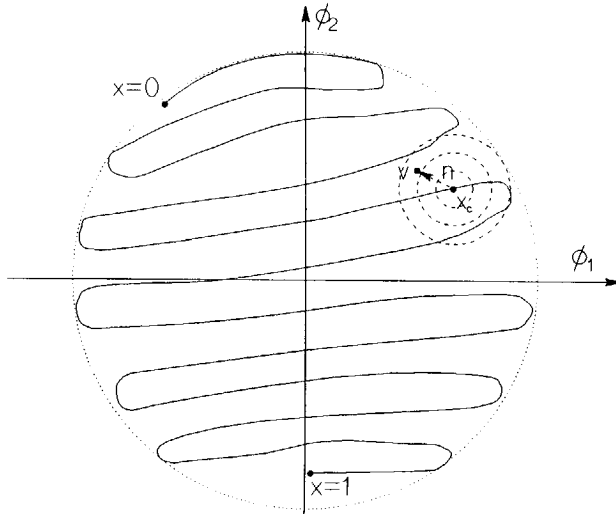


Fig. 7.1: Threshold effect in non-linear modulation of continuous signal $x(t)$. For a signal below the threshold, the received signal $v(t)$ frequently falls on a different fold of the signal locus than the transmitted signal x_c because of the noise $n(t)$.

The latter condition means that the random noise vector in Fig. 7.1 very rarely becomes so large that the detector decodes a signal on another fold of the locus of $x(t)$ [1]. If, on the other hand, the noise power is relatively large, the detector is likely to detect a signal on another fold of the locus, and thus a large error occurs. In this case the message is likely to be lost completely, and the received signal is said to be 'below the receiver threshold'.

The transition from 'below the threshold' to 'above the threshold', may occur with a only minor increase, say of a few dB, of the pre-detection C/I or C/N ratio. Still, the effect of the end-to-end performance of the link is large. The signal appears to 'capture' the receiver in such an event. In studies of collision-type random-access networks for data communication, the description 'capture effect' has been adopted to describe the fact that any practical radio receiver is resistant against noise and interference at least to some extent. If packets compete for successful reception at the receiver, one signal may 'capture' the receiver. In this context, however, the word 'effect' has lost its meaning of indicating a remarkably rapid impairment of the quality of reception with only a slight decrease of the C/I -ratio.

An ad-hoc definition of receiver capture is now proposed. It may not immediately provide a rigid base for analytical evaluation, but it represents an extension of the FM-capture effect for the case of discrete messages, such as data packets:

- *'Receiver capture' is the phenomenon that the decisions required from the receiver to estimate or reconstruct a (potential) message transmitted on the channel, are dominated by a single signal, despite the presence of interfering (contending) signals and noise.*

Hence, receiver capture essentially is an event (denoted as c_j) of the physical layer of the communication system. Conditions relevant to the probability of capture are, among other things, received power levels, the type of modulation, robustness of receiver synchronisation, characteristics of the interference and channel fading. According to this definition, the type of channel coding does not directly affect the probability of capture.

This means that capture probabilities do not uniquely determine the throughput of the channel. Taking also the effects of channel coding into consideration, 'transmitter success' (event s_j) and 'receiver success' (event $s_1 \vee s_2 \vee \dots \vee s_N$) can be defined as follows [12]:

- *'transmitter success' is the event that the bit sequence of the packet from transmitter j is identical to the bit sequence decoded by the receiver, i.e., the packet*

from terminal j is received error-free or contains only a correctable number of bit errors.

- *'receiver success' is the event that the receiver decodes a bit sequence that has been transmitted by (at least) one of the competing transmitters.*

Evidently, 'receiver success' and 'receiver capture' are highly correlated, but the two events do not necessarily coincide: Receiver capture does not necessarily imply correct decoding of the test packet. Also, in some fortuitous cases, receiver success may occur while receiver capture does not occur: The receiver may, by chance, take a right decision even if all signals are buried in noise or mutual interference [12], [13].

Because of the discrete nature of digital messages, events of receiver or transmitter success can be sharply defined. In contrast to this, the distinction between capture and non-capture is more arbitrary. However, exact computation of probabilities of success is always a complicated task. This is illustrated by the large number of a different (approximate) models used to compute these probabilities. In theoretical analyses, basically two classes of models exist:

- A* Models focusing on the physical layer: It is assumed that successful reception occurs if (and only if) a limited number of conditions are fulfilled, under which *capture* is likely to occur. Mostly, received signal powers are considered as the criterion for capture. In these models, the probability of bit errors and the effect of coding are not addressed explicitly. These models can be acceptable if $Pr(s_j | c_j) \rightarrow 1$ and $Pr(s_j | \text{NOT } c_j) \rightarrow 0$, e.g. if a clear receiver threshold is present.
- B* Models focusing at the data-link layer. To confine the complexity of the calculations, such models often simplify receiver behaviour, e.g. by assuming perfect carrier and bit synchronisation. Such a model is acceptable if the receiver manages to capture a signal even if the C/I ratio is small. Practical coherent narrowband receivers, however, often fail to acquire synchronisation to a short (burst-type) transmission if the interference is strong. In the latter case, theoretical block error rates for perfectly synchronised detectors tend to overestimate the probability of successful reception.

Examples of models that belong to category *A* are

- A1* *classification of power level*: Metzner [3] proposed to intentionally divide the participating terminals into two groups: one group employs substantially higher transmit power than the other group. He assumed that receiver capture occurs either if a collision involves exactly one packet from an high-power terminal and an arbitrary number of packets from low-power terminals, or if only a packet from the low-power group is present.
- A2* In 1977 Abramson [4] proposed the *vulnerability circle* to evaluate capture in a radio network: A packet transmitted from distance r is received correctly if and only if no other packet is generated within a (vulnerability) circle of radius $c_z r$, with c_z a system constant [4],[16].
- A3* Kuperus and Arnbak [19] considered a *capture ratio*: A packet is received correctly if and only if its received power exceeds the joint interference power by at least a threshold factor (or capture ratio) z . During the capture period, the received power is considered constant [5],[17]-[31],[40].
- A4* Sinha and Gupta [6] assumed correct packet reception if the entire packet is received during a *non-fade interval*. In [6], [7], [8], the model was considered for noise limitations of the network performance, rather than contention limitations.
- A5* If colliding data packets arrive not exactly simultaneously, the receiver may lock on the *first arriving* signal. This initiating packet is (only) received correctly if the joint interference power that accumulates during the following vulnerable period is sufficiently low. Packets arriving at a busy receiver are considered to be lost. For instance, a spread spectrum receiver may lock to an arriving packet, and effectively reject interference from later packets [36],[41].

Models of the second category, based on bit error probabilities, are

- B1* *Error-free detection*: a receiver is considered that always achieves perfect carrier and bit synchronisation to the test packet. If no error occurs in this packet, transmitter success is assumed [9],[13].
- B2* *Error-free detection of a random packet*: Zhang, Pahlavan and Ganesh [10] considered a receiver that randomly selects a favourite test packet, and

acquires perfect bit and carrier synchronisation to this packet. If the test packet happens to be selected by the receiver and detection occurs without any error, it is assumed to be received successfully.

- B3** *Correctable errors*: The previous two models can be extended by considering a coding scheme that can correct up to t errors [12].

Hybrid models

- D1** Habbab, Kavehrad and Sundberg [11] assumed a receiver to lock to the signal from the *nearest* transmitter. If this packet is received with less than t bit errors, receiver capture is assumed.
- D2** Sheikh, Yao and Wu [35] considered a bit error to occur if the received signal power fails to exceed the power contained in the bit of the strongest interfering packet. Transmitter success occurs if less than t out of L bits are in error.

In the following sections, we study the probability $q_n(r)$ that a test packet from a terminal located at a distance r is received successfully, conditional on the presence of n competing transmissions from terminals with unknown locations. Moreover, results from chapter 6 are used to express the unconditional probability $Q(r)$ of successful transmission, taking account of the influence of the access protocol.

7.2 QUASI-UNIFORM SPATIAL DISTRIBUTION

In this chapter, the near-far effect and Rayleigh fading are considered, while shadowing is ignored ($\sigma_s=0$, $P_j=\bar{P}_j$). The normalised local-mean power $\bar{P}_j$ received from the j -th mobile terminal at a (normalised) distance r_j from the central receiver is assumed to be on the form $\bar{P}_j=r_j^{-4}$, thus the attenuation exponent in (2.13) is taken as

$\beta=4$. If the position of the terminal is unknown, the probability density function (pdf) of the mean power is found from

$$\left| f_{\bar{p}_j}(\bar{p}_j) d\bar{p}_j \right| = \left| \frac{2\pi r_j G(r_j)}{G_t} dr_j \right| \quad (7.3)$$

with $G(r)$ the offered traffic per unit area and G_t the total offered traffic (see chapter 6). Throughout this chapter, we assume the quasi-uniform spatial distribution of the offered channel traffic suggested in [5], namely

$$G(r) = \frac{G_t}{\pi} \exp\left(-\frac{\pi}{4} r^4\right). \quad (7.4)$$

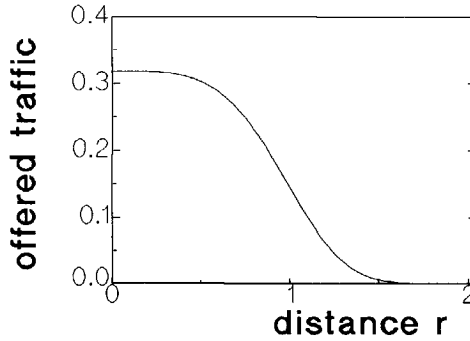


Fig. 7.2: *Quasi-uniform distribution of the offered traffic $G(r)$ versus distance r normalised for $G_t = 1$ ppt.*

The median of this distribution is $r_M \approx 0.734$, since

$$\frac{1}{G_t} \int_0^{r_M} 2\pi r G(r) dr = \operatorname{erf}\left(\frac{\sqrt{\pi}}{2} r_M^2\right) = \frac{1}{2} \quad (7.5)$$

where the error function erf is defined as [15]

$$\operatorname{erf}(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\lambda^2) d\lambda. \quad (7.6)$$

For $\beta=4$, the median received power is $\bar{p}_M=r_M^{-4}\approx 3.545$. Fig. 7.2 shows that (7.4) is an approximation of the exact uniform distribution

$$G(r) = \begin{cases} \frac{1}{\pi} G_p, & 0 < r < 1 \\ 0, & r > 1 \end{cases} \quad (7.7)$$

by a smooth analytical function. The reason to adopt this quasi-uniform spatial distribution is the convenient analytical expression found for the pdf of the joint power of n uncorrelated signals after incoherent addition [5]: n -fold convolution of (7.3) with (7.4) gives

$$f_{\bar{p}_t}(\bar{p}_t | n) = \begin{cases} \frac{n-1}{2} \bar{p}_t^{-\frac{3}{2}} \exp\left\{-\frac{\pi n^2}{4\bar{p}_t}\right\}, & n=1,2,\dots \\ \delta(\bar{p}_t), & n=0, \end{cases} \quad (7.8)$$

where $\bar{p}_t$ ($\bar{p}_t = \Sigma \bar{p}_j$) is the local-mean power of the joint interference signal.

Further, Rayleigh fading in a narrowband channel is assumed for each signal. The instantaneous amplitude ρ_j of the j -th carrier is Rayleigh distributed with mean $\bar{p}_j$, so the corresponding in-phase and quadrature carrier components ζ_j and ξ_j are independently Gaussian distributed, with mean $\bar{p}_j$. Combining the statistical fluctuations caused by the spatial distribution of the mobile terminals (7.4) and those caused by Rayleigh fading, the unconditional pdf's of the amplitude ρ_j and the in-phase component ζ_j are [10],[14]

$$f_{\rho_j}(\rho_j) = \frac{\sqrt{2\pi} \rho_j}{\left(\frac{\pi}{2} + \rho_j^2\right)^{\frac{3}{2}}} \quad (7.9)$$

and

$$f_{\zeta_j}(\zeta_j) = \frac{1}{\sqrt{2\pi} \left(\frac{\pi}{2} + \zeta_j^2 \right)}, \quad (7.10)$$

respectively. Any retransmitted or rescheduled packet is assumed to experience uncorrelated fading and path loss.

7.3 MODEL BASED ON VULNERABILITY CIRCLE

7.3.1 REVIEW

In 1977, Abramson [4] suggested the vulnerability-circle as a model for receiver capture: A (test) packet transmitted from a terminal at a distance r_j is received correctly if no other packet is transmitted within a circle of radius $c_z r_j$, with c_z a system constant (model A2). This is equivalent to assuming that capture occurs if and only if the area-mean power $\bar{p}_j$ of the test packet exceeds the largest area-mean power $\bar{p}_k$ among the interfering signals by at least a factor of $z = c_z^\beta$, thus if $\bar{p}_j \geq c_z^\beta \bar{p}_i$ for all $i = 1, \dots, n$. For slotted ALOHA, the probability of successful transmission is,

$$\begin{aligned} Q(r_j) &= Pr \left(\begin{array}{l} \text{no interfering arrivals} \\ \text{from } 0 < r < c_z r_j \end{array} \right) \\ &= \exp \left\{ - \int_0^{c_z r_j} 2\pi r G(r) dr \right\}. \end{aligned} \quad (7.11)$$

The ALOHA-net at the University of Hawaii [4] employed fixed transmitters at islands located at ranges of several tens of kilometres. Rayleigh fading and shadowing, which are ignored in (7.11), may not have played a mayor role in the signal attenuation, so the propagation model $p_j = r_j^{-\beta}$ might have been appropriate in this case. The vulnerability-circle has also been used in many other studies, particularly in the evaluation of multi-hop packet radio systems, e.g. [16],[42].

7.3.2 ANALYSIS FOR QUASI-UNIFORM OFFERED TRAFFIC

Abramson [4] considered (exactly-) uniform offered traffic with infinite extension of the service area, thus $G(r)=G_0$ for all $r>0$. With our choice of a quasi-uniform spatial distribution (7.4), one finds for slotted ALOHA

$$\begin{aligned} Q(r_j) &= \exp \left\{ - \int_0^{c_j r_j} 2r G_r \exp \left\{ -\frac{\pi}{4} r^4 \right\} dr \right\} \\ &= \exp \left\{ -G_r \operatorname{erfc} \left(\frac{\sqrt{\pi}}{2} c_z^2 r_j^2 \right) \right\}, \end{aligned} \quad (7.12)$$

To compute the throughput for ISMA, we initially consider a test packet that arrives during the vulnerable period of an initiating packet (event H_d). The probability that the initiating signal (with index 1) arrives from outside the vulnerability circle is

$$q_1(r_j | H_d) = \Pr(r_1 < c_z r_j) = \int_{c_z r_j}^{\infty} 2r \exp \left\{ -\frac{\pi}{4} r^4 \right\} dr = \operatorname{erfc} \left(\frac{\sqrt{\pi}}{2} c_z^2 r_j^2 \right), \quad (7.13)$$

with $\operatorname{erfc}(\cdot)$ the complementary error function, i.e., $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$. The probability $H(r_j)$ that no further interfering transmission starts during the vulnerable period from within the vulnerable circle is, analogous to (7.12),

$$H(r_j) = \exp \left\{ -d_1 G_r \operatorname{erfc} \left(\frac{\sqrt{\pi}}{2} c_z^2 r_j^2 \right) \right\}. \quad (7.14)$$

Using (6.23), the probability of a successful transmission becomes for nonpersistent ISMA,

$$Q(r_j) = H(r_j) \frac{1 + d_1 G_r \operatorname{erfc} \left(\frac{\sqrt{\pi}}{2} c_z^2 r_j^2 \right)}{G_r (1 + d_1 + d_2) + \exp(-d_1 G_r)}. \quad (7.15)$$

For unslotted p -persistent ISMA without signalling delays, one finds

$$\begin{aligned}
 Q(r_j) &= \frac{1 + p G_i \exp(p G_i) \Pr \left(\begin{array}{l} \text{no interferers} \\ \text{from } 0 < r < c_z r_j \end{array} \right)}{1 + p G_i \exp(p G_i)} \\
 &= \frac{1 + p G_i \exp \left\{ + p G_i \operatorname{erfc} \left(\frac{\sqrt{\pi}}{2} c_z^2 r_j^2 \right) \right\}}{1 + G_i \exp(p G_i)}.
 \end{aligned} \tag{7.16}$$

Thus for the vulnerability circle model, $Q(r)$ can be expressed without explicit derivation of $q_n(r)$. Numerical results for $Q(r)$ are included later in Fig 7.15.

7.4 MODEL BASED ON CAPTURE RATIO

Model A3 assumes a test packet to capture the receiver in the base station if, and only if, its instantaneous power p_j exceeds the instantaneous *joint* interference power p_i by at least a threshold factor (or 'capture ratio') z . The received power is assumed constant during the reception of a packet ($T_s < T_\phi$, see section 2.1.7).

7.4.1 REVIEW

The effect of cumulation of interference power if more than one interfering signal is present was considered in a number of papers. Kuperus and Arnbak [19] and Fronczak [40] derived receiver capture probabilities in a Rayleigh-fading channel for the particular spatial distribution that gives an exponential distribution of the signal power at the receiver. Namislo [17] reported the probability of capture for a slotted ALOHA network with near-far effect. The results were obtained by means of Monte Carlo simulation. Verhulst et al. [18] studied slow frequency hopping in cellular telephone networks. Their results included an analytical expression for the throughput

of slotted ALOHA in a channel with slow Rayleigh fading, but without shadowing and near-far effect. Arnbak and Van Blitterswijk [5] elaborated the method in [19] for a channel with combined Rayleigh fading and near-far effect caused by the quasi-uniform spatial distribution (7.4). Coherent and incoherent addition of interference signals were compared. The author [20] showed that the use of integral transforms of spatial distributions can greatly facilitate the analysis of the throughput of mobile ALOHA networks with Rayleigh fading and near-far effect. This technique was used to quantify the effect that remote terminals have to perform more retransmissions than nearby terminals [21]. An investigation of various spatial distributions and a further comparison of the vulnerability circle (42) with the capture-ratio model (43) has been presented by Lau and Leung in [22] focusing on channels without Rayleigh fading.

The influence of shadowing was discussed by Prasad and Arnbak in [23]. The technique by Schwartz & Yeh (see chapter 3) was employed to compute the joint interference power cumulated from multiple log-normal signals. In [24] and [25], this technique was extended for combined Rayleigh fading and shadowing with coherently and incoherently adding signals, respectively. The influence of combined near-far effect and shadowing on the throughput of the ALOHA channel was studied by Linnartz and Prasad in [26]. The author expressed capture probabilities for channels with combined Rayleigh fading, shadowing and near-far effect, using integral transforms [28]. Van der Plas and Linnartz employed the results to study the stability of a mobile slotted ALOHA network.

The total throughput of nonpersistent ISMA-networks with capture in Rayleigh-fading channels was reported by Zdunek et al. [29]. Prasad further extended the analysis to include the effects of shadowing [34] and combined shadowing and near-far effect [27]. Linnartz [30] reported the probability of successful transmission of an a priori selected 'test' packet. This allowed computation of the probability of successful transmission as a function of the distance between the terminal and the receiving base station. Near-far effects caused by an exactly uniform spatial distribution and Rayleigh fading were considered. Analytical expressions in closed-form were derived for $Q(r)$ in the case of slotted ALOHA, unslotted nonpersistent ISMA and unslotted 1-persistent ISMA.

Analogous to the results reported by the author for an *exactly* uniform spatial distribution in [30], the next section gives the probability of capture for the *quasi*-uniform spatial distribution (7.4).

7.4.2 ANALYSIS FOR QUASI-UNIFORM OFFERED TRAFFIC

Similar to the evaluation of the outage probability in cw communication in section 3.2, the probability of capture (event c_j) conditional on the local-mean power of the test packet is expressed as a Laplace image, viz.,

$$\begin{aligned}
 \Pr(c_j | \bar{p}_j) &\triangleq \Pr\left(\frac{p_t}{p_j} > z \mid \bar{p}_j\right) \\
 &= \int_{z=0}^{\infty} \int_{y=0}^{\infty} \frac{1}{\bar{p}_j} \exp\left(-\frac{xy}{\bar{p}_j}\right) f_{p_t}(x) x dx dy \\
 &= \int_{0-}^{\infty} \exp\left\{-\frac{xz}{\bar{p}_j}\right\} f_{p_t}(x) dx = \mathcal{L}\left\{f_{p_t}, \frac{z}{\bar{p}_j}\right\},
 \end{aligned} \tag{7.17}$$

where $f_{p_t}(\cdot)$ is the pdf of the joint interference power p_t caused by cumulation of n contending packets. The lower limit of the integral over p_t (with dummy variable x) is taken '0-' to ensure that (7.17) is also valid if, with some non-zero probability, interference is absent in a noiseless channel ($n=0$, $N_A=0$). In the special case

$$f_{p_t}(p_t) = \delta(p_t), \tag{7.18}$$

Eq. (7.17) becomes

$$\Pr(c_j) = \mathcal{L}\left\{\delta(\cdot), \frac{z}{\bar{p}_j}\right\} = 1, \tag{7.19}$$

i.e., capture always occurs if no interference and noise is present, which agrees with the assumptions of model A3.

For the case of multiple interfering signals ($n=2, \dots$), coherent and incoherent cumulation of interference signals will be distinguished in chapter 8. Here we address the case that the interference power p_i is caused by *incoherent* cumulation of n independently fading signals. The pdf of joint interference power is the n -fold convolution of the pdf of the individual signal powers. Laplace transformation results in the multiplication of n factors (see chapter 3), so, if all interfering packets arrive from random positions in the service area and if $N_A=0$,

$$q_n(r) = \mathcal{L}\left\{f_{p_i, \frac{z}{p_0}}\right\} = \mathcal{L}^n\left\{f_{p_i, \frac{z}{p_0}}\right\} = \{q_1(r)\}^n. \quad (7.20)$$

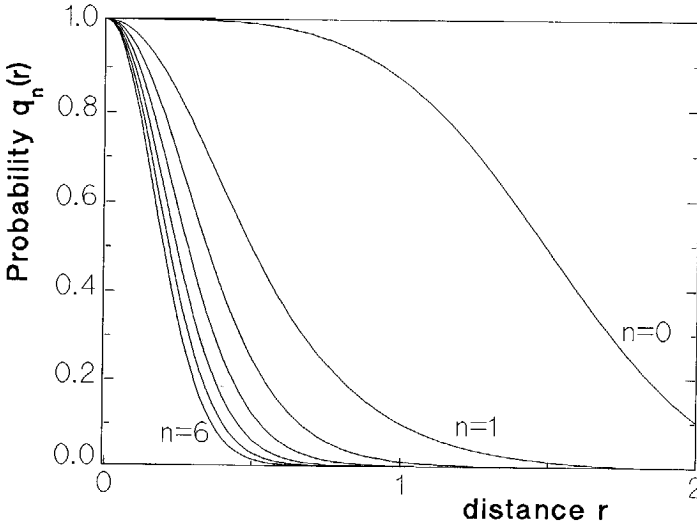


Fig. 7.3: Probability of capture $q_n(r)$ versus distance r for $n=0, \dots, 6$, according to capture-ratio model. $z=4$ (6 dB), Median $C/N=20$ dB ($N_A \approx 0.035$).

For the quasi-uniform spatial distribution (7.4), one finds after applying Eq. (7.4.1) in [34],

$$q_n(r) = \left[1 - \frac{\pi}{2} \sqrt{z} r^2 \operatorname{erfc} \left(\frac{\sqrt{z\pi}}{2} r^2 \right) \exp \left(\frac{z r^4 \pi}{4} \right) \right]^n. \quad (7.21)$$

The effect of noise was also considered in chapter 3. If additive noise with power $p_N = N_A = N_0 B_T$ is present, the probability of successful transmission (7.21) is to be multiplied by a factor of $\exp\{-z N_A r^\beta\}$. Fig. 7.3 portrays $q_n(r)$ for $n=0,1,\dots,6$ and a receiver threshold of 6 dB ($z=4$). The median signal-to-noise ratio is 20 dB, i.e., the noise power is $N_A \approx 0.035$ (see section 7.2).

Using the expressions presented in chapter 6 and the series expansion of the exponential function, the probability of a successful transmission for slotted ALOHA is found on the form of

$$\begin{aligned} Q(r) &= \exp\{-z N_A r^\beta\} e^{-G_t} \sum_{n=0}^{\infty} \frac{G_t^n}{n!} q_n(r) \\ &= \exp\{-G_t(1 - q_1(r)) - z N_A r^\beta\}. \end{aligned} \quad (7.22)$$

with $q_1(r)$ on the form of (7.21) with $n=1$. Analogously, the probability of correct reception for nonpersistent ISMA becomes

$$Q(r) = \exp\{-d_1 G_t(1 - q_1(r)) - z N_A r^\beta\} \frac{1 + q_1(r) d_1 G_t}{G_t(1 + d_1 + d_2) + e^{-d_1 G_t}}. \quad (7.23)$$

For unslotted p -persistent ISMA with zero signalling delay, $Q(r)$ is

$$Q(r) = \exp\{-z N_A r^\beta\} \frac{1 + p G_t \exp(p G_t q_1(r))}{1 + G_t \exp(p G_t)} \quad (7.24)$$

Numerical results for $Q(r)$ in these cases are shown in Fig. 7.16.

7.5 MODEL BASED ON NON-FADE INTERVAL

In section 7.4, the duration of a packet was assumed to be short with respect to the rate of channel fading. This is reasonable if mobile terminals move less than a fraction of the wave length during the transmission of a packet ($T_5 \ll T_6$). For usual vehicle speeds, this requires that data packets are shorter than a few milliseconds [32]. In practical narrowband UHF networks, this condition is not always satisfied. During the transmission of a packet, the received signal power may fade. To investigate this effect, approximations suggested in chapter 4 will now be considered.

For a wanted Rayleigh fading signal with local mean power $\bar{p}_j$ in the presence of Nakagami fading interference with mean power $\bar{p}_i$, the average non-fade duration (expressed in seconds) was found on the form of (4.35). In analogy with section 4.10, the probability that a test packet is located in a non-fade interval can be found if a number of additional assumptions are made:

- (i) Threshold crossings are memory-less, so that non-fade durations are exponentially distributed. It may be argued that non-fade durations corresponding to the maximum Doppler shift prevail in mobile reception (section 4.8). However, if the speed of the vehicle in an urban environment is also seen as a stochastic variable, non-fade interval lengths tend to be spread over a wider range; lacking further details, an exponential distribution yields convenient mathematical expressions and appears plausible.
- (ii) The joint interference-plus-noise signal behaves as a non-fading signal. This is correct in the limiting case $n \rightarrow \infty$, but underestimates the duration of non-fade intervals when the interference is dominated by a single fading signal ($m+1$) by about 13% (see (4.36)).

Similar to the analysis in [6]-[8] for noise-limited channels, the probability of successful reception is found from the requirements that C/I is larger than z at the start of the packet and that the packet duration must be shorter than the time towards the next fade. Rewriting Eq. (4.56) gives

$$Pr(c_j | \bar{p}_i \bar{p}_j) = \exp \left\{ -z \frac{\bar{p}_i + N_A}{\bar{p}_j} - \sqrt{2\pi T_s} f_m \sqrt{z \frac{\bar{p}_i + N_A}{\bar{p}_j}} \right\}. \quad (7.25)$$

The probability of capture $q_n(r_j)$ becomes, after averaging over the joint interference power (7.8) and substituting $\bar{p}_i = s^{-4}$,

$$\begin{aligned} q_n(r_j) &= \int_0^\infty Pr(c_j | \bar{p}_i \bar{p}_j) f_{\bar{p}_i}(\bar{p}_i | n) d\bar{p}_i \\ &= 2 \int_0^\infty n s \exp \left\{ -z(s^{-4} + N_A) r_j^4 - \frac{\pi}{4} n^2 s^4 - r_j^2 \sqrt{2\pi z(s^{-4} + N_A)} f_m T_s \right\} ds \end{aligned} \quad (7.26)$$

for $n=1,2,3,\dots$. For $n=0$, $q_0(r_j)$ is found from (7.25) with $\bar{p}_i=0$.

Fig. 7.4 confirms that the effect of the packet duration T_s is significant. This suggests that during a collision, the fade margin can be relatively small. In such cases the assumption of constant received power (as in section 7.4) appears optimistic and will lead to overestimating channel performance if packet durations are larger than or in the same order of magnitude as the time constants of the fading.

The case $f_m T_s = 0.0166$ corresponds to, e.g., packets of $L=16$ bits transmitted at a bit rate of $r_b=16$ kbit/s in a mobile radio channel at $f_c=900$ MHz by a terminal with velocity $v=5.55$ m/s (20 km/h), thus with a maximum Doppler shift of $f_m=16.6$ Hz. Computer simulation of this case will be reported in section 7.7 for noncoherent detection of DPSK signals.

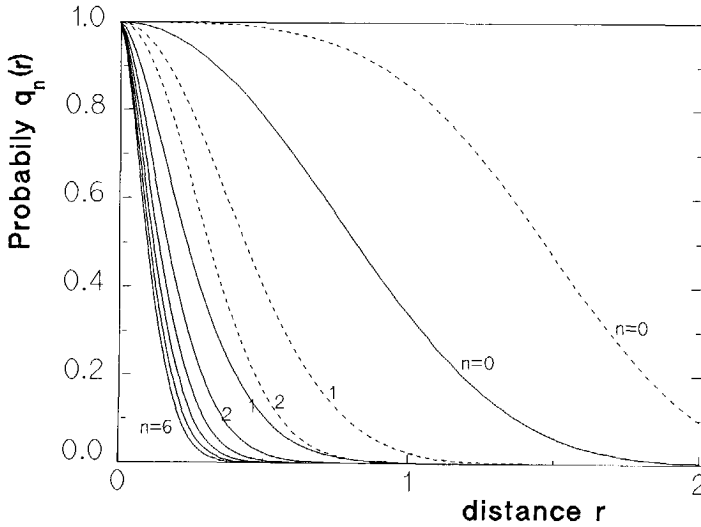


Fig. 7.4 Probability of successful reception $q_n(r_j)$ versus distance r_j for packets of normalised duration (---) $f_m T_s = 0.0166$ and (—) $f_m T_s = 1$. Median $C/N = 20$ dB ($N_A \approx 0.035$). Number of interfering signals $n = 0, 1, \dots, 6$. Receiver threshold $z = 4$ (6 dB).

The probability of successful access, $Q(r_j)$, in a slotted ALOHA network is found by considering a Poisson distributed number of interferers with unknown positions. So

$$Q(r_j) = e^{-G_i} \Pr(c_j | \bar{p}_j = r_j^{-\beta}, \bar{p}_i = 0) + \sum_{n=1}^{\infty} \frac{G_i^n}{n!} e^{-G_i} \int_0^{\infty} 2ns \cdot \exp\left\{-z(s^{-4} + N_A)r_j^4 - \frac{\pi}{4}n^2s^4 - r_j^2\sqrt{2\pi z(s^{-4} + N_A)}f_m T_s\right\} ds, \quad (7.27)$$

where we substituted $\bar{p}_i = s^{-4}$.

Fig. 7.5 shows that (7.27) with $f_m T_s = 0$ is pessimistic compared to the exact result (7.22) for packets of infinitely short duration. A possible explanation of this discrepancy is that in the event of very short packet durations ($T_s \rightarrow 0$) in slots with one (or few) interfering signal(s), fades of the test packet may coincide with fades of the interference. In such events, our assumption (ii) of non-fading interference gives relatively pessimistic estimates of the capture probabilities. However, considering the

arguments raised in section 4.10, one may conclude that the assumption of packets of zero duration ($T_s \rightarrow 0$) is always optimistic, even if ($T_s \ll T_6$).

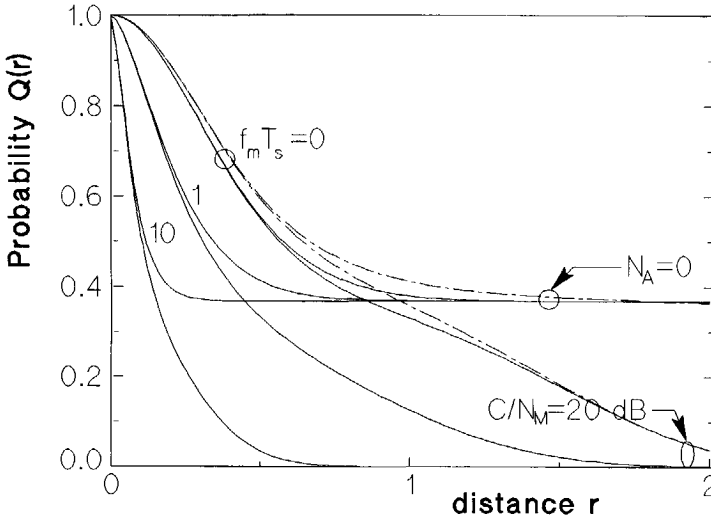


Fig. 7.5 Probability of successful access $Q(r)$ (—) versus distance r for slotted ALOHA with packets of normalised duration $f_m T_s = 0, 1$ and 10 . Noise-free channel ($N_A = 0$) and noise power $N_A \approx 0.035$ ($C/N_M = 20$ dB). Receiver threshold $z = 4$ (6 dB). Exact solution (7.22) for packets of zero duration ($\cdots$). Quasi-uniform offered traffic with $G_t = 1$ ppt.

For $f_m T_s > 1$ ($T_s > T_6$), the performance is substantially degraded by fades during packet reception. Fig. 7.5 indicates that for relatively long packets ($T_s > 10 f_m^{-1}$) in a channel with a median C/N -ratio of 20 dB, the majority of terminals experience a failure of almost every transmission attempt ($Q(r) \approx 0$ if $r > 0.5$). This is in sharp contrast to the case for shorter packets, when all terminals may have an acceptable probability of success ($Q(r) > 0.35$ if $T_s \rightarrow 0$ and $0 < r < 1$).

In Fig 7.17, numerical results for (7.27) and similar results for $Q(r)$ in the case of unslotted ISMA are given.

7.6 MODEL BASED ON PROBABILITY OF TRANSMITTER SUCCESS

We now address the models of the second category (category *B*). A test packet of L bits is assumed to be received successfully if and only if the bit sequence detected by the receiver entirely matches the bit sequence in the test packet. Error detection and correction coding [12] are not considered explicitly in this section.

7.6.1 REVIEW

Habbab, Kavehrad and Sundberg presented the throughput of slotted ALOHA nets with BPSK modulation in [33]. The analysis was later extended in [11]. The model for successful reception was based on criteria of terminal position and packet success rates. Slow and fast Rayleigh fading of the test packet were considered. With slow fading, the amplitude and phase of each signal are assumed constant for the entire duration of the packet ($T_5 < T_6$, see section 2.1.7). Fast fading, on the other hand, causes substantial fluctuations of the signal amplitudes and phases during packet reception. In an extreme case, the received signal fading is uncorrelated from bit to bit. Although a practical receiver is likely to lose carrier synchronisation under such circumstances ($T_3 \approx T_6$), the expressions proposed for modelling fast fading may be reasonable if $T_3 < T_6 < T_5$.

In [11], the joint interference signal was approximated as Gaussian noise. Zhang, Pahlavan and Ganesh [10] also studied an ALOHA network with BPSK and fast Rayleigh fading. A number of improvements to their model were suggested by Linnartz, Goossen and Hekmat in [13]. Zhang and Pahlavan [14] addressed slow fading channels with interference samples of constant envelope during successive bits of a packet. In [10] and [14], a receiver was assumed to lock perfectly to a randomly chosen signal out of the $n+1$ colliding packets, and all other packets were assumed to

have zero probability of capturing the receiver, even though they might arrive with substantially higher power. Other synchronisation models will be addressed in the next sections.

7.6.2 FORMULATION OF RECEIVER MODEL

The model described in chapter 5 is reconsidered. We shall assume no bit synchronisation offset between contending signals ($\alpha_i=0$). This overestimates the effect of interference and may yield slightly pessimistic results for perfectly synchronising receivers (see section 5.6). Moreover, in the case of random α_i , the worst-case interfering BPSK bit sequence $\kappa_{i,\rightarrow}=\kappa_{i,\leftarrow}$, i.e., no phase reversal, is expected to occur during 50% of the bits in the test packet. This event gives the same error probability as the case $\alpha_i=0$.

The received joint signal $v_i(t)$ has the form

$$v_i(t) = \rho_j \kappa_j \cos(\omega_c t + \theta_j) + \sum_{i=1}^n \rho_i \kappa_i \cos(\omega_c t + \theta_i) + n(t), \quad (7.28)$$

where κ_i ($\kappa_i = \pm 1$) represents the phase reversals arising from BPSK modulation of the i -th carrier, and $n(t)$ is the AWGN process. The phasor diagrams Fig. 7.6 - Fig. 7.8 present the in-phase and quadrature signal component after (fictitious) correlation with $\cos \omega_c t$ and $\sin \omega_c t$, thus $2/T_b \cdot T_b \int v_i(t) \cos(\omega_c t) dt$, and $2/T_b \cdot T_b \int v_i(t) \sin(\omega_c t) dt$, respectively. AWGN, although not strictly bandlimited at the receiver front-end, can also be depicted in the in-phase / quadrature diagram because of the bandpass nature after the integration over the bit duration T_b .

The decision variable for synchronous bit extraction is obtained by multiplying the received (composite) signal $v(t)$ by a locally generated cosine $2\cos(\omega_c t + \theta_r)$, where θ_r ,

denoted the phase of the local oscillator in the receiver, and integrating over the entire bit duration T_b . In Fig. 7.6 - Fig. 7.8, this corresponds to determining the projection of the received resultant vector on a line with direction θ_r . For a test packet j in the presence of n interferers, the decision variable is

$$v = \frac{1}{T_b} \int_0^{T_b} 2v(t) \cos(\omega_c t + \theta_r) dt = \rho_j \kappa_j \cos(\theta_j - \theta_r) + \sum_{i=1}^n \rho_i \kappa_i \cos(\theta_i - \theta_r) + n_I. \quad (7.29)$$

Three idealised cases for the phase of the local oscillator in the coherent detector are compared, namely a receiver locked to the test signal ($\theta_r = \theta_j$; Fig. 7.6), a receiver locked to one of the interferers ($\theta_r = \theta_k$; Fig. 7.7) and a receiver with arbitrary, but constant, phase ($\theta_r - \theta_i = \text{constant}$; Fig. 7.8). The latter two events correspond to extreme cases of carrier phase errors in the receiver, caused by signals competing with the test packet.

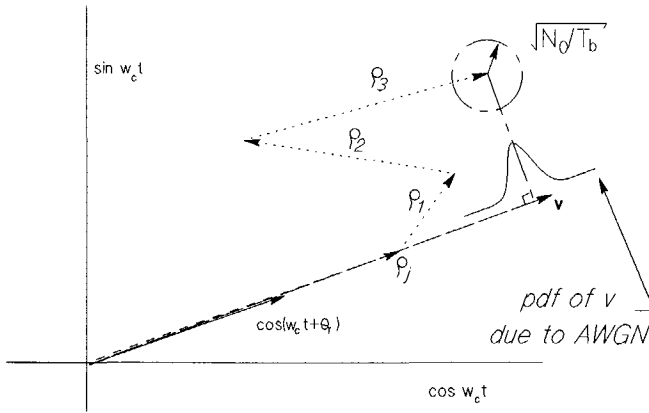


Fig. 7.6: Phasor diagram for coherent detection for a test signal in the presence of three interferers and AWGN. (—) test signal, (···) interfering signal. Receiver locked to the test packet.

In a Rayleigh-fading channel, the in-phase components ζ_i ($\zeta_i = \rho_i \cos(\theta_i - \theta_r)$) of the n interferers (with random phase relative to the local oscillator in the receiver), and the noise sample n_I are all Gaussian variables without mutual correlation. The rate of fading determines the correlation of amplitude and phase in successive bits. Fast and

slow fading are now distinguished.

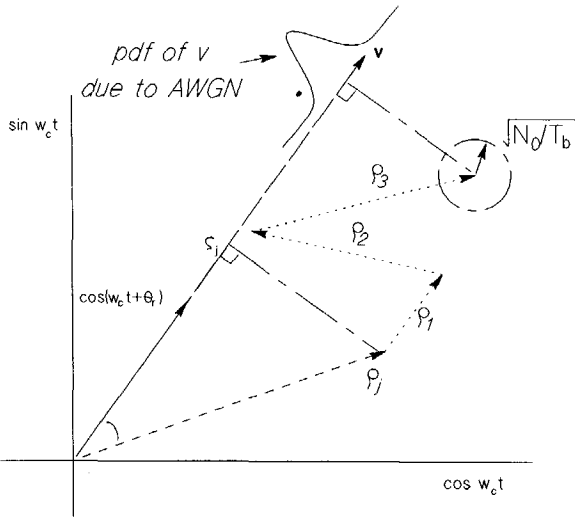


Fig. 7.7: Phasor diagram for coherent detection for a test signal in the presence of three interferers and AWGN. (---) test signal, (....) interfering signal. Receiver locked to an interferer.

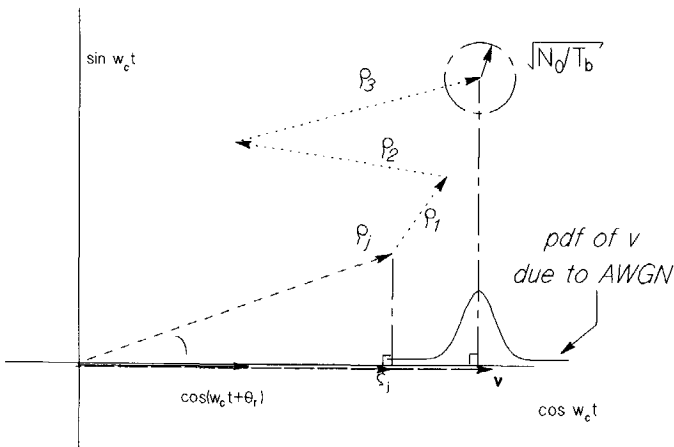


Fig. 7.8: Phasor diagram for coherent detection for a test signal in the presence of three interferers and AWGN. (---) test signal, (....) interfering signal. Arbitrary but constant receiver phase.

7.6.3 ANALYSIS FOR QUASI-UNIFORM OFFERED TRAFFIC AND FAST FADING

With fast Rayleigh fading, the duration of a packet is substantially longer than the time constants of the multipath fading. Further, we assume that during one bit time the channel characteristics do not change ($T_3 < T_6 < T_5$). During reception of a packet, each signal is expected to experience several fades. If it can be assumed that the received amplitude and phase of all signals are statistically independent from bit to bit even though the receiver remains perfectly locked to the test packet j ($\theta_r = \theta_j$), the transmitter success probability for BPSK is obtained from

$$q_n(r_j) = \int_0^\infty f_{\bar{p}_t}(\bar{p}_t | n) \left[1 - P_b(e | \bar{p}_j = r_j^{-4}, \bar{p}_t) \right]^L d\bar{p}_t, \quad (7.30)$$

where the bit error probability is taken on the form of (5.13). Inserting (5.13) in (7.30) and considering a quasi-uniform spatial distribution of offered traffic (7.8) with $\bar{p}_t = s^{-4}$, the probability of correct reception is found to be

$$q_n(r_j | \theta_r = \theta_j) = \int_0^\infty 2ns \exp\left(-\frac{\pi}{4}n^2s^4\right) \left[\frac{1}{2} + \frac{1}{2} \sqrt{\frac{T_b r_j^{-4}}{T_b r_j^{-4} + T_b s^{-4} + N_0}} \right]^L ds \quad (7.31)$$

for $n=1, 2, \dots$. For $n=0$, $q_0(r)$ is found from Eq. (5.13) with $\bar{p}_t=0$. This probability is depicted in Fig. 7.9 for $n=0, 1, 2, \dots, 6$. The median C/N -ratio is taken 20 dB: the corresponding spectral power density of the noise is $N_0 = 0.01 \bar{p}_M T_b \approx 0.035 T_b$.

Two capture models are now defined:

MODEL BF1: This model assumes perfect synchronisation for the test packet (Fig. 7.6):

$$q_n(r_j) = q_n(r_j | \theta_r = \theta_j) \quad \forall j, \quad (7.32)$$

so

$$C_{n+1} = (n+1)q_n|_{\theta_r=\theta_j} = (n+1) \int_0^{\infty} 2\pi r q_n(r_j | \theta_r=\theta_j) G(r_j) dr_j. \quad (7.33)$$

This model is based on the two assumptions that (i) in the case of error-free reception of the (rapidly fading) test signal, the signal is also sufficiently strong to maintain perfect receiver synchronisation during the entire packet while (ii), for a weak signal, the probability of error-free reception is negligible irrespective of the carrier phase synchronisation, provided that L is sufficiently large.

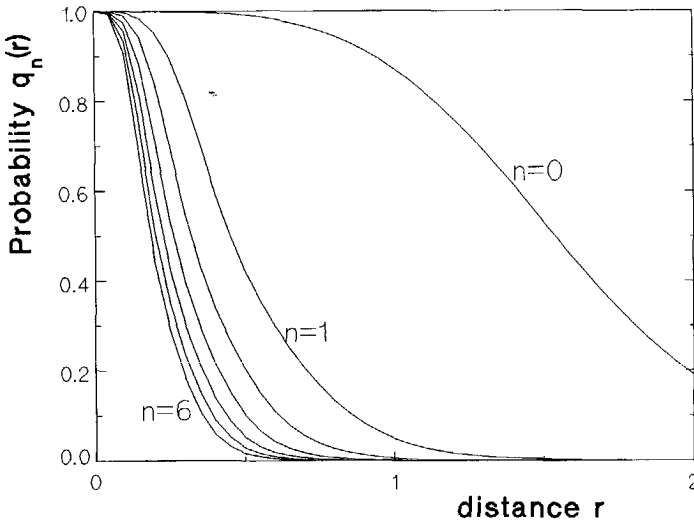


Fig. 7.9: Probability of success $q_n(r)$ versus the distance r for fast fading and $n=0,1,\dots$ interfering signals. Perfect carrier synchronisation $\theta_r=\theta_j$, $C/N_M=20$ dB: $N_A \approx 0.035$.

MODEL BF2: The receiver is assumed to lock to a randomly selected signal k out of the $n+1$ competing packets, where k does not necessarily represent the test packet or the strongest contending packet. Correct reception of the test packet occurs if and only if the receiver locks to the test packet, i.e., if the receiver selects $k=j$ (Fig. 7.6), and no bit error occurs [8]. This produces the (pessimistic) relations

$$q_n(r_j) = \Pr(k=j) q_n(r_j | \theta_r = \theta_j) = \frac{1}{n+1} q_n(r_j | \theta_r = \theta_j) \quad \text{and} \quad C_{n+1} = q_n |_{\theta_r = \theta_j}. \quad (7.34)$$

For a numerical comparison of model *BF1* and *BF2* we refer to [13]. The results for the two models appeared essentially different for large offered traffic loads. Numerical results for $Q(r)$ for model *BF1* are included in Fig. 7.18.

7.6.4 ANALYSIS FOR QUASI-UNIFORM OFFERED TRAFFIC AND SLOW FADING WITH GAUSSIAN INTERFERENCE

As noted earlier, packets should be short with respect to the channel fading to fully exploit the benefits from receiver capture. For packets of sufficiently short duration, the received amplitude and carrier phase may be assumed to be constant throughout the duration of a packet for each of the $n+1$ signals. This condition is satisfied if the motion of each transmitter during a packet time is negligible compared to the wave length ($T_s \ll T_\phi$). Packet error rates for a slow fading (test) signal were reported in [38] for channels with AWGN.

The probability of error-free reception of a data packet of L bits from terminal j with a known local-mean received signal power, is found by averaging $\Pr(s_j)$ over the Rayleigh fading of the wanted signal and over the local-mean interference power, with

$$q_n(r_j | \theta_r = \theta_j) = \int_0^\infty \int_0^\infty 2nsr_j^4 \exp\left\{-\frac{\pi}{4}n^2s^4 - pf_j^4\right\} \cdot \left[1 - \frac{1}{2} \operatorname{erfc}\left\{\sqrt{\frac{p_j T_b}{N_0 + s^{-4} T_b}}\right\}\right]^L ds dp_j. \quad (7.35)$$

Here we assumed that the receiver is locked to the test packet ($\theta_r = \theta_j$; Fig. 7.6), that the interference sample is Gaussian distributed with mean $\bar{p}_i$ ($s^{-4} = \bar{p}_i$) and that

interference samples during successive bits are statistically independent. This 'Gaussian assumption', suggested in [11], is reasonable if the number of interfering signals is large ($n \rightarrow \infty$), or if the interfering signal is fast fading. However, the case of a slow fading test signal with fast fading interference is at odds with the fact that *all* signals are competing for receiver capture and a priori have identical characteristics. The principal difference between (7.30) for fast fading and (7.35) for slow fading is the sequence of the mathematical operation of averaging over the Rayleigh fading of the envelope of the test packet and taking the L -th power. Numerical results from (7.35) are in Fig. 7.10 for $n=0, \dots, 5$. Since computation time for numerical evaluation of (7.35) is significantly larger than with (7.31), results for $n=6, \dots$ have not been obtained.

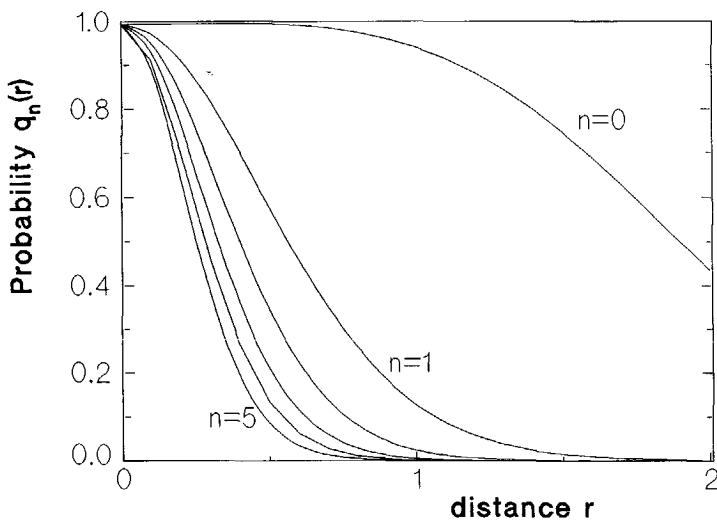


Fig. 7.10: Probability of transmitter success $q_n(r)$ versus the distance r for slow fading test packet and $n=0, 1, \dots, 5$. 'Gaussian' interfering signals. Perfect carrier synchronisation $\theta_r = \theta_j$, $C/N_M = 20$ dB ($N_A \approx 0.035$).

Various models for receiver synchronisation can be defined:

MODEL BSG1: Perfect synchronisation is assumed: $q_n(r_j) = q_n(r_j | \theta_r = \theta_j)$

MODEL BSG2: The receiver randomly locks to a favourite packet:

$$q_n(r_j) = (n+1)^{-1} q_n(r_j | \theta_r = \theta_j).$$

MODEL BSG3: The receiver locks to the signal from the nearest transmitter, while more remote packets are always lost in a collision [11].

Here, we confine ourselves to model *BSG1*. Corresponding results for $Q(r)$, taking account of the behaviour of the random access protocol, are in Fig. 7.14.

7.6.5 ANALYSIS FOR QUASI-UNIFORM OFFERED TRAFFIC AND SLOW FADING WITH CONSTANT-ENVELOPE INTERFERENCE

For angle-modulated signals with constant envelope, a Gaussian approximation of the pdf of the interference signal (as in (7.35)) may become inaccurate: successive interference samples in the decision variable occur with constant in-phase amplitude $\zeta_1, \dots, \zeta_n$ throughout the entire packet. Zhang and Pahlavan [10] studied correlated interference samples in successive bits. Their model is extended here. In particular, the distinction between the a priori selected test packet and the packet that acquires receiver synchronisation is further developed.

As illustrated by Fig. 7.6, for known amplitudes ρ_i , phases θ_i and bit contents κ_i of the test signal ($i=j$) and all interference signals ($i=1, \dots, n$), the decision variable ν has the Gaussian pdf

$$f_\nu(\nu | \{\rho_i, \theta_i, \kappa_i\}_{i=j, i=0}^n) = N \left(\sum_{i=0}^n \rho_i \kappa_i \cos(\theta_i - \theta_j), \frac{N_0}{T_b} \right). \quad (7.36)$$

If the test packet contains a '1' ($\kappa_j=1$), the detector makes a bit error if the decision variable happens to be less than zero. For reasons of symmetry of the events $\kappa_j = \pm 1$, the bit error probability, conditional on the set of bits $\{\kappa_i\}$ in the interfering packets and on the envelope and phase of each signal, is

$$P_b(e | \{\rho_p \theta_p \kappa_i\}) = \frac{1}{2} \operatorname{erfc} \left(\frac{\rho_f \cos(\theta_r - \theta_f) + \sum_{i=1}^n \kappa_i \rho_i \cos(\theta_r - \theta_i)}{\sqrt{2N_0 T_b}} \right). \quad (7.37)$$

The integrated noise n_f is uncorrelated during each successive bit. Further, the mean value of (7.36) may change because of (possible) phase reversals. With n interfering signals, there are 2^n different sets $\{\kappa_i\}$ and all are assumed to be equally likely, so

$$P_b(e | \{\rho_p \theta_i\}_{i=j,i=0}^n) = 2^{-n-1} \sum_{\kappa_i=\pm 1} \dots \sum_{\kappa_n=\pm 1} \operatorname{erfc} \left(\frac{\rho_f \cos(\theta_r - \theta_f) + \sum_{j=1}^n \kappa_j \rho_j \cos(\theta_r - \theta_j)}{\sqrt{2N_0 T_b}} \right). \quad (7.38)$$

Each in-phase component ζ_i ($\zeta_i = \rho_i \cos(\theta_r - \theta_i)$) either has a Gaussian probability density (if θ_i is random) or has a Rayleigh probability density (if the receiver is locked to the i -th signal).

For BPSK, the capture probability conditional on the carrier phasors $\{(\rho_p, \theta_i)\}$ received from the interference signals is

$$Pr(s_j | \{\rho_p \theta_{ij}\}_{i=j,i=0}^n) = \left[1 - P_b(e | \{\rho_p \theta_{ij}\}_{i=j,i=0}^n) \right]^L. \quad (7.39)$$

with (7.38). For differentially encoded BPSK, phase reversal of the carrier of the test packet (and thus inverting all L bits) does not affect the probability of a bit error. Consequently, the conditional packet error probability for coherent detection of differentially encoded BPSK follows from

$$Pr(s_j | \{\rho_p \theta_{ij}\}_{i=0}^n) = \left[1 - P_b(e | \{\rho_p \theta_{ij}\}_{i=0}^n) \right]^L + \left[P_b(e | \{\rho_p \theta_{ij}\}_{i=0}^n) \right]^L. \quad (7.40)$$

The effect that one extra bit is required for differential encoding is ignored here; in fact, with coherent detection of BPSK a reference carrier phase is also required, which may necessitate a preamble of a least one bit. The three cases of carrier synchronisation of the receiver in Fig. 7.6 - Fig. 7.8 are considered: (a) a receiver

locked to the test packet, (b) a receiver locked to another packet and (c) a receiver with a constant but arbitrary phase with respect to each signal.

(a) The capture probability $q_n(r|\theta_r=\theta_j)$ for a receiver locked to the test packet (Fig. 7.6) is obtained by integrating over the in-phase amplitudes $\{\rho_j, \zeta_1, \dots, \zeta_n\}$. So

$$q_n(r_j|\theta_r=\theta_j) = \int_0^\infty d\rho_j \int_{-\infty}^\infty d\zeta_1 \dots \int_{-\infty}^\infty d\zeta_n \Pr(s_j | \{\rho_i, \theta_i\}, \theta_r=\theta_j) f_{\rho_j|\tau_j} f_{\zeta_1} \dots f_{\zeta_n}. \quad (7.41)$$

with ρ_j Rayleigh distributed and $\zeta_1, \dots, \zeta_n$ distributed according to (7.9).

(b) If the carrier recovery circuit in the receiver happens to be locked to the k -th interference signal ($k=1, 2, \dots, n$; $k \neq j$; Fig. 7.7), the decision variable, evaluated for a test packet with a random (but constant) phase relative to the receiver ($\theta_r=\theta_j=\text{constant}$), goes into

$$v = \kappa_j \zeta_j + \kappa_k \rho_k + \sum_{i=1, i \neq k}^n \kappa_i \zeta_i + n_I. \quad (7.42)$$

The capture probability is found by integration

$$q_n(r|\theta_r=\theta_k) = \int_{-\infty}^\infty d\zeta_j \int_{-\infty}^\infty d\zeta_1 \dots \left(\int_0^\infty d\rho_k \right) \dots \int_{-\infty}^\infty d\zeta_n \cdot \Pr(s_j | \{\rho_i, \theta_i\}_{i=j, i=0}^n, \theta_r=\theta_k) f_{\zeta_j|\tau_j} f_{\zeta_1} \dots (f_{\rho_k}) \cdot f_{\zeta_n}. \quad (7.43)$$

with ζ_j Gaussian distributed, $\zeta_1, \dots, \zeta_n$ distributed according to (7.9), and ρ_k distributed according to (7.8).

(c) The phase reference θ_r of the coherent detector is assumed fixed but arbitrary, i.e., it is not related to the phase of any of the $n+1$ signals (Fig. 7.8). Each signal produces a Gaussian in-phase component. The capture probability of error-free detection turns into

$$q_n(r_j | \text{no lock}) = \int_{-\infty}^{\infty} d\zeta_j \int_{-\infty}^{\infty} d\zeta_1 \dots \int_{-\infty}^{\infty} d\zeta_n \Pr(s_j | \{\rho_i, \theta_i\}_{i=j, i=0}^n) f_{\zeta_j | r_j} f_{\zeta_1} \dots f_{\zeta_n}. \quad (7.44)$$

The probability $q_n(r)$ is depicted in Fig. 7.11 for a single interfering signal ($n=1$), for the various cases of receiver synchronisation. Fig. 7.12 reports the effect of the number of interferers for a receiver perfectly locked to the test packet, computed from (7.41) with (7.39). These results have been obtained by numerical computation of the above $n+1$ dimensional integrals. Due to the wide dynamic range of the signals, numerical results obtained from the computations become less accurate for $r < 0.1$. Since the computation time exponentially increases with n , results for $n=4,5,\dots$ could not be obtained.

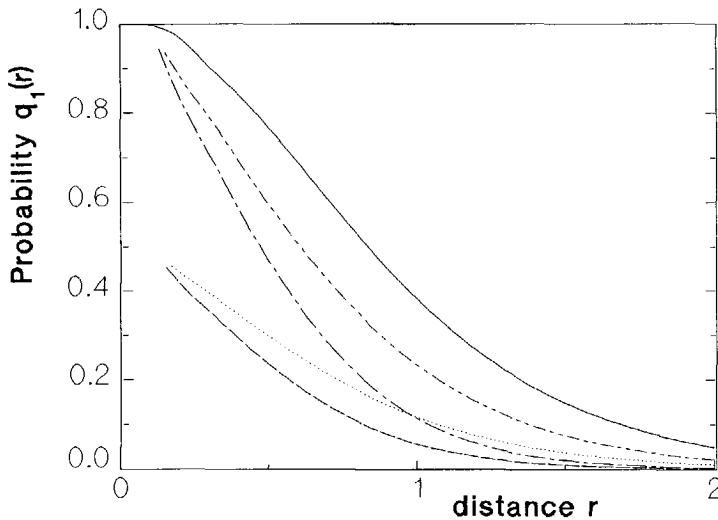


Fig. 7.11: Probability success $q_1(r)$ for a test packet in the presence of $n=1$ interfering signal versus distance r for slow fading BPSK signals. (—) receiver locked to test packet, receiver locked to interfering packet, (— · —) with and (— —) without differential encoding, random but constant phase of the local oscillator (— · · —) with and (· · ·) without differential encoding. $C/N_M = 20$ dB ($N_A \approx 0.035$). Packet length $L = 16$ bit.

For a coherent receiver locked to the test packet, BPSK and DPSK are seen to give almost equal capture probabilities. Fig. 7.11 suggests that if large synchronisation errors can occur, e.g. if the receiver locks to an interfering signal or if the receiver has an arbitrary phase, differential encoding increases the probability of correct reception by almost a factor of 2.

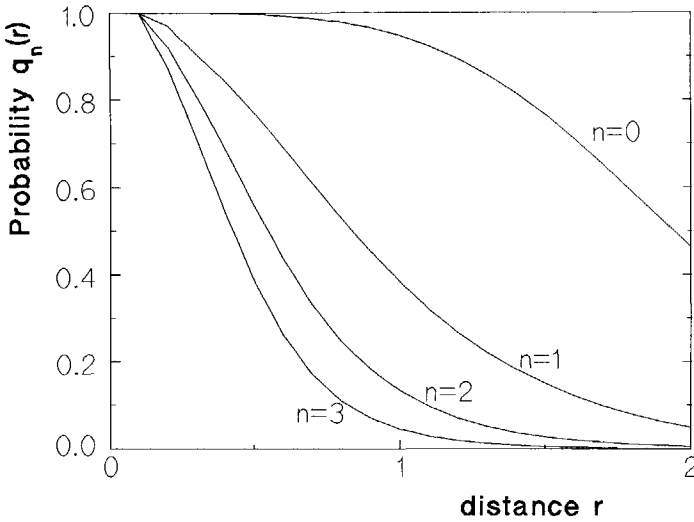


Fig. 7.12: Model BS1: Probability success $q_n(r)$ for a receiver perfectly locked to the test packet, in the presence of $n=0, 1, 2$ and 3 interfering signals, versus distance r for slow fading BPSK signals. $C/N_M=20$ dB ($N_A \approx 0.035$). Packet length $L = 16$ bit.

We now postulate five capture models, based on the three idealised cases of carrier synchronisation in Fig. 7.6 - Fig. 7.8.

MODEL BS1: Analogous to model BF1 and BSG1, perfect carrier synchronisation is always assumed for the test packet. We apply (7.32) for each of the signals in the collision. Particularly if two signals with approximately equal power are competing, this assumption is optimistic. Results for $q_n(r)$ ($n=1, 2$ and 3) in Fig. 7.12 are substantially more optimistic than the results for Gaussian interference in Fig. 7.10.

MODEL BS2: The receiver locks to any one of the $n+1$ contending signals. Capture occurs if and only if the receiver locks to the test packet *and* the detected bit sequence is identical to the bit sequence of the test packet. The capture probability is found from (7.34), (7.39) and (7.41); numerical results for $q_n(r)$ are a factor $(n+1)$ smaller than results from model BS1.

The models BS3- BS5 address the effect of inaccuracies in receiver synchronisation with respect to the phase of the test packet, caused by interfering signals. Extreme cases are considered: the receiver is assumed to lock perfectly to the carrier of one of the $n+1$ signals, but this does not exclude other packets from being received correctly. The probability of correct detection of the test packet is evaluated for events of reception with and without carrier phase offset. To this end we use (7.41) to (7.44). Carrier synchronisation to an interferer may be reasonable in some unslotted networks: the receiver may maintain carrier synchronisation to the initiating packet. The capture probability for this packet is found from

$$q_n(r_j | H_1) = q_n(r_j | \theta_r = \theta_j) \quad (7.45)$$

with (7.41), whereas for each successive (test) packet (j) in the vulnerable period, the capture probability is taken to have the form

$$q_n(r_j | H_d) = q_n(r_j | \theta_r = \theta_1) \quad (7.46)$$

with (7.43), where θ_1 denotes the phase of the initiating packet.

MODEL BS3: The receiver locks to one of the $n+1$ BPSK signals. The probability that one of $n+1$ packets is received correctly is $C_{n+1} = q_n(r_j | \theta_r = \theta_j) + nq_n(r_j | \theta_r = \theta_k, k \neq j)$.

MODEL BS4: This model is identical to BS3 except that we assume coherent detection of differentially encoded BPSK.

MODEL BS5: The carrier phase reference of the coherent detector is constant but arbitrary, i.e., it is not related to the phase of any of the $n+1$ signals. With DPSK, the probability of correct packet reception follows from (7.44) and $C_{n+1} = (n+1) q_n$.

The models *BS3-BS5* require constant carrier phases, unaffected by mobile fading or by differences in carrier frequencies during a packet time ($T_5 < T_6$). Consequently, these models become inappropriate with fast fading. A comparison of these five models is given in Fig. 7.21 for $Q(r)$ in the event of nonpersistent ISMA.

7.7 COMPUTER SIMULATION

Rayleigh fading in a narrowband mobile radio channel has been simulated by splitting the signal into an in-phase component, on the form $\kappa_i \cdot \cos(\omega_c t)$ and a quadrature component, on the form $\kappa_i \cdot \sin(\omega_c t)$, and multiplying these components by independent Gaussian variables ζ_i and ξ_i , respectively. The factors ζ_i and ξ_i are low-pass filtered according to the Doppler spectrum in a typical mobile channel [32]. The block diagram of the fading generator as implemented by Pluymers [9] in computer software is given in Fig. 7.13.

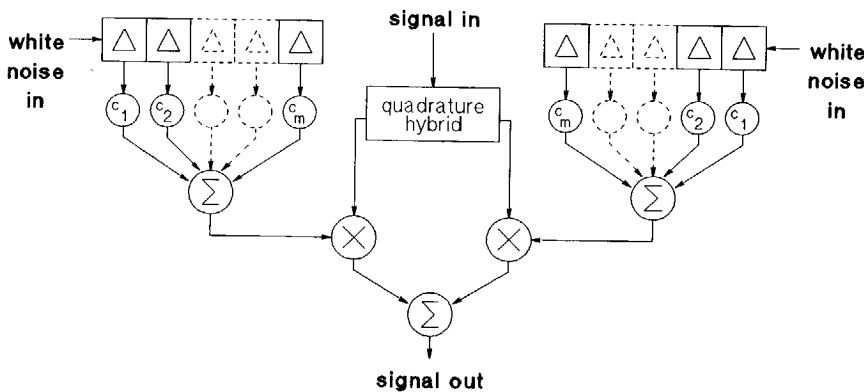


Fig. 7.13: Channel simulator for frequency nonselective Rayleigh fading, based on tapped-delay line filters.

Packets of $L=16$ bit with DPSK modulation with a bit rate of $r_b=16$ kbit/s on a carrier frequency of $f_c=900$ MHz are considered. The speed of all vehicles is 20 km/h ($v=$

5.55 m/s). Hence, the maximum Doppler shift is $f_m = v f_c / c = 16.6$ Hz, so the Rayleigh fading is relatively slow compared to a packet duration of $T_s = 1$ ms ($f_m T_s = 0.0166$). For the quasi-uniform distribution of the interfering traffic, the probability $q_n(r_i)$ of successful transmission of a test packet from r_i in the presence of $n=1$ and 2 interfering packets is depicted in Fig. 7.14 (marked by Δ and $\square$, respectively).

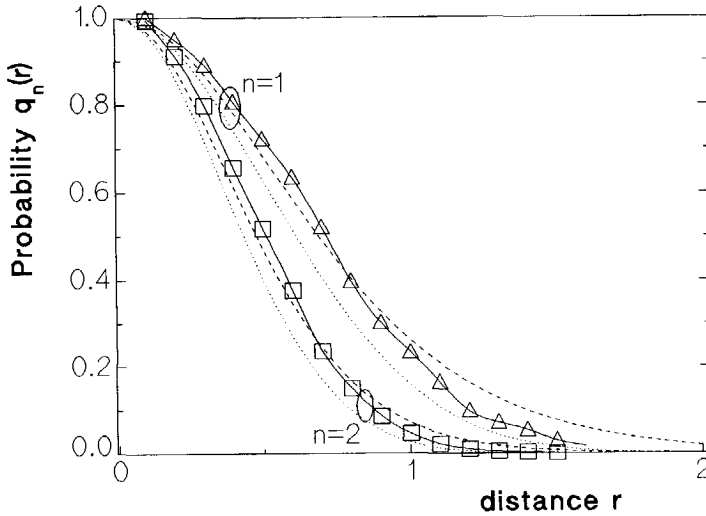


Fig. 7.14: Probability of capture $q_n(r)$ versus distance r for non-coherent detection of DPSK. Packet length $L=16$ bit at a bit rate of $r_b=16$ kbit/s. Δ : one interferer; $\square$: two interferers. Vehicle speed 20 km/h ($v=5.55$ m/s). Carrier frequency $f_c=900$ MHz. Median C/N is 20 dB ($N_A \approx 0.035$). Corresponding results for capture-ratio model (—) and for the non-fade model ($\cdots$), assuming $z=1$ (0 dB).

The capture probability found from the simulation (see Fig. 7.14) is lower than the mathematical results from the model for transmitter success for coherent detection of BPSK (see Fig. 7.12). Possible explanations can be that (i) non-coherent detection is simulated, whereas coherent detection is considered in (7.41), and (ii) despite the slow terminal speeds, signal levels still fluctuate during a packet.

In Fig. 7.14, simulation results are compared with model A3 and A4 with receiver threshold $z=1$ ('perfect capture'). Both models appear relatively pessimistic for $r < 0.7$, but model A3 is relatively optimistic for $r > 0.8$. A possible explanation of the high

simulated probabilities of capture for $r < 0.7$ might be the assumption that the receiver is in perfect bit-synchronisation with the test packet, with non-aligned bit timing for interfering signals.

7.8 COMPARISON OF MODELS AND TOTAL THROUGHPUT

In the previous sections, various models to compute $q_n(r)$ have been proposed. The probabilities $q_n(r)$ have been inserted in expressions for the probability $Q(r)$ of a successful transmission for various protocols, as derived in chapter 6. Next, the models *A2*, *A3*, *A4*, *BF1*, *BSG1* and *BS1* are compared in Fig. 7.15 - Fig. 7.20. The (local-mean) signal-to-noise ratio for a signal from r_M is taken 20 dB ($C/N_M = 20$ dB), thus $N_A \approx 0.035$ ($\beta = 4$). The offered traffic is quasi-uniformly distributed with $G_t = 1$ ppt. Results are indicated by solid lines (—) for slotted ALOHA, by broken lines (— —) for unslotted nonpersistent ISMA, and by dots (· · ·) for unslotted p -persistent ISMA

The effect of packet length appears significant (compare Fig. 7.16 and Fig. 7.17); appropriate system design may require careful selection of the number of bits per packet and the bit rate. Longer packets experience a larger probability of being lost because of channel fades. On the other hand, shortening packet duration by increasing r_b may lead to intersymbol interference, may require larger guard times to ensure 'clean' slotted arrival of packets from various locations and may also increase the relative duration of propagation delays of feedback signals in ISMA.

If a capture model assumes constant received signal amplitudes, the event of a deep fade of the interference is accounted for as a contribution to $q_i(r)$: Model *A3* appeared more optimistic than model *A4* with $T_s \rightarrow 0$. However, capture of a test packet during fades of the interference is possible only with extremely short packet lengths (see section 4.10).

For a slow fading channel, the Gaussian approximation of the interference signal is inappropriate for $n=1$: $q_1(r)$ is underestimated by more than 30% for $r > r_M$. For $r > 1$ the models *BS1* and *BSG1* differ by more than a factor 2. Based on observations to be discussed in section 7.9, it is the impression of the author that if error correction coding is applied, the differences between the exact model *BS1* and the Gaussian approximation *BSG1* may become less significant.

For any n , the probability of transmitter success in a fast fading channel is seen to decrease rapidly beyond a certain distance. Such a 'knee' is less apparent for slow fading: distant terminals then experience a relatively high probability of success. With slow Rayleigh fading, a packet from a distant terminal might, under certain circumstances, even survive a collision with a packet from a more nearby terminal. However, if the slow fading channel is evaluated at a receiver threshold of $z=4$, such high probabilities of capture are not confirmed. This indicates that in (7.41) a substantial portion of the successful packets captures the receiver at remarkably low C/I -ratios, whereas a practical narrowband receiver may lose synchronisation in such case. Results for a capture ratio of few dB, to account for a synchronisation margin, may be more realistic. This suggests that considering packet error rates for a perfectly synchronised receiver are presumably optimistic.

Slotted ALOHA is seen to result in the most significant unfairness because of prevailing packets from nearby users. In contrast to this, nonpersistent ISMA without delay ($d=0$) provides a uniform chance of access for all terminals, although $Q(r)$ degrades for terminals beyond $r=1$ because of noise. A signalling delay ($d>0$) is known to degrade the average performance of ISMA networks for receivers with capture [24], [27], [29] and without capture [39]. Nonetheless, it appears that nearby users benefit from a signalling delay. This is explained by a high probability of capture for strong packets arriving during the inhibit signalling delay.

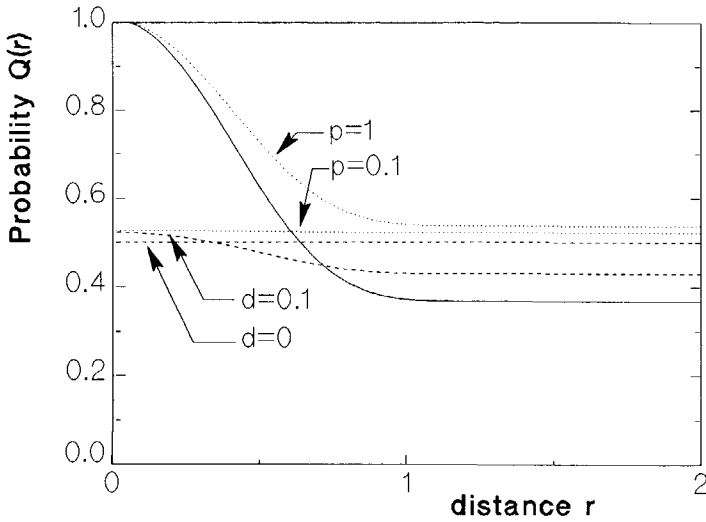


Fig. 7.15: Model A2: 'Vulnerability circle'. Probability $Q(r)$ of successful transmission versus distance r for (—) slotted ALOHA, (— —) unslotted nonpersistent ISMA, and (···) unslotted p -persistent ISMA. vulnerability parameter $c_z = \sqrt{2}$ ($c_z^4 = z = 4$; 6dB)

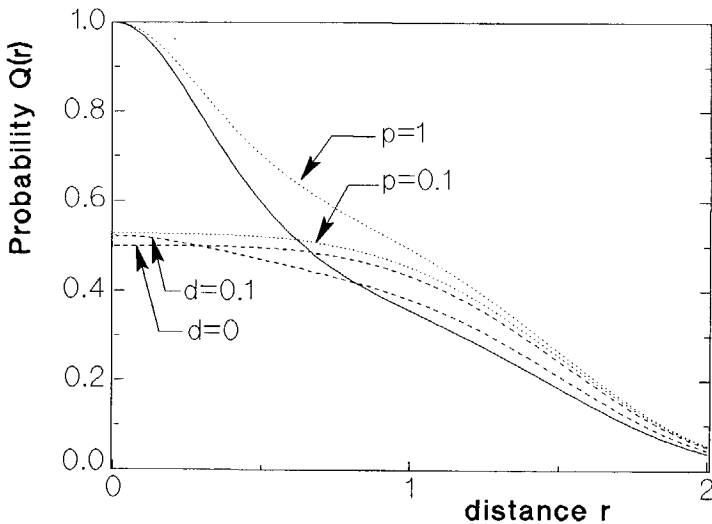


Fig. 7.16: Model A3: 'Capture-ratio for slow fading'. Probability $Q(r)$ of successful transmission versus distance r . Receiver threshold $z = 4$ (6 dB).

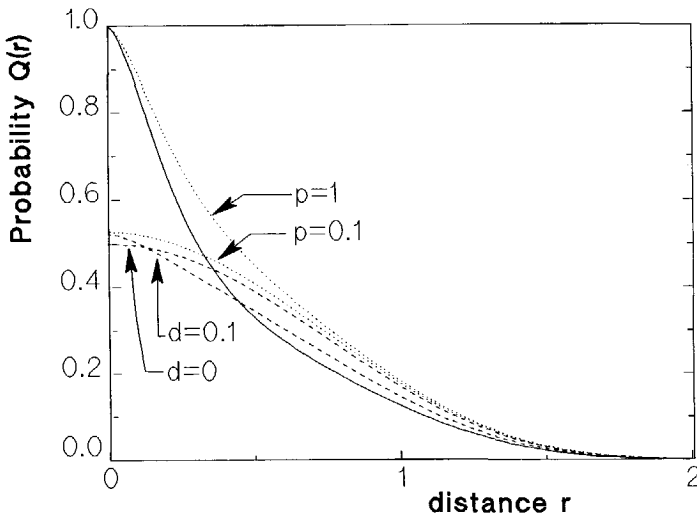


Fig. 7.17: Model A4: 'Non-fade duration.' Probability $Q(r)$ of successful transmission versus distance r . Receiver threshold $z=4$ (6 dB). Normalised packet duration $f_m T_s = 1$

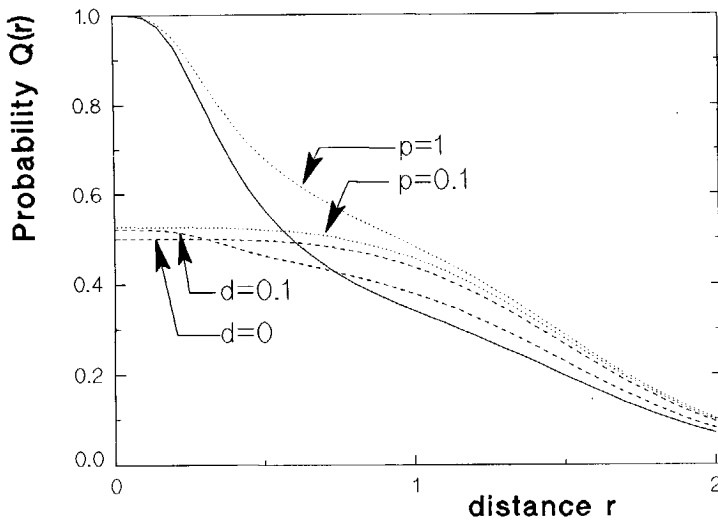


Fig. 7.18: Model BF1: 'Transmitter success for fast fading'. $Q(r)$ versus distance r . Packet length $L=16$ bit.

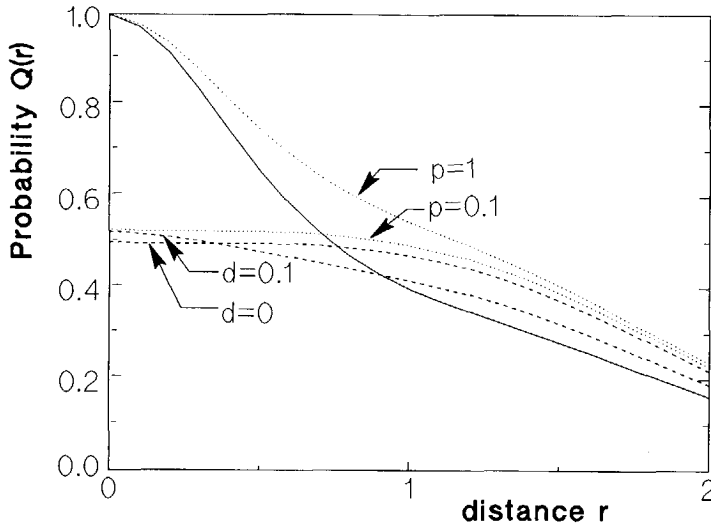


Fig. 7.19: Model BSG1: 'Transmitter success for slow fading with Gaussian interference'. $Q(r)$ versus distance r . Packet length $L=16$ bit.

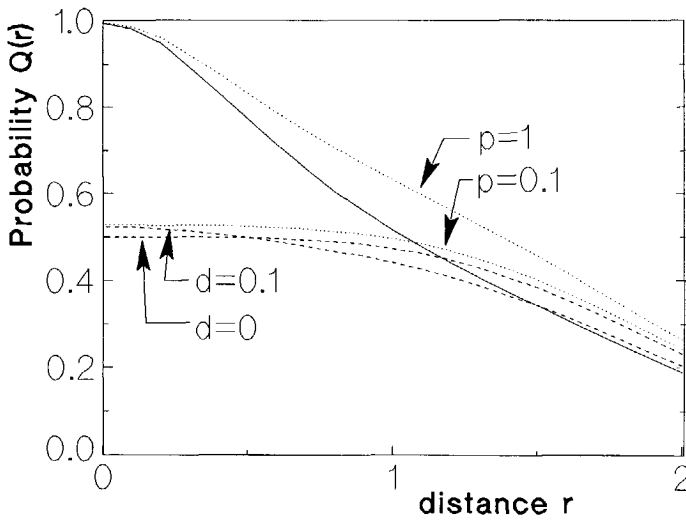


Fig. 7.20: Model BS1: 'Transmitter success for slow fading'. $Q(r)$ versus distance r . Packet length $L=16$ bit.

For nonpersistent ISMA in slow-fading channels, the probability of a successful transmission according to the models *BS1-BS5* are compared in Fig. 7.21.

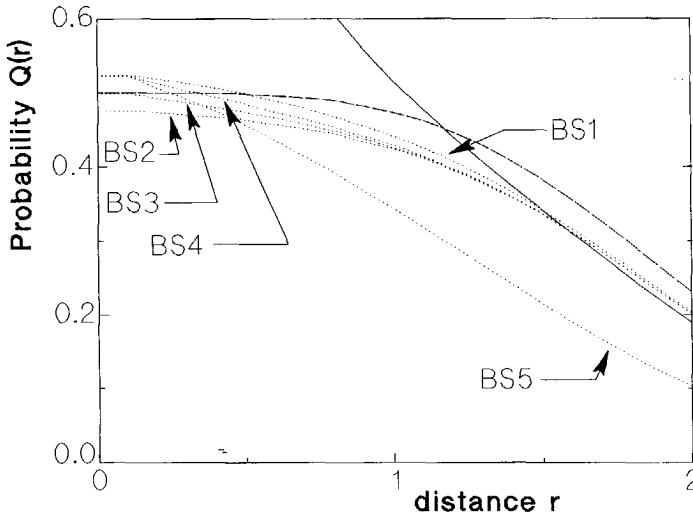


Fig. 7.21 Probability of transmitter success $Q(r)$ versus distance r for nonpersistent ISMA ($\cdots$) in a slow fading channel, Comparison of model *BS1-BS5*. Inhibit signalling delay $d=0.1$. (—) Slotted ALOHA according to Model *BS1*, and (---) nonpersistent ISMA without delay ($d=0$). Quasi-uniform offered traffic with $G_t=1$ ppt. Median C/N is 20 dB. Slow fading. Packet length $L=16$ bit.

For terminals near the receiver, ignoring capture in the event H_{db} , as occurs in model *BS2*, gives pessimistic estimates for the probability of successful transmission. Even if a packet that arrives during the vulnerable period fails to seize full carrier synchronisation, as e.g. is assumed in the models *BS3-BS5*, it nonetheless has a significant probability of being received correctly. Results are compared with slotted ALOHA and with ISMA without inhibit signalling delay (thus without collisions).

Results for the various capture models considered here mainly differ for small r . For weak signals arriving from remote terminals, capture is generally limited to cases with no contenders at all, so

$$Q(r) \rightarrow \frac{I}{I+B} \exp\{-d_1 G_t\} q_0(r) \quad \text{for } r \rightarrow \infty. \quad (7.47)$$

Hence, probabilities for model *BS1-BS4* converge for large r . (Gardiner and Jabbar [8] considered CSMA in a channel without capture ($q_n(r)=0$ for $n=1,..$), though with limitations by multipath fading if $n=0$. Their results correspond to (7.47) with (7.25) and $p_i=0$.)

Fig. 7.22 summarises the probability C_{n+1} that one out of $n+1$ packets captures the receiver for the models *A3*, *BF1*, *BF2*, *BS1*,...,*BS5*. If a data packet arrives without interference ($n=0$), a median C/N -ratio of 20 dB causes an average 'outage' probability of less than 4% ($q_0 \approx 0.96$) in a slow-fading channel and 8% ($q_0 \approx 0.92$) in a fast-fading channel. If interference is present ($n \geq 1$), the models *BS1-BS5* produce widely different estimates of C_{n+1} .

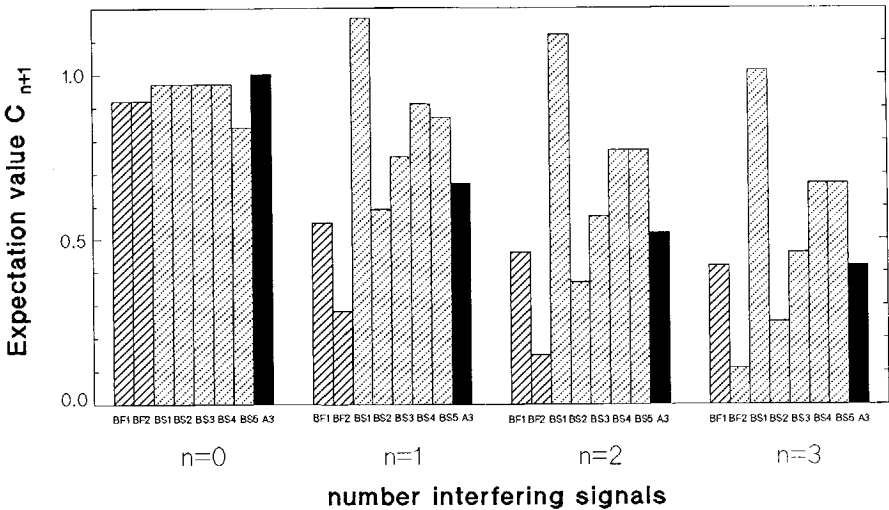


Fig. 7.22: Expectation value C_{n+1} for model *A3*, *BF1*, *BF2*, *BS1-BS5*. Quasi-uniform offered traffic. Median C/N -ratio 20 dB. Packet length $L=16$ bit. Receiver threshold (model *A3*) $z=4$.

It is seen that model *BS1* becomes unacceptable for determining C_{n+1} : it would appear far too optimistic to ignore carrier synchronisation errors during the detection of the test packet. Receiver synchronisation impairments in a practical receiver are likely to be a more significant cause of packet loss than excessive interference samples

in the coherent detector. The other extreme, model *BS2* is believed to underestimate capture performance since it ignores the fact that any narrowband receiver is likely to lock to the strongest signal rather than to a random signal. The threshold model *A3* with $z=4$ (6 dB) tends to produce results which represent a good compromise of estimates for C_{n+1} according to the all models *BS1-BS5*. For $n>1$, particularly the estimates of C_{n+1} by model *BS3* closely agree with a capture ratio of $z=4$ (6 dB).

The steady-state throughput S_t of the network in its entirety is studied in Fig. 7.23 and Fig. 7.24 for model *A3* and *BF1*, respectively. Similar curves for the models *BS1-BS5* are not available because of the large computational effort required to evaluate $q_n(r)$ for large n .

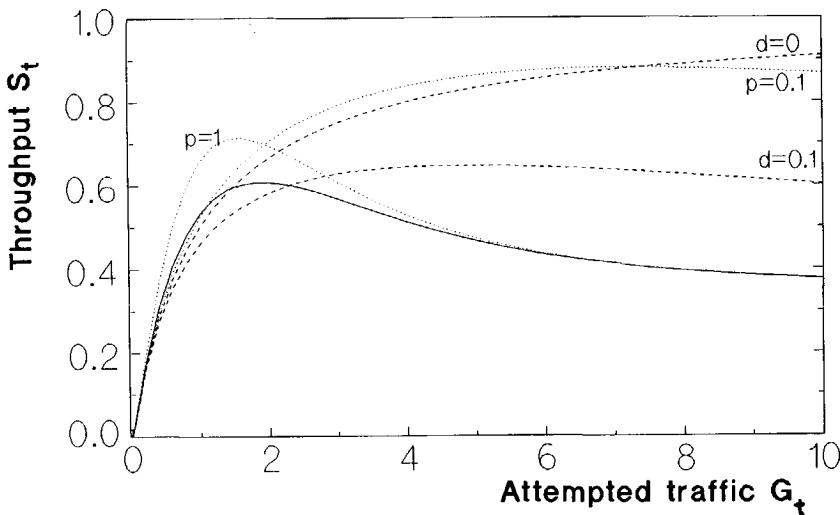


Fig. 7.23: Total throughput S_t versus G_t for fast fading model *BF1*. Packet length $L = 16$ bit. Quasi-uniform offered traffic. Median C/N -ratio 20 dB.

As seen in these Figures for low offered traffic loads ($G_t < 1$ ppt), slotted ALOHA and ISMA, have almost equal total throughput. 1-persistent ISMA yields higher throughput than nonpersistent ISMA. For reasonably high traffic loads ($3 < G_t < 10$ ppt), nonpersistent ISMA with a small signalling delay, say $d < 0.1$, outperforms slotted ALOHA

and 1-persistent ISMA. Anticipating a further discussion of model A3 in chapter 8, the theoretical limits for $G_t \rightarrow \infty$ are

$$\lim_{G_t \rightarrow \infty} S_t = \lim_{i \rightarrow \infty} i q_{i-1} = \frac{2}{\sqrt{\pi z}} \quad (7.48)$$

for slotted ALOHA and unslotted p -persistent ISMA with $0 < p \leq 1$ and without signalling delays ($d_1 = d_2 = 0$, $t_p = 0$). For nonpersistent ISMA with signalling delays,

$$S_t \rightarrow \frac{2}{\sqrt{\pi z}} \frac{1}{1 + d_1 + d_2}. \quad (7.49)$$

Thus in the limit $G_t \rightarrow \infty$, the use of feedback signalling decreases the throughput if delays occur. However, Fig. 7.24 shows that for reasonably high offered traffic, say $G_t < 10$ ppt, nonpersistent ISMA with $d_1 = d_2 = 0.1$ has a higher (steady-state) throughput than slotted ALOHA. At these traffic loads, collisions occur frequently, particularly in slotted ALOHA. Their effect is, however, significantly less disastrous than in channels without capture.

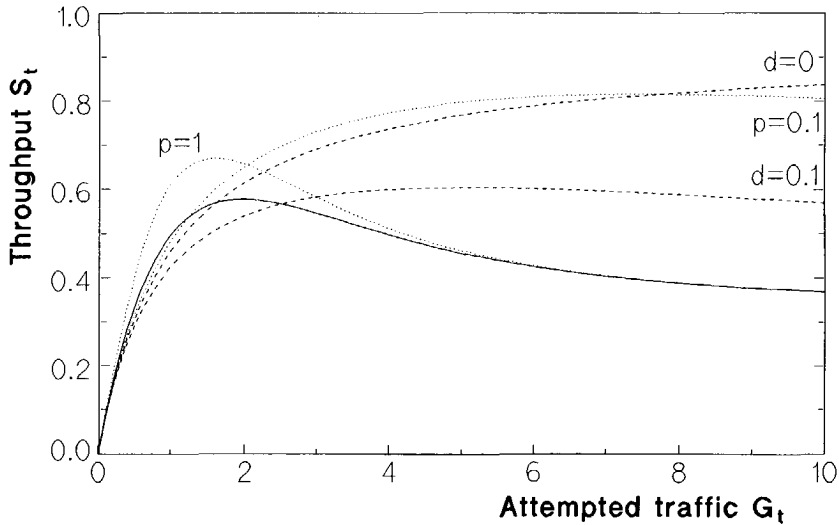


Fig. 7.24: Total throughput S_t versus offered traffic G_t for the capture ratio model A3. Receiver threshold $z=4$ (6 dB). Quasi-uniform offered traffic. Median C/N-ratio 20 dB.

7.9 DISCUSSION

Several models for the probability of successful reception have been proposed. The capture-ratio model proved a convenient technique to compute the performance of mobile random-access networks. The fact that the probability of capture is expressed in closed form suggests that refinement of the analysis, e.g. to include the effect of shadowing, might also be feasible. For the other models studied in this chapter, analysis for multiple co-existing types of fading will presumably become substantially more complicated. Nonetheless, much attention is paid in technical literature to the effects of the detection process for particular modulation and coding techniques, whereas the effect of different received power levels is often highly simplified. It is the author's strong impression that this attention, though appropriate for wideband spread-spectrum systems with adaptive power control, may not result in a realistic assessment of the performance of systems with narrowband channel with multipath fading, shadowing and near-far effects, without power control. It was argued previously [5],[37] that adaptive power control degrades the performance of a random-access channels because it reduces the probability of capture. Moreover, power control may be difficult to implement in random access channels because each mobile terminal only occasionally transmits a data packet, so no continuous feedback information on the attenuation in the inbound channel is available.

We now propose a technique to evaluate the suitability of the capture ratio model. The probability of successful reception is on the general form

$$q_n = \int_0^\infty \int_0^\infty \dots \int_0^\infty Pr(s_j | p_j, p_1, \dots, p_n) f_{p_j}(p_j) f_{p_1}(p_1) \dots f_{p_n}(p_n) dp_j dp_1 \dots dp_n. \quad (7.50)$$

The capture-ratio model A3 simply assumes

$$Pr(s_j | p_j, p_0, \dots, p_n) = U\left(p_j - z \sum_{i=1}^n p_i\right) \quad (7.51)$$

with $U(\cdot)$ the unit step function ($U(x)=0$ if $x<0$, and 1 otherwise). For a realistic radio receiver, a smoother transition than this step function may exist, depending on the type of modulation, synchronisation capabilities and on the character of the interference. The immunity of a perfectly locking receiver to a single (constant-envelope) interfering signal leads to a very abrupt transition (see (7.39) and (7.38) with $N_0 \rightarrow 0$), so a step function near a C/I -ratio of 0 dB ($z=1$ in 7.5.1) may be considered. In practice the transition may be mainly determined by the ability of the receiver to lock on the test signal.

However, for Gaussian distributed interference, the error probability (5.10) (see also section 7.6.4) leads to a smoother transition. If an error correction code is used, that can correct up to t errors, the probability of correct reception of a data packet of L bits from terminal j (denoted as event s_j), given the instantaneous received signal power, is

$$Pr(s_j | p_j, \bar{p}_t) = \sum_{m=0}^t \binom{L}{m} (1 - P_b)^{L-m} P_b^m \quad (7.52)$$

where we assumed that bit error are statistically independent from bit to bit because of 'Gaussian' interference. Fig. 7.25b illustrates the packet success probability (7.52) versus the C/I -ratio for packets of $L=16$ bits for various t .

To find the inaccuracy arising from the capture-ratio model A3 the probability of transmitter success $Pr(s_j)$ is considered to satisfy the bounds

$$\left\{ \begin{array}{ll} 0 \leq Pr(s_j | C/I) < \epsilon_1(z_1) & C/I < z_1 \\ 0 < Pr(s_j | C/I) < 1 & z_1 \leq C/I \leq z_2 \\ 1 - \epsilon_2(z_2) < Pr(s_j | C/I) \leq 1 & C/I > z_2 \end{array} \right. \quad (7.53)$$

where $\epsilon_1(z_1)$ and $\epsilon_2(z_2)$ are small ($\epsilon_1, \epsilon_2 < 1$) and depend on the choice of z_1 and z_2 , respectively. Such bounds can be computed from (7.52) for Gaussian or constant-envelope interference using the bit error probabilities (5.10) and (7.37), respectively. For a mixed-type of interference, worst-case bounds can be defined similar to (7.53).

For slotted ALOHA, model A3 computes the performance measures $Q(r)$ and S_i from $Pr(C/I > z)$, while the probability of transmitter success $Pr(s_j)$ might be more appropriate. The unconditional probability $Pr(s_j)$ can be found by averaging $Pr(s_j | C/I)$ over the pdf of the signal-to-interference ratio:

$$Pr(s_j) = \int_0^{\infty} Pr(s_j | C/I = z_1) \frac{dPr(C/I < z)}{dz} \Big|_{z=z_1} dz_1. \quad (7.54)$$

In most cases $Pr(s_j | C/I)$ is not exactly known because the character of the interference is a stochastic variable, depending among other things on n . Nonetheless, the absolute error ϵ in $Pr(s_j)$ can be bounded by

$$\begin{aligned} |\epsilon| &\leq e_1 Pr(C/I < z_1) + e_2 Pr(C/I > z_2) + Pr(z_1 \leq C/I \leq z_2) \\ &= e_1 (1 - Pr(C/I > z_1)) + e_2 Pr(C/I > z_2) + Pr(C/I > z_2) - Pr(C/I > z_1) \\ &\leq \max(e_1, e_2) + Pr(C/I > z_1) - Pr(C/I > z_2) \quad \forall z_1 < z_2. \end{aligned} \quad (7.55)$$

Thus, the capture-ratio model may be considered to yield accurate results if the effect of the choice of z on $Q(r)$ is limited. Fig. 7.25a compares the probability of successful access $Q(r)$ for a terminal at the median distance $r=r_M=0.734$, versus the capture ratio. Channels with Rayleigh fading and quasi-uniform offered traffic (7.6) (—) and channels with Rayleigh fading only (- -) are considered. In the latter case, $Q(r) = \exp\{z(z+1)^{-1}G_i\}$ [5][18]. The total offered traffic is $G_i=0.2, 1$ and 3 ppt.

Results in Fig. 7.25 indicate that the capture-ratio model becomes increasingly appropriate with increasing packet length L and error-correction distance t . Particularly if the offered traffic is modest ($G_i < 1$), the capture ratio models gives relatively accurate results. In channels with large fluctuations of the received power the model is more accurate than in channels with Rayleigh fading only. This can be understood intuitively since $Pr(z_1 < C/I < z_2)$ is small if large fluctuations of the received signal powers occur. In channels with Rayleigh fading only, instantaneous differences in received signal power are smaller. Nonetheless, the effect of z appeared small even for channels without near-far effect and shadowing: in narrowband communication,

capture is unlikely to occur if multiple signals arrive with almost identical power, irrespective of z ($z > 1$), so $Q(r)$ is then largely determined by the probability that no contending packet is present ($Q(r) \rightarrow \Pr(n=0)$).

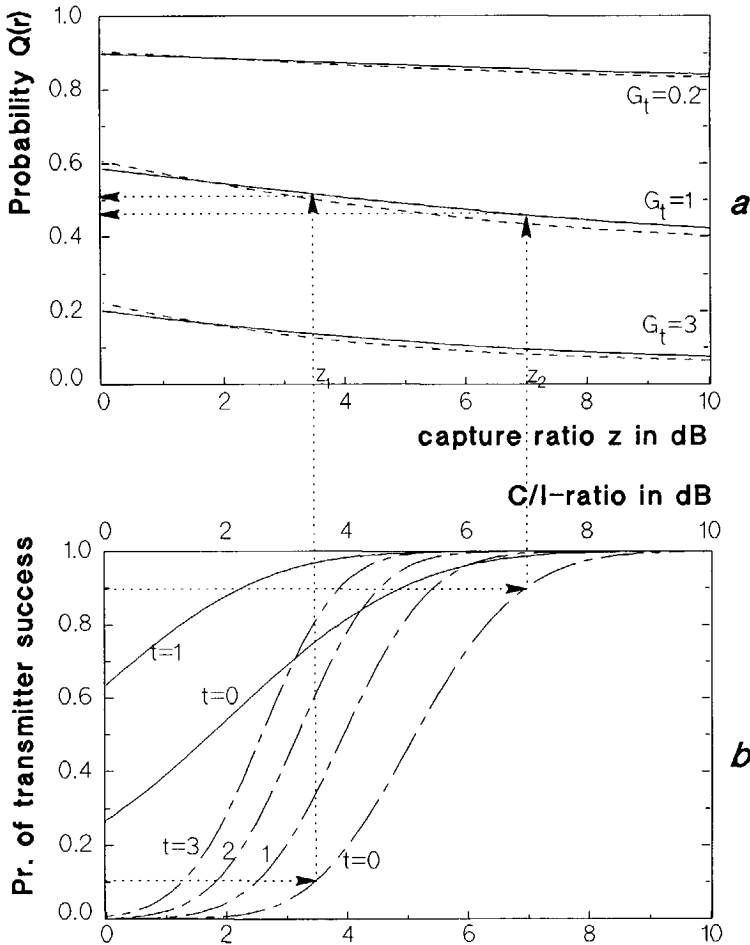


Fig. 7.25a (above): Model A3: Probability of capture $Q(r_M)$ for slotted ALOHA versus receiver threshold z . (—) Quasi-uniform offered traffic (test packet transmitted at $r=r_M \approx 0.734$), and (- -) Rayleigh fading only, with $G_t = 0.2, 1, 3$ ppt. Noise-free channel $N_A = 0$.

Fig. 7.25b (below): Probability of correct reception of a packet of $L=16$ bit (—) and $L=128$ bit (- -) versus the C/I-ratio. BPSK modulation with Gaussian interference for various error-correction distances t .

In most analyses in the next two chapters, numerical results given for $z=4$ (6 dB), which is more pessimistic than the values suggested by Fig. 7.25b. This is believed to be a more realistic value for practical receivers for BPSK-signals in narrowband channels, because imperfect synchronisation may impair the capture performance, compared to idealised model as *BSI*. For analogue FM modulation by an FFSK subcarrier, the threshold model is believed reasonable even if fluctuations in received power are relatively small, because of the classical FM capture effect. In this case, practical values for z are about 10-20 dB, corresponding to the FM threshold. Then results for $z=4$ may tend to be optimistic.

7.10 CONCLUSIONS

A large number of channel aspects influence the probability of successful packet reception. Complete investigation of the combined effects of all relevant aspects appears prohibitively complex. However, a number of simplified models have been investigated and compared.

Mobile slotted ALOHA and 1-persistent ISMA networks exhibit a significant near-far effect: packets from nearby terminals have a substantially higher probability of being received successfully than from remote terminals. This effect is much reduced in networks employing nonpersistent ISMA. However, at light offered traffic loads, slotted ALOHA and 1-persistent ISMA offer higher throughput.

The relative packet duration, compared to the rate of multipath fading, has a substantial effect on the probability of successful access in a mobile network. The performance rapidly deteriorates if the packet duration becomes larger than the time constant of the multipath fading. The throughput for fast fading channels, i.e. with

independent amplitudes for successive bits in a packet, is substantially smaller than for slow fading with constant signal amplitudes during packet reception.

The approximation of (one or multiple) contending (constant-envelope) signals by a Gaussian-distributed joint interference signal appeared pessimistic in a random-access network with narrowband channels. This seems in contrast to the fact that this approximation is frequently applied in spread-spectrum networks.

The capture-ratio model produced convenient mathematical expressions for the probability of successful reception. However, the accuracy of simplifying the detection process was also disputed. The discussion in this chapter showed that in *narrowband* mobile channels, this simplification is acceptable since relatively large fluctuations of the received signal power occur. Some approximation of the detection process in a small range of the C/I -ratio near the threshold appeared acceptable. This is a major distinction between narrowband and spread-spectrum communication; in the latter case, the capture ratio model may be unrealistic. These observations give a strong justification and motivation to study the capture-ratio model for narrowband channels in more detail in the next chapter. Comparison with models for transmitter success probabilities showed that $z=4$ may be a reasonable choice for the receiver threshold.

In slow fading channels, receiver synchronisation may become a more critical issue than bit errors caused by excessively strong interference samples. Synchronisation impairments has been studied from a number of simplified models; further investigation is recommended but is thought outside the scope of this dissertation.

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CHAPTER 8

SPATIAL DISTRIBUTION OF TRAFFIC IN MOBILE SLOTTED ALOHA NETWORKS

Chapter 7 showed that differences in received signal power largely influence the probability of capture and thus the performance of the network. A special case has been addressed: packets were assumed to be offered to the channel according to a (quasi-) uniform distribution of transmitting terminals over the service area, and shadowing was not considered. These assumptions facilitated the investigation of capture or packet-success probabilities according to various models for fast and slow fading channels. In the present chapter, the detection process will be simplified by considering a receiver capture 'threshold', but the spatial distribution of traffic over the service area and the power fluctuations caused by shadowing, the near-far effect

and Rayleigh fading will now be treated in more detail. Also, network stability and delay will be addressed.

It has been demonstrated already, particularly in chapter 3 and 7, that Laplace image functions, introduced in section 2.1.5, facilitate the analysis of the threshold model. The motivation to study these images in more detail has been the two observations that (i) Laplace images have a close analogy with 'characteristic functions' or 'moment generating functions' and fully describe the statistical behaviour of the fluctuations of the received power, in particular the linear moments of the pdf can easily be obtained once the image function is known and (ii) Laplace transforms appear explicitly in the expression for receiver capture in a Rayleigh-fading channel. In our case, inverse transformation, which is a difficult mathematical problem [2],[24], is not required.

Section 8.1 discusses the properties of the Laplace transform of the pdf of received signal power. Expressions for the probability of successful transmission and for total channel throughput are given in section 8.2 and 8.3 for incoherent and coherent cumulation of interference power, respectively. Section 8.4 studies a number of special cases of the spatial distribution of offered traffic. Stability and delay are addressed in section 8.5. Section 8.6 concludes this chapter.

8.1 LAPLACE IMAGE FUNCTIONS AND INTEGRAL TRANSFORMS OF SPATIAL DISTRIBUTION

Since Laplace images of the pdf of received interference power play a key role in the subsequent analysis, a number of properties are summarised in this section. First we introduce the image function $\psi_j(s)$, defined as the one-dimensional, one-sided Laplace image of the pdf of the *local-mean* power of the j -th signal, with

$$\psi_j(s) \triangleq \int_{0-}^{\infty} \exp\{-s\bar{p}_j\} f_{\bar{p}_j}(\bar{p}_j) d\bar{p}_j. \quad (8.1)$$

We also introduce the image function $\phi_j(s)$, defined as the Laplace transform of the *instantaneous* packet power f_{p_j} . So,

$$\phi_j(s) \triangleq \int_{0-}^{\infty} \exp\{-sp_j\} f_{p_j}(p_j) dp_j. \quad (8.2)$$

As distinct from $\psi_j(s)$, $\phi_j(s)$ thus incorporates the effect of Rayleigh fading. The image function $\phi_\sigma(s)$, defined in section 2.1.5 as the Laplace transform of the pdf of the power of a signal subjected to multipath fading and shadowing, is an example of (8.2).

A number of properties of these images are given below:

The k -th derivative of the images in the point $s=0$ equals the k -th linear moment μ_k of the original pdf except for a sign-factor $(-1)^k$, since

$$\phi_j^{(k)}(0) = \int_{0-}^{\infty} (-x)^k e^0 f_{p_j}(x) dx = (-1)^k E[p_j^k] \triangleq (-1)^k \mu_k. \quad (8.3)$$

This illustrates the relation between the Laplace transform and the 'moment-generating function'. The defining integrals of the Laplace images (8.1) and (8.2) will converge (at least) for $\text{Re}(s) \geq 0$, because any pdf integrated over its domain yields unity, and $0 \leq \exp\{-x\} \leq 1$ for x real and positive. The relations

$$0 \leq \phi_j(s) \leq \phi_j(0) = 1 \quad (8.4)$$

apply for $\text{Re}(s) \geq 0$. Similar relations hold for $\psi_j(s)$. From the definitions, it follows that the image and all even derivatives of the image of a pdf of a real, positive random variable are real, positive, and decreasing functions of their argument s along the positive real axis, i.e., for integer k and $s > 0$:

$$\begin{aligned}
 0 &< \phi_j^{(2k)}(s) < \phi_j^{(2k)}(0) & \forall s>0, k \in \mathbb{N} \\
 \phi_j^{(2k+1)}(0) &< \phi_j^{(2k+1)}(s) < 0 & \forall s>0, k \in \mathbb{N}
 \end{aligned}
 \tag{8.5}$$

Further, if $f_{p_i}(p_j)$ is bounded for $p_j \geq 0$,

$$\lim_{s \rightarrow \infty} \phi_j(s) = 0. \tag{8.6}$$

Eq. (8.6) shows that mostly the image vanishes for real s tending to infinity. The case of no interference, i.e., $f_{p_i}(p_j) = \delta(p_j)$, is an exception to this (see sect. 7.4.2).

The relation between $\psi_j(s)$ and $\phi_j(s)$ follows from Lemma I.

Lemma I

Assume two functions $f_1(x)$ and $f_2(w)$, with Laplace images, $\phi_1(q)$ and $\phi_2(p)$, respectively. Let the relation between $f_1(x)$ and $f_2(w)$ be

$$f_2(w) = \int_0^{\infty} \frac{1}{x} \exp\left\{-\frac{w}{x}\right\} f_1(x) dx. \tag{8.7}$$

The Laplace image $\phi_2(p)$ then equals

$$\phi_2(p) = \int_0^{\infty} \frac{1}{p} \exp\left\{-\frac{q}{p}\right\} \phi_1(q) dq. \tag{8.8}$$

A typical pair of $f_1(x)$ and $f_2(w)$ are the pdf's of received local-mean and instantaneous power in a Rayleigh-fading channel, respectively.

Proof

Using the definition of the Laplace transform gives

$$\begin{aligned}\phi_2(w) &= \int_0^\infty \int_0^\infty \frac{1}{x} \exp\left\{-\frac{w}{x} - wp\right\} f_1(x) dx dw \\ &= \int_0^\infty \frac{1}{x} \frac{f_1(x)}{p + \frac{1}{x}} dx.\end{aligned}\tag{8.9}$$

A substitution of $y = x + p^{-1}$ gives

$$\phi_2(p) = \int_{p^{-1}}^\infty \frac{1}{yp} f_1\left(y - \frac{1}{p}\right) dy.\tag{8.10}$$

This can be written as a integral over q , viz.,

$$\begin{aligned}\phi_2(p) &= \frac{1}{p} \int_0^\infty \int_{p^{-1}}^\infty f_1\left(y - \frac{1}{p}\right) \exp\{-yq\} dy dq \\ &= \frac{1}{p} \int_0^\infty \int_0^\infty f_1(x) \exp\left\{-q\left(x + \frac{1}{p}\right)\right\} dx dp.\end{aligned}\tag{8.11}$$

The integral over q corresponds to a Laplace transform from x into q domain, so

$$\phi_2(p) = \frac{1}{p} \int_0^\infty \phi_1(q) \exp\left\{-\frac{q}{p}\right\} dq. \quad \text{Q.E.D.}\tag{8.12}$$

We now address the relation between the image functions $\phi(s)$ and $\psi(s)$. Eq. (8.8) can be rewritten as

$$\phi_f(s) = \int_0^\infty \psi_f(sx) e^{-x} dx\tag{8.13}$$

Generalizing, the k -th derivative of $\phi_j(s)$ is

$$\phi_j^{(k)}(s) = \int_0^{\infty} x^k e^{-x} \psi_j^{(k)}(xs) dx. \quad (8.14)$$

In the limit $s \rightarrow 0$ and applying integral formula (3.351.3) in [3], this becomes

$$\phi_j^{(k)}(0) = \psi_j^{(k)}(0) \int_0^{\infty} x^k e^{-x} dx = k! \psi_j^{(k)}(0), \quad (8.15)$$

provided that the limit operation ($s \rightarrow 0$) and integration may be interchanged. A formal proof of the existence of the limit (8.15) is not given as the same result can be obtained by comparing the derivatives $\phi_j^{(k)}(0)$ and $\psi_j^{(k)}(0)$ [4]. Equation (8.15) is a trivial result for $k=0$, and $k=1$, but it shows that higher order linear moments v_k of the instantaneous power ($v_k \Delta E[p_j^k]$) equal $k!$ times the linear moments μ_k of the pdf of the local-mean power ($\mu_k \Delta E[\bar{p}_j^k]$). This property was used by Prasad in [6] to compute the moments of the pdf of a signal with combined shadowing and Rayleigh fading. Furthermore, we conclude that the image $\phi_j(s)$, which at first glance seems only 'an unfocused picture' of $\psi_j(s)$ ('blurred' by the spreading of the received power caused by Rayleigh fading), does uniquely specify all moments of the mean power pdf. Moreover, as seen via a Taylor expansion of $\psi_j(s)$, $\phi_j(s)$ also uniquely specifies the pdf of the local-mean interference power

$$\psi(s) = \sum_{k=0}^{\infty} \mu_k \frac{(-s)^k}{k!} = \sum_{k=0}^{\infty} v_k \frac{(-s)^k}{(k!)^2}, \quad (8.16)$$

provided that the two series converge.

8.1.1 IMAGE FUNCTIONS IN CHANNELS WITHOUT SHADOWING

In channels without shadowing ($\sigma_s=0$), the local-mean power is assumed to be normalised on the form

$$\bar{p}_j = \bar{\bar{p}}_j = r_j^{-\beta} . \quad (8.17)$$

If the position of the terminal is unknown, the pdf of its mean power is found by transformation of the stochastic variable from the r (normalised distance) domain into the $\bar{p}$ (normalised power) domain

$$\left| f_{\bar{p}_j}(\bar{p}_j) d\bar{p}_j \right| = \left| \frac{2\pi r_j G(r_j)}{G_t} dr_j \right| . \quad (8.18)$$

In this case, the image $\psi_j(s)$, together with the multiplicative factor G_t for the total offered traffic, uniquely describes the spatial distribution of the packet traffic offered to the channel. We do not consider any specific information about individual (interfering) signals, so we may assume that the pdf of received power is identical for each terminal. Therefore, we omit the index j in the notation of the images in this case.

The image $\psi(s)$ can be obtained directly from the spatial distribution of the offered traffic $G(r)$ by substituting (8.18) in the definition (8.1), so

$$G_t \psi(s) = \int_0^{\infty} 2\pi r G(r) \exp\{-sr^{-\beta}\} dr . \quad (8.19)$$

If we denote this transform pair as $G(r) \leftrightarrow G_t \psi(s)$, standard properties of the Laplace transform can be rewritten as

$$\begin{aligned}
G(r) &\rightsquigarrow G_t \psi(s) \\
aG(r) &\rightsquigarrow a G_t \psi(s) \\
-r^{-\beta} G(r) &\rightsquigarrow G_t \psi'(s) \\
G(r) \exp(-\nu r^{-\beta}) &\rightsquigarrow G_t \psi(s+\nu)
\end{aligned} \tag{8.20}$$

Examples of image functions of typical spatial distributions of the offered traffic have been given by the author in his master's thesis and in [4]:

Model	$G(r)=$	$G_t \psi(s)=$
ring	$\frac{1}{2\pi} \delta(r-1)$	e^{-s}
quasi-uniform	$\frac{1}{\pi} \exp(-\frac{\pi}{4} r^2)$	$e^{-\sqrt{\pi s}}$
uniform	$\frac{1}{\pi}$ for $0 < r < 1$	$e^{-s} - \sqrt{\pi s} \operatorname{erfc} \sqrt{s}$
circular belt	$\frac{2}{\pi r^6} \exp(-r^{-4})$	$\frac{1}{s+1}$

(8.21)

It is seen that the images of the second and third example have an unbounded derivative at $s=0$ ($\psi'(s) \rightarrow \infty$ if $s \downarrow 0$). This is caused by unbounded mean received powers: Any spatial distribution with non-zero offered traffic infinitely close to the receiver, i.e. with $G(r)=G_\epsilon > 0$ for $r \rightarrow 0$, has unbounded moments, since

$$\begin{aligned}
\mu_k &= (-1)^k \psi^{(k)}(s) \\
&= \int_0^\infty 2\pi r^{1-k\beta} \frac{G(r)}{G_t} dr \\
&> 2\pi \frac{G_\epsilon}{G_t} \int_0^\epsilon r^{1-k\beta} dr
\end{aligned} \tag{8.22}$$

if ϵ is sufficiently small to assume $G(r)$ to be constant for $0 < r < \epsilon$. This integral diverges for $k \geq 1$ if $\beta \geq 2$. This result indicates that with distributions of the offered traffic that appear uniform at least in the vicinity of the receiver, the unconditional mean-received power is unbounded ($\mu_1 \Delta E[\bar{p}_j] \rightarrow \infty$). As will be shown later, this

significantly affects the estimate of the throughput of random-access nets at high traffic loads.

On the other hand, assuming all traffic to be generated beyond a distance of at least ϵ , with $0 < \epsilon < 1$, the moments increase no faster than exponentially with k , since

$$\begin{aligned}\mu_k &= \int_{\epsilon}^{\infty} 2\pi r^{1-k\beta} \frac{G(r)}{G_t} dr \\ &\leq \epsilon^{-k\beta} \int_{\epsilon}^{\infty} 2\pi r \frac{G(r)}{G_t} dr = \epsilon^{-k\beta} .\end{aligned}\quad (8.23)$$

We conclude this section showing that for channels without shadowing any of the functions $G(\cdot)$, $f_{\bar{p}}(\cdot)$, $f_p(\cdot)$, $\phi(\cdot)$ or $\psi(\cdot)$ uniquely specifies all other functions, except possibly for the multiplicative factor G_t . In the case of channels without shadowing, the image $\phi(s)$ is found as

$$\phi(s) = \int_0^{\infty} \int_0^{\infty} \frac{1}{p_j} \exp\left\{-sp_j - \frac{p_j}{p_j}\right\} f_{\bar{p}_j}(\bar{p}_j) d\bar{p}_j dp_j . \quad (8.24)$$

Similar to (8.19), for channels without shadowing ($\sigma_s=0$), the definition of $\phi(s)$ as an integral transform of the received power can also be stated in terms of an integral transform of the spatial distribution $G(r)$, viz.,

$$\phi(s) = \int_0^{\infty} \int_0^{\infty} 2\pi \lambda^{\beta+1} \exp\{-(\lambda^{\beta}+s)w\} \frac{G(\lambda)}{G_t} d\lambda dw . \quad (8.25)$$

Interchanging the order of integration yields

$$\phi(s) = \frac{1}{G_t} \int_0^{\infty} \frac{r^{\beta}}{r^{\beta}+s} 2\pi r G(r) dr . \quad (8.26)$$

We have now established the relations illustrated in Fig. 8.1, where arrows indicate definitions by the equations quoted. It is seen that any distribution or image defines all other functions.

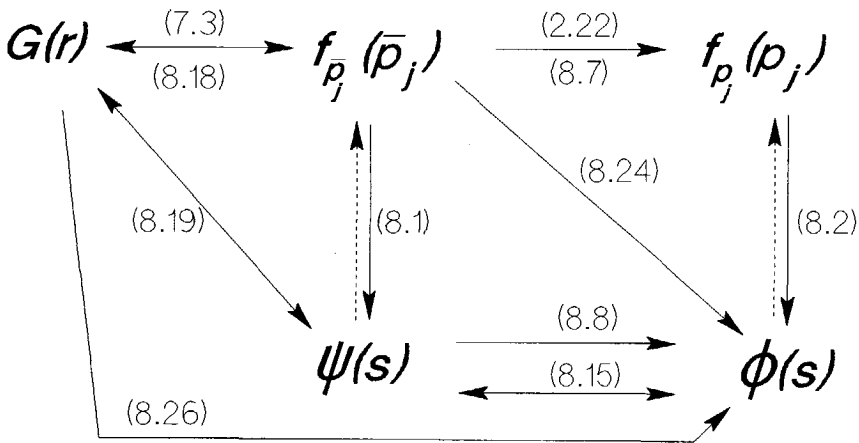


Fig. 8.1: Relations between pdf's and image functions.

After this general introduction to image functions, these images will now be used to compute the probability of capture in Rayleigh-fading channels. Coherent and incoherent cumulation of interference signals are distinguished.

8.2

CAPTURE PROBABILITY FOR COHERENT CUMULATION

Coherent cumulation was discussed in chapter 3. If interference signals would add coherently, the amplitude of the joint signal of n Rayleigh-fading signals is again Rayleigh distributed with mean $\bar{p}_i = \Sigma \bar{p}_i$. Hence, the conditional probability that a

signal from terminal j captures receiver A (event A_j) is

$$\begin{aligned} Pr(A_j|\bar{p}_j, n) &= \mathcal{L} \left\{ f_{p_t}(p_t|n), \frac{z}{p_j} \right\} \\ &= \mathcal{L} \left\{ \int_0^{\infty} \frac{1}{p_t} \exp\left(-\frac{p_t}{p_j}\right) f_{p_t}(\bar{p}_t|n) d\bar{p}_t, \frac{z}{p_j} \right\}. \end{aligned} \quad (8.28)$$

Assuming a Poisson distributed number of interfering signals and using lemma I, we find

$$Pr(A_j|\bar{p}_j) = \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} \int_0^{\infty} \frac{\bar{p}_j}{z} \exp\left(-\frac{q\bar{p}_j}{z}\right) \psi(q|n) dq, \quad (8.29)$$

where we used $\psi(s|n)$ to denote the Laplace image of the conditional pdf of the joint local-mean interference power, given the presence of n interfering signals. This joint power is the sum of the local mean powers of the individual contributing signals, so the image of the pdf of the local-mean power of the joint signal $\psi(q|n)$ equals $\psi''(q|1)$. If one substitutes $x = q\bar{p}_j/z$ and uses the series expansion of the exponential function, (8.29) becomes

$$\begin{aligned} Pr(A_j|\bar{p}_j) &= \int_0^{\infty} \exp\left\{-x - G_t + G_t \psi\left(z \frac{x}{\bar{p}_j}\right)\right\} dx \\ &= r_j^{-\beta} \int_0^{\infty} \exp\left\{\lambda r_j^{-\beta} - G_t + G_t \psi(z\lambda)\right\} d\lambda, \end{aligned} \quad (8.30)$$

where we used (8.17) to derive the expression on the second line of (8.30). This can be evaluated numerically by the Gauss-Laguerre integration method [1]. In this approximation, the probability of success $Q(r_j)$ is expressed as

$$Q(r_j) \triangleq Pr(A_j|\bar{p}_j = r_j^{-\beta}) = \sum_{k=1}^m w_k \exp\{-L(zr_j^{\beta} x_k)\} + R_m. \quad (8.31)$$

Here, w_k are weight factors in the sample points x_k for an M -point quadrature, R_m denotes the remainder, and $L(s)$ is defined as

$$\begin{aligned}
 L(s) &\triangleq -G_t + G_t \psi(s) \\
 &= \int_0^{\infty} [\exp\{-s\lambda^{-\beta} - 1\} - 1] 2\pi r G(r) dr .
 \end{aligned}
 \tag{8.32}$$

The author showed in [4] that this result can be used to calculate the throughput for an arbitrary distribution of $G(r)$. Specific properties of the performance of typical channels with coherent cumulation will be addressed in later sections.

The probability of successful transmission q_n , conditional on the number of interferers n , but averaged over all expected positions of the terminal transmitting the test signal is

$$\begin{aligned}
 q_n &= \int_0^{\infty} 2\pi r G(r) r^{-\beta} \int_0^{\infty} \exp\{\lambda r^{-\beta}\} \psi(z\lambda | n) d\lambda dr \\
 &= - \int_0^{\infty} \psi'(\lambda) \psi^n(\lambda z) d\lambda ,
 \end{aligned}
 \tag{8.33}$$

where we used (8.19) and (8.20). Numerical examples will be discussed in section 8.2.3 and 8.2.4. In the special case of a perfect-capture receiver, i.e., if $z=1$ (0dB), one finds

$$q_n = \int_0^1 \psi^n d\psi = \frac{1}{n+1}
 \tag{8.34}$$

for any spatial distribution of the offered traffic.

8.2.2 IMAGE OF THE SPATIAL DISTRIBUTION OF THE THROUGHPUT

Analogous to the image $\psi(s)$ of received power of *attempted* packets, a Laplace image function of the received signal power of *successful* packets can be defined. For channels without shadowing, we define

$$\begin{aligned}
 \chi(s) &\triangleq \mathbb{E}\left\{f_{\bar{p}_j}(\bar{p}_j|A_j, s)\right\} \\
 &= \frac{1}{S_t} \int_0^{\infty} 2\pi r S(r) \exp\{-sr^{-\beta}\} dr,
 \end{aligned} \tag{8.35}$$

where A_j denotes that test packet j captures receiver A . Inserting (8.30) gives

$$\begin{aligned}
 S_t \chi(s) &= \int_0^{\infty} 2\pi r r^{-\beta} G(r) \Pr(c_0 | \bar{p}_0 = r^{-\beta}) dr \\
 &= \int_0^{\infty} 2\pi r r^{-\beta} G(r) \exp\{-(s+\lambda)r^{-\beta} - G_t + G_t \psi(z\lambda)\} dr \\
 &= -G_t \exp(-G_t) \int_0^{\infty} \psi'(\lambda+s) \exp\{G_t \psi(z\lambda)\} d\lambda.
 \end{aligned} \tag{8.36}$$

The last step is based on the properties summarised in (8.20). In the special case of a perfect-capture receiver, i.e., a receiver with a capture ratio of $z=1$ (0 dB), the integral can be solved for $s=0$. One finds

$$S_t = -\exp\{-G_t(1-\psi(\lambda))\} \Big|_{\lambda=0}^{\infty} = 1 - \exp\{-G_t\} \tag{8.37}$$

Since the number of packets in a time slot is assumed to be Poisson distributed, precisely the part $1-\exp\{-G_t\}$ of all slots is occupied by one or more packets. Thus all non-empty slots *on the average* contribute one successful packet. However, (8.37) does not guarantee that every occupied slot contributes exactly one received packet: some occupied slots may contribute none, while other slots might, in principle, contribute, more than one packet. Examples of the latter event can easily be found. For instance, Fig. 8.2 depicts three signals, each on the form $v_i(t) \approx \cos(\omega_c t + \theta_i + \phi_{\Delta} \kappa_i)$, with $i=1, 2$ or 3 , with small phase modulation index ($\phi_{\Delta} \ll 1$) to ensure coherent cumulation. In Fig. 8.2, $\theta_1 \approx 0$, $\theta_2 \approx 0$ and $\theta_3 \approx \pi$ rad., so signal $v_1(t)$ and $v_2(t)$ satisfy the condition for receiver capture, because $v_2(t) - v_3(t) \approx 0$ and $v_1(t) - v_3(t) \approx 0$, respectively.

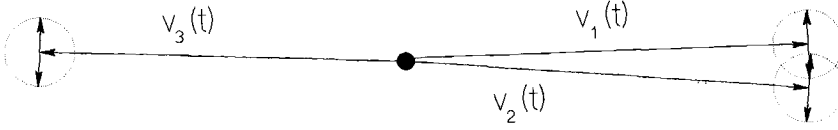


Fig. 8.2: Phasor diagram of the typical event (for coherent cumulation) that more than one signal can capture the receiver.

An asymptotic expansion for high traffic loads can be obtained by integration by parts,

$$S_r \chi(s) = -e^{-G_r} \int_0^{\infty} \frac{\Psi'(\lambda + \nu)}{z \Psi'(z\lambda)} \frac{d}{d\lambda} \exp\{-G_r \Psi(z\lambda)\} d\lambda. \quad (8.38)$$

This technique, and conditions for its application are discussed in [15]. Repeated integration by parts of (8.38) is worked out in [4]. An asymptotic expansion of $\chi(s)$ has been found on the form of

$$S_r \chi(s) = -\frac{\Psi'(s)}{z\mu_1} - \frac{1}{G_r} \left[\frac{\Psi''(s)}{z^2 \mu_1^2} + \frac{\mu_2}{\mu_1^3} \frac{\Psi'(s)}{z} \right] + \dots \quad (8.39)$$

with μ_k the k -th linear moment of the local mean-power of received packets. If $s=0$ is inserted, one finds

$$S_r = \frac{1}{z} + \frac{1}{G_r} \frac{\mu_2}{\mu_1^2} \frac{z^{-1}}{z^2} + \dots \quad (8.40)$$

provided that the moments (8.3) of the pdf of received power exist. It is seen that, for $G_r \rightarrow \infty$, $S_r \rightarrow 1/z$. This is a generalisation of the limit $S_r \rightarrow 1/z$ found by Arnbak and v. Blitterswijk [5] for Rayleigh-fading channels without near-far effect. So, for any realistic spatial distribution of the offered traffic, the throughput does not decrease to zero for $G_r \rightarrow \infty$, if interfering signals would indeed add coherently. The limit z^{-1} is

approached increasingly rapid if the spread μ_2 of the received signal powers is small. If moments are unbounded ($\mu_k \rightarrow \infty$), e.g. in the case of the (quasi-) uniform distribution (8.21), different limits can occur, as in [5].

8.2.3 RING MODEL OF OFFERED TRAFFIC

If all signals arrive with identical mean power ($\bar{p}_j = 1$), say from mobiles moving on a ring around the central receiver, the image function $\psi(s)$ equals $\exp\{-s\}$. The probability q_n of successful reception (see (8.30)) becomes

$$q_n = \int_0^{\infty} \exp\{-(nz+1)\lambda\} d\lambda = \frac{1}{nz+1} \quad (8.41)$$

which agrees with results reported by Arnbak and Van Blitterswijk [5]. Using the techniques embodied in (8.36), the total throughput is found as

$$\begin{aligned} S_t &= S_t \chi(0) = -G_t e^{-G_t} \int_0^{\infty} \exp\{-\lambda + G_t e^{-z\lambda}\} d\lambda \\ &= G_t \int_0^1 e^{-G_t x} (1-x)^{\frac{1}{z}-1} dx. \end{aligned} \quad (8.42)$$

The author has shown in his master's thesis and in [4] that this can be expressed in terms of the incomplete gamma function [1] by application of integral expression (3.331.1) in [3] or in terms of Kummer's function [1], with $S_t = G_t M(1, 1/z + 1, -G_t)$. Moreover, the series expansion

$$S_t = G_t - \frac{G_t^2}{\frac{1}{z} + 1} + \frac{G_t^3}{\left(\frac{1}{z} + 1\right)\left(\frac{1}{z} + 2\right)} - \dots \quad (8.43)$$

was developed in [4]. Numerical results for S_t are in Fig. 8.3.

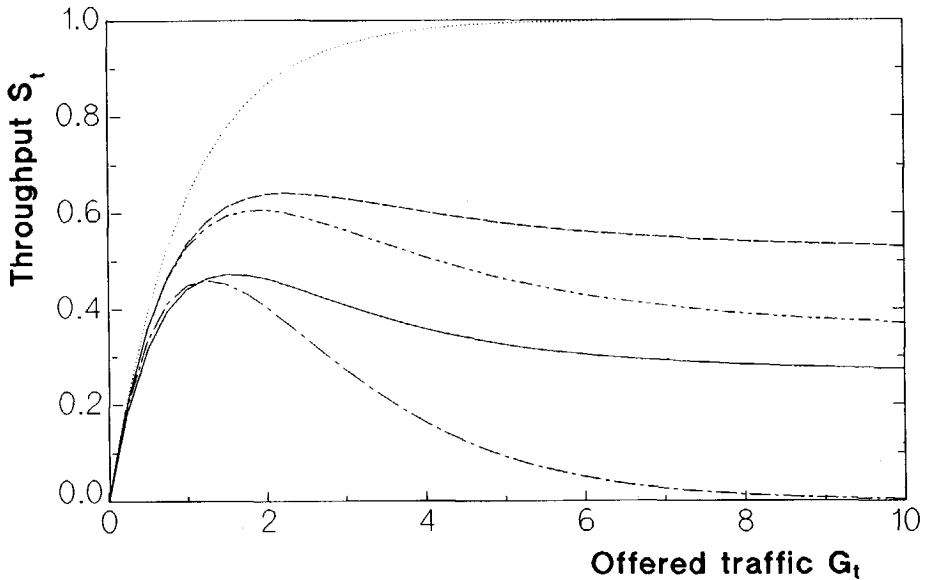


Fig. 8.3: Throughput S_t versus offered traffic G_t with coherent cumulation for (—) ring model and (---) quasi-uniform offered traffic and with incoherent cumulation for (-·-) ring model and (-·-) quasi-uniform offered traffic. Receiver threshold $z = 4$ (6 dB).

(...): one successful packet per slot ($S_t = 1 - \exp\{-G_t\}$).

It is seen that at high offered traffic loads, say $G_t > 2$ ppt, coherent cumulation gives substantially higher throughput than incoherent cumulation (if the receiver threshold is the same in both cases). Particularly, if $G_t \rightarrow \infty$, the ring model gives $S_t \rightarrow 0$ for incoherent cumulation (see section 8.4.1) but $S_t \rightarrow 1/z$ for coherent cumulation.

8.2.4 QUASI-UNIFORM OFFERED TRAFFIC

The quasi-uniform distribution (see chapter 7 and Table (8.21)) of offered traffic has the image function $\psi(s) = \exp\{-\sqrt{\pi s}\}$. The probability of successful transmission is

$$\begin{aligned}
 q_n &= \int_0^{\infty} \sqrt{\pi} \exp\{-\sqrt{\pi\lambda} (n\sqrt{z} + 1)\} d\lambda \\
 &= \frac{e^{-t}}{n\sqrt{z} + 1} \bigg|_{t=0}^{\infty} = \frac{1}{n\sqrt{z} + 1} .
 \end{aligned}
 \tag{8.44}$$

It is seen that (8.44) is on the same form as (8.41), though with z replaced by $\sqrt{z}$. This implies that the total throughput for a quasi-uniform spatial distribution must be on the form of (8.42), with $\sqrt{z}$ inserted for z [5].

It is concluded that the analysis reported here, based on image functions, yields closed-form expressions, in terms of known mathematical functions for the problem studied by Arnbak and van Blitterswijk [5]. This is in contrast to the final results reported in [5] which were obtained by numerical analysis.

Numerical results for Fig. 8.3 show that coherent cumulation gives substantially higher throughput than incoherent cumulation. However, with the modulation techniques used in practical mobile radio system signal are likely to add incoherently. Further, signals as depicted in Fig. 8.2, with a relatively large carrier component (because $\phi_{\Delta} < 1$), are vulnerable to interference (see section on "minimum energy signals" in sect. 4.5 of ref [25]). Thus, comparison between coherent and incoherent cumulation may not be fair if identical receiver thresholds are considered.

8.3 CAPTURE PROBABILITY FOR INCOHERENT CUMULATION

With coherent cumulation, individual fluctuations of carrier phases are assumed to be negligibly small during the reception of each packet. In contrast to this, this section addresses *incoherent* cumulation, which occurs as soon as modulation causes substantial phase deviations. For further discussion of incoherent cumulation, we refer to chapter 3.

Initially a finite population of N terminals with known positions is considered. The probability $Pr(k_{ON})$ that terminal k ($k=1,2,...,N$) transmits during a particular time slot is assumed stationary. This probability includes the transmission of a newly arrived packet as well the retransmission of previously collided packets. Similar to the expression for outage probability in chapter 3, the conditional probability that a test packet (with index j , from a terminal at distance r_j) captures the receiver is

$$Pr(A_j | \bar{p}_j) = \prod_{\substack{k=1 \\ k \neq j}}^N \mathcal{Q} \left\{ f_{p_k}, \frac{z}{p_j} \right\}, \quad (8.45)$$

where f_{p_k} represents the unconditional pdf of received interference power, taking into account the probability $Pr(k_{OFF})$ that the terminal remains idle ($p_k=0$ if k_{OFF}).

8.3.1 VULNERABILITY WEIGHT FUNCTION

We continue to consider channels without shadowing in this section. If the local-mean power of the signal from each terminal is known, the pdf of the local mean-power becomes a exponential distribution. In this case,

$$\begin{aligned} Pr(A_j | r_j, \{r_k\}_{k=1}^N) &= \prod_{\substack{k=1 \\ k \neq j}}^N 1 - \left(1 - \mathcal{Q} \left\{ r_k^\beta \exp(-r_k^\beta p_k), z r_j^\beta \right\} \right) Pr(k_{ON}) \\ &= \prod_{\substack{k=1 \\ k \neq j}}^N \{ 1 - W(r_j, r_k) Pr(k_{ON}) \}. \end{aligned} \quad (8.46)$$

For ease of notation we defined the weight function $W(r_j, r_k)$ ($0 < W(.,.) < 1$) as

$$\begin{aligned}
 W(r_j, r_k) &\triangleq \frac{z r_j^\beta}{z r_j^\beta + r_k^\beta} \\
 &= 1 - \frac{1}{1 + z \left(\frac{r_j}{r_k} \right)^\beta} .
 \end{aligned}
 \tag{8.47}$$

This function is illustrated in Fig. 8.4 for $r_j=1$ and for a receiver threshold of 0 dB ($z=1$). Other threshold values can be considered by scaling the curve appropriately.

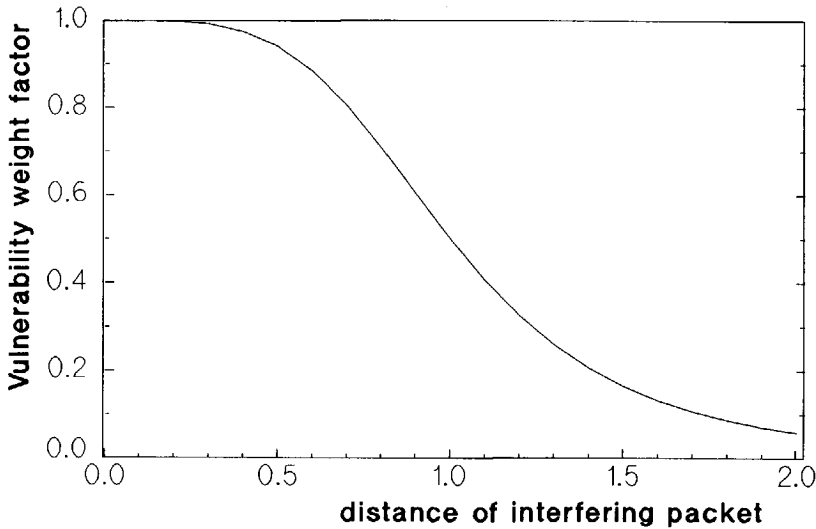


Fig. 8.4 Factor $W(1, r_k)$ to weigh the vulnerability of a test packet from unity distance ($r_j=1$) to an interfering signal from r_k . Receiver threshold $z=1$ (0 dB).

The following observations can be made:

A signal from a distant terminal ($r_j \rightarrow \infty$) is most vulnerable to interference from terminal k ($r_k < r_j$). This is represented by a weight factor close to unity ($W(r_j, r_k) \rightarrow 1$). Moreover, for channels without capture ($z \rightarrow \infty$), the weight factor goes to unity ($W(\cdot, \cdot) \rightarrow 1$) for all ranges. Then

$$Pr(A_j | \{r_{ij}\}_{i=1}^N) = \prod_{\substack{k=1 \\ k \neq j}}^N 1 - Pr(k_{ON}) \quad (8.48)$$

which agrees with the well-known results for wired networks without capture [8]. On the other hand, for a test packet arriving at the receiver from a nearby terminal ($r_j \rightarrow 0$), interference from remote terminals may be neglected, since the weight factor tends to zero ($W(r_j r_k) \rightarrow 0$) for $r_j < r_k$.

We now address an infinite population of independently operating mobile terminals by letting $N \rightarrow \infty$. Moreover, the exact location of each terminal is not considered to be known. Taking account of Rayleigh fading, the pdf of the instantaneous power received from an individual (interfering) terminal is

$$\begin{aligned} f_{p_k}(p_k) &= \int_0^\infty f_{p_k}(p_k | \bar{p}_k) f_{\bar{p}_k}(\bar{p}_k) d\bar{p}_k \\ &= \int_0^\infty r_k^\beta \exp(-r_k^\beta p_k) \frac{2\pi r_k G(r_k)}{G_t} dr_k. \end{aligned} \quad (8.49)$$

From this, the capture probability $q_n(r_j)$ is determined as

$$\begin{aligned} q_n(r_j) &= \mathcal{L}^n \left\{ \int_0^\infty r^\beta \exp(-r^\beta p_k) \frac{2\pi r G(r)}{G_t} dr, z r_j^\beta \right\} \\ &= \left[\int_0^\infty \int_0^\infty r^\beta \exp(-(r^\beta + z r_j^\beta)x) \frac{2\pi r G(r)}{G_t} dx dr \right]^n \\ &= \left[\frac{1}{G_t} \int_0^\infty \frac{r^\beta}{r^\beta + z r_j^\beta} 2\pi r G(r) dr \right]^n \end{aligned} \quad (8.50)$$

where the Laplace transformation and the integral over r have been interchanged.

Moreover, (8.50) with (8.26) verifies that $q_n(r_j) = \phi^n(z r_j^{-\beta})$.

Since $q_n(r) = q_1^n(r)$, the unconditional probability of capture $Q(r)$ is, for a Poisson distributed number of interfering packets,

$$\begin{aligned}
 Q(r) &= \sum_{n=0}^{\infty} \frac{G_t^n}{n!} \exp(-G_t) q_n(r) \\
 &= \exp\{-G_t(1 - q_1(r))\} \\
 &= \exp\left\{-\int_0^{\infty} W(r, x) G(x) 2\pi x dx\right\}.
 \end{aligned} \tag{8.51}$$

Thus, the probability of a successful transmission is on the form of a traffic intensity $G(r)$ multiplied by a weight factor, which is entirely determined by the mean propagation attenuation and the receiver capture ratio z . It is seen that interfering traffic from remote areas ($r_i > r_j$) does not play a major role since $W(r_j, r_i) \rightarrow 0$, whereas nearby interfering traffic causes more destructive collisions, and must be weighted by a factor approaching unity ($W(r_j, 0) = 1$).

If the weight function is replaced by a step function, the vulnerability circle proposed by Abramson [7] is recovered. This model [7] ignores cumulation of interference power and Rayleigh fading. However, active terminals outside this circle do introduce some interference. Accordingly, the integral in (8.51) has its upper limit at infinity, rather than at $c_z r_j$. Apparently, Rayleigh fading of the interfering packet traffic change the circle adopted by Abramson into the softer transition region described by the vulnerability weight function.

8.4 CAPTURE PROBABILITIES FOR SPECIAL CASES OF SPATIAL DISTRIBUTION

This section summarises results from an evaluation of a number of examples of the spatial distribution of the offered traffic. Incoherent cumulation of interference power is assumed.

8.4.1 RING DISTRIBUTION OF OFFERED TRAFFIC

For channels without near-far effect and shadowing, the local-mean power of all received signals is identical ($\bar{p}_i=1$ for all i). A similar situation arises if adaptive power is used to ensure equal local-mean power for all terminals. The probability of capture is evaluated by inserting a spatial distribution on the form of

$$G(r) = \frac{G_t}{2\pi r} \delta(r-1) \quad (8.52)$$

in (8.50). This gives

$$q_n = q_n(1) = \frac{1}{(z+1)^n} \quad (8.53)$$

and the total throughput (see Fig. 8.3) is

$$S_t = G_t \exp\left\{-\frac{z}{z+1} G_t\right\}. \quad (8.54)$$

These results were previously presented in [5] and [9] with another method of derivation, thus without explicit use of the weight function (8.46), (8.48) and (8.47).

8.4.2 UNIFORM DISTRIBUTION WITH INFINITE EXTENSION

For (globally) uniformly distributed offered traffic with intensity $G(r)=G_0$ over a service area with infinite extension ($0 < r < \infty$), the total offered traffic G_t is unbounded. As will be shown now, nonetheless a non-zero throughput S_t is found for $\beta > 2$. The probability of a successful transmission from distance r is, using (8.51),

$$Q(r) = \exp \left\{ -2\pi G_0 \int_0^\infty \frac{z r^\beta \lambda}{\lambda^\beta + z r^\beta} d\lambda \right\}. \quad (8.55)$$

After substitution of $t = \lambda r^{-1/\beta}$, this gives

$$Q(r) = \exp \left\{ -2\pi G_0 z^{\frac{2}{\beta}} \int_0^\infty \frac{t dt}{t^\beta + 1} \right\}. \quad (8.56)$$

The integral is given as Eq. (3.241.2) in [3] and in [26]

$$Q(r) = \exp \left\{ -\frac{2\pi^2 G_0 z^{\frac{2}{\beta}}}{\beta \sin \frac{2\pi}{\beta}} r^2 \right\}. \quad (8.57)$$

In the event of the propagation law $\beta=4$ one finds

$$Q(r) = \exp \left\{ -\frac{\pi^2}{2} G_0 \sqrt{z} r^2 \right\}. \quad (8.58)$$

Fig. 8.7 (---) gives the probability of capture versus distance r for globally uniform offered traffic.

For a minimum success rate Q_{MIN} to be realised at any position within the unit circle, thus for $Q(r) \geq Q(1) = Q_{MIN}$ for any $0 \leq r \leq 1$, the packet traffic per unit of area offered to the network must be bounded by

$$G_0 < -\frac{2}{\pi^2 \sqrt{z}} \ln Q_{MIN}. \quad (8.59)$$

Fig. 8.5 gives the maximum allowed traffic intensity per unit of area as a function of the required Q_{MIN} .

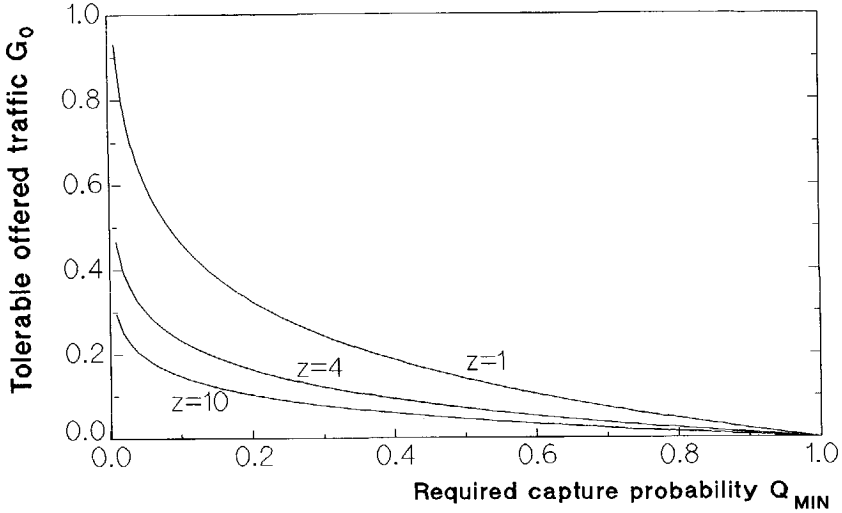


Fig. 8.5: Maximum allowed offered traffic per unit of area for a given minimum probability of capture Q_{MIN} . Uniform offered traffic with infinite extension

If we also account for additive noise with power N_A (see section 3.3.3 or 7.4), the total throughput S_t is found on the form

$$S_t = \int_0^{\infty} 2\pi r_j G_0 Q(r_j) \exp\{-zN_A r_j^4\} dr_j$$

$$= \begin{cases} \frac{\pi}{2} G_0 \sqrt{\frac{\pi}{zN_A}} \exp\left\{\frac{\pi^4 G_0^2}{16N_A}\right\} \operatorname{erfc}\left(\frac{\pi^2 G_0}{4\sqrt{N_A}}\right) & \text{if } N_A > 0 \\ \frac{2}{\pi\sqrt{z}} \approx \frac{0.64}{\sqrt{z}} & \text{if } N_A = 0 \end{cases} \quad (8.60)$$

The total throughput is shown in Fig. 8.6a versus the signal-to-noise ratio for terminals at normalised distance $r=1$ ($C/N=N_A^{-1}$), with total offered traffic within the unit circle $\pi G_0 = 1$ ppt. The effect of the propagation exponent β is depicted in Fig. 8.6b, with, for $N_A=0$,

$$S_i = \int_0^{\infty} 2\pi r G_0 \exp \left\{ -\frac{\frac{2}{\beta} \pi^2 G_0 \frac{z^2}{\beta}}{\sin \frac{2\pi}{\beta}} r^2 \right\} dr = \frac{\beta}{2\pi z^{\frac{2}{\beta}}} \sin \frac{2\pi}{\beta} .$$

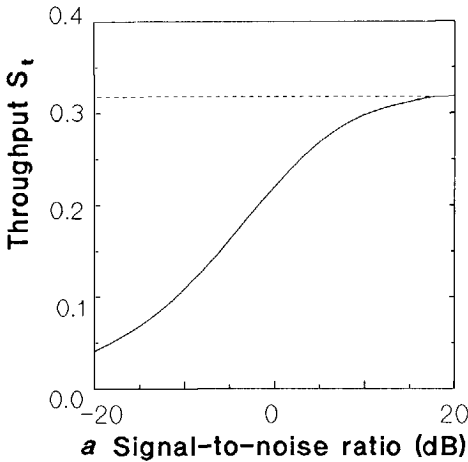


Fig. 8.6a: Total throughput S_i versus signal-to-noise ratio at $r=1$. (--) limit for noise-free channel. Globally uniform offered traffic with $\pi G_0 = 1$ ppt. Receiver threshold $z=4$ (6 dB).

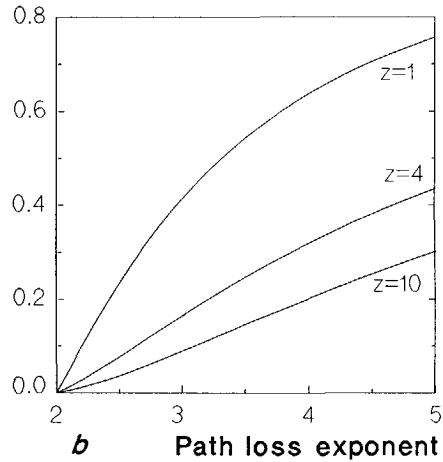


Fig. 8.6b: Total throughput S_i versus propagation exponent β . Globally uniform offered traffic. Receiver threshold $z=1, 4$ and 10 . Noise-free channel.

In the special case of offered traffic with infinite extension, the throughput appears to be sensitive to the path loss law β . For channels with free-space propagation ($\beta=2$), the throughput even degrades to zero. An explanation for this behaviour is the effect that the expected number of interferers in a circular band at distance r_i from the base station is proportional to r_i while the signals from these interfering terminals are attenuated proportionally to r_i^2 . Hence, the expected interference power per band decreases with r_i^{-1} . If during a certain time slot, the nearest active terminal is at a distance r_o , the total interference power experienced from more remote terminals is

unbounded because the integral over r_i^{-1} from r_0 to infinity diverges. Hence, for $\beta=2$, $S_i \rightarrow 0$. In fact, the above discussion can be extended to conclude that any form of cellular radio with regular frequency re-use is possible only because signal powers degrade on earth far more rapidly than in free space. For finite offered traffic loads, the effect of β is presumably much less pronounced.

8.4.3 UNIFORM DISTRIBUTION IN CIRCULAR BAND

If the offered traffic is uniformly distributed with intensity G_0 between two circles with the radius r_1 and r_2 , respectively, i.e.,

$$G(r) = \begin{cases} G_0 = \frac{G_t}{\pi(r_2^2 - r_1^2)}, & r_1 < r < r_2 \\ 0, & \text{elsewhere} \end{cases} \quad (8.61)$$

the probability of a successful transmission becomes

$$Q(r) = \exp \left\{ -\pi G_0 \int_{\lambda=r_1}^{r_2} \frac{z r^\beta}{\lambda^\beta + z r^\beta} d\lambda^2 \right\}. \quad (8.62)$$

For $\beta=4$, the integral can be expressed in terms of the arctangent function. Using Eq. (4.4.34) in [1] or (1.625.9) in [3] to subtract the resulting 'arctan'-terms, one obtains

$$Q(r) = \exp \left\{ -\sqrt{z} \pi r^2 G_0 \arctan \left(\frac{\sqrt{z} r^2 (r_2^2 - r_1^2)}{z r^4 + r_1^2 r_2^2} \right) \right\}. \quad (8.63)$$

Numerical results are shown in Fig. 8.9 by broken lines (- -). In the limit for increasing offered traffic ($G_t \rightarrow \infty$), the probability $Q(r)$ tends to zero for all r ($0 < r_1 < r < r_2$). The total throughput also reduces to zero ($S_t \rightarrow 0$) for any positive (and fixed) inner boundary r_1 ($r_1 > 0$). Moreover,

$$\lim_{r_1 \rightarrow 0} \lim_{G_t \rightarrow \infty} S_t = 0 . \quad (8.64)$$

However, as will be shown in the next section,

$$\lim_{G_t \rightarrow \infty} \lim_{r_1 \rightarrow 0} S_t = \frac{2}{\pi \sqrt{z}} . \quad (8.65)$$

As already seen in (8.60), (8.65) also holds for an infinitely extended distribution of offered traffic ($r_2 \rightarrow \infty$).

8.4.4 UNIFORM DISTRIBUTION WITHIN UNIT CELL

The uniform spatial distribution within the unit cell, viz.,

$$G(r) = \begin{cases} G_0 = \frac{G_t}{\pi} & 0 < r < 1 \\ 0 & \text{elsewhere} \end{cases} \quad (8.66)$$

is a special case of the distribution addressed in the previous section. Inserting $r_2=1$ and taking the limit $r_1 \rightarrow 0$, the throughput for homogeneous offered traffic within the unit cell is found on the form

$$Q(r) = \exp \left\{ -\sqrt{z} r^2 G_t \arctan \left(\frac{1}{\sqrt{z} r^2} \right) \right\} . \quad (8.67)$$

For a minimum success rate Q_{MIN} required at the cell boundary, the packet traffic offered to the net must be bounded by

$$G_0 < \frac{-\ln Q_{MIN}}{\pi\sqrt{z} \arctan\left(\frac{1}{\sqrt{z}}\right)} . \quad (8.68)$$

Since $\arctan(1) = 1/4\pi$, for near-perfect capture ($1 < z < 4$) this is slightly less than twice the maximum allowed traffic in the event of uniform spatial distribution with infinite extension ($r_2 \rightarrow \infty$, as in Fig. 8.5).

If the offered traffic G_i is increased without limit, the probability of successful reception for any individual terminal with finite (non-zero) distance to the base station necessarily tends to zero. Nonetheless, packets from the immediate vicinity of the receiver continue to contribute to the throughput. The probability C_{i+1} of correct reception of one out of $i+1$ packets (see Fig. 8.13) can be found from

$$\begin{aligned} C_{i+1} &= (i+1) \int_0^1 q_i(r) 2r dr \\ &= (i+1) \int_0^1 \left[\int_0^1 \frac{2x^5 dx}{x^4 + zr^4} \right]^i 2r dr \\ &= \frac{(i+1)}{\sqrt{z}} \int_0^{\sqrt{z}} \left[1 - y \arctan \frac{1}{y} \right]^i dy . \end{aligned} \quad (8.69)$$

For large i , small values of y offer the principal contribution to the integral. We now apply integration by parts to evaluate the asymptotic behaviour for $i \rightarrow \infty$:

$$\lim_{i \rightarrow \infty} \int_a^1 \frac{d\{f(y)\}^{i+1}}{f'(y)} = -\lim_{i \rightarrow \infty} \frac{\{f(a)\}^{i+1}}{f'(a)} + \frac{\{f(1)\}^{i+1}}{f'(1)} + \lim_{i \rightarrow \infty} \int_a^1 \frac{\{f(y)\}^{i+1}}{\{f'(y)\}^2} f''(y) dy , \quad (8.70)$$

where, for convenience of notation, $f(y) \triangleq 1 - y \arctan(y^{-1})$. For large i , the integral and the term for $y=1$ on the right hand side vanish and we find $C_{i+1} \rightarrow 2/(\pi\sqrt{z})$. For large G_0 , the probability of less than i_ϵ packets in any finite interval of time can be made arbitrarily small. Thus, for slotted ALOHA, the throughput at high traffic loads must

approach $S_f \rightarrow 2/(\pi/z)$. This agrees with (8.60) and with the limit observed by Lau and Leung in [10] for channels without Rayleigh fading.

8.4.5 ONE-DIMENSIONAL UNIFORM DISTRIBUTION

We address a slotted ALOHA net supporting packet traffic transmitted by vehicles located on a highway, with intensity $\frac{1}{2}G_0$ packets per slot per unit of distance, passing by the base station with angle θ_0 . The offered traffic per unit of area is on the form

$$G(r, \theta) = \frac{1}{2} G_0 [\delta(\theta - \theta_0) + \delta(\theta - \theta_0 + \pi)], \quad (8.71)$$

so, by averaging over a circular ring with radius r ($0 \leq \theta < 2\pi$), $G(r)$ is found as

$$G(r) = \int_0^{2\pi} \frac{G(r, \theta)}{2\pi r} dr = \frac{1}{2\pi r} G_0. \quad (8.72)$$

Then, if $\beta=4$,

$$\begin{aligned} Q(r) &= \exp \left\{ -G_0 \int_0^\infty \frac{zr^4}{x^4 + zr^4} dx \right\} \\ &= \exp \left\{ -\frac{1}{4} \sqrt{2} \pi G_0^4 \sqrt{z} r \right\}, \end{aligned} \quad (8.73)$$

where the integral was solved using [26]

$$\int_0^\infty \frac{x^{m-1}}{1+x^n} dx = \frac{\pi}{n} \frac{1}{\sin\left(\frac{m\pi}{n}\right)}. \quad (8.74)$$

Numerical results are shown in Fig. 8.7 by (---). It is seen that, compared to the two-dimension globally uniform distribution (see (8.58) and (- -) in Fig. 8.7) packets from

relatively remote terminals still have a significant probability of capture. The total throughput becomes

$$S_t = \int_0^{\infty} G_0 \exp\left\{-\frac{\sqrt{2}}{4} \pi G_0^4 \sqrt{z} r\right\} dr = \frac{2\sqrt{2}}{\pi^4 \sqrt{z}} \approx \frac{0.90}{\sqrt[4]{z}}. \quad (8.75)$$

for any (positive, non-zero) value of G_0 . Since we confine ourselves to realistic narrowband receivers with $z > 1$, this throughput is higher than the throughput with uniform offered traffic in two dimensions (cf. (8.60)).

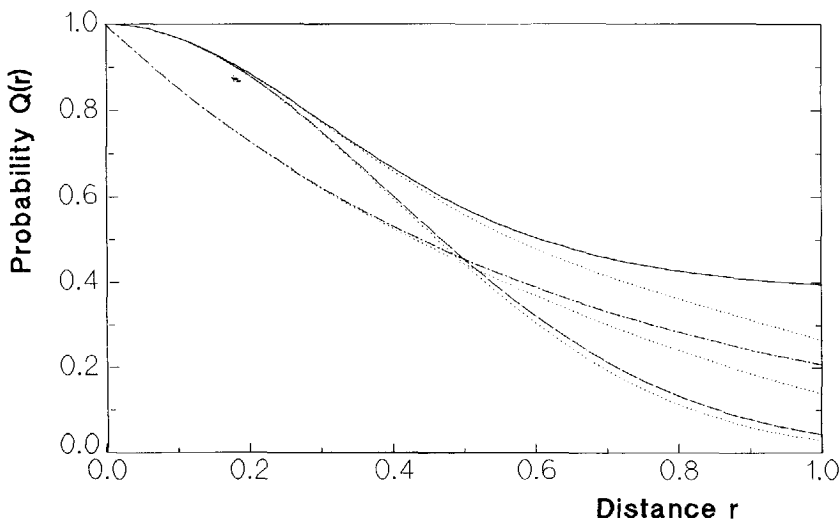


Fig. 8.7: Probability of successful transmission $Q(r)$ versus distance r for (---) two-dimensional uniform distribution with infinite extension, (—) uniform distribution in unit circle and (— · —) one-dimensional uniform distribution with infinite extension. Receiver threshold $z=4$ (6 dB). Total traffic offered within unit circle is 1 ppt. (···) corresponding results for noisy channel ($N_A=0.1$)

8.4.6 NON-UNIFORM DISTRIBUTIONS

Exactly or quasi-uniform offered traffic were studied in the previous sections. Such distributions may be reasonable in a number of cases. For instance, if vehicle speeds are sufficient to ensure that the position of a terminal becomes uncorrelated between each (re-) transmission, a uniform offered packet traffic may be reasonable. Moreover, in certain vehicle location systems or telemetry applications, a (routine) status report lost in a collision may not need to be retransmitted. This leads to a uniform distribution if the participating terminals are uniformly distributed around the base station. However, major deficiencies of the uniform distribution are: *i*) uniform distributions allow an incidental transmitter to be unrealistically close to the receiver, which seriously hinders realistic assessment of the throughput, and *ii*) uniform distributions ignore that retransmissions are likely to arrive predominantly from areas with poor propagation. The first aspect is addressed in the next section concerning log-normal spatial distributions, the second aspect will be considered in section 8.5.3 and in chapter 9.

8.4.7 LOG-NORMAL SPATIAL DISTRIBUTION

Linnartz and Prasad suggested in [11] to approximate the uniform spatial distribution by a two-parameter 'log-normal spatial distribution' on the form of

$$G(r) = \frac{G_t \beta}{(2\pi)^{3/2} r^2 \sigma_d} \exp\left\{-\frac{\beta^2 \ln^2 \frac{r}{r_m}}{2\sigma_d^2}\right\}, \quad (8.76)$$

with r_m the logarithmic mean distance. We normalise distances by taking the parameter $r_m=1$. The parameter σ_d defines the spatial spread of the offered traffic.

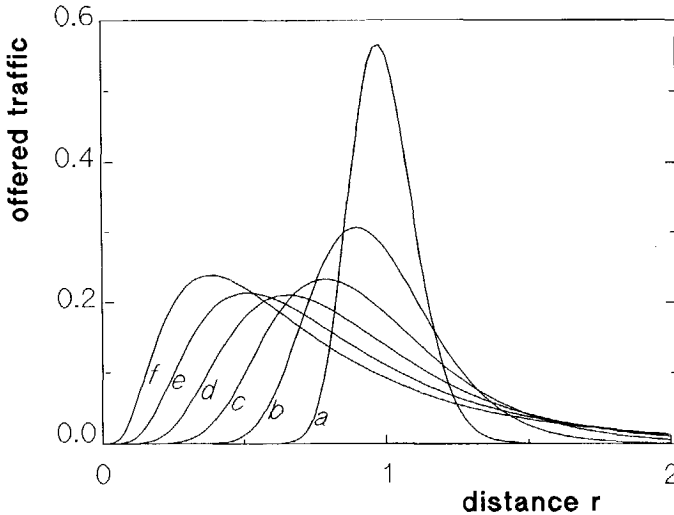


Fig. 8.8: Normalised 'log-normal' spatial distribution of offered traffic $G(r)/G_0$, for $\beta=4$ for various spatial spreads σ_d : $s_d=2, 4, 6, 8, 10$ and 12 dB, (a)..(f), resp.

Basically, this approximation has two effects:

- (i) it smooths the transition from high offered traffic intensity inside the cell to zero offered traffic outside the cell, and
- (ii) the density of users unrealistically close to the receiver is sharply diminished.

Moreover, the distribution (8.76) produces a log-normal pdf of the area-mean power, so study of the effects of combined shadowing and near-far effects becomes mathematically tractable [11]. Fig. 8.8 illustrates this spatial distribution for $\beta=4$ and various σ_d .

Initially we assess an appropriate choice of σ_d to approximate the exactly uniform distribution. To this end, the logarithmic moments of the uniform and the log-normal distribution are compared. Analogous to definitions in chapter 2, the k -th logarithmic moment μ_k of the mean received signal power is defined as

$$\begin{aligned}
 \mu_k &\triangleq \int_0^{\infty} [-\ln \bar{p}_j]^k f_{\bar{p}_j}(\bar{p}_j) \bar{p}_j \\
 &= \frac{1}{G_i} \int_0^{\infty} [-\beta \ln r]^k 2\pi r G(r) dr
 \end{aligned} \tag{8.77}$$

provided that the integral exists. The logarithmic variance is found from $\sigma_d^2 = \mu_2 - \mu_1^2$. For the uniform spatial distribution, the logarithmic moments (8.77) exist, namely

$$\mu_k = (-\beta)^k \int_0^1 \ln^k r r dr = (-\beta)^k \frac{k!}{2^k}. \tag{8.78}$$

This is in contrast to the *linear* moments of the (exactly) uniform distribution, which diverge, as seen from (8.22). Eq. (8.78) shows that the logarithmic standard deviations of the uniform distribution and the log-normal distribution become equal if one takes $\sigma_d = \frac{1}{2}\beta$. For UHF groundwave propagation over a smooth, idealised earth ($\beta=4$), the logarithmic standard deviation of the uniform distribution equals $\sigma_d=2$ ($s_d=8.68$ dB).

As discussed in sect. 2.1.2, the log-normal shadowing for the model by Egli [13] was reported to be in the order of $s_s=12$ dB. Remarkably, since we found $\sigma_d < \sigma_s$ for all realistic values of β , other parameters, for instance obstacles to propagation, would appear to have a more pronounced influence on the standard deviation of the signal power fluctuations than the distance between transmitter and receiver.

Prasad and Arnbak [12] presented the throughput for ALOHA networks over channels with log-normal fading and developed an approximate technique for channels with combined shadowing and Rayleigh fading [6]. In contrast to this, here we will adopt a formally exact technique, similar to the derivation of outage probabilities as presented in chapter 3. Using the weight function with (8.76), the throughput of slotted ALOHA with Rayleigh fading and shadowing is found on the form

$$S_t = \int_0^{\infty} \frac{G_t \beta}{r_j \sigma_d \sqrt{2\pi}} \exp \left\{ -\frac{\beta^2 \ln^2 r_j}{2\sigma_d^2} - \int_0^{\infty} \frac{z r_j^\beta}{r^\beta + z r_j^\beta} \frac{G_t \beta}{\sqrt{2\pi} r \sigma_d} \exp \left(\frac{\beta^2 \ln^2 r}{2\sigma_d^2} \right) dr \right\} dr_j. \quad (8.79)$$

Numerical results are obtained from the Hermite polynomial method [1], with

$$S_t = \frac{G_t}{\sqrt{\pi}} \sum_{j=1}^M w_j \exp \left\{ -\frac{G_t}{\sqrt{\pi}} \sum_{i=1}^M w_i \frac{z \exp(\sqrt{2} x_j \sigma_d)}{\exp(\sqrt{2} x_i \sigma_d) + z \exp(\sqrt{2} x_j \sigma_d)} \right\} \quad (8.80)$$

where w_j and w_i are weight factors for samples of the integrand in the point x_j and x_i [1]. Results are shown in Fig. 8.9.

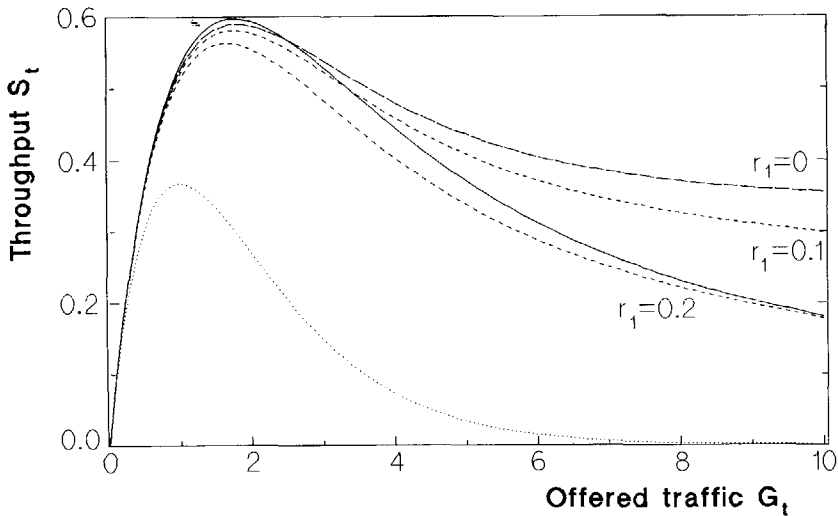


Fig. 8.9 Throughput S_t of slotted ALOHA in a Rayleigh fading channel with (—) a log-normal spatial distribution ($\sigma_d=2$), and (---) exactly uniform distribution for $r_1 < r < 1$ and (---) exactly uniform within the unit circle $0 < r < 1$ ($r_1=0$). Receiver threshold $z=4$ (6 dB) and (···) no capture ($S_t=G_t \exp\{-G_t\}$).

It is seen that at high traffic loads, say, $G_t > 3$ ppt, the assumption of log-normally distributed offered traffic leads to a substantially lower estimate of the throughput than uniform offered traffic within $0 < r < 1$. We conclude the following: Although for

the main part of coverage area the near-far effect does not lead to large power fluctuations compared to shadowing, the possibility that occasionally a transmitter may be unrealistically close to the receiver substantially influences the average throughput.

8.5 CAPTURE PROBABILITY IN CHANNELS WITH RAYLEIGH FADING, SHADOWING AND NEAR-FAR EFFECT

In order to investigate combined Rayleigh fading, shadowing and near-far effects, the weight-function method is now extended. The probability of capture $q_n(r)$, given the area-mean power $\bar{p}_j = r^{-\beta}$ of the test packet and given the presence of n interferers, is

$$\begin{aligned} q_n(r) &= \int_0^{\infty} f_{\bar{p}_j}(\bar{p}_j | \bar{p}_j = r^{-\beta}) Pr(A_j | n, \bar{p}_j) d\bar{p}_j \\ &= \int_0^{\infty} \frac{1}{\sqrt{2\pi}\sigma_s \bar{p}_j} \exp\left\{-\frac{1}{2\sigma_s^2} \ln^2(\bar{p}_j r^\beta)\right\} \phi_n\left(\frac{z}{\bar{p}_j}\right) d\bar{p}_j. \end{aligned} \quad (8.81)$$

The Laplace image $\phi(s)$ of the pdf of the power received from an interfering mobile terminal experiencing combined groundwave path loss, shadowing and Rayleigh fading is

$$\phi(s) = \frac{1}{G_i} \int_0^{\infty} \int_0^{\infty} \frac{1}{1+s\bar{p}_i} \frac{2\pi r G(r)}{\sqrt{2\pi}\sigma_s \bar{p}_i} \exp\left\{-\frac{\ln^2(\bar{p}_i r^\beta)}{2\sigma_s^2}\right\} dr d\bar{p}_i. \quad (8.82)$$

This can be written as an integral transform of the spatial distribution by interchanging the order of integration. Rewriting the integral over $\bar{p}_i$ using the substitution

$$y^2 \triangleq \frac{1}{2\sigma_s^2} \ln^2(r_i^\beta \bar{p}_i) \quad (8.83)$$

the image (8.82) can be interpreted as weighing the vulnerability of the test packet to interference at distance r_i , i.e.,

$$G_i \Phi(s) = \int_0^{\infty} 2\pi r_i W_o(s, r_i) f_r(r_i) dr_i \quad (8.84)$$

where the vulnerability weight function $W_o(\cdot)$ ($0 < W_o(\cdot) < 1$) is defined by

$$W_o(r_j, r) \triangleq \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2) dx}{1 + z r_j^{\beta} r^{-\beta} \exp(\sqrt{2} \sigma_s x)} = \phi_o(z r_j^{\beta} r^{-\beta}) \quad (8.85)$$

where $\phi_o(\cdot)$ is defined in section 2.1.5 as the Laplace image of the pdf of the received power from a 'Suzuki-fading' signal. For channels without shadowing ($\sigma_s \rightarrow 0$), the simpler weight function defined in (8.47) is recovered. Fig. 8.10 compares the weight function for channels including shadowing with the case of no shadowing ($\sigma_s \rightarrow 0$).

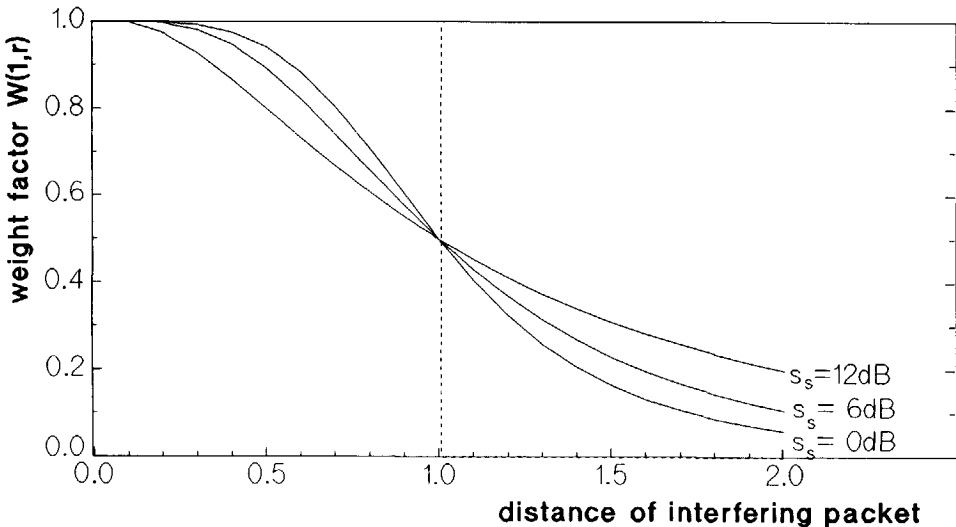


Fig. 8.10 Weight factor $W_o(1, r)$ (—) and approximation by a step function (---), describing the vulnerability of a test packet (at $r_j=1$) to interference at distance r_k . Shadowing is $s_s=0, 6$ and 12 dB. UHF groundwave propagation with $\beta=4$.

For uniform spatial density of offered traffic, the Laplace image of the (unconditional) pdf of received interference power p_i becomes

$$\Phi(s) = \int_0^1 2r_i \phi_o(sr_i^{-\beta}) dr_i = \int_0^1 \frac{2r_i}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{r_i^4 \exp(-y^2)}{r_i^4 + s \exp(\sqrt{2}\sigma_s y)} dy dr_i. \quad (8.86)$$

After substitution of

$$t \triangleq \frac{r_i^\beta}{s \exp\{\sqrt{2}\sigma_s y\}}, \quad (8.87)$$

the image becomes, for $\beta=4$,

$$\Phi(s) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \int_0^{t_{\max}} \frac{\sqrt{t}}{1+t} dt \sqrt{s} \exp\left\{-y^2 + \frac{1}{2}\sqrt{2}\sigma_s y\right\} dy, \quad (8.88)$$

with $t_{\max}$ defined as

$$t_{\max} \triangleq \frac{1}{s} \exp\{-\sqrt{2}\sigma_s y\}. \quad (8.89)$$

Using Eq. (14.115) in [14], the image function becomes

$$\begin{aligned} \Phi(s) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \left[1 - \sqrt{s} \exp\left\{\frac{1}{2}\sqrt{2}y\sigma_s\right\} \right. \\ \left. \cdot \arctan\left(\sqrt{\frac{1}{s}} \exp\left\{-\frac{1}{2}\sqrt{2}y\sigma_s\right\}\right) \right] e^{-y^2} dy, \end{aligned} \quad (8.90)$$

which is to be inserted in (8.81). In the latter, the integral over the local-mean power $\bar{p}_j$ of the test packet is rewritten, using the substitution

$$x^2 \triangleq \frac{1}{2\sigma_s^2} \ln(\bar{p}_j r^\beta). \quad (8.91)$$

Hence,

$$q_n(r) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \exp(-x^2) \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x,y) \exp(-y^2) dy \right]^n dx \quad (8.92)$$

where for shortness of notation we introduced $f(x,y)$, defined as

$$f(x,y) \triangleq 1 - \sqrt{z} r^2 \exp\left\{\frac{\sqrt{2}}{2} \sigma_s(y-x)\right\} \arctan\left\{\frac{1}{z r^4} \exp\left\{\frac{1}{2} \sqrt{2} \sigma_s(x-y)\right\}\right\}. \quad (8.93)$$

It is seen that in the event of combined near-far effect, shadowing and Rayleigh fading, capture probabilities $q_n(r)$ can be expressed as a two-dimensional integral.

Capture probabilities $\{C_i\}$ with $C_i = i q_{i-1}$, thus unconditional on the location of the particular mobile transmitting the test packet, will be used in section 8.6 to study the stability of the network in its entirety. To this end, probability (8.92) is averaged over the pdf of the unknown area-mean power of the test packet. The capture probability q_n , given that n packets interfere, is

$$q_n = \frac{2}{\sqrt{\pi}} \int_0^1 \int_0^{\infty} r \exp(-x^2) \left[\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(x,y) \exp(-y^2) dy \right]^n dx dr. \quad (8.94)$$

Numerical results for this capture probability can be obtained using the Hermite polynomial method [1] for both infinite integrals in (8.94). Results are included later in Fig. 8.13 for various propagation models. The non-zero limit of C_i for $i \rightarrow \infty$ is due to the unrealistic traffic assumed in the immediate vicinity of the base station ($r \rightarrow 0$).

8.6 DYNAMIC PERFORMANCE OF MOBILE ALOHA NETS

It has been discussed in chapter 6 that data packets experience delays because of a number of queuing mechanisms. Here we address the delay caused by the fact that packets unsuccessfully offered to the random access channel have to be retransmitted after a random waiting time. During the analysis of throughput S_r and the probability of success $Q(r)$ it was assumed that unsuccessful packets are rescheduled, so that each new attempt is seen as a new contribution to the offered traffic G_r . Unsuccessful packets remain in the buffer of the terminal for retransmission. In this section we address a queueing model based on a Markov Chain. This model offers the possibility to evaluate the effect that after an unsuccessful transmission attempt, the terminal retransmits the packet with a minimum (random) waiting time to reduce the delay. If multiple terminal have packets for retransmission, destructive collisions may continue to collide. The backlog, packet delay and also the stability of the network are studied from a Markov Chain.

A well-known procedure for evaluation of the stability of the ALOHA channel [16]-[18] considers a finite population of N terminals. Each terminal has a buffer of length one to store the last packet, which may need retransmission. The local behaviour of such a terminal is modelled by a tri-state Markov chain (see Fig. 8.11) .

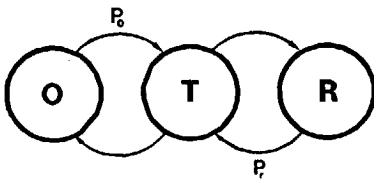


Fig. 8.11: Markov chain model for a terminal

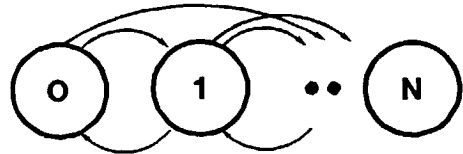


Fig. 8.12: Markov chain model for the ALOHA network

In the origination state (O), a packet is generated (for transmission in the next available time slot) with probability P_o . In the transmission state (T), the terminal is busy with either transmitting a new packet or retransmitting a collided packet. The

mobile terminal returns to the O-state if it receives a positive acknowledgement at the end of the time slot during which a packet is transmitted; otherwise it enters the retransmission or 'backlogged' state (R). From the latter state, retransmission (i.e., transition into the T-state) occurs with probability P_r . A backlogged terminal is blocked in the sense that no new messages can arrive when the terminal is in state R. This tri-state model implies the absence of a buffer for more than one packet in a mobile terminal.

The global behaviour of the entire slotted ALOHA network is also modelled by means of a finite Markov chain (Fig. 8.12). The state of the network represents the number of terminals m in the state R at the start of the time slot. This is also called the system 'backlog'. With a population of N terminals, we have $N+1$ states. During a time slot, i ($i=j+k$) packets are transmitted whenever j terminals in the O state and k terminals in the R-state change into the T-state at the same time. The probability of a transition into another state may be obtained from the probability distribution of j and k and the capture probability C_{j+k} that one of the packets survives the collision. Here, j and k are binomial random variables. For a given backlog, j and k are mutually independent. In the original work [16], [17], channels without capture were evaluated: the probability C_i that one out of the i colliding packets is successfully received was taken equal to unity for $i=1$ and zero otherwise.

These investigations take into account that the system backlog m and the probability of successful transmission are statistically correlated: With a high system backlog, frequent retransmissions occur (if $P_r > P_o$), and collisions are very likely. Hence, the probability of success generally reduces with increasing system backlog (if $C_k < C_m$ for $k > m$), which may lead to saturation or instability of the network if terminals in backlog employ (too) short retransmission waiting times (too high P_r). If the number of terminals N is finite, the network depicted in Fig. 8.12 may become bi-stable, i.e., the network oscillates between low and high backlog states [16]-[18].

8.6.1 MEASURES OF DYNAMIC PERFORMANCE

The throughput S_m in state m is found from the probability of successfully transmitting a packet in a time slot, and is determined by averaging the probability of i ($i=j+k$) colliding packets, given a backlog of m terminals, i.e.

$$S_m = \sum_{k=0}^{N-m} \binom{N-m}{N-m-k} P_o^k (1-P_o)^{N-m-k} \sum_{j=0}^m \binom{m}{j} P_r^j (1-P_r)^{m-j} C_{j+k} \quad (8.95)$$

Stability is studied from the expected drift d_m in each state, defined as the difference between the expected 'input' traffic $(N-m)P_o$ and the expected 'output' traffic S_m in that state, i.e.,

$$d_m = (N-m)P_o - S_m \quad (8.96)$$

expressed in states per time slot. In a simulation to be described in section 8.6.4, this drift is obtained directly as the average motion towards another (higher or lower) state, when the network is in state m . The network is expected to operate in a state near an equilibrium point, i.e., where the expected drift (8.96) crosses zero with negative derivative [16]-[17].

The average total network throughput S_t is found by averaging (8.95) over the state probabilities π_m , viz.,

$$S_t = \sum_{m=0}^N \pi_m S_m \quad (8.97)$$

where the probabilities $\{\pi_m\}$ that the network is in state m ($m=0,1,..N$) can be obtained recursively from the probabilities of transition from state m_1 to m_2 , which depend on P_o , P_r , $\{C_i\}$, N , m_1 and m_2 [18],[22].

In a well-functioning network in equilibrium, the channel throughput S_t must equal the newly generated traffic, so

$$S_t = \left(N - \sum_{m=0}^N m \pi_m \right) P_o \quad (8.98)$$

For stable (and bi-stable) systems, the access delay can be evaluated from Little's formula [23]

$$D_a = \frac{E[m]}{S_t} = \frac{\sum_{m=0}^N m \pi_m}{\sum_{m=0}^N S_m \pi_m} \quad (8.99)$$

Namislo [22] reported drifts, stability and delays for a network with capture in terms of N , P_o , $\{C_i\}$ and P_r . The probabilities of capture were obtained from Monte-Carlo simulation, taking account of path losses; fading and shadowing were not considered. The stability of mobile ALOHA channels including Rayleigh fading and shadowing in addition to near-far effects will be reported in the next section.

8.6.2 NETWORK STABILITY

Fig. 8.13 summarises the probability that one out of i packets survives a collision, for various propagation models. Inserting the capture probabilities $\{C_i\}$ in (8.95) and (8.96) gives the expected dynamic behaviour of the network. In Fig. 8.14, the dynamic behaviour of the mobile network is shown for various propagation models in terms of the expected drift (8.96). A population of N ($N=100$) terminals, spread with uniform distribution over coverage area, and contending for a common receiver with a threshold of $z=4$, is assumed. The assumed probability of packet generation is $P_o=0.0055$ and the probability of retransmission is $P_r=0.08$.

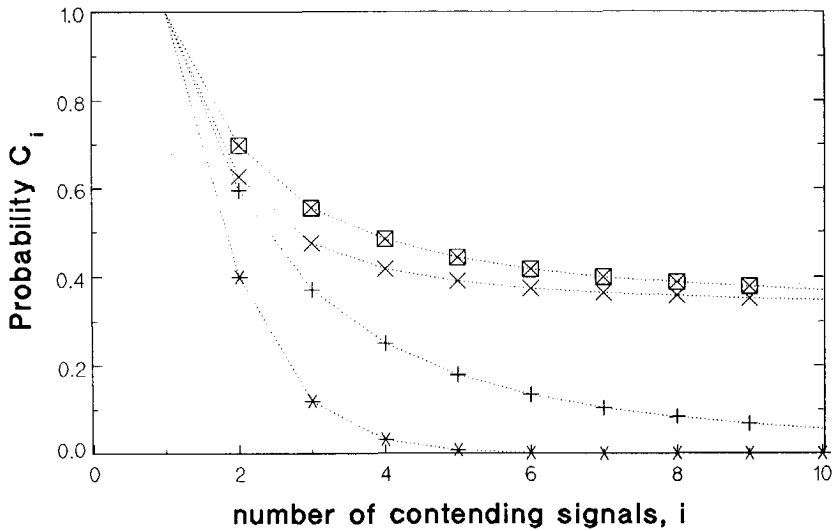


Fig. 8.13: Probability that one out of i packets successfully captures the base station, for (*) Rayleigh fading, (+) Rayleigh fading and shadowing ($\sigma_s=1.36$ or 6 dB), (x) near-far effect and Rayleigh fading, and (X) near-far effect, Rayleigh fading and shadowing. Receiver threshold $z=4$ (6 dB).

If capture never occurs in the presence of other packets ($C_i=0$ for $i>1$), entire saturation ($E[m]\approx N=100$) leads to extraordinarily high delay ($D_a\approx 4.6 \cdot 10^5$ slots). For the traffic parameters studied here, network performance is not essentially improved by Rayleigh fading only, e.g. if adaptive power control is applied to compensate slow power variations: steady-state operation is found at a backlog of nearly $E[m]\approx 100$ terminals, leading to an average delay of $D_a\approx 7000$ slots. The curves (c,d,e) in Fig. 8.14 for Rayleigh fading combined with shadowing or near-far effects all show a single, almost identical equilibrium with low average delay in the order of 10 to 15 time slots. Even if the near-far effect is ignored, i.e., all terminals are located at an identical distance from the central receivers, as in curve (c), the network is stable and terminals experience little backlog. Therefore, it may be concluded that for the reported traffic parameters, the stability of the network does not rely on terminals unrealistically close to the base station.

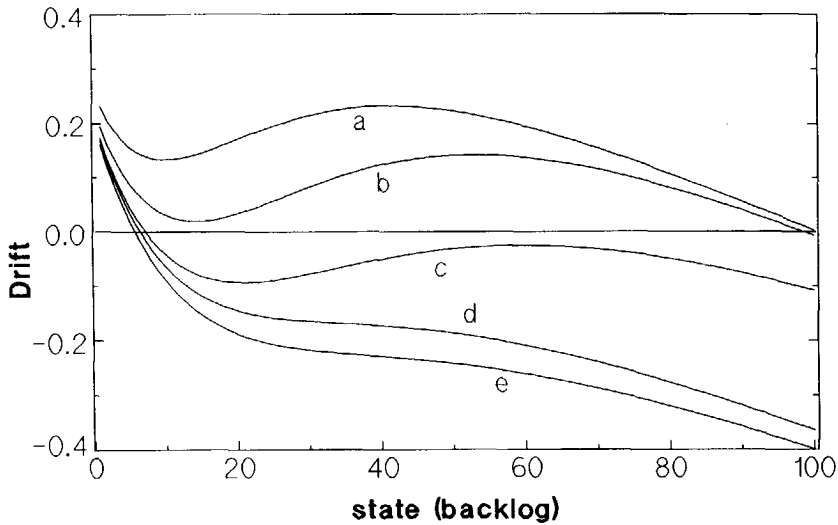


Fig. 8.14: Drift for a network of $N=100$ users for a receiver without capture (a), and for a receiver with threshold of 6 dB ($z=4$) for a channel Rayleigh fading only (b), shadowing ($\sigma_s=1.36$ or 6 dB) and Rayleigh fading (c), near-far effect and Rayleigh fading (d), near-far effect, shadowing ($\sigma_s=1.36$) and Rayleigh fading (e).

The drift at high backlog, especially d_N , is mainly determined by the probability of capture realised in the event of many colliding terminals (C_∞). The probability that one out of the i Rayleigh fading signals is sufficiently stronger than the joint interference is low, as compared to other propagation mechanisms: the power level of a Rayleigh-fading signal is usually relatively constant with occasional deep fades, whereas up-fades are rare. Hence, the probability that one signal is substantially stronger than the joint interference power is rather low. Consequently, the drift from full saturation d_{100} is almost zero in curve (b). The log-normal distribution, due to shadowing, exhibits a higher probability of upfades of the signal power, resulting in a negative drift of $d_{100} \approx -0.1$ away from the situation with all terminals in backlog. The uniform spatial distribution, with non-zero traffic in the immediate vicinity of the receiver, has a fast recovery of about $d_m \approx -0.4$ backlogged terminals per time slot away from states with high backlog ($m \rightarrow 100$). In this event, $-d_N$ and S_N roughly equal the capture probability C_∞ , since C_i is almost constant for large i .

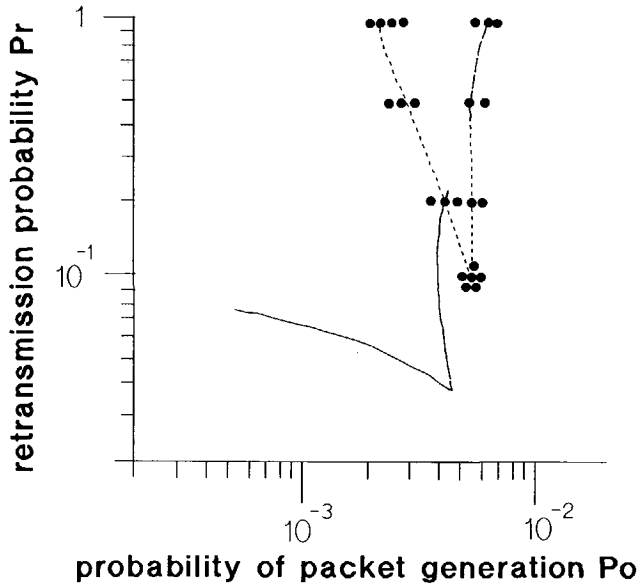


Fig. 8.15: Bistability area for a network with $N=100$ users without capture (—) [19] and with imperfect capture (- -) ($z=10$) in mobile channels. The latter bifurcation set has been estimated by trial in 23 points (●).

Fig. 8.15 gives the region for P_o and P_r where the network with $N=100$ terminals is bistable. A conservative receiver threshold of 10 dB ($z=10$) is considered, so these results may be somewhat pessimistic. The locus (—) of the bifurcation set for ALOHA without capture ($C_i=0$ for all $i>0$) is taken from Onozato and Noguchi [19]. Recently, additional bifurcations sets were reported, mapping the stability of a slotted ALOHA network with capture where the users are divided into two groups [20]. A similar 'catastrophe theory' for a full propagation model including Rayleigh fading, shadowing and the near-far effect, has not yet been developed. Our bifurcation set (- -) was estimated by trial and error with the technique used to obtain drift curves as in Fig. 8.14. A uniform spatial distribution, shadowing ($s_s=6$ dB) and Rayleigh fading are considered. The mobile network exhibits bistability at substantially higher packet traffic loads when receiver capture occurs, even for a receiver threshold of 10 dB ($z=10$).

It appears that the network is always stable, irrespective of the retransmission probability P_r , if $P_o < P_{o,max}$ with roughly $P_{o,max} \approx 2 \cdot 10^{-3}$. This agrees with the observation by Ghez, Verdú and Schwartz [21], that for

$$NP_o < \lim_{n \rightarrow \infty} C_{n+1} \quad (8.100)$$

the channel is stable if the probability of capture C_{n+1} is independent from slot to slot. It has been shown in section 8.4 that, for noise-free channels with uniform offered traffic without shadowing and $\beta=4$,

$$\lim_{G_r \rightarrow \infty} S_r = \lim_{n \rightarrow \infty} C_{n+1} = \frac{2}{\pi\sqrt{z}} \quad (8.101)$$

whereas for channels with shadowing, C_∞ is believed to be at least as large as $2/\pi\sqrt{z}$ (see Fig. 8.13). For $z=10$ and $N=100$ this indicates that the net is stable as long as $P_o < 2/(100\pi/10) \approx 0.002$. The region of bistability in Fig. 8.15 is relatively small: the transition through this area from a single (low-backlog) equilibrium into saturation occurs for relatively small increments of the packet generation probability P_o . In contrast to this, since the bifurcation set of the mobile net is almost parallel to the P_r -axis, a change in the retransmission probability will have less effect on the stability of the net. For a packet generation probability up to $P_o=0.002$, even a persistent retransmission schedule ($P_r=1$) leads to a stable network. Decreasing the generation probability P_o always has a positive effect on the network performance, since the drift towards lower backlog increases for all n .

Simulations [22] indicated that reducing the probability of retransmission P_r has a *positive* effect on the network performance in saturated networks (increasing throughput, decreasing backlog and delay), but a *negative* effect in stable, unsaturated networks (lower throughput, increasing backlog and delay). In bistable nets, appropriate reduction of P_r can remove bistability, but Fig. 8.15 suggests that a relatively drastic reduction of P_r may be required. Simulation runs, however, indicated that after reducing P_r the newly attained stable network usually is in a state of relatively high backlog. By reducing the probability of retransmission, the effective

time each terminal spends in the origination mode also reduces, which indirectly resulted in a low input traffic load. This would suggest that mobile channels might as well be managed by directly controlling the input traffic, for instance by limiting the number of terminals N that are allowed to be signed on simultaneously. Simulations gave the impression that area where the network is bistable further decreases if z becomes smaller.

8.6.3 INFLUENCE OF NEAR-FAR EFFECT

Packet transmissions considered so far were assumed to be uniformly distributed over the coverage area. In this event, the probability of correct reception of one out of $n+1$ colliding packets, C_{n+1} , has some non-zero limit for $n \rightarrow \infty$ (see section 8.4.3). As an alternative, the log-normal spatial distribution (8.76) is now considered. This allows more realistic modelling of the packet traffic very close to the central receiver. The true uniform distribution is compared with a log-normal distribution with $\sigma_d=2$ ($s_d=8.68$ dB). The state probability π_m and average drift d_m are depicted in Fig. 8.16 for a receiver threshold of $z=10$ (10 dB). The traffic parameters ($P_o=0.0055$, $P_r=0.08$) and the pessimistic receiver threshold ($z=10$) were chosen such that the two spatial models studied give different assessments of the stability of the network with $N=100$ terminals. While the assumption of a uniform (continuous) distribution of the offered traffic suggests a stable network with little backlog, the log-normal model predicts bistable performance.

Judging from the results of a simulation by Van der Plas [22], who took into account the effect that most retransmissions originate from the boundary of the service area, both the uniform and the log-normal distributions lead to optimistic estimates of the network performance. Initially the software simulation program randomly distributes N ($N=100$) terminals over the area $0 < r < 1$ and estimates the corresponding local-mean powers for every terminal according to $p_j = \bar{p}_j = r_j^{-4}$. Shadowing is ignored in this

experiment ($\sigma_s=0$). All terminal states are modeled as a tri-state variable. In the O state, the program performs a random experiment to simulate generation of a packet with probability P_o . In the R state, permission for retransmission of a previously collided packet is granted with probability P_r . In both cases, the terminal enters the T state where the transmission and (multipath) propagation of a packet is simulated by the random generation of an instantaneous received power p_j , according to an exponential distribution with local-mean $r_j^{-\beta}$. By accounting for all the received interference packets, the program determines whether any of the colliding packets is strong enough to capture the receiver.

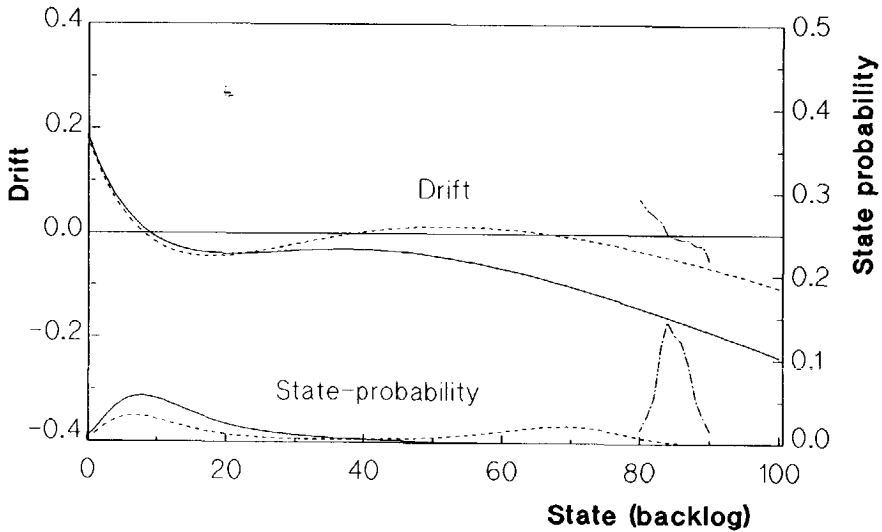


Fig. 8.16: Expected drift and state probability for a slotted ALOHA network with $N=100$ users and a receiver threshold of $z=10$, (—) according to a uniform distribution, (- -) a log-normal spatial distribution with $\sigma_d=2$ ($s_d \approx 8.68$ dB) and (···) a simulation. Probability of a new packet generation $P_o=0.0055$. Probability of retransmission $P_r=0.08$.

Because of to the near-far effect, the average time a terminal is in the retransmission mode increases with distance. Since $P_o < P_r$, the traffic offered per unit of area increases with distance. This has a disadvantageous influence on the network performance: colliding signals are more often received from far away and hence more

with nearly equal (low) powers. Consequently, the probability C_i that the power of one of these signals sufficiently exceeds the joint interference is relatively low.

8.7 CONCLUSIONS

The capture-ratio model and Laplace transforms of received signal have proved a most fruitful method to assess capture probabilities and throughput of mobile slotted ALOHA nets. Results confirmed observations previously reported in literature that the throughput of the ALOHA channel is substantially enhanced by receiver capture. The throughput of a number of spatial distributions has been compared, including a ring model, quasi- and exactly uniform distributions in one and two dimensions, and a 'log-normal' spatial distribution. The way terminals are distributed over the coverage area has a substantial influence on the throughput of the net. Also, the effect of combined Rayleigh fading shadowing and near-far effect has been studied. A general impression from these results is that the larger the differences in received power, the larger the probability that one packet is received correctly in a collision of many competing packets. This enhances the throughput and the stability of the network.

Uniform distribution of offered traffic, although frequently adopted in literature, gives an optimistic assessment of the network behaviour under high offered traffic loads.

This popular model fails in two respects:

- (i) It allows terminals to be infinitely close to the base station. For high traffic loads, stability may therefore rely upon terminals unrealistically close to the base station;
- (ii) Another effect is the interaction between the attempted traffic and the system performance: almost all retransmissions are clustered in (distant) parts of the coverage area with poor propagation. Since these transmissions may arrive with almost equal, low power, mutual collisions usually result in the loss of all packets involved.

Stability and delay performance have been investigated from a Markov chain model. The calculations reveal that bistability of the mobile networks considered occurs only for a very limited range of transmission parameters. The transition from low-backlog to saturation, i.e., the transition through the area of bistability, is very abrupt. This suggests that, in contrast to the case for wired networks, radio protocol design should focus on continuously assuring sufficient throughput and acceptable delay, for instance by exercising (slow) centralised control over the (number of) terminals allowed to sign on. In mobile ALOHA networks, the issues of (dynamically) repairing instability and collision resolution are presumably less critical compared with networks without capture. However, the near-far effect may not always be modelled by a (continuous) uniform spatial density of the traffic.

Analysis of various alternative distributions revealed that throughput and delay are less enhanced by capture than suggested by the theoretical investigations of (quasi-) uniform spatial distributions. Similarly, the estimation of stability areas made in section 8.6 may be further refined by investigating the drift at high backlog from a non-uniform and dynamically changing spatial distribution of the backlogged terminals. To this end, the spatial distribution of offered traffic would have to be assessed as a function of the number of terminals in backlog.

A similar effect as under (ii) is believed to occur in networks with stationary terminals, such as in narrowband indoor wireless office communication. The main cause of fading, transmitter mobility, is not present for terminals at a fixed location. Transmitters accidentally placed near a multipath null generate the main part of all retransmissions. Simulation revealed that area-mean powers remaining fixed for each terminal due to virtually constant propagation distances degrades the performance for mobile networks. Presumably, constant shadow and multipath attenuation during retransmissions in fixed networks further degrades overall performance. A more detailed study of the network performance for stationary subscribers is recommended.

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CHAPTER 9

FREQUENCY RE-USE IN WIDE-AREA PACKET-SWITCHED NETWORKS

In present mobile radio networks, high spectrum efficiency has been achieved by extensive frequency reuse. In the late 70's, cellular engineering was developed to accommodate the rapid growth of mobile telephony. Since in numerous mobile communication systems mostly stereotyped and short messages are sent, data packet transmission can provide more efficient use of the available bandwidth. The possibility of frequency re-use in wide-area random-access networks is studied in this chapter by extending the results in chapter 8 to include the effect of inter-cell interference. Study of wide-area data networks with cellular frequency re-use requires distinction between the inbound channel, which is randomly accessed by a large population of transmitters, and the outbound channel, where the base station multiplexes data

packets for various receiving mobile terminals [1]. Here we confine ourselves to the inbound (contention-type) channel.

In this case, a large number of terminals can operate within one cell and inbound messages are sent to the base station over a common radio channel, according to an appropriate random-access scheme. The access to the radio channels has a 'bursty' character, and packets lost due to interference are automatically retransmitted. In these networks, therefore, the need to continuously safeguard the traffic in one cell from excessive interference by co-channel transmissions in other cells is less demanding than in mobile (cw) telephony. Thus, frequency re-use distances and cell cluster sizes may be smaller. Moreover, a packet-switched cellular radio network might exploit the diversity resulting from packet reception by base stations outside the particular cell in which the terminal is located. Two methods to increase the system capacity are considered in this chapter: cellular frequency re-use and site diversity.

Most of the recent studies of the performance of mobile ALOHA networks address the case of a single base station, whereas cellular frequency re-use and site diversity has received relatively little attention. Only few previous reports on wide-area data networks with frequency re-use patterns are known to the author. The Nordic Mobitex system [2] is an example of a public mobile data network. The channel access resembles reservation ALOHA, and its frequency re-use strategy is similar to cellular voice networks. Goodman describes a cellular packet communication network for mixed traffic in [3]. Packet Reservation Multiple Access (PRMA) is proposed for channel access; this combines aspects of framed ALOHA and TDMA. Spatial frequency re-use in the proposed network is principally based on cellular techniques, designed similar to networks for cw voice communication. Hafez and Nehme [4] discussed some of the aspects of spatial re-use of spectrum resources in mobile data networks. A single inbound frequency is applied; within a frame of C slots, each time slot is only used once in a cluster of C cells to avoid co-channel interference. Wilson and Rappaport [5] addressed multichannel CSMA schemes in a cellular communication environment. Each base station is assigned a set of channels, and users access any one of the nearest base stations according to a multichannel CSMA protocol. Further, a

number of papers consider frequency re-use in (military) multi-hop packet radio nets, without fixed base stations. A major distinction from public data networks is the use of fixed nodes, i.e., base stations, and centralised frequency management in civil networks. Because most available publications offer only a first exploration of frequency re-use in wide-area data networks, further investigation of multi-cell multiple access networks appears important for the successful design of future mobile data communications networks.

This chapter addresses the possible application of (slotted) ALOHA in wide-area networks. Section 9.1 describes wide-area data networks and a number of assumptions made for analysis. The model for the mobile radio channel and for receiver capture are a combination of models used earlier in this thesis, e.g. in chapter 3 and 8. Section 9.2 summarises the relevant expressions, verifies the suitability of the capture-ratio model and establishes a notation appropriate to multi-receiver ALOHA networks. Cellular ALOHA networks are addressed in section 9.4 on the assumption that each base station only accepts packets from terminals within its cell, although interference may be caused from packets in other cells. Section 9.5 accounts for the effect that most retransmissions of previously unsuccessful packet transmissions occur in areas with poor propagation to the common receiver. The analysis is extended to site diversity in section 9.6 to 9.9. Section 9.10 concludes this chapter.

9.1 MODEL FOR CELLULAR DATA NETWORKS

In contrast to cellular cw communication, in a cellular packet data network, each inbound channel supports intermittent traffic from multiple users. We assume an infinite population of terminals, transmitting data packets with a spatial distribution $G(\underline{x})$, where $\underline{x}$ is the position in the cell area. If for a terminal location j only the distance a_j to receiver A is relevant, we write $G(a_j)$. The cellular frequency re-use pattern is depicted in Fig. 9.1.

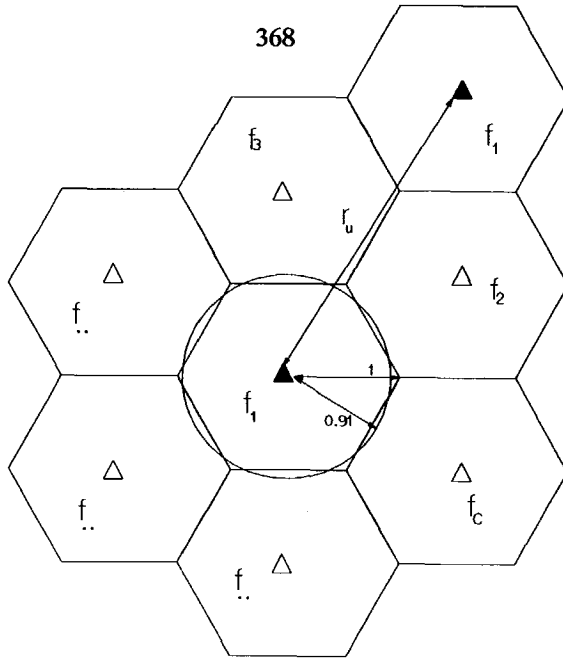


Fig. 9.1: Cellular frequency re-use in a mobile ALOHA network. Hexagonal cells are approximated by circular cells of identical surface.

In the analysis of hexagonal cellular networks, it is common practice to normalise the length of the sides of each hexagon (and the radius) to unity. Normalisation of sides of the hexagon to unit length in Fig. 9.1 is in contrast to the normalisation adopted in chapter 8, where the radius of a circular cell was normalised to unity. A hexagonal cell with sides of unit length has surface $\frac{1}{2} \cdot 3\sqrt{3}$. This corresponds to the surface of a circular cell with radius $R = 3^{3/4} / \sqrt{2\pi} \approx 0.91$.

If the offered traffic per unit area is constant throughout the entire network, the traffic offered from a cell to its base station A has the spatial distribution

$$G(a_j) = \begin{cases} G_0 = \frac{1}{\pi R^2} G_c, & 0 < a_j < R \\ 0, & \text{elsewhere,} \end{cases} \quad (9.1)$$

with $G(a_j)$ expressed in packets per packet time per unit of area, where a_j is the distance to base station A and G_c is the total traffic offered per cell, expressed in

packets per packet time. Besides $G(a_i)$, interfering packet signals from co-channel cells may also be received at the base station.

If C different carrier frequencies are used in each cluster of cells, the frequency re-use distance is $r_u = \sqrt{3C}$, normalised to the radius of a hexagonal cell (see Eq. (1.1)). Interfering signals from co-channel cells arrive from distances in a subinterval of $r_u - 1 < a_i < r_u + 1$, with the main body of the distribution in $(r_u - R, r_u + R)$. If adjacent cells use different inbound frequencies, inter-cell interference arrives only from distant terminals. If the frequency re-use distance r_u is relatively large ($r_u \gg 1$, $a_i \gg 1$), all interfering signals from co-channel cells arrive with almost the identical mean power $\bar{p}_{Ai} \approx r_u^{-\beta}$. This approximation will be used here also for relatively small re-use distances.

In a slotted network with global slot synchronisation and negligible propagation delays, the total packet traffic interfering with a test packet has the spatial distribution

$$G(a_i) = G_0 [U(a_i) - U(a_i - R)] + 6 \frac{G_c}{2\pi r_u} \delta(a_i - r_u), \quad (9.2)$$

where $U(x)$ is the unit step function ($U(x) = 1$ if $x > 0$ and zero otherwise).

Random access for mobile terminals by (pure or slotted) ALOHA is considered. If, in practical applications, relatively long messages are to be transmitted or if messages arrive according to some predictable pattern, other access techniques may be more appropriate, but the evaluation of mixed traffic is outside the scope here. For slotted wide-area networks, we assume that time slot synchronisation is perfect throughout the entire network and known to all terminals; propagation delays and technical difficulties of implementation are thus neglected. Despite guard times required in practical networks, all data packets are assumed to be of uniform duration, equal to the slot length, which is taken as the normalised unit of time. (The duration of a packet, expressed in seconds, is denoted by T_s .) The feedback channel for acknowledgements of correctly received packets is assumed to be perfect. This implies

that transmissions by the base stations are coordinated to guarantee that messages to a mobile terminal experience no harmful interference. The network protocol requires the presence of a suitable control network interconnecting all base stations. Here, we do not address the design of this (wired) network and of the supporting protocols.

Global slot synchronisation might prove feasible in networks of limited size employing a relatively low bit rate, for instance $r_b = 1200$ bit/s [2][11]. If the bit rate is chosen higher, as is increasingly the case in mobile data nets, propagation delays will become of the same order of magnitude as the duration of a data packet. For instance, a call request by a mobile station in the GSM system ($r_b \approx 270$ kbit/s) is made by means of slotted ALOHA. During the telephone call, in TDMA operation, timing advance and retard is possible for each mobile to ensure slotted arrival of blocks of bits. In contrast to this, isolated data packets are offered to the channel in random-access networks so providing feedback information on variable propagation delays is not feasible.

If the bit rate in a random-access channel is increased, packets become of shorter duration. The effective throughput can ultimately be severely impaired by the necessary guard times to ensure that, despite random propagation delays, received packets fit within prescribed time slots. In section 9.3, it will be verified that the throughput advantages of slotted ALOHA over pure (unslotted) ALOHA vanish if required guard times exceed the duration of a packet. That section also discusses that the results derived in this chapter for slotted ALOHA can be adapted to (pessimistically) approximate the throughput for pure ALOHA by considering interfering traffic with doubled intensity.

Time slotting may, nonetheless, be required to allow implementation of collision resolution schemes to ensure the stability of the network. If propagation delays become very large, a practical solution may be to split the inbound channel into a number of parallel channels, each with smaller bit rates and correspondingly longer packets.

9.2 CHANNEL AND CAPTURE MODEL

UHF pathloss and Rayleigh fading are considered, whereas shadowing is ignored for ease of analysis. The received packet power from two successive transmissions of the terminal is assumed to be entirely uncorrelated, because of Rayleigh fading. Note that this assumes that the terminal is moving. We will distinguish the cases that, during a retransmission waiting time, the mobile terminal moves sufficiently far to assume uncorrelated distances a_j during each retransmission ($T_g < T_g$), and, alternatively, that, for each packet, the distance a_j remains constant until a (re-) transmission attempt is eventually successful ($T_g < T_g$).

The threshold model for receiver capture (see chapter 7 and 8) will be used. Thus, a packet is assumed to capture the receiver in a base station if, and only if, its instantaneous power exceeds the instantaneous joint interference power by a certain margin (factor) z , called the receiver threshold. Although this over-simplifies the detection process in the receiver, we found in chapter 7 that the capture-ratio model may be considered appropriate if the probability that the C/I ratio is in the transition range ($z_1 < C/I < z_2$) is relatively low; z_1 is the C/I -ratio at which the probability of successful reception becomes significant and z_2 the C/I -ratio at which the probability of excessive bit errors in the packet becomes negligible. The accuracy of the capture-ratio model depends highly on the probability that during a certain time slot the C/I ratio is in the transition range, which is found as the throughput for z_2 minus the throughput for z_1 . Typically this transition was found for C/I ratios ranging from 0 to 6 dB ($z_1 \approx 1$; $z_2 \approx 4$).

Fig. 9.2b illustrates the packet success probability for Gaussian interference versus the C/I ratio for various error correction distances t , as reported in chapter 7. A more rapid transition for the case of a single (constant-envelope) interfering signal is likely to occur near a C/I -ratio of 0 dB. The throughput of the ALOHA channel versus the threshold z is depicted in Fig. 9.2a. We used the results (8.60) for $G_c \rightarrow \infty$ and (8.67) with (6.12) for $G_c = 1$ and 2 ppt. This figure confirms that the variations in received signal powers may be sufficiently high to ensure that the critical probability

$Pr(z_1 < C/I < z_2)$ is relatively small, although the effect of the choice of z is more significant than seen in section 7.9 for $Q(r)$. It is seen that the accuracy of the model would decrease somewhat if the offered traffic G_c becomes large ($G_c \gg 1$).

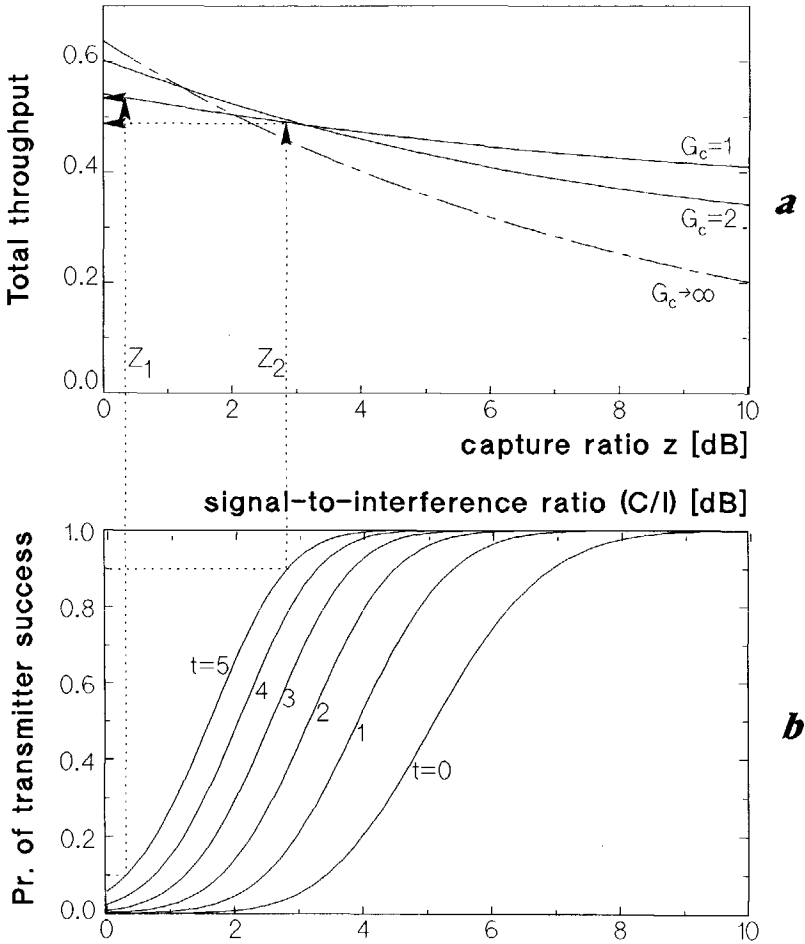


Fig. 9.2a: (above) Throughput S_t versus receiver threshold z . Uniform offered traffic with $G_c=1, 2$ and infinity.

Fig. 9.2b: (below) Probability of correct reception of a packet of $L=127$ bits with BPSK modulation versus the C/I ratio with Gaussian interference for various error correction capabilities $t=0, 1..5$.

In this chapter, numerical results are presented for $z=4$ (see chapter 7). During the reception of a packet, the received signal powers (thus also the C/I ratio) are assumed to be constant: packets are considered to be short compared to the time constants of the multipath fading. As already discussed in chapter 7, the results presented here may be optimistic if, in a practical system, packet durations are not small compared to the time constants of the fading. On the other hand, the fact that we neglect propagation delays and guard times limits the suitability of the model to packet durations that are long compared to the propagation delays. In other words, in this chapter, the packet duration is assumed shorter than the time the vehicle takes to move over a distance larger than a fraction of the wavelength, but longer than the time an electromagnetic signal takes to cross the service area. This condition has been denoted as $T_4 < T_5 < T_6$ in section 2.1.7.

9.2.1 SUMMARY OF WEIGHT-FUNCTION TECHNIQUE

The notation adopted here is a combination of the notation introduced in chapter 3 and 8. Results relevant to the analysis here are briefly summarised in this section. In a random-access net, the distinction between 'wanted' and 'interfering' signals vanishes. We consider one particular signal, called the 'test signal', which is denoted with index j . The event that the C/I -ratio of a packet signal from terminal j at receiver A is above the receiver threshold z is denoted as A_j . The probability of capture $Pr(A_j | \bar{p}_{A_j})$ can be expressed on the form of Eq. (8.45). Three cases of contaminating signals will be considered: (i) Interference caused by terminals occasionally transmitting a packet in the same cell, terminals occasionally transmitting a packet in a co-channel cell, and the receiver noise floor. (ii) The probability that terminal k , either in the same cell or in a co-channel cell, offers an interfering packet during the time slot in which the 'test' packet from terminal j is transmitted is denoted as $Pr(k_{ON})$. This probability is assumed stationary for each terminal. (iii) The receiver noise floor, caused by additive noise with power N_A , is described by the pdf $\delta(p_n - N_A)$, with Laplace image $\exp\{-sN_A\}$.

We denote the probability that the signal from terminal j sufficiently exceeds the noise floor at receiver A as P_{NA} .

For a finite population of N terminals, the probability $Pr(A_j | \{a_i\})$ that a 'test' packet transmitted by terminal j captures the receiver in base station A , given all distances a_i ($i=1,...,N$) and a_j was shown to be (Eq. (8.46))

$$Pr(A_j | \{a_i\}_{i=1}^N) = P_{NA} \prod_{i=1, i \neq j}^N [1 - W(a_j, a_i) Pr(i_{ON})], \quad (9.3)$$

with the vulnerability weight function $W(a_j, a_i)$ ($0 \leq W(.,.) \leq 1$) defined as $W(a_j, a_i) = za_j^\beta (za_j^\beta + a_i^\beta)^{-1}$.

For an infinite population, the probability of successfully receiving signal j from a known distance a_j in the presence of a Poisson distributed number of n interfering signals ($j \notin \{1, .., n\}$) with unknown positions was expressed in (8.51) as

$$Pr(A_j | a_j) = P_{NA} \exp \left\{ - \int_0^\infty 2\pi a_i W(a_j, a_i) G(a_i) da_i \right\}. \quad (9.4)$$

The throughput per unit of area $S(a_j)$, defined as the average number of successful packets per unit area per time slot, is obtained from $S(a_j) = G(a_j)Pr(A_j | a_j)$. The total throughput at receiver A , denoted as S_A and expressed in successful packets per packet time, is obtained by polar integration of $S(a_j)$.

9.3

GUARD TIMES AND UNSLOTTED ALOHA

This section refines the discussion on propagation delays and guard times that was initiated in section 9.1. We assume a guard time T_g required to ensure synchronised

arrival of packets. For a network with cell radius of D metres, $T_g = 2D/c$, while for slot synchronisation of interfering cells a larger T_g is required. The duration each slot has to be at least $T_s + T_g$, with T_s and T_g expressed in seconds. We consider the situation that the offered packet traffic λ_c per *second* is independent of the guard time. Consequently, in a system where guard times are required, the offered traffic G_{tg} expressed in packets per *time slot*, is larger than the offered traffic per slot G_t in a theoretical network without guard times, i.e.,

$$\lambda_c = \frac{G_{tg}}{T_s + T_g} = \frac{G_t}{T_s}. \quad (9.5)$$

If required guard times become excessively large, say if $2D > T_s c$, unslotted ALOHA might be considered. Exact analysis of unslotted ALOHA is complicated by the fact that the number of interfering signals changes during the reception of a packet. We now propose a model that somewhat overestimates the effect of interference in pure ALOHA. A test packet is assumed to capture the receiver if the received power exceeds z times the total power accumulated from *all* signals that are present during at least a part of the duration of reception of the test packet. For an infinite population of terminals, the number of packets overlapping with the test packet is Poisson distributed with mean $2G_t$, so one may use the expression presented in the next sections if the contending or interfering traffic per cell G_c is replaced by $2G_c$, and similarly, if the traffic per area $G(x)$ is replaced by $2G(x)$. This also corresponds to the case $G_{tg} = 2G_t$ in (9.5). Hence, the probability of capture can be estimated from

$$\begin{aligned} Pr(A_j | n, \bar{p}_{A_j}) > P_{UA} &= P_{NA} \prod_{i=1}^n \mathcal{L} \left\{ f_{p_{A_i}}, \frac{z}{p_{A_j}} \right\} \\ &= P_{NA} \prod_{i=1}^{n_1} \mathcal{L} \left\{ f_{p_{A_i}}, \frac{z}{p_{A_j}} \right\} \prod_{i=n_1+1}^n \mathcal{L} \left\{ f_{p_{A_i}}, \frac{z}{p_{A_j}} \right\}, \end{aligned} \quad (9.6)$$

where $1, \dots, n_1$ denote the interfering packet signals present at the instant of arrival of the test packet j , and $n_1 + 1, \dots, n$ denote packets present at the end of the test packet.

One may conclude that unslotted ALOHA becomes favourable if T_g approaches or exceeds T_s .

9.4 SPECTRUM EFFICIENCY FOR GLOBALLY UNIFORM OFFERED TRAFFIC

If cellular frequency re-use is employed in mobile *data* networks, the outage probability (as expressed in chapter 3) may not be the most appropriate performance measure. Spectrum efficient design requires more appropriate criteria. Data mostly requires highly reliable communication without outages, but is significantly less vulnerable to delay than voice communication. Data packets lost during a signal outage are, if necessary for the application, retransmitted. In a cellular data network, the spectrum efficiency is affected by the choice of the cluster size in two mutually conflicting ways. The bandwidth available per cell or per base station decreases proportionally to the cluster size if the total bandwidth allocated to the system is fixed. On the other hand, small cluster sizes lead to large mutual interference between cells. This in turn leads to poor reception, requiring many retransmissions, so the effective throughput may reduce if the cluster size is chosen too small.

A first step to understand the spectrum efficiency of a mobile ALOHA network uses a criterion based on the attempted traffic that can be supported with a certain prescribed probability of capture. In analogy with assumptions in chapters 6, 7 and 8, it is assumed that the *offered* traffic is known. A uniformly distributed offered traffic is reasonable, for instance, if the packet traffic load is kept relatively low to ensure that few packets are lost in collisions. If vehicle speeds are sufficiently large to ensure that the positions of a terminal become uncorrelated between each (re-) transmission, uniform offered packet traffic also may be a reasonable assumption. Moreover, in certain vehicle location systems or telemetry applications, a (routine) status report lost in a collision may not need to be retransmitted. For instance, the proposed SOCRATES

system, which is intended for road traffic management, gathers information on urban traffic flows that is randomly transmitted by vehicles participating in the road traffic [7]. This leads to uniform offered traffic if the road traffic is uniformly distributed around the location of the base station.

Cellular frequency re-use is considered. A base station is assumed to accept packets only from terminals inside its cell. Packets from co-channel cells are considered interference. The following measure of the spatial bandwidth (or spectrum) efficiency of a cellular data network, expressed in bit/s/Hz per base station, is proposed

$$SE_1 \triangleq \frac{\lambda_c L}{B_c} = \frac{\lambda_c L}{CB_T}, \quad (9.7)$$

where L is the packet size in bits, λ_c is the maximum offered traffic expressed in packets per second ($\lambda_c = G_c/T_s$) that can be supported with a satisfactory grade of service, i.e., with a probability of success larger than Q_{MIN} for a worst-case location of the mobile terminal ($a_j=R$). B_c denotes the bandwidth occupied by the cellular network with C frequencies, each of bandwidth B_T . For a given type of modulation, one can define η_r as the bit rate per occupied bandwidth ($\eta_r = r_b/B_T$). So, the spectrum efficiency SE_I can be expressed as

$$SE_1 \triangleq \frac{G_c}{LT_b} \frac{L}{CB_T} = \pi R^2 G_0 \frac{\eta_r}{C}. \quad (9.8)$$

We assume that the parameters B_c , η_r , L and z are known and that $R \approx 0.91$. The spectrum efficiency, or equivalently the arrival rate λ_c that can be supported, is to be optimised. Both the spectrum efficiency and the probability of success are functions of G_0 and C . Thus efficient spectrum use requires that the total traffic offered within the cell ($G_c = \pi R^2 G_0$) and the tolerated co-channel interference from outside the cell are optimised jointly. The resulting optimum cluster size C_{OPT} then determines the system parameters B_T , r_b and T_b . This two-dimensional optimisation problem is in contrast to the case for optimum spectrum efficiency in cellular telephony [9], where the traffic

within each cell consists of only one signal. In the latter case, efficient spectrum efficiency only requires a one-dimensional optimisation.

We now use our weight-function technique to express the influence of G_0 and C on the probability of success. The interfering packet traffic has the spatial distribution (9.2). Using (9.4), the probability of capture is found as

$$Pr(A_j | a_j) = P_{NA} \exp \left\{ - \int_0^R 2\pi \lambda W(a_j, \lambda) G_0 d\lambda - 6G_c \frac{za_j^\beta}{za_j^\beta + r_u^\beta} \right\}. \quad (9.9)$$

The worst-case probability of successful access occurs at $a_j = R \approx 0.91$, with, for $\beta = 4$,

$$Pr(A_j | a_j = R) = \exp \left\{ -zN_A R^4 - \pi \sqrt{z} R^2 G_0 \arctan \left(\frac{1}{\sqrt{z} R^2} \right) - 6 \frac{zR^4 R^2 \pi G_0}{zR^4 + r_u^4} \right\}, \quad (9.10)$$

where we used (8.67). The maximum value of G_0 that satisfies $Pr(A_j | a_j = R) > Q_{MIN}$ is found from

$$G_{0,MAX} = \frac{-\ln Q_{MIN} - zN_A R^4}{\pi \sqrt{z} R^2 \arctan \left(\frac{1}{\sqrt{z} R^2} \right) + \frac{6z\pi R^6}{zR^4 + 9C^2}}, \quad (9.11)$$

with $Q_{MIN} < \exp\{-zN_A R^4\}$. Eq. (9.11) is inserted in the definition of spectrum efficiency (9.7):

$$SE_1 = \frac{\eta_r}{C} \frac{-\ln Q_{MIN} - zN_A R^4}{\sqrt{z} \arctan \left(\frac{1}{\sqrt{z} R^2} \right) + 6 \frac{zR^4}{zR^4 + 9C^2}}. \quad (9.12)$$

Thus SE_1 depends on Q_{MIN} , N_A , C , z and η_r . Fig. 9.3 portrays Eq. (9.12) versus the cluster size C for $\eta_r = 1$ bit/s/Hz and receiver thresholds of $z = 4$ (6 dB) and $z = 100$ (20 dB). The criterion Q_{MIN} is taken equal to 0.5 and 0.9 for a noise-free channel

($N_A=0$). For a noisy channel, the curves represent the case $Q_{MIN}=0.5\exp\{-(0.91)^2zN_A\}$ and $Q_{MIN}=0.9\exp\{-(0.91)^2zN_A\}$, respectively.

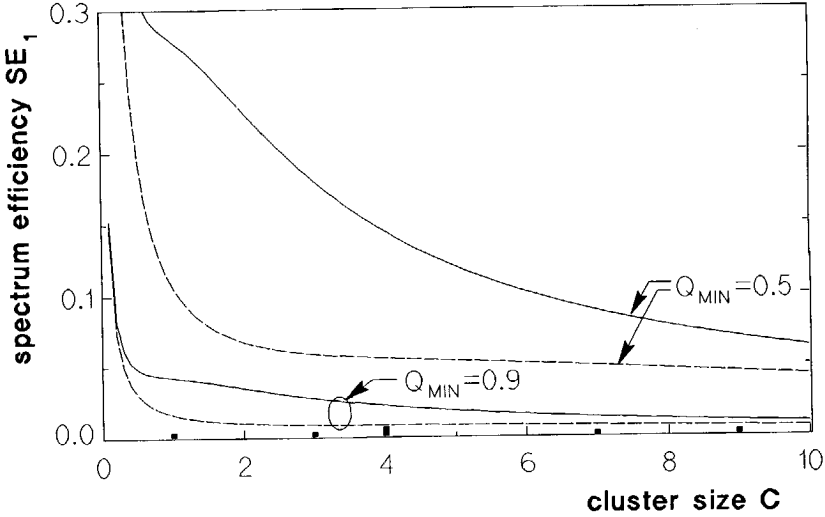


Fig. 9.3: Spatial bandwidth efficiency SE_p , in bit/s/Hz per base station, versus cluster size C for a minimum probability of capture $Q_{MIN}=0.9$ and 0.5 . $\eta_r=1$ bit/s/Hz. Receiver threshold: (—) $z=4$ (6 dB) and (- -) $z=100$ (20 dB). Noise-free channel ($N_A=0$). Path loss law $\beta=4$.

As the numerical examples of (9.12) suggest, optimum spectrum efficiency is thus achieved if all cells use the same inbound frequency. This can be understood intuitively from earlier results for uniform offered traffic, within a single cell and with infinite extension. If $C=1$, the case for infinitely extended uniform offered traffic (8.59) applies: $G_{0,MAX}=2\pi^{-2}z^{-1/2}Q_{MIN}$.

Alternatively, if one would take $C>1$ ($C=3, 4, 7, 9, \dots$), the bandwidth per cell is reduced by a factor C . In the most optimistic scenario, the effect of co-channel interference from other cells vanishes. According to (8.68), in this case the offered traffic $G_{0,MAX}$ could approximately be doubled. However, for doubled G_0 and increased C , the arrival rate λ is multiplied by a factor $2/C$, because $\lambda = G_0/T_s = G_0\eta_r B_c/(CL)$. Thus any value of $C \geq 2$ is likely to lead to lower tolerable offered traffic than in the case with $C=1$.

Hence, in the ALOHA network studied here, the reduction of interference from co-channel cells does not justify the choice of different inbound frequencies for adjacent cells. In practical networks with mixed traffic, the cluster size may still have to be chosen larger to ensure reliable operation during transfer of longer messages for priority traffic. In this case, the results in Fig. 9.3 can be of assistance to estimate the improved spectrum efficiency from separating the channels for inbound (randomly arriving) shorts packets and reserved transfer of longer messages.

9.5 TRAFFIC TO BE OFFERED TO ACHIEVE UNIFORM THROUGHPUT

In contrast to the previous case of a known distribution of the *offered* packet traffic, in practical random access networks, most retransmissions can be expected to occur in areas with poor propagation to the base stations. We now assume that retransmissions are always transmitted from the same distance as where the first transmission was attempted. This corresponds to a prescribed *throughput* per unit of area. A reasonable assumption is uniform throughput, with spatial distribution

$$S(a_j) = \begin{cases} S_0 = \frac{1}{\pi R^2} S_r, & 0 < a_j < R \\ 0, & \text{elsewhere} \end{cases} \quad (9.13)$$

We address the effect of interference from outside the coverage area of the base station in more detail.

Analogous to (9.8), the spectrum efficiency can be defined as

$$SE_2 \triangleq \frac{S_A \eta_r}{C}, \quad (9.14)$$

where S_A is the maximum uniform throughput on the form of (9.13) that can be achieved. Prescribed throughput and hexagonal frequency re-use are considered by rewriting (9.4) and (9.2) as

$$G(a_j) = \frac{S_0}{Pr(A_j | a_j)} \quad (9.15)$$

$$= S_0 \exp \left\{ +zN_A a_j^8 + \int_0^{\infty} 2\pi a_i W(a_j, a_i) (G(a_j) + 6G_I(a_i)) da_i \right\},$$

where $G(a_j) = G(a_j)$ denotes the offered traffic within the cell. Interference from a co-channel cell is denoted as $G_I(a_j)$. It is also assumed that the total traffic offered within each co-channel cell is identical to the traffic G_c offered in the cell served by receiver A . A solution for the offered traffic is obtained by means of an iterative computational technique with (9.15): Each new estimate of $G(a_j)$ on the left hand-side of (9.15) is computed by inserting the previous estimate of $G(a_j)$ and $G_I(a_j)$ in the integral, with $G_I(a_j) = G_c \delta(a_j - r_u) / (2\pi r_u)$. The traffic per cell G_c is obtained by polar integration of $G(a_j)$ from 0 to $R \approx 0.91$. Analogous to the second member of the right-hand side of (9.2), $G_I(a_j)$ is then found from $G_c \delta(a_j - r_u) / (2\pi r_u)$. The author has shown in his Master's thesis (see also [12]) that, if a (single) solution of (9.15) exists and numerical (rounding) errors are negligible, the iteration scheme converges to the solution $G(a_j) = G(a_j)$ from a first estimate $G(a_j) = S_0$.

Fig. 9.4 gives the traffic $G(a_j)$ to be offered to achieve uniform throughput of $S_A = 0.4$ ppt per cell for various cluster sizes ($C = 4, 9$ and 100), although for small cluster sizes, approximation (9.2) may lose its accuracy because it simplifies the pdf of interference power from co-channel cells. Table 9.1 and Table 9.2 give the expected number of required (re-) transmission attempts for terminals at the boundary of the cell and averaged over all positions in the cell ($0 < a_j < R$), to achieve $S_A = 0.4$ and 0.2 ppt, respectively. Also, the effect of a noise floor $N_A = 0.1$, which corresponds to a worst-case local-mean C/N -ratio of 11.6 dB for a terminal at $a_j = R \approx 0.91$, is considered.

Table 9.1: Expected number of required transmission attempts in a cellular ALOHA data network for various cluster sizes C and noise levels N_A . Throughput per cell is $S_t=0.4$ ppt. Receiver threshold $z=4$ (6 dB).

C	$r_u = \sqrt{3C}$	N_A	G_t	number of attempts <i>average, fringe of cell</i>	
1	1.73	0	no solution found		
1	1.73	0.1	no solution found		
4	3.46	0	0.64	1.61	1.94
4	3.46	0.1	0.80	2.00	2.97
9	5.20	0	0.62	1.56	1.79
9	5.20	0.1	0.75	1.88	2.68
100	17.3	0	0.62	1.54	1.76

Table 9.2: Expected number of required transmission attempts in a cellular ALOHA data network for various cluster sizes C and noise levels N_A . Throughput per cell is $S_t=0.2$ ppt. Receiver threshold $z=4$ (6 dB).

C	$r_u=\sqrt{3C}$	N_A	G_t	number of attempts average, fringe of cell	
1	1.73	0	0.29	1.45	1.96
1	1.73	0.1	0.35	1.76	2.96
4	3.46	0	0.24	1.21	1.28
4	3.46	0.1	0.27	1.36	1.74
9	5.20	0	0.24	1.19	1.25
9	5.20	0.1	0.27	1.35	1.69
100	17.3	0	0.24	1.19	1.25
100	17.3	0.1	0.27	1.34	1.68

UHF groundwave propagation with $\beta=4$ is considered. For a receiver threshold of 6 dB, a fade margin of 5.6 dB remains at $a_j=R\approx 0.91$. Interestingly, the effect of intercellular interference is relatively small, even for $C=1$ or 4, compared to the effect of noise. The observation that noise may substantially diminish the performance in mobile ALOHA channels is in contrast to observations in [8], where a prescribed *offered* traffic was assumed. It is seen in Fig. 9.4 that packets from the remote terminals are much more likely to be lost in noise, which causes a significant increase in the number of (re-) transmission attempts at larger ranges. It was seen in [9] that poor reception at the fringe of the cell may also threaten the stability of the network. A higher fade margin appears desirable.

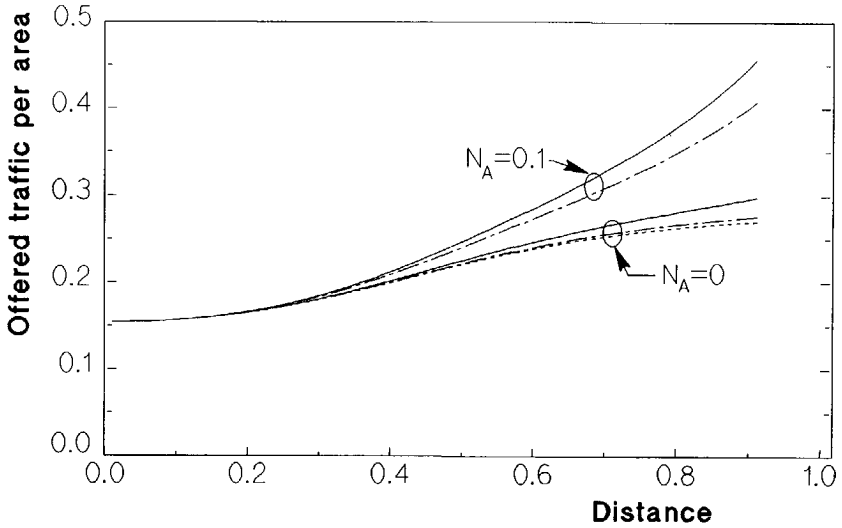


Fig. 9.4: Traffic per unit of area $G(a_j)$ to be offered to achieve throughput $S_t=0.4$ ppt with uniform distribution. Receiver threshold $z=4$ (6 dB). Cluster size $C=4$ (—), 9 (— · —) and 100 (— — —). Effect of noise: $N_A=0$ and 0.1. Path loss law $\beta=4$.

For a cluster size of $C=1$ and a receiver threshold of $z=4$, the maximum uniform throughput of slotted ALOHA is $S_A \approx 0.25$ ppt for $N_A=0.1$ and $S_A \approx 0.3 \dots 0.35$ ppt for a noise-free channel ($N_A=0$). It turned out from our computations that the traffic to be offered for the cases $C=4$ with $S_A=0.4$ ppt and $N_A=0.1$ is almost identical with the case $C=1$ with $S_A=0.2$ ppt and $N_A=0.1$. In both cases, the expected number of required transmission attempts is about three for terminals near the fringe of the cell. A slightly higher required throughput caused divergence of the iteration scheme (9.15). The spectrum efficiency is thus about $SE_2 \approx 0.2\eta_r$ for $C=1$ and $SE_2 \approx 0.1\eta_r$ for $C=4$. For larger cluster sizes, the spectrum efficiency SE_2 further decreases. It can also be concluded that, if spectrum utilisation is optimised, distant terminals may have to perform many retransmissions. SE_2 is seen to be in the same order of magnitude as SE_1 with $Q_{MIN}=0.5$.

9.6 MODEL FOR TWO-BRANCH SITE DIVERSITY

It has been illustrated that relatively short frequency re-use distances can be tolerated without substantial sacrifices to the throughput of each cell. In the extreme case $C=1$, adjacent cells use the same frequency and slot synchronisation, so packets transmitted near the boundary of the two cells can be received successfully at more than one base station. This offers the perspective of 'site diversity'. Although some results are available for antenna (or micro-) diversity at a single base station, e.g. [10], the effect of applying multiple receiver sites has received little attention in technical literature. In [1], Chang studied the throughput of a random access network with multiple receiver sites by extensive evaluation of expected events of collisions. Here, the vulnerability-weighting technique is extended to produce analytical expressions for the throughput of slotted ALOHA with site diversity.

In the next two subsections (9.6.1 and 9.6.2), results for site diversity in chapter 3 are simplified by neglecting the effect of shadowing ($\sigma_s=0$). This allows study in section 9.7 of the spatial distribution of an infinite population of terminals.

9.6.1 ACTIVITY OF INTERFERING TERMINALS

The probability $Pr(k_{ON}|A_j)$ that terminal k has transmitted a packet, given the (a posteriori) knowledge A_j that a packet from terminal j captures base station A , was derived in chapter 3 from Bayes' rule:

$$Pr(k_{ON}|A_j, \{a_k\}) = \frac{1 - W(a_j, a_k)}{1 - W(a_j, a_k) Pr(k_{ON})} Pr(k_{ON}), \quad (9.16)$$

where in contrast to Eq. (3.30) shadowing has been ignored ($\sigma_s=0$).

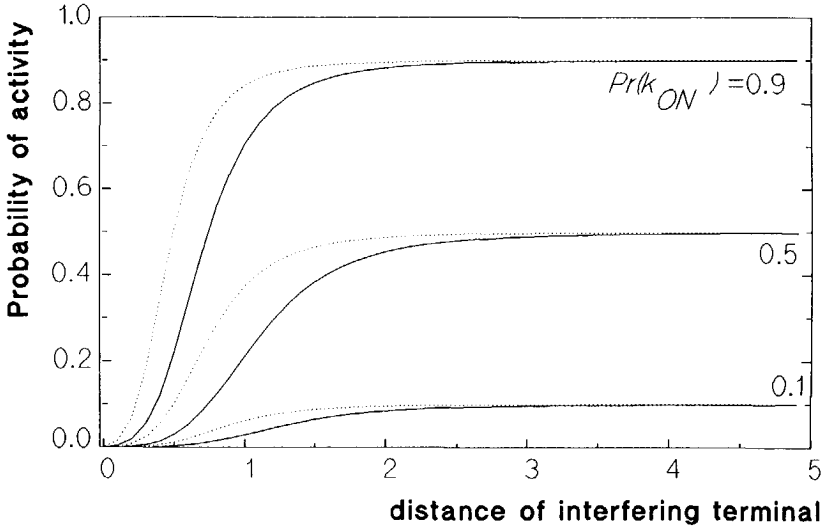


Fig. 9.5: *A posteriori probability $P(k_{ON}|A_j)$ that an interfering terminal k at distance a_k transmitted a packet, given that a packet from terminal j at distance $a_j=1$ captures receiver A in the same slot. The a priori probability of transmission is $Pr(k_{ON})=0.1, 0.5$ and 0.9 . Receiver threshold $(\cdot \cdot)$ $z=1$ and $(—)$ $z=4$.*

Fig. 9.5 agrees with the intuitive understanding that the probability of an active interferer in the range $0 < a_k < \beta \sqrt{z} a_j$ must be very small. In cellular telephone networks, generally it is *a priori* unlikely that this range contains an interferer, because re-use distance are usually relatively large. It was seen in chapter 3 that considering a posteriori probabilities of active interferers is relevant only if substantial shadowing is present and relatively small re-use distances are employed. In random-access networks, however, (9.16) singles out a significant area for interfering contenders, which may be relevant even if no shadowing occurs $\sigma_s=0$.

9.6.2 CAPTURE PROBABILITY FOR FINITE POPULATION OF TERMINALS

The positions of the N terminals are denoted as $\underline{x}_i$. Since terminal i is at (normalised) distances a_i to base station A and at distance b_i to base station B , $|\underline{x}_i - \underline{x}_A| = a_i$ and

$|\underline{x}_A - \underline{x}_B| = b_i$ (see Fig. 3.5). The normalised distance between the two base stations is R_d . Similar to (9.3), the conditional probability that the packet from terminal j captures base station B given that it also captures base station A is found as

$$\begin{aligned} Pr(B_j | A_j, \{\underline{x}_i\}_{i=0}^N) &= P_{NB} \prod_{k=1, k \neq j}^N 1 - W(b_j, b_k) Pr(k_{ON} | A_j, a_j, a_k) \\ &= P_{NB} \prod_{k=1, k \neq j}^N 1 - \frac{1 - W(a_j, a_k)}{1 - W(a_j, a_k) Pr(k_{ON})} W(b_j, b_k) Pr(k_{ON}), \end{aligned} \quad (9.17)$$

where P_{NB} is the probability that the received signal fails to exceed the noise floor at receiver B . The probability that a packet from terminal j captures at least one of the two base stations (A or B), given the position of each terminal i ($i=1, 2, \dots, N$), equals

$$Pr(A_j \vee B_j | \{\underline{x}_i\}_{i=1}^N) = Pr(A_j | \{\underline{x}_i\}) + Pr(B_j | \{\underline{x}_i\}) - Pr(A_j | \{\underline{x}_i\}) Pr(B_j | A_j, \{\underline{x}_i\}). \quad (9.18)$$

Inserting (9.3) and (9.17) in (9.18) gives

$$\begin{aligned} Pr(A_j \vee B_j | \{\underline{x}_i\}_{i=1}^N) &= P_{NA} \prod_{i=1, i \neq j}^N 1 - W(a_j, a_i) Pr(i_{ON}) \\ &\quad + P_{NB} \prod_{i=1, i \neq j}^N 1 - W(b_j, b_i) Pr(i_{ON}) - P_{NA} P_{NB} \prod_{i=1, i \neq j}^N 1 - W_{AB} Pr(i_{ON}), \end{aligned} \quad (9.19)$$

where we introduced the joint weight function

$$\begin{aligned} W_{AB} &\triangleq W(a_j, a_k) + W(b_j, b_k) - W(a_j, a_k) W(b_j, b_k) \\ &= 1 - \frac{a_k^\beta b_k^\beta}{(za_j^\beta + a_k^\beta)(zb_j^\beta + b_k^\beta)}. \end{aligned} \quad (9.20)$$

Here, the factor W_{AB} weighs the disturbance caused by a interfering packet signal from position $\underline{x}_k$ to a reception of a data packet by terminal j at the two base stations A and B simultaneously. For large receiver threshold ($z \rightarrow \infty$) or for an interferer relatively close to either of the base stations ($a_k \ll a_j$ or $b_k \ll b_j$), the weight factor tends to unity. This represents the fact that the interference signal is very likely to disturb duplicated reception of the test packet from terminal j at both stations. For a

remote interferer ($b_k, a_k \rightarrow \infty$), W_{AB} tends to zero: despite interference from k , the test packet is likely to be received correctly at both base stations simultaneously.

9.7 CAPTURE PROBABILITY FOR INFINITE POPULATION OF TERMINALS

For a packet transmitted from a position $\mathbf{x}_j$ in the presence of n interfering packets, the probability of capturing at least one of the two base stations is found by rewriting (9.19) for a population of n interfering terminals known to transmit, and averaging over all possible positions of the n interferers. So

$$Pr(A_j \vee B_j | \mathbf{x}_j, n, \{i_{ON}\}) = \int \int_{area} \dots \int \int_{area} Pr(A_j \vee B_j | \mathbf{x}_j, \{\mathbf{x}_i, i_{ON}\}_{i=1}^n, n) f_{\mathbf{x}_1}(\mathbf{x}_1) \dots f_{\mathbf{x}_n}(\mathbf{x}_n) d\mathbf{x}_1 \dots d\mathbf{x}_n \quad (9.21)$$

Inserting $Pr(i_{ON} | i_{ON}) = 1$ for $i = 1, 2, n$ in (9.19) and (9.21), gives

$$\begin{aligned} Pr(A_j \vee B_j | n, \mathbf{x}_j, \{i_{ON}\}) &= \int \int_{area} \dots \int \int_{area} \left\{ P_{NA} \prod_{i=1}^n [1 - W(a_j, a_i)] \right. \\ &\quad \left. + P_{NB} \prod_{i=1}^n [1 - W(b_j, b_i)] - P_{NA} P_{NB} \prod_{i=1}^n [1 - W_{AB}] \right\} f_{\mathbf{x}_1}(\mathbf{x}_1) \dots f_{\mathbf{x}_n}(\mathbf{x}_n) d\mathbf{x}_1 \dots d\mathbf{x}_n. \end{aligned} \quad (9.22)$$

Since each of the integration variables $\mathbf{x}_i$ occurs only in one factor of each of the products, one may interchange product and integration. Hence,

$$\begin{aligned} Pr(A_j \vee B_j | n, \mathbf{x}_j, \{i_{ON}\}) &= P_{NA} \left[\int \int_{area} \{1 - W(a_j, a_i)\} f_{\mathbf{x}_i}(\mathbf{x}_i) d\mathbf{x}_i \right]^n \\ &\quad + P_{NB} \left[\int \int_{area} \{1 - W(b_j, b_i)\} f_{\mathbf{x}_i}(\mathbf{x}_i) d\mathbf{x}_i \right]^n - P_{NA} P_{NB} \left[\int \int_{area} \{1 - W_{AB}\} f_{\mathbf{x}_i}(\mathbf{x}_i) d\mathbf{x}_i \right]^n. \end{aligned} \quad (9.23)$$

For n Poisson distributed, this becomes a sum of exponential functions, viz.,

$$\begin{aligned}
 Pr(A_j \vee B_j | \underline{x}) &= \sum_{n=0}^{\infty} \frac{G_i^n}{n!} e^{-G_i} Pr(A_j \vee B_j | n, \underline{x}, \{i_{ON}\}) \\
 &= P_{NA} \exp \left\{ - \int_{area} W(a_j, a_i) G(\underline{x}_i) d\underline{x}_i \right\} \\
 &\quad + P_{NB} \exp \left\{ - \int_{area} W(b_j, b_i) G(\underline{x}_i) d\underline{x}_i \right\} - P_{NA} P_{NB} \exp \left\{ - \int_{area} W_{AB} G(\underline{x}_i) d\underline{x}_i \right\}
 \end{aligned} \tag{9.24}$$

with $G(\underline{x}_i)$ the offered traffic per unit of area at $\underline{x}_i$. Eq. (9.24) offers a mathematical expression for the probability of capturing at least one receiver, given an arbitrary spatial distribution of the offered traffic, described by $G(\underline{x}_i)$. The expression contains three terms: the first and second term are on the form (9.4) for the individual receivers A and B ; the third terms corresponds to successful reception at both receivers simultaneously.

9.8 CAPTURE PROBABILITY FOR GLOBALLY UNIFORM OFFERED TRAFFIC

We assume uniform offered packet traffic of $G(\underline{x}) = G_0$ for all $\underline{x}$. The first two terms in (9.24) are on the analytic form of (8.59). Using the substitutions $r = a_k$ and $b_k^2 = r^2 + R_d^2 - 2R_d r \cos \phi$, the third term in (9.24) equals, for a noise-free system,

$$Pr(A_j \wedge B_j | \underline{x}) = \exp \left\{ - \int_0^{2\pi} \int_0^{\infty} \left[1 - \frac{r^4 (r^2 + R_d^2 - 2rR_d \cos \phi)^2}{(za_j^4 + r^4)(zb_j^4 + (r^2 + R_d^2 - 2rR_d \cos \phi)^2)} \right] G_0 r dr d\phi \right\} \tag{9.25}$$

which has been evaluated numerically.

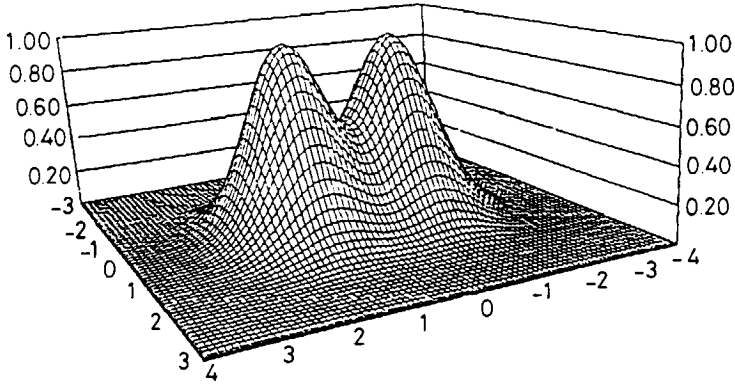


Fig. 9.6: Probability of capture $\Pr(A_j \vee B_j | x_j)$ as a function of the position of the terminal. Base stations are located at $(1,0)$ and $(-1,0)$. The receiver threshold is 6 dB ($z=4$). The offered traffic per unit of area is $G_0 = 0.1$ ppt.

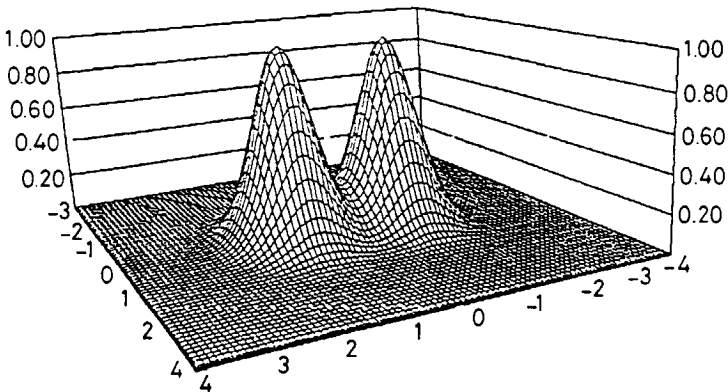


Fig. 9.7: Probability of capture $\Pr(A_j \vee B_j | x_j)$ as a function of the position of the terminal. Base stations are located at $(1,0)$ and $(-1,0)$. The receiver threshold is 6 dB ($z=4$). The offered traffic per unit of area is $G_0 = 0.2$ ppt.

Fig. 9.6 and Fig. 9.7 give numerical results of the probability of capturing at least one of two geographically separated base stations as a function of the (two-dimensional) position of the transmitting terminal. Base station A is located at $(-1,0)$ and B is located at $(+1,0)$. Fig. 9.8 presents the cross-section of Fig. 9.6 along the axis through both base stations. The exact result for the probability $Pr(A_j \vee B_j | x_j)$ of capture at one of the two receivers is compared with the approximation $Q_M(x_j)$, calculated by

$$Q_M(x_j) \triangleq Pr(A_j | x_j) + Pr(B_j | x_j) - Pr(A_j | x_j) Pr(B_j | x_j) . \quad (9.26)$$

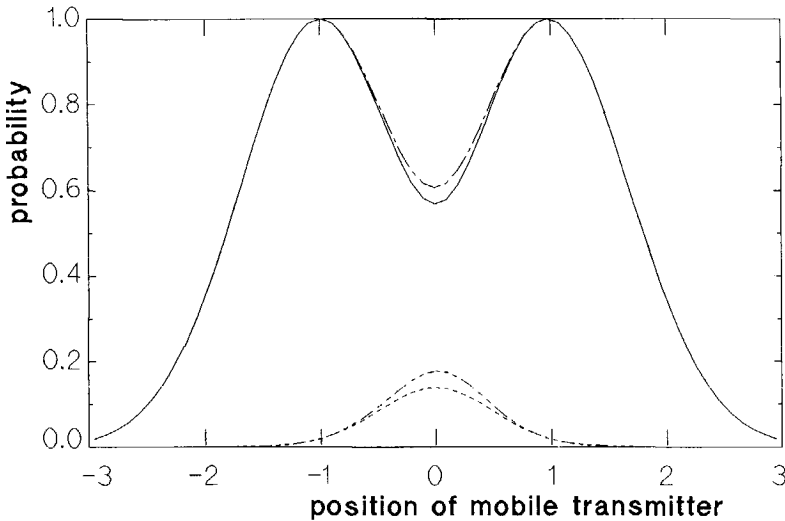


Fig. 9.8: Probability of successful reception in at least one receiver $Pr(A_j \vee B_j | x_j)$ and at both receivers simultaneously $Pr(A_j \wedge B_j | x_j)$ for a terminal located on the line through the base stations at $(1,0)$ and $(-1,0)$. The total offered traffic per unit of area is $G_0 = 0.1$ ppt. The receiver threshold is 6 dB ($z=4$).

(—) reception at one receiver; (---) approximation
 (-·-) reception at both receivers; (····) approximation

Also, the probability $Pr(A_j \wedge B_j)$ of successful reception at both receivers simultaneously is compared with $Pr(A_j) \cdot Pr(B_j)$. These approximations correspond to the case that the interference level at the two receivers would be uncorrelated at both sites, which

underestimates $Pr(A_j \wedge B_j)$ and overestimates $Pr(A_j \vee B_j)$. At the traffic load studied ($G_0=0.1$ ppt), probability (9.25) of reception at both base stations simultaneously is seen to be underestimated by about 20% in the worst case, i.e., for a terminal located halfway between the two receivers ($\underline{x}_j=(0,0)$). The effect on the probability of capture $Pr(A_j \vee B_j | \underline{x}_j=(0,0))$ is less than 5%.

9.9 TOTAL THROUGHPUT AND APPROXIMATION TECHNIQUES

The total throughput of the two base stations is given by the integral

$$S_{A \vee B} = \int \int_{\text{area}} G_0 Pr(A_j \vee B_j | \underline{x}_j) d\underline{x}_j \quad (9.27)$$

which, taking account of the three terms in (9.24), may also be written as

$$S_{A \vee B} = S_A + S_B - S_{A \wedge B} . \quad (9.28)$$

The common throughput of the two base stations can be computed from

$$S_{A \wedge B} = S_A \int_0^\infty f_{b_j}(b_j | A_j) Pr(B_j | b_j, A_j) db_j . \quad (9.29)$$

This might be approximated by

$$S'_{A \wedge B} \triangleq S_A \int_0^\infty f_{b_j}(b_j | A_j) Pr(B_j | b_j) db_j . \quad (9.30)$$

The approximation $S'_{A \wedge B}$ neglects that during each slot, the number of interferers and the interference power level at receiver A and B are dependent, and the vulnerabilities of the test packet to these interferers are correlated at the two receivers. This causes B_j and A_j to be mutually correlated, as is considered in (9.29).

The exact total throughput depicted in Fig. 9.9 is obtained by 4-fold integration, arising from combining (9.25) with (9.27), in order to take due account of the correlated interference experienced at the two base stations. We now address the approximate and much simpler technique based on (9.30) to estimate the common throughput $S'_{A \wedge B}$.

As derived in chapter 8, the probability of capture in a noise-free channel with infinitely extended uniform offered traffic ($G(a_j)=G_0$) and $\beta=4$, is

$$Pr(A_j | a_j) \triangleq Q(a_j) = \exp \left\{ -\frac{\pi^2}{2} G_0 \sqrt{z} a_j^2 \right\} \quad (9.31)$$

and the total throughput per receiver is $S_t = 2/(\pi/z)$. The a posteriori pdf of the propagation distance a_j of a correctly received packet is Rayleigh distributed, with pdf

$$f_{a_j}(a_j | A_j) = \frac{2\pi a_j S(a_j)}{S_A} = \frac{2\pi a_j G_0 Pr(A_j | a_j)}{S_A} = \frac{a_j}{\sigma^2} \exp \left\{ -\frac{a_j^2}{2\sigma^2} \right\} \quad (9.32)$$

with variance

$$\sigma^2 = \frac{1}{\pi^2 \sqrt{z} G_0} . \quad (9.33)$$

We now consider distances to base station B , which has a distance R_d from base station A . Transforming the Rayleigh distributed a_j into the statistical variable of the distance b_j to base station B , one obtains the Ricean distribution

$$f_{b_j}(b_j | A_j) = \frac{b_j}{\sigma^2} \exp \left\{ -\frac{R_d^2 + b_j^2}{2\sigma^2} \right\} I_0 \left(\frac{R_d b_j}{\sigma^2} \right) \quad (9.34)$$

where $I_0(\cdot)$ is the zero-th order modified Bessel function of the first kind. Inserting (9.34) and using capture probabilities on the form of (9.4) to describe the probability of capture at B , (9.30) becomes

$$S'_{A \wedge B} = \frac{2}{\pi\sqrt{z}} \int_0^{\infty} \exp\left\{-\frac{b_j^2}{2\sigma^2}\right\} \frac{b_j}{\sigma^2} \exp\left\{-\frac{R_d^2 + b_j^2}{2\sigma^2}\right\} I_0\left(\frac{R_d b_j}{\sigma^2}\right) db_j. \quad (9.35)$$

The approximation (9.30) is seen to allow analytical solution, viz. by substituting

$$x^2 \Delta 2b_j^2 \quad \text{and} \quad y^2 \Delta \frac{1}{2}R_d^2, \quad (9.36)$$

one finds

$$S'_{A \wedge B} = \frac{1}{\pi\sqrt{z}} \exp\left(-\frac{R_d^2}{4\sigma^2}\right). \quad (9.37)$$

The approximate total throughput $S'_{A \vee B}$ of the two base stations is found from

$$S'_{A \vee B} = S_A + S_B - S_{A \wedge B}. \quad (9.38)$$

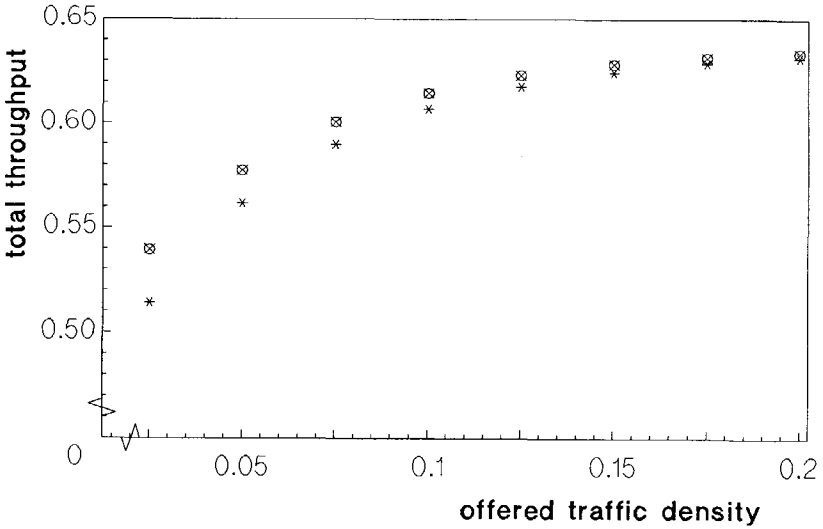


Fig. 9.9: Exact total throughput (*) of two base stations versus the traffic G_0 per unit of area, and approximation (⊗). The receiver threshold is $z=4$ (6 dB) and the receiver separation is $R_d=2$. UHF path loss law $\beta=4$.

The above approximate joint throughput (9.38) (indicated by + in Fig. 9.9) has been compared with the results found from numerical integration of the exact expression (9.27) with (9.24) and (9.25) (indicated by * in Fig. 9.9). Comparison with exact results confirmed that the approximation technique underestimates the number of packets that capture both receivers simultaneously; however, it is noted that the accuracy of the approximation is better than a few percent for sufficiently spaced receivers, say $R_d > \sigma$.

9.10 CONCLUSIONS

The performance of wide-area slotted ALOHA networks has been addressed using the 'receiver threshold' model. A measure for the spectrum efficiency of a cellular network with cluster size C has been proposed. This measure is particularly relevant to some information gathering systems that do not necessarily need retransmission of lost packets, such as in the SOCRATES system [7] for road traffic data. It appeared for uniform offered traffic that, $C=1$, i.e., the design where all cells use the same frequency, gives the highest spectrum efficiency. If $C>1$, the bandwidth per receiving base station decreases accordingly, which appears to limit the throughput more significantly than the compensation achieved from reduced interference from co-channel cells. The spectrum efficiency was found to depend largely on the required minimum probability of successful access. If a probability of access of about 0.5 is acceptable for remote terminals, the spectrum efficiency can be in the order of 0.3 bit/s/Hz per receiver, for robust and efficient types of modulation ($z=4$; $\eta_r=1$ bit/s/Hz).

Systems that require retransmission of each lost packet have also been considered. The spatial distribution of the packet traffic to be offered to the channel in order to achieve a uniform throughput has been evaluated. Corresponding spectrum efficiencies were found to be in the order of 0.2 bit/s/Hz per receiver or less for

robust and efficient types of modulation ($z=4$; $\eta_r=1$ bit/s/Hz). This is significantly larger than the spectrum efficiency of cellular telephone networks with narrowband channels and usual cluster sizes of about $C=9$, which cannot accommodate more than η_r/C bit/s/Hz per receiver.

It was seen that the effect of noise can be large: many packets from remote terminals are lost in noise, so these terminals are likely to perform many retransmissions, which increases the number of collisions and degrades the total system performance. Nonetheless, it appeared that if high spectrum utilisation is to be achieved, remote terminals have to perform many retransmissions. Thus, considering the observations in section 8.6.4, spectrum-efficient networks in which lost packets are retransmitted from the same location may require some measures to ensure stability.

Results confirm that in (the packet-switched) ALOHA networks, frequency re-use distances can be substantially smaller than in (circuit-switched) networks for cw radio telephony. This suggests that if frequency re-use patterns are designed for mixed traffic, e.g. to support circuit-switched voice for emergency calls [2], spectrum usage during normal packet-switched operation is far from optimum. Moreover, results motivate the use of wide-area networks with 'contiguous frequency re-use', thus using the entire available bandwidth in the entire area rather than avoiding the use of the same channel in adjacent cells, as is conventional in cellular telephony networks. In this case, packets can be received outside the cell where the transmitting terminal is located. This type of site diversity may enhance the system throughput.

A technique has been proposed to compute the probability of successful reception of data packet if site diversity is employed. A two-receiver configuration has been studied analytically. Numerical results have been obtained for the capture probability and the network throughput, for the case of a uniform spatial distribution of the total packet traffic offered to the network. It can be concluded that site diversity, which is inherent to the optimum cluster size $C=1$ in a random-access network, significantly increases the probability of successful access for the terminals near the boundary between two 'cells'.

9.11 REFERENCES IN CHAPTER 9

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CHAPTER 10

DISCUSSION AND CONCLUSIONS

This dissertation has addressed a number of technical problems in modern mobile communication. The increased possibility to exchange information with mobile users will render new services feasible which may have a large impact on business activities and possibly also on society at large. Mobility enhances the need for exchange of information, and communication can be used to enhance the efficiency of mobility.

Efficient use of the limited radio spectrum resource is of key importance in the design and planning of mobile radio systems. Spectrum efficiency was previously studied in many other types of communication systems. Often a purely noise-limited situation was assumed where the quality of reception was determined mainly by the local

signal-to-noise ratio at the fringe of the service area. Modulation was generally chosen such that the occupied bandwidth per transmitter was as small as possible. Digital modulation techniques with a high bit rate per occupied bandwidth have been developed (large r_b/B_T). Many of these bandwidth-efficient techniques are also applicable in trunk connections between telephony switches or nets for distribution of radio and television broadcast signals to the home (CATV). A standard measure of the spectrum efficiency of radio voice links is expressed in erlang/Hz.

In present cellular radio systems, bandwidth efficiency is greatly improved by the dense re-use of available frequencies in the service area. Most mobile networks are interference-limited, rather than noise-limited. This, however, requires careful consideration of the effect of mutual interference between co-channel users. Therefore, not only the occupied bandwidth, but also the size of the area in which it cannot be re-used, is of importance. Hence, spectrum-efficient design must consider the spatial pattern of the communication infrastructure. The *spatial* bandwidth efficiency of mobile telephone networks is expressed in erlang/Hz/km².

Accordingly, this dissertation addressed the effects of interference, given a certain spatial layout of the network. Propagation channel impairments, such as multipath reception and shadowing occasionally threaten the performance of mobile communication networks. Fluctuations of the received power necessitate the inclusion of fade margins in the maximum tolerable (mean) signal-to-interference ratio. Thus, the spatial bandwidth efficiency depends on the propagation characteristics of the mobile channel.

In chapter 1, the design, planning and control of radio networks to accommodate the required number of users within the allocated bandwidth was discussed in the relatively broad scope of a layered system model. Chapters 2-9 were intended to contribute to the understanding of various factors at the lower OSI layers that determine the performance of spectrum-efficient narrowband mobile networks. This concluding chapter summarises the results most relevant to the spatial bandwidth efficiency of mobile radio networks.

10.1 DESIGN AND PLANNING OF CELLULAR VOICE NETWORKS

Limitations caused by propagation effects are of major concern in mobile radio. Most of the computations reported in this dissertation have addressed generic system design. Propagation effects were treated in a stochastic way. Idealised hexagonal or circular cell structures have been considered. In practical systems planning, however, tailor-made solutions for particular propagation environments and for specific spatial distributions of the tele-traffic intensity may be required to achieve sufficient bandwidth efficiency. In this case, terrain data is usually necessary. If the area-mean power can be forecasted from this data with sufficient accuracy, margins to overcome propagation uncertainties can be kept reasonably small (see section 3.4.5). Hence, the availability of reliable propagation models is of key importance to efficient cellular design (see discussion in section 3.2.1).

UHF field measurements of path losses in rural areas have been reported and analysed. The linear-regression technique proposed in section 2.2.4.2 and used in section 2.3 differs from most studies of propagation, where 'scatter diagrams' are used. Such diagrams show each sample of measured path loss versus a certain selected parameter, plus a least-square-error estimate of the effect of the parameter. Often the measurements show large deviations from the least-square-error estimate. In chapter 2, scatter diagrams were not included; instead, standard deviations and the remaining error dependence on *other* path parameters were reported. The regression method adopted proved useful in determining shortcomings of the various models studied. Thus, the accuracy of the propagation prediction model could be improved by refining the model step by step (sect. 2.3.1 - 2.3.6).

The combined occurrence of ground reflections, local scatters and propagation by diffraction renders exact forecasting of propagation losses impossible. Two empirical methods for estimating the path loss, namely, (i) linear addition of losses as they

would occur if the propagation mechanisms occurred independently and (ii) an empirical method proposed by Blomquist [8], have been verified and some empirical refinements have been proposed.

One counter-intuitive conclusion drawn from the propagation measurements was that macro-cellular networks may be more spectrum-efficient in open area than in an area with dense vegetation: Because of ground reflections, the signal power was found to decay relatively fast in open area (see Fig. 2.2). In wooded areas, a larger separation of co-channel users may thus be required to avoid harmful interference caused by signals propagating by diffraction over tree tops.

The performance of cellular networks for voice communication is often expressed in terms of the probability of a signal outage. An exact analytical technique was presented in chapter 3 for the computation of outage probabilities in multi-user mobile radio networks. Numerical results for idealised hexagonal cell structures confirmed the accuracy of previously proposed approximate techniques. It is a common frustration in scientific research that exact computations require longer (computer) calculations than approximations. In contrast, the exact method presented in chapter 3 turned out to be faster than previous approximations. This method allows faster evaluation of solutions for realistic cell layouts, so it increases the probability that the selected solution is close to optimum.

The performance analysis was extended to two-branch site (or macro-) diversity in a cellular telephone network (see sections 3.3 and 3.4). In channels with shadow fading, site diversity substantially enhances the quality of the end-to-end circuit. For a prescribed network performance, this implies that higher interference power levels may be tolerated with site diversity, which improves the spectrum efficiency of the network.

Next to improving design and planning of cell layouts, fast computational techniques for calculating outage probabilities may be advantageous in real-time dynamic channel

assignment (DCA) algorithms or in simulation of the performance algorithms for decentralised management and handovers initiated by mobile terminals.

10.2 PERFORMANCE OF DIGITAL CELLULAR NETWORKS

The recently developed more efficient voice-coding algorithms render spectrum-efficient digital cellular telephony feasible. In such networks, voice data is often split in frames or blocks. A block of data is likely to be received correctly unless a fade is experienced during the reception of the block. In chapter 4, the average non-fade duration has been studied analytically for the case of a Rayleigh-fading signal in the presence of multiple interfering signals with Rayleigh fading. The results showed out that the probability of a block being received within a non-fade interval is often significantly smaller than the outage probability. This suggests that blocks should be short compared to the time constant of the fading. The numerical results in chapter 4 quantify this observation that was previously concerned for noise-limited rather than real interference-limited networks (see also section 7.1).

For digital communication, the average bit error rate is also a relevant measure of the performance. Previous analyses of the BER in fading channels with co-channel interference appeared too complicated for use in studies of system performance. A simplified model was proposed in chapter 5. Numerical results and closed-form asymptotic expressions were generated for the local-mean and area-mean BER in micro- and macro-cellular networks, and the model allowed analysis of the performance of random-access networks (chapter 7). The analysis showed that increased shadowing has a degrading effect on the average BER. Rician K -factor (ratio of specular and scattered signal power) also appeared to have a substantial influence.

In stationary, i.e., non-fading, AWGN channels, the BER rapidly decreases if signal power is increased. This possibility is lost in mobile channels with fading. Even for high signal-to-interference-plus-noise ratios, the average BER remains relatively poor. This effect, plus the observation in sections 4.9 and 4.10 that the average duration of fade and non-fade intervals is in the order of many hundreds or thousands of bits in typical mobile radio networks, lead to the conclusion that mobile channels require special error control. Bit interleaving and powerful error-correction coding may not suffice. Some remaining problems for research on channel coding will be discussed further in section 10.6.

10.3 PERFORMANCE OF CELLULAR DATA NETWORKS

At the time of completion of this dissertation, first generation cellular telephone networks were in operation in many countries. The digital pan-European GSM system is due to be implemented, but practical introduction of this second generation system has been delayed.

The behaviour and the performance of cellular telephone networks have been determined by researchers during the past two decades. Results, or at least approximations, are now known for a number of performance aspects. Further refinements, particularly the extension to interference-limited, rather than noise-limited, cases and new computational techniques have been offered in this dissertation in chapters 2-5.

On the other hand, for mobile *data* communication, only few systems are in operation and the discussion about efficient designs are still being conducted. Two aspects have been addressed in this dissertation, namely (i) the performance of the access schemes, as determined by mutual competition by terminals inside the cell (chapters 6-8), and (ii) co-channel interference by signals from other cells (chapter 9).

10.3.1 EFFECT OF COMPETING PACKETS INSIDE THE CELL

Standard results for the performance of (wired) data communication are not appropriate for mobile data systems. One of the problems is that, in fading channels, the performance experienced by individual mobile terminals may differ widely from the mean performance averaged over all terminals. Two types of near-far effects have been distinguished: effects caused by the fact that propagation attenuation increases with distance (chapters 7-8), and effects caused by propagation delays (chapter 6). The latter effects have been studied for ISMA networks with propagation delays in the inbound random access and feedback channel. It was found in section 6.7.6 that the approximation in terms of fixed, i.e., distant-independent, propagation delays leads to somewhat pessimistic results for the data throughput. For a worst-case investigation of an ISMA network the delay in dissemination of the channel status is to be based on the two-way propagation time, rather than the one-way propagation time.

For reasonable packet traffic loads, the near-far effect of varying propagation delays appeared smaller than that of varying attenuation. A large number of channel aspects play a role in the conditional probability of successful access for a signal received from a certain distance, such as the type of modulation and coding, packet duration, speed of the terminal and depth of shadow fading. A number of simplified models, each dealing with only a limited number of effects, have been investigated and compared in chapter 7. In mobile slotted ALOHA networks, packets from nearby terminals have substantially higher probability of successful reception than those from remote terminals. This effect is much smaller in networks employing ISMA, though nonpersistent ISMA does not necessarily improve the overall throughput compared to slotted ALOHA (see section 7.8).

Results in chapter 7 and 8 confirmed the previous reports of enhanced throughput of the ALOHA channel by receiver capture. However, it was found that the way in which

terminals are distributed over the coverage area has a large influence on the throughput of the net. Various spatial distributions have been compared in chapter 8 and the effect that almost all retransmissions occur in distant parts of the coverage area with poor propagation was addressed in section 9.5.

Stability and delay performance of mobile ALOHA networks have been investigated with a Markov chain model. The calculations reveal that bi-stability occurs only for a very limited range of traffic parameters. In contrast to the case for wired networks, this suggests that protocol design may focus on continuously assuring sufficient throughput and acceptable delay, for instance by exercising (slow) centralised control over the number of mobile terminals allowed to sign on. However, effective techniques exist to ensure stability, such as dynamic frame-length ALOHA or stack algorithms, and appear relatively easy to implement. These may ensure the required performance in a wider range of traffic situations. Moreover, these techniques may be relevant if frequent retransmissions by a few terminals at the fringe of the cell threaten the stability of the entire network.

The result in section 8.6 that mobile slotted ALOHA (without more sophisticated retransmission schemes) remain stable in most conditions was a motivation to study system behaviour using steady-state analysis of the access protocol. This simplification allowed further extension of the study. For instance, the impact of noise, (non-) fade durations and cellular frequency re-use were computed in chapter 7 and 9.

10.3.2 EFFECT OF INTERFERENCE FROM CO-CHANNEL CELLS

Beside the performance of 'isolated' random-access networks, wide-area networks with frequency re-use have also been addressed, with a model recognising that most retransmissions occur at the fringe of each cell. The spatial distribution of the packet traffic to be offered to the channel in order to achieve a uniform throughput has been

determined. It was found that the effect of noise can be substantial. Since packets from remote terminals are more prone to noise, they are likely to perform more retransmissions which increase the number of collisions and degrade the total system performance.

Results in chapter 9 showed that in ALOHA networks, frequency re-use distances can be made substantially smaller than in networks for cw radio telephony. The computational results even motivate the use of a wide-area packet-switched network with contiguous frequency re-use, i.e., using the entire available bandwidth in the entire area with co-channel receiving base stations at different locations, rather than splitting the bandwidth in a number of channels, as in cellular telephone networks. In this case, packets can be received outside the cell where the transmitting terminal is located. This type of site diversity enhances the system throughput (see section 9.9). This conclusion is in sharp contrast to the spatial design of present mobile data networks, which appear to be based on cell layouts with cluster sizes optimised for circuit-switched voice communication.

10.4 NARROWBAND VERSUS WIDEBAND COMMUNICATION

This thesis is limited to narrowband mobile communication over fading ground-wave radio channels. Wideband communication would require fundamental reconsideration of a number of topics. For signals with a bandwidth exceeding the coherence bandwidth of the propagation environment, significant intersymbol interference (ISI) will occur and techniques to combat these effects, such as adaptive channel equalisation must be considered.

Also, narrowband and wideband radio links react differently to motion of the antenna. The former experience fades during movement when bursts of bit errors occur. On the other hand, for a stationary terminal, the channel properties remain 'frozen'. In

wideband communication, however, deep fades of the received power over the entire bandwidth of the signal are very unlikely. After appropriate equalisation, the bit error rate will be nearly stationary and ergodic, irrespective of whether the terminal is in motion or stationary. This also suggests that in networks with only limited terminal mobility, such as indoor wireless systems, wideband communication can assure fairer channel performance for all users, though at the cost of increased system complexity.

Spread-spectrum and CDMA techniques are currently proposed for indoor and micro-cellular networks. However, broadband signals received from various users are often optimistically assumed to arrive with identical mean power. If this is not the case, the performance will severely be limited. Because of path loss and shadowing, adaptive power control [9] may be required to fulfil this restriction. This, however, may not be practical in a wide-area network with multiple cooperating receivers at different locations.

In the random-access networks that take advantage of feedback information on the channel state, such as ISMA, the propagation delay relative to the packet duration should be relatively small to ensure high performance. In wideband networks, the round-trip delay may become too large to effectively employ CSMA or ISMA if the range of the cell is large (see chapter 6).

Another issue is the power consumption of a handheld terminal. Wideband communication generally requires substantially more signal processing, e.g. for channel equalisation, spread-spectrum decoding or simply to receive a few blocks of data embedded in a continuous signal with a high symbol rate.

10.5 MATHEMATICAL TECHNIQUES EMPLOYED IN THIS THESIS

The 'capture-ratio' or 'threshold' model (chapter 3, 4, 8 and 9) produced convenient mathematical expressions for the probability of successful reception. Discussion showed that this simplification is acceptable in the case of *narrowband* mobile channels. Since large fluctuations of the received signal power occur, the accuracy of the results may not depend critically on a detailed consideration of the behaviour in a small range of the C/I -ratio near the 'threshold'. This is in contrast to recently reported observations for CDMA and spread-spectrum communication [4],[5].

Narrowband mobile propagation is of a stochastic nature. This requires a statistical evaluation of propagation measurements, hence the study of the performance of mobile communication systems relies heavily on probability theory. It appeared that the 'threshold' model can be analysed with the Laplace transform (also related to the 'characteristic function') of the probability density functions of the received interference power. The Laplace transform is widely used in electrical engineering [6] because it changes differential equations describing signals in electronic circuits into algebraic equations which are generally easier to solve. In the situation covered in this dissertation, the lure of the Laplace transform is its ability to map the complicated operation of convolution into multiplication. If the interfering power from multiple terminals cumulates incoherently, the probability density of the joint interference power is the convolution of the pdfs of the interference received from the individual terminals. Laplace transformation, however, changes this into a multiplication of image functions.

Moreover, the exponentially distributed power received from a Rayleigh-fading signal was seen to match with the kernel of the Laplace image (see section 3.2). This observation paved the way for relatively convenient mathematical expressions for outage probabilities in macro-cellular telephony and for capture probabilities in random-access channels. In particular, the performance of slotted ALOHA was addressed in chapter 8. In this case, a Poisson-distributed number of colliding packets leads to expressions that also include the exponential function. Interestingly, the

vulnerability circle, proposed by Abramson [7] to describe the effect of colliding packets, could be modified to take exact account of Rayleigh fading and cumulation of interference signals. It thus transpired that, because of the matching mathematical properties of Rayleigh fading, Laplace transforms and Poisson distributions, the intensity of contending packet traffic may simply be multiplied by a propagation-dependent factor to describe the effect of the interference caused to a selected test packet. In this way, the technique was refined in chapter 8 by interpreting the Laplace image functions as integral transforms of the spatial distribution of the offered packet traffic. In section 8.3.1, the kernel of this integral transform was called the 'vulnerability weight function'.

In particular, the Laplace image of the pdf of the power received over a channel with Rayleigh fading and shadowing appeared relevant to the computations. In chapter 2, this image was found on the general form

$$\Phi_o(s) \triangleq \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2} dx}{1 + se^{\sqrt{2}\sigma x}}, \quad (10.1)$$

which was studied in section 2.1.5 and Appendix C.

10.6 SCOPE OF FUTURE WORK

At the time of completion of this thesis, the relative advantages of narrowband, wideband and spread-spectrum transmission were not fully assessed, despite the fact that many separate investigations have been conducted. Since many new systems are being developed and standards are drafted for future global or regional networks, a better insight into the relative performance is acute. Further study to compare the two techniques from a point of spectrum efficiency is recommended.

Most of the results presented here were confined to macro-cellular networks. Similar studies for communication over shorter range are recommended, for reasons summarised in section 1.5.2.

The technique proposed in chapter 3 for computation of outage probabilities might be further refined, using expressions cited in section 2.1.5, to provide a method for rapid computation of outage probabilities in practical networks with customised cell layout. Even in some 'state-of-the-art' planning tools for cellular telephony, the effect of multiple interfering signals is ignored simply because network planning would otherwise become prohibitively time consuming. This illustrates the need for more convenient computational techniques.

The effect of correlated shadow attenuation has hitherto received relatively little attention. Further study is recommended.

Frequency management in cellular systems becomes increasingly dynamic [1], [2]. This allows assignment of channels according to actual demand and spatial distribution of active terminals, counteracting traffic variations caused by unpredictable events like traffic jams, accidents, etc., and dynamically assigning spectrum resources according to daily patterns of mass movement. Decentralised management and handovers initiated by the mobile terminals are alternative techniques which also receive attention in the development of new systems. To the knowledge of the author only few results exist for the performance, the spectrum efficiency or the stability of cellular systems with decentralised frequency management or with DCA. Further study of these techniques is recommended.

Error-correction and error-detection coding in a mobile fading channel with multiple users, turned out to provide interesting problems for further investigation, including some aspects not encountered in single-user non-fading AWGN channels. Detailed study was considered outside the scope of this dissertation, but Werry and the author report tentative results on block success, error and failure probabilities for an ALOHA channel in [3]: The block error probability in mobile networks with error correction coding but

without error detection does not vanish for increasing signal-to-noise ratios. Signal fading and mutual interference substantially contribute to the probability of block errors. This is in contrast to the case for single-user AWGN channels. The common use of the radio channel and receiver by multiple terminals also affects the choice of code words within the Hamming space, and so influences success, error and failure probabilities [3]. Further investigation is recommended.

Refinement of the technique to study stability of networks with *non-ergodic* channels is also left as a recommendation. If, because of the near-far effect, some terminals always transmit over relatively poor channels and other terminals always transmit over relatively good channels, the network is significantly less stable than if time-average and ensemble-average capture probabilities would be identical, as in networks with ergodic channels. An appropriate and mathematically tractable technique to study the stability of networks with different channels for different users has not yet been developed.

Particularly from the results in chapter 9 it appeared that further development of efficient strategies for frequency re-use in wide-area packet-switched mobile data networks may largely enhance spectrum efficiency. The inclusion of a refined model for mixed traffic in the analyses and development of a technique for (dynamic) control of frequency re-use according to the traffic pattern near each base station are recommended.

The expressions derived in chapter 3 for the a posteriori probability that an interferer was active despite the fact that the test signal captured the receiver may be an interesting starting point for a number of the aforementioned topics. These expressions might be relevant particularly to the study of stability of random-access networks with near-far effect, Bayesian control algorithms for random-access channels [10], throughput of collision resolution algorithms, terminal behaviour in decentralised frequency management and spectrum efficiency wide-area packet data networks.

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APPENDIX A: VARIANCE OF NOISE SAMPLES IN A CORRELATION RECEIVER

This appendix calculates the variance of the noise sample in the decision variable (5.6). As seen in Fig. 5.1., the received signal is correlated with the locally generated signal $h(t) = 2/T_b \cdot \cos \omega_c t$. For ease of analysis, it is assumed that $\omega_c T_b = 2\pi M$. In practice, the carrier frequency ω_c will be many orders of magnitude larger than the bit rate ($\omega_c T_b \gg 1$) and the inaccuracy with M non-integer is negligible. The noise component $y(t)$ at the output of the integrator is offered to the sampling device. This signal $y(t)$ is on the form of

$$y(t) = \frac{2}{T_b} \int_{t-T_b}^t \cos \omega_c t \, n(t) \, dt \quad (\text{A.1})$$

The auto-correlation is defined as $R_y(t, t+\tau) = E[y(t) \cdot y(t+\tau)]$. If the AWGN signal $n(t)$ is offered to the receiver, one obtains

$$\begin{aligned} R_y(t, t+\tau) &= E \left[\int_{t-T_b}^t n(t_1) h(t_1) dt_1 \int_{t-T_b}^t n(t_2+\tau) h(t_2+\tau) dt_2 \right] \\ &= \int_{t-T_b}^t \int_{t-T_b}^t E[n(t_1) n(t_2+\tau)] h(t_1) h(t_2+\tau) dt_1 dt_2 \\ &= \int_{t-T_b}^t \int_{t-T_b}^t R_n(t_2+\tau-t_1) h(t_1) h(t_2+\tau) dt_1 dt_2 . \end{aligned}$$

For AWGN with one-sided spectral power density N_0 , the auto-correlation is $R_n(t, t+\tau) = \frac{1}{2} N_0 \delta(\tau)$. Thus

$$\begin{aligned} R_y(t, t+\tau) &= \int_{t-T_b}^t \int_{t-T_b}^t \frac{N_0}{2} \delta(t_2+\tau-t_1) h(t_1) h(t_2+\tau) dt_1 dt_2 \\ &= \frac{N_0}{2} \int_{t-T_b}^t h(t_1) h(t_1-\tau) dt_1 \end{aligned}$$

So, $R_y(0) = N_0/T_b$. This result is used in (5.7).

APPENDIX B: NON-STATIONARY (POISSON) ARRIVALS

This Appendix addresses stochastic arrival processes with a quasi-stationary arrival rate $\lambda(t)$, i.e., $\lambda(t)$ is assumed constant during an infinitesimal interval of duration Δt , but may fluctuate during a longer observation interval T . Nonetheless, the number of arrivals in a certain interval T is Poisson distributed. The expected number of arrivals equals the *mean* arrival rate multiplied by the duration of the interval (see Eq. (6.7) and (6.8)). A proof is given in Appendix B.2. Initially, the familiar mathematical relation

$$\lim_{J \rightarrow \infty} \left(1 + \frac{\xi}{J}\right)^J = e^\xi \quad (\text{B.1})$$

is generalised.

B.1 LEMMA

Let $\xi(\tau)$ be a bounded function, with $\xi(\tau) \leq M$ for any τ in $[0, a]$. Then

$$\lim_{J \rightarrow \infty} \prod_{j=0}^{J-1} \left(1 + \xi\left(j \frac{a}{J}\right) \frac{a}{J}\right) = \exp \left\{ \int_0^a \xi(\tau) d\tau \right\}. \quad (\text{B.2})$$

Proof

For ease of notation, the proof is given for the special case $a=1$, but no essential differences occur for other values of a . We define M as the maximum of $\xi(\tau)$, so

$$M \triangleq \max_{x \in [0,1]} \xi(x).$$

The interval $(0,1)$ of the domain of $\xi(\tau)$ is partitioned into J sub-intervals of equal length. To derive an expression for (B.1), we consider the graph of $y=1/x$ in Fig. B.1. The shaded area is estimated by the two rectangular areas. For any j ($j \in \{0, \dots, J-1\}$) one may write the inequalities

$$\frac{J}{J+M} \frac{\xi(j/J)}{J} \leq \frac{\xi(j/J)}{J + \xi(j/J)} \leq \ln \left(1 + \frac{\xi(j/J)}{J} \right) \leq \frac{\xi(j/J)}{J}. \quad (\text{B.3})$$

After summing from $j=0$ to $J-1$, and applying the exponential function, one finds

$$\exp \left\{ \frac{J}{J+M} \sum_{j=0}^{J-1} \frac{\xi(j/J)}{J} \right\} \leq \prod_{j=0}^{J-1} \left(1 + \frac{\xi(j/J)}{J} \right) \leq \exp \left\{ \sum_{j=0}^{J-1} \frac{\xi(j/J)}{J} \right\}. \quad (\text{B.4})$$

In the limit for $J \rightarrow \infty$, equation (B.4) implies that

$$\lim_{J \rightarrow \infty} \prod_{j=0}^{J-1} \left(1 + \frac{\xi(j/J)}{J} \right) = \exp \left\{ \int_0^1 \xi(\tau) d\tau \right\} \quad \text{Q.E.D.} \quad (\text{B.5})$$

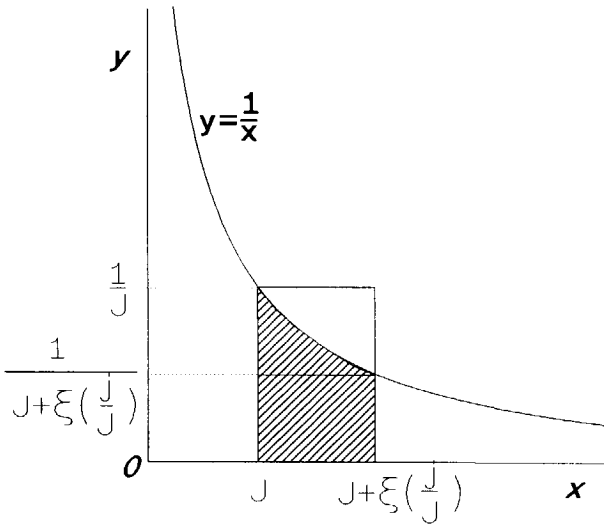


Fig. B.1: Enclosure of the graph $y=1/x$ considered in the proof of the Lemma.

B.2 NON-STATIONARY ARRIVAL PROBABILITIES

Let $N_{\langle t, t+T \rangle}$ denote the random variable that records the number of arrivals of a non-stationary Poisson process during a time interval $[t, t+T]$. The Poissonian assumptions can be rewritten for the time-variant arrival rate $\lambda(t)$ ($\lambda(t) > 0$):

$$\begin{aligned} P_{N_{\langle t, t+\Delta t \rangle}}(n=0) &= 1 - \lambda(t)\Delta t + o(\Delta t) \\ P_{N_{\langle t, t+\Delta t \rangle}}(n=1) &= \lambda(t)\Delta t + o(\Delta t) \\ P_{N_{\langle t, t+\Delta t \rangle}}(n \geq 2) &= o(\Delta t) \end{aligned} \quad (\text{B.6})$$

where $\lambda(t)$ is considered constant in $\langle t, t+T \rangle$. The interval $\langle t, t+T \rangle$ is partitioned into J subintervals, with j_k ($0 \leq j_k \leq J-1$) the sequence number of the subinterval where the k -th arrival is located. The probability of n arrivals during $\langle t, t+T \rangle$ is

$$P_{N_{\langle t, t+T \rangle}}(n) = \lim_{J \rightarrow \infty} \sum_{j_1=0}^{J-1} \sum_{j_2=0}^{J-1} \cdots \sum_{j_n=0}^{J-1} Pr \left\{ \begin{array}{l} 1 \text{ arrival in } j_1 \Delta t, \\ \text{and 1 arrival in } j_2 \Delta t, \\ \vdots \\ \text{and 1 arrival in } j_n \Delta t \end{array} \right\}. \quad (\text{B.7})$$

By inserting the first-order approximations of probabilities of arrival during infinitesimal periods, one finds

$$\begin{aligned}
 P_{N_{(t,T)}}(n) &= \lim_{J \rightarrow \infty} \frac{1}{n!} \sum_{j_1=0}^{J-1} \cdots \sum_{j_n=0}^{J-1} \prod_{k=1}^n \mu(j_k \Delta t) \Delta t \prod_{\substack{l=1 \\ l \neq j_1, \dots, j_k}}^{N-1} 1 - \lambda(l \Delta t) \Delta t \\
 &= \lim_{J \rightarrow \infty} \frac{1}{n!} \sum_{j_1=0}^{J-1} \cdots \sum_{j_n=0}^{J-1} \prod_{k=1}^n \frac{\lambda(j_k \Delta t) \Delta t}{1 - \lambda(j_k \Delta t) \Delta t} \prod_{l=0}^{J-1} 1 - \mu(j_l \Delta t) \Delta t.
 \end{aligned} \tag{B.8}$$

Using the Lemma (B.2), the latter product is written as the exponent of an integral, viz.,

$$P_{N_{(t,T)}}(n) = \frac{e^{-\bar{\lambda}T}}{n!} \lim_{J \rightarrow \infty} \sum_{j_1=0}^{J-1} \cdots \sum_{j_n=0}^{J-1} \prod_{k=1}^n \frac{\lambda(j_k \Delta t) \Delta t}{1 - \lambda(j_k \Delta t) \Delta t}, \tag{B.9}$$

where the average arrival rate is defined as in (6.8). For $J \rightarrow \infty$, thus $\Delta t \rightarrow 0$, each summing in (B.8) goes into an integration, viz.,

$$\begin{aligned}
 P_{N_{(t,T)}}(n) &= \exp\left\{\frac{-\bar{\lambda}T}{n!}\right\} \int_0^T \cdots \int_0^T \lambda(t_1) \cdots \lambda(t_n) dt_1 \cdots dt_n \\
 &= \frac{(\bar{\lambda}T)^n}{n!} \exp\{-\bar{\lambda}T\},
 \end{aligned} \tag{B.10}$$

which proves Eqs. (6.7) and (6.8).

APPENDIX C FURTHER DISCUSSION OF SUZUKI IMAGE FUNCTION

In section 2.1.5, the Laplace transform of a Suzuki pdf was found to be

$$\phi_o(s) \triangleq \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-x^2) dx}{1 + s \exp(\sqrt{2}x\sigma_s)}. \tag{C.1}$$

A general closed-form expression for $\phi_o(s)$ with arbitrary σ_s and s has not been found, but the author received a number of valuable suggestions for mathematical evaluation of this integral after 'posting' it on the electronic news service "usenet sci.math".

By substitution of $y \triangleq x + \frac{1}{2}\sqrt{2}\sigma_s$ in $\phi_o(s^{-1}) = 1 - \phi_o(s)$, Meertens (C.W.I., Amsterdam) showed that

$$\phi_o(s) = \frac{e^{\frac{\sigma_s^2}{2}}}{s\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp\{-y^2\} dy}{1 + \frac{1}{s}e^{-\sqrt{2}\sigma_s y + \sigma_s^2}} = \frac{e^{\frac{\sigma_s^2}{2}}}{s} \phi_o\left(\frac{e^{\sigma_s^2}}{s}\right) \tag{C.2}$$

Together with $\phi_o(s^{-1}) = 1 - \phi_o(s)$, this gives solutions in closed form for $s = \exp\{k\sigma_s^2\}$ with $k = 1, 2, \dots$, viz.,

$$\begin{aligned}
\phi_\sigma(e^{\sigma_s^2}) &= \frac{1}{2} \exp\left\{-\frac{1}{2}\sigma_s^2\right\}, \\
\phi_\sigma(e^{2\sigma_s^2}) &= \exp\left\{-\frac{3}{2}\sigma_s^2\right\} - \frac{1}{2} \exp\left\{-2\sigma_s^2\right\}, \\
\phi_\sigma(e^{3\sigma_s^2}) &= e^{-\frac{5}{2}\sigma_s^2} - e^{-4\sigma_s^2} + \frac{1}{2} e^{-\frac{9}{2}\sigma_s^2}, \quad \text{etc.}
\end{aligned} \tag{C.3}$$

Montgomery (U.C.L.A.) suggested studying

$$f(A, B, C) \triangleq \int_{-\infty}^{\infty} \frac{e^{-Ax^2 - Bx - C}}{\cosh(x)} dx = e^{-C} \int_{-\infty}^{\infty} \frac{e^{Ax^2 - C} \cosh(Bx)}{\cosh(x)} dx \tag{C.4}$$

with $A > 0$. The image $\phi_\sigma(s)$ can be expressed in terms of $f(A, B, C)$ by a linear change of variable. The latter integral (C.4) is well-known for $B=1$, but is also easily computed for $B=3, 5, \dots$, by expressing the quotient $\cosh(Bx)/\cosh(x)$ as a sum of exponentials. This technique appeared to produce the same results as the method proposed by Meertens, i.e., (C.3) was recovered.

Olde Daalhuis (C.W.I., Amsterdam) rewrote $\phi_\sigma(s^{-1}) = 1 - \phi_\sigma(s)$ and (C.2) as

$$\phi_\sigma(s) = 1 - s e^{\frac{1}{2}\sigma_s^2} \phi_\sigma(s e^{-\sigma_s^2}) \tag{C.5}$$

Repeated substitution of $\phi_\sigma(\cdot)$ then gives the series representation

$$\phi_\sigma(s) = \sum_{n=0}^{\infty} \exp\left\{-\frac{1}{2}\sigma_s^2 n^2\right\} \left(-s \exp\{\sigma_s^2\}\right)^n \tag{C.6}$$

Olde Daalhuis also showed that $\phi_\sigma(s)$ can be written in terms of a theta function, viz.,

$$\phi_\sigma(s) = \frac{e^{\frac{1}{2}\sigma_s^2}}{s - \frac{1}{s}} \left[1 - \frac{e^{-\frac{1}{2}\sigma_s^2}}{s} + \theta_4\left(\frac{\ln(s)}{2j}, e^{\frac{1}{2}\sigma_s^2}\right) \right], \tag{C.7}$$

with the theta function

$$\theta_4(x, q) \triangleq \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2} e^{2jnx} \tag{C.8}$$

The relation with Jacobi theta-functions was also noted by Walsh (Rutgers University). He suggested to regard s in (C.1) as a complex number, and replace σ_s by a pure imaginary $j\sigma_s$. This allows to expand $1/(1 + \exp(j\sqrt{2} \cdot \sigma_s x))$ into a geometric series; Theta functions are obtained by integrating this series term-by-term.

Vvedenskaia suggested to write $\phi_\sigma(\cdot)$ as a series of error functions: the integral (C.1) can be split into a part from $-\infty$ to $B-$ and a part from $B+$ to $+\infty$, such that both parts

can be written as an integral over a geometric series, each term containing an error function.

Watson (Oxford University) suggested the alternative representation

$$\phi_o(s) = \frac{1}{2} - \int_0^{\infty} \frac{\exp(-\frac{1}{2}\sigma_s^2 x^2) \sin(x \ln(s))}{\sinh(\pi x)} dx \quad (\text{C.9})$$

As Craddock (Univ. of New South Wales, Australia) pointed out, this is a Fourier sine transform evaluated in the point $\ln(s)$. The expression of the transform can be found from the transform of $\exp\{-j\pi q x^2\} \text{cosech}(\pi x)$ with $\text{Im}\{q\} < 0$ [1], but becomes lengthy.

It is the impression of the author that these suggestions, in particular the series (C.6), can be relevant in developing rapid computational techniques for planning of practical cellular networks.

- [1] "Tables of integral transforms Vol. 1", Bateman Manuscript Project, McGraw Hill, 1954, page 92, Eq. 38.

APPENDIX D: LIST OF MAJOR SYMBOLS

symbol	description [unit]	introduced in sect.:
a	duration of a cycle in CSMA or ISMA [*]	6.6
A	average duration of a cycle in CSMA or ISMA [*]	6.6
A_B	index indicating base station	3.2
A_{Bu}	diffraction loss according to model by Bullington [dB]	2.2.4
A_d	theoretical diffraction loss [dB]	2.1.1
A_{Dg}	diffraction loss according to model by Deygout [dB]	2.2.4
A_{fs}	free space loss [dB]	2.1.1
a_j	distance between terminal j and receiver A [*]	3.2
$A_j B_j$	event of successful reception of a packet from terminal j at receiver A	3.2
A_K	propagation attenuation in dB as predicted by model K ($K=1,2,\dots$) [dB]	2.1.1
A_{loc}	shadow attenuation [dB]	2.3.5
A_m	measured propagation attenuation [dB]	2.2.4
A_{mh}	theoretical diffraction loss over main hill [dB]	2.2.4
A_R	ground-reflection loss [dB]	2.1.1
B	average duration of the busy period [*]	6.6
B	backlog, number of terminals in retransmission mode	8.6
b_j	see a_j [*]	3.2
B_c	coherence bandwidth [Hz]	2.1.3
B_T	occupied bandwidth [Hz]	3.1.1
c	speed of light [m/s]	2.1.3.2
C	cluster size in cellular frequency re-use	1.5.1
C_i	probability that one out of i packets capture the receiver	6.3
c_j	event that signal j captures receiver A	4.10 / 7.1
$\hat{C}_j$	amplitude of the dominant (specular) component of Rician fading signal with index j [*]	2.1.3
c_z	vulnerability parameter	6.7.7 / 7.3
c_1, c_2	empirical parameters in propagation model, c_1 in [dB]	2.3.5
d	distance between transmitter and receiver [m]	2.1.1
D_a	access delay [*]	8.6.1
d_m	drift of the backlog of packets in a multiple access net [*]	8.6.1
D_q	queueing delay in mobile terminal [*]	6.4.2
d_r, d_t	receiver, transmitter distance to obstacle [m]	2.3.1
d_0	event that signal 0 experiences a drop-out	4.9
d_1, d_2	inhibit signalling delays [*]	6.6
e	$2.71828\dots$ ($\ln e = 1$)	
e	error	2.2.4
$E[\cdot]$	expectation value	2.2.4
$erf(x)$	error function	7.2
$erfc(x)$	complementary error function ($1-erf(x)$)	5.1
f_c	carrier frequency [Hz]	2.1.1
f_m	Doppler shift [Hz]	2.3.1.2 / 4.1
$f_x(x)$	probability density function of x	2.1.2

N.B.: The mathematically more correct notation $f_X(x)$ is not adopted here for practical reasons.

$F_x(x)$	cumulative distribution function	6.7.2
$G(r)$	offered packet traffic per unit of time per unit of area at a distance r [*]	6.1.7
G_t	total offered (attempted) packet traffic per unit of time [*]	6.2
h	duration of the inhibited period [*]	6.7.1
H	average duration of the inhibited period [*]	6.7.1
H_B	event that test packet arrives when busy signal is received	6.6
H_d	event that test packet is bound to arrive in the vulnerable period of an initiating packet	6.6
H_I	event that test packet is bound to arrive in at idle receiver	6.6
h_m	(effective) height of an obstacle [m]	2.1
h_r	height of the receive antenna [m]	2.1.1
h_t	height of the transmit antenna [m]	2.1.1
i	interfering signal index	2.1 / 3.2
I	integral	5.6
I	average duration of idle period [*]	6.6
$i(r)$	duration of the idle period [*]	6.7.1
$I(r)$	average duration of the idle period [*]	6.7.1
i_{ON}	event that transmitter i is active	3.3.2
$I_0(x)$	Modified Bessel function of the first kind and zero order	2.1.3
j	imaginary number $\sqrt{-1}$	
j	test signal index	2.1
J	number of subintervals	App. B
$J_0(x)$	Bessel function of first kind and zero order	2.1.3
K	index indicating propagation model, used only in sections	2.2 / 2.3
K	Rician factor: ratio of the power of the dominant (specular) component and the power of the scattered component	2.1.3 / 5.3
K_i, K_n	constants in expression for BER	5.2
k_{ON}	event that transmitter k is active	3.3.2
L	Number of bits in a packet	7.6
$\log(\cdot)$	logarithm with base 10	
$\ln(\cdot)$	natural logarithm	
m	backlog of a multiple access net	8.6
m	Nakagami m -parameter	4.1
m_K	mean power expressed [dB]	2.3
n	total number of interfering terminals	3.1.2
$n(t)$	noise signal [*]	5.2
$N(z)$	threshold crossing rate [s^{-1}]	4.5
N	number of terminals in the population	3.3.2 / 6.2
N_A	noise floor of receiver A [*]	3.3.3
n_I	noise sample in decision variable [*]	5.2
N_0	one-sided spectral power density of additive white Gaussian noise [*, Hz^{-1}]	5.1
$o(x)$	zero of the order x	6.2
O	outage probability	3.4.3
p	persistence of terminal in ISMA and CSMA protocols	6.1.5
p_{A_s}	instantaneous power received from terminal s at base station A [*]	3.2
$\bar{p}_{A_s}$	local-mean power received from terminal s at base station A [*]	3.2

$\bar{p}_{A_s}$	area-mean power received from terminal s at base station A [*]	3.2
$P_b(e)$	bit error probability	5.1
P_c	packet error probability, block error rate	3.1.1
p_j	power received from terminal j [*]	2.1.3
$\bar{p}_j$	local-mean received power [*]	2.1.2
$\bar{p}_j$	area-mean received power [*]	2.1.1
$P_n(n)$	probability on n arrivals	6.2
P_{NA}	probability that the wanted signal is below the noise floor at receiver A ($p_{A_i} < zN_A$)	3.3.3
$Pr()$	probability	
P_r	probability of packet retransmission	6.4.2 / 8.6
P_t	joint interference power [*]	3.1.2
P_0	probability of packet arrival	6.4.2 / 8.6
p_0	power of wanted signal (with index 0) [*]	3.2
$Q(r)$	probability of successful transmission of a data packet given the distance of the mobile terminal to the receiver	6.4.1
$\bar{q}_j$	local-mean scattered power [*]	2.1.3
q_n	capture probability conditional on n interfering signals	6.3
$q_n(r)$	capture probability for a packet from distance r conditional on n interfering signals	6.3
Q_t	probability of successful transmission of a data packet	6.4.1
r, r_j	distance between (j -th) mobile terminal and the base station [*]	2.1.1 / 6.3
R	normalised distance, cell radius [*]	6.7.2 / 9.1
R_c	reflection coefficient	2.1.1
$R[\cdot, \cdot]$	linear regression	2.2.4
r_b	bit rate [s^{-1}]	4.10
$R_k R_m$	remainder in numerical computation technique	2.1.5
r_u	normalised frequency re-use distance (distance between the centres of two co-channel cells) [*]	1.5.1
s	variable of Laplace transform in image domain [*]	2.1.5
$s_p s_s$	standard deviation of shadowing of individual signal	2.1.2
s_j	event of success of the j -th transmitter	7.1
s_K	standard deviation of received power [dB]	2.2.4
S_m	throughput of a random-access net with backlog m [*]	8.6.1
S_t	channel throughput [*]	6.4.2
s_t	(logarithmic) standard deviation of joint interference signal	3.1.2.1
t	time [s]	2.1.3 / 5.4
t	time [*]	6.7
t	error correction distance	7.9
t_B	instant when initiating packet arrives at base station [*]	6.7.1
T	observation interval [s]	2.1.7
T_b	bit duration [s]	5.1
t_I	instant when base station starts transmitting idle signal[*]	6.7.1
t_p	round-trip delay [*]	6.7.1
T_s	duration of a data packet or block [s]	4.10
T_0	criterion (minimum duration) of a signal drop-out [s]	4.9

$T_r..T_{10}$	time scale [s]	2.1.7
u	normalised speed of light [*]	6.7.1
v, v_i	terminal velocity [m/s]	4.1
v	(sect. 2.2. and 2.3:) diffraction parameter	2.1.1
$v(r)$	duration of the vulnerable period as a function of terminal distance [*]	6.7.1
$V(r)$	average duration of the vulnerable period $v(r)$ [*]	6.7.1
$v(t)$	received signal [*]	2.1.3
$v_i(t)$	joint received signal [*]	3.1.2
$W(\cdot, \cdot)$	weight function	8.3.1
X	theoretical propagation loss according to a certain model [dB]	2.2.4
y	amplitude-ratio between wanted and joint interference signal	4.5
$\dot{y}$	(time-) derivative of amplitude-ratio between wanted and joint interference signal [s^{-1}]	4.5
z	receiver threshold	3.1.1
α_i	bit synchronisation offset of the i -th interfering signal	5.6
β	attenuation exponent	2.1.1
γ	0.57721.. (Euler's constant)	2.1.3
$\gamma(\cdot, \cdot)$	incomplete gamma function	6.7.3
$\Gamma(\cdot)$	gamma function	4.1
$\delta(\cdot)$	Dirac-function	3.3.2
Δ	phase difference between direct and ground-reflected wave	2.1
Δ_t	delay spread [s]	2.1.3
ϵ	displacement of the antenna [m]	2.1.3
ζ_j	in-phase component of the signal amplitude [*]	2.1.3
η	fade margin (ratio between r.m.s. amplitude of wanted signal and r.m.s. amplitude of interference and noise)	4.5
θ	carrier phase	3.1.2
θ_r	phase of local oscillator in coherent receiver	7.6.2
κ_i	value of bit in i -th signal in balanced notation ($\kappa = \pm 1$)	5.6
$\kappa_{i \leftrightarrow} (\kappa_{i \rightarrow})$	value of overlapping earlier (later) bit in the i -th interfering signal	5.6
λ	wave length [*]	2.1.1
$\lambda, \lambda(\cdot)$	Poisson arrival rate [*]	6.2
ν	receiver decision variable	5.2
ξ_j	quadrature phase component [*]	2.1.3
π	3.141592....	
π_m	probability of a backlog m	8.6.1
Π	product	3.2
ρ_j	amplitude of signal j ($\rho^2 = \zeta^2 + \xi^2$) [*]	2.1.3
$\dot{\rho}, \rho_i$	(time-) derivative of amplitude of signal i [* , s^{-1}]	4.1
ρ_t	amplitude of joint interference signal [*]	4.3
σ	standard deviation	
$\sigma_{\rho j}$	st. dev. of (time-) derivative of the i -th Rayleigh fading signal [* , s^{-1}]	4.1
σ_i	st. dev. of shadowing of the i -th signal	2.1.2
σ_t	logarithmic st. dev. of the power of the joint interference signal	3.1.2.1
$\sigma_{\rho t}$	standard deviation of $\dot{\rho}_t$ [* , s^{-1}]	4.5

σ_s, σ_i	logarithmic standard deviation of shadowing	2.1.2
σ_p	logarithmic standard deviation of model error	2.2.4
Σ	sum	
τ	interarrival time [*]	6.2
$\tau_F(z)$	fade duration [s]	4.9
$\tau_{NF}(z)$	non-fade duration [s]	4.10
$\phi(A, i)$	Laplace image of a Suzuki pdf of power of the i -th interferer at receiver A , taken in the point $s = z\bar{p}_{A0}$	3.3.2
$\phi_\sigma(s)$	Laplace transform of a Suzuki pdf	2.1.5
ϕ_I	Laplace image of power of the joint interference signal	4.2
ϕ_i	Laplace image of power of interference from terminal i	4.2
χ	factor in expression for average non-fade duration	4.6
$\psi_i(t)$	phase modulation of the i -th carrier	3.1.2
$\psi(\cdot)$	laplace image function	8.1
$\psi(A, B, i)$	function	3.4.2
Ψ	empirical vegetation parameter	2.3.3
ω_c	angular carrier frequency ($2\pi f_c$) [rad s ⁻¹]	2.1.3
ω_m	angular Doppler frequency ($2\pi f_m$) [rad s ⁻¹]	4.1
I, II System I or II		1.2.1
$\mathcal{L}\{\}$	Laplace transform	2.1.5
$\otimes$	convolution	3.2

[*] Note on normalisation, units and dimensions

All variables and parameters concerning *physical* transmission aspects are expressed in SI-MKSA dimensions, i.e., based on the second, in contrast to most parameters concerning *logical* protocol issues, where for convenience packet duration is taken as the normalised unit of time.

Analogously, distances relevant to the *architecture* of frequency re-use patterns are normalised to the cell radius and denoted by the letter r , whereas distances relevant to physical transmission are expressed in SI-MKSA dimensions, i.e., based on the metre, and denoted by the letter d .

Received signal amplitudes and powers are normalised according to Eq. (2.13), i.e., normalised to unity received (area-mean) power at the fringe of the cell.

APPENDIX E: LIST OF ACRONYMS

ATF-1 · · ATF-4	(Dutch) vehicular telephone networks
AWGN	additive white Gaussian noise
BCMA	Busy Channel Multiple Access
BER	bit error rate
BPSK	Binary Phase Shift Keying
BFSK	Binary Frequency Shift Keying
BS	Base Station
CCIR	International Radio Consultative Committee
CCITT	International Telegraph and Telephone Consultative Committee
cdf	cumulative distribution function
<i>C/I</i>	carrier-to-interference ratio
<i>C/N</i>	carrier-to-noise ratio
CSMA	carrier sense multiple access
CW	continuous wave (as opposite to burst communication)
DPSK	Differentially Encoded Phase Shift Keying
DSI	Digital Speech Interpolation
EDI	Electronic Data Interchange
FEC	Forward Error Correction coding
FSK	Frequency Shift Keying
GSM	Groupe Speciale Mobile
GMSK	Gaussian Minimum Shift Keying
IEE	Institution of Electrical Engineers
IEEE	Institute of Electrical and Electronics Engineers
ISI	Inter-Symbol Interference
ISMA	Inhibit Sense Multiple Access
ITU	International Telecommunication Union
MKE	multiple knife edge
MS	Mobile Station
MSK	Minimum Shift Keying
OSI	Open Systems Interconnection (reference model)
pdf	probability density function
pps	packets per slot (in slotted access schemes)
ppt	packets per unit of time (in slotted or unslotted access schemes)
SKE	single knife edge
TASI	Time-Assigned Speech Interpolation
TEM	Transversal Electromagnetic (field)
WOS	wireless office (communication) system

APPENDIX F: LIST OF RELATED PUBLICATIONS BY THE AUTHOR

JOURNAL PAPERS

- [1] J.P.M.G. Linnartz and R. Prasad, "Threshold crossing rate and average non-fade duration in a Rayleigh-fading channel with multiple interferers", *Archiv für Elektronik und Übertragungstechnik (AEÜ)*, pp. 345-349. Vol. 43, No.6, Nov/Dec. 1989.
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- [L1] J.P.M.G. Linnartz, R. Prasad and J.C. Arnbak, "Spatial distribution of traffic in a cellular ALOHA network", *Archiv für Elektronik und Übertragungstechnik (AEÜ)*, Vol. 42, No. 1, Jan./Feb. 1988, pp 61-63.
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- [c5] J.P.M.G. Linnartz, "Near-Far effects in some random-access radio systems with fading", 1990 Bilkent Int. Conf. on New Trends in Comm., Control, and Signal Processing, Ankara, July 2-5, 1990, pp. 504-510.
- [c6] A. Safak, R. Prasad and J.P.M.G. Linnartz, "Outage probability analysis for mobile radio systems due to correlated log-normal interferers", 1990 Bilkent Int. Conf. on New Trends in Comm., Control, and Signal Processing, Ankara, July 2-5, 1990, pp.518-524.
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- [R2] J.P.M.G. Linnartz, "Een literatuurstudie naar propagatie in de VHF en UHF banden voor mobiele en transportabele netwerken" (in Dutch), FEL-TNO report 1988-53, Physics and Electronics Laboratory TNO, The Hague, NL.

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SAMENVATTING (Summary in Dutch)

Door de eeuwen heen heeft de mens bijzonder belang gehad bij goede communicatie. De behoefte aan communicatie is nauw verweven met mobiliteit. Mogelijk gemaakt door recente technologische ontwikkelingen is thans een sterke groei van 'mobiele communicatie' te bespeuren. Mobiele communicatie behelst (radio-) communicatie waarbij minstens een der deelnemers in beweging kan zijn. Congestie van het beschikbare radiospectrum stelt hoge eisen aan de efficiëntie van het 'ethergebruik' van systemen voor mobiele communicatie.

Dit proefschrift onderzoekt de prestatie van systemen voor smalbandige mobiele radiocommunicatie. Met name de invloed van 'fading' ten gevolge van meerwegontvangst en schaduwwerking en van wederzijdse interferentie tussen mobiele gebruikers wordt onderzocht. Daartoe worden in eerste instantie relevante propagatieverschijnselen belicht, waarbij het accent ligt op statistische kanaalmodellen. Ten behoeve van de planning van praktische systemen wordt ook de bruikbaarheid van enkele propagatiemodellen die terreingegevens in beschouwing nemen empirisch geëvalueerd. De statistische kanaalbeschrijving wordt vervolgens gebruikt om de kwaliteit van mobiele radioverbindingen te bepalen. De kans op uitval van de verbinding ('outage') en de gemiddelde duur hiervan worden bepaald, rekening houdend met storingen veroorzaakt door andere gebruikers in het net. Een vereenvoudigd model voor de kans op bitfouten bij digitale transmissie wordt voorgesteld. Numerieke resultaten worden gegeven voor mobiele telefonienetten.

Teneinde de prestatie van pakket-geschakelde mobiele netten te kunnen onderzoeken worden de statistieken van willekeurig arriverende databerichten (pakketten) bij enkele toegangsprotocollen besproken. Gebruik makend van de gevonden eigenschappen van de fysieke radioverbinding wordt de capaciteit van mobiele datanetten bepaald, waarbij bijzondere aandacht uit gaat naar de ruimtelijke verdeling van gebruikers over het verzorgingsgebied van het netwerk. De prestatie van het net wordt bepaald in relatie tot de afstand tussen gebruiker en het vaste basisstation. Hiermee kan ook de kans op succesvolle verzending van een datapakket voor de slechtst bediende gebruiker worden bepaald.

Methoden zijn ontwikkeld om, in een net van gegeven dimensionering, de invloed van onderlinge storingen tussen de mobiele terminals te bepalen. Hiermee is de mogelijkheid geschapen tot het kiezen van het (vanuit frequentiegebruik gezien) meest efficiënte ontwerp waarbij een voorgeschreven prestatie van het mobiele net gewaarborgd is. Daardoor zijn de fundamentele resultaten beschreven in deze dissertatie toepasbaar in het spectraal-efficiënt ruimtelijk ontwerp (en planning) van mobiele communicatie-infrastructuren.

VERANTWOORDING (Account of scientific results)

Dit proefschrift kwam tot stand gedurende een periode van drie en een half jaar waarin de auteur werkzaam was als universitair docent aan de vakgroep Telecommunicatie en Verkeersbegeleidings-Systemen (T.V.S.) aan de T.U. Delft. Sommige van de beschreven resultaten zijn bereikt met steun van andere leden van de vakgroep, waarbij met name studenten hebben bijgedragen tot de (numerieke) uitwerking van enkele modellen. Deze bijdragen worden hieronder beschreven.

- Hoofdstuk 1:

De introductie tot dit proefschrift bevat enkele gedachten die oorspronkelijk zijn verwoord in een gezamenlijke publicatie van de auteur met Prof. Dr. J.C. Arnbak.

- Hoofdstuk 2:

De auteur gebruikte in 2.3 meetresultaten verkregen door de Materiaal Voorzienings Afdeling MBA-2 van de Koninklijke Landmacht te Den Haag. Hiertoe werd een meetopstelling gebruikt die door FEL-TNO was ontwikkeld.

- Hoofdstuk 4:

Resultaten zijn deels ontleend aan het artikel "Threshold crossing rate and average non-fade duration in a Rayleigh-fading channel with multiple interferers". Mede-auteur Prof. Dr. R. Prasad droeg bij tot de modelvorming.

- Hoofdstuk 5:

Resultaten in voor interferentie met willekeurige bit timing in 5.6 zijn ontleend aan het werk van A.J. 't Jong gedurende zijn laboratoriumtaak, begeleid door de auteur.

- Hoofdstuk 6:

De methode voor bepaling van de exacte duur van de 'inhibited period' in 6.7.5 is ontwikkeld door R.-J. Venema gedurende diens afstudeerperiode, waarin hij begeleid werd door Ir. G.A. Awater en de auteur.

- Hoofdstuk 7:

De modellen in 7.6.5 zijn geanalyseerd door R. Hekmat gedurende zijn de afstudeerperiode, waarin hij begeleid werd door de auteur. De numerieke resultaten in 7.6.3 komen voort uit het afstudeerwerk van H. Goossen, begeleid door Prof. Dr. R. Prasad en de auteur.

Simulatie-resultaten in 7.7 zijn gegenereerd door P.P.C. de Klerk tijdens zijn laboratoriumtaak. De software was grotendeels ontwikkeld door R. Pluymers.

- Hoofdstuk 8:

Deze resultaten zijn grotendeels ontleend aan het afstudeerwerk van de auteur (1986), destijds in Eindhoven begeleid door Prof. Dr. J.C. Arnbak.

De stabiliteit van mobiele ALOHA kanalen met capture 8.6 is onderzocht door C. van der Plas in zijn afstudeerproject, begeleid door de auteur.

- Appendix B:

De analyse van niet-stationaire Poisson processen is voorgesteld door Ir. G.A. Awater.

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CURRICULUM VITAE

Jean-Paul M.G. Linnartz was born in Heerlen, in the province of Limburg (The Netherlands), on September 10, 1961. He attended Gymnasium β at the Scholengemeenschap St. Michiel in Geleen.

He received his Ir. (M.Sc.) Degree in Electrical Engineering Cum Laude from Eindhoven University of Technology, in December 1986. His thesis addressed the spatial distribution of packet traffic in a mobile ALOHA network.

During his military service (January 1987- April 1988), he was with The Netherlands Organisation for Applied Scientific Research, Physics and Electronics Laboratory (F.E.L-T.N.O.), The Hague, where he was involved in research on UHF propagation and frequency assignment for mobile and transportable radio networks.

He joined Delft University of Technology in May 1988 as 'universitair docent' (assistant professor) in the Telecommunications and Traffic-Control Systems Group. After defending his PhD dissertation, he will join the Department of Electrical Engineering and Computer Sciences (EECS) at the University of California at Berkeley.

During his academic studies, he worked with local radio stations in Geleen, Bilzen (Belgium) and Delft, and with "Regionale Omroep Zuid"¹ in Maastricht. He is the author of a number of magazine articles on technical aspects of radio broadcasting. Further special interests are in art, photography, sailing and travelling.

¹Currently: "Omroep Limburg"