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SYNCHRONOUS MC-CDMA IN DISPERSIVE, MOBILE RAYLEIGH CHANNELS.

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Time variations of the mobile radio channel cause InterCarrier Interference (ICI) to Orthogonal Frequency Division Multiplexing (OFDM) and Multi-Carrier CDMA (MC-CDMA) radio links. This paper evaluates the effect of Doppler and delay spreads on the performance of an MC-CDMA receiver. An MMSE receiver is analyzed for MC-CDMA transmission over a mobile channel with Doppler. A simplified, pseudo MMSE receiver is proposed that does not need to perform real-time inversion of a large matrix. It performance is shown to be better than for (uncoded) OFDM.

1. Summary

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation method designed in the 1960's and 1970's in which multiple user symbols are transmitted in parallel using different subcarriers. These subcarriers have overlapping spectra, but their signal waveforms are specifically chosen to be orthogonal. Compared to modulation methods such as BPSK, QPSK or MSK, OFDM transmits symbols, which have relatively long time duration, but a narrow bandwidth. Mostly, OFDM systems are designed such that each subcarrier is small enough in bandwidth to experience frequency-flat fading. This also ensures that the subcarriers remain orthogonal when received over a (moderately) frequency selective but time-invariant channel. If the OFDM signal is received over such a channel, each subcarrier experiences a different attenuation. In Coded-OFDM, errors which are most likely to occur on subcarriers which are most severely attenuated, are repaired by error correction codes. To this end, the redundancy in the error correction code is typically spread over many different subcarriers.

While its robustness against frequency selectivity is seen as an advantage of OFDM, any time-varying character of the channel is known to pose limits to the system performance. Time variations are known to corrupt the orthogonality of the OFDM subcarrier waveforms [1]. In such case, InterCarrier Interference (ICI) occurs because signal components from one subcarrier cause interference to other, mostly to neighboring subcarriers. The effect of ICI has been analyzed for carrier frequency errors and Wiener phase noise in [2]. In this paper, we model mobile radio propagation to study the effect of user mobility. In a Rayleigh fading channel, Doppler spreading caused by the mobile channel cannot be compensated (or modeled) by a common frequency correction for all subcarriers. Because of multipath propagation, the receiver sees a large collection of incoming waves, each with it own angle of arrival and corresponding Doppler frequency offset. Only in the special case that the delay spread among these multiple reflections is negligible compared to inverse of the total transmit bandwidth (so all subcarriers see the same channel fading), the Doppler spread can be shown to be equivalent to a common frequency offset. The effect of Doppler spread on OFDM was previously addressed, e.g. in [3] and [4].

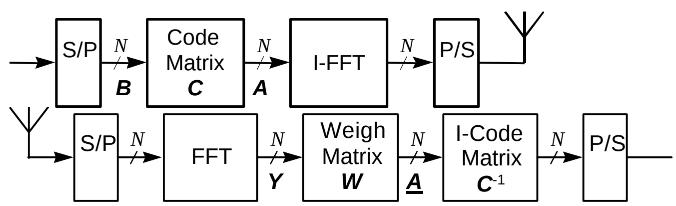

Figure 1: OFDM and MC-CDMA Transmit System and MMSE Receiver architecture

This paper reviews the classic OFDM modulation, and introduces new results for a CDMA-type of transmission which is an extension of the basic OFDM principle. At PIMRC 1993, this form of Orthogonal Multi-Carrier CDMA was proposed [5], [8]. Basically it applies an OFDM-type of transmission to a multiuser synchronous DS-CDMA signal. In DS-CDMA, each user bit is transmitted as many sequential chips, each of which is of short duration, thus of wide bandwidth. In contract to this in MC-CDMA, chips are long in time duration, but narrow in bandwidth. Multiple chips are not-sequential, but transmitted in parallel on different subcarriers. Several other MultiCarrier CDMA schemes have also been proposed, but we restrict our analysis here the above one.

The outline of the paper is as follows: Section 2 combines the OFDM transmit model with models by Clarke and Aulin for multipath channels. It gives expressions for the ICI under Doppler spreading. The effect of a Rayleigh Doppler spread on the BER is calculated for a conventional OFDM receiver. Section 3 addresses MC-CDMA and derives receiver settings for an MMSE receiver and a channel with delay and Doppler spread. A simplification is proposed which mitigates the need for accurate channel estimation and adaptive filtering. Its performance is analyzed. Numerical and simulation results are in Section 5.

2. Channel model

For OFDM, vector $\boldsymbol{A}$ of length N carries a 'frame' of user data, with $\boldsymbol{A} = [a_0, a_1, \dots a_{N-1}]^T$, where the elements a_n are user symbols. In MC-CDMA, $\boldsymbol{A} = \boldsymbol{CB}$, where $\boldsymbol{C}$ is an N by N code matrix and $\boldsymbol{B} = [b_0, b_1, \dots b_{N-1}]^T$ represents a frame of user data. The k-th column of $\boldsymbol{C}$ represents the spreading code of user data stream k, and will be denoted as $(c_k[0], .. c_k[N-1])^T$. We use $\boldsymbol{C} = N^{-1/2}$ WH_N where WH_N is the Walsh-Hadamard matrix of size N by N. In that case, $\boldsymbol{C} = \boldsymbol{C}^{-1} = \boldsymbol{C}^H$, so $\boldsymbol{C}\,\boldsymbol{C} = \boldsymbol{I}_N$ with $\boldsymbol{I}_N$ the N by N identity matrix. For ease of analysis, we normalize the modulation depth as $\mathrm{E}\, b_i b_j^* = \delta_{ij}$, or equivalently $\mathrm{E}\boldsymbol{BB}^H = I_N$. Then $\mathrm{E}[\boldsymbol{AA}^H] = \mathrm{E}\boldsymbol{C}[\boldsymbol{BB}^H]\boldsymbol{C}^H = \boldsymbol{CC}^H = I_N$.

The Wide Sense Stationary Uncorrelated Scattering multipath channel is modeled as a collection of I_w reflected waves. Each wave has its particular Doppler frequency offset ω_i, path delay T_i and amplitude D_i, each of which is assumed to be constant. Vector $\boldsymbol{Y}$ describes the outputs of the FFT at the receiver, with $\boldsymbol{Y} = [y_0, y_1, .., y_{N-1}]^T$, with $y_m = \Sigma_n a_n \beta_{m,n} T_s$ where $\beta_{m,n}$ is the 'transfer' for a signal transmitted at subcarrier n and received at subcarrier m,

$$b_{m,n} = \sum_{i=0}^{I_w-1} \frac{D_i}{2} \operatorname{sinc}\left(n - m + \frac{\omega_i}{\omega_s} \right)$$
$$\exp\{-j(\omega_c + \omega_i + n\omega_s)T_i - \tfrac{1}{2} j\omega_i T_s + jp(n-m)\}$$

The OFDM frame duration T_s is related to the intercarrier spacing $f_s = \omega_s/(2\pi)$, according to $\omega_s T_s = 2\pi$. For uniform angles of arrivals of reflected waves one can show that the variance of the ICI signal spilled from transmit subcarrier n into received subcarrier $m = n + \Delta$ equals [3]

$$P_\Delta = \mathrm{E}_{ch}\, b_{n+\Delta,n} b^*_{n+\Delta,n} = \frac{P_T}{8p} \int_{-1}^{1} \frac{\operatorname{sinc}^2\left(\Delta + \frac{f_\Delta}{f_s} x\right) dx}{\sqrt{(1-x^2)}}$$

where f_D is the maximum Doppler shift and P_T the local mean received power, per subcarrier (see Fig. 2).

For engineering applications with small Doppler spreads, a rule of thumb can be derived. It appeared necessary to consider higher order tiers of neighboring subcarriers, but it was permissible to use a first-order approximation for the sinc. For arguments near zero, we take $\operatorname{sinc}(f f_s^{-1}) \approx 1$, so we find that $P_0 \approx P_T$. For $\Delta = k$ (and for $f << f_s$), we approximate $\operatorname{sinc}(k + f f_s^{-1}) \approx \operatorname{sinc}(k) + (k + f f_s^{-1} - k)\operatorname{sinc}'(k) = (-1)^k k^{-1} f f_s^{-1}$. Moreover we observe that $P_k = P_{-k}$. Inserting these, we find

$$P_k \approx \frac{2P_T}{pf_\Delta k^2 f_s^2} \int_0^{f_\Delta} \frac{f^2 df}{\sqrt{1 - f^2 f_\Delta^{-2}}} = \frac{f_\Delta^2 P_T}{pf_s^2 k^2}\left[\arcsin 1 - \sqrt{1 - \frac{f^2}{f_\Delta^2}} \right]$$

or $P_k = f_\Delta^2 / (2 f_s^2 k^2) P_T$. We use that $\Sigma_{k=1} k^{-2} = \zeta(2) = \pi^2/6$. So, for BPSK OFDM

$$B_1 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{g}{\frac{p^2 f_\Delta^2}{6 f_s^2} g + 1}}$$

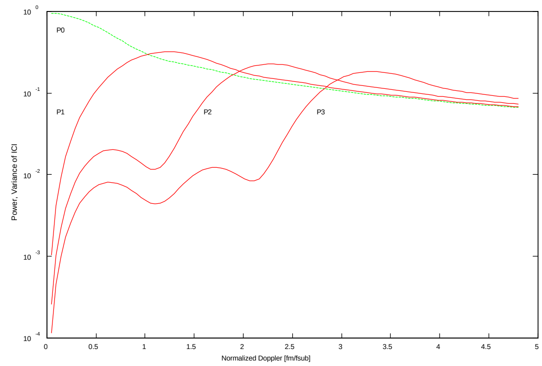

Figure 2: Received power P_0, and the variances P_1, P_2, and P_2 of the ICI versus the normalized Doppler spread λ for $p_T = 1$.

3. Receiver model

The Minimum Mean-Square Error Estimate of the user data is equal to the conditional expectation $\mathrm{E}[\boldsymbol{B}|\boldsymbol{Y}] = \mathrm{E}[\boldsymbol{C}^{-1}\boldsymbol{A}|\boldsymbol{Y}] = \boldsymbol{C}^{-1}\mathrm{E}[\boldsymbol{A}|\boldsymbol{Y}]$. Let the estimate $\underline{\boldsymbol{A}}$ be a linear combination of $\boldsymbol{Y}$, namely $\underline{\boldsymbol{A}} = \boldsymbol{WY}$. The optimum choice of matrix $\boldsymbol{W}$ follows from the orthogonality principle that the estimation error is uncorrelated with the received data, viz., $\mathrm{E}[(\boldsymbol{A} - \underline{\boldsymbol{A}})\boldsymbol{Y}^H] = 0_N$ with 0_N the all-zero matrix of size N by N. Thus we arrive at $\boldsymbol{W} = \mathrm{E}[\boldsymbol{AY}^H]\, \mathbf{R}_{YY}^{-1}$, for the optimum estimation matrix. Here $\boldsymbol{Y} = \boldsymbol{HA} + \boldsymbol{N}$, where channel matrix $\boldsymbol{H}$ has the components $\boldsymbol{H}_{nm} = T_s \beta_{nm}$. So,

$$\begin{aligned} \mathrm{E}[\boldsymbol{AY}^H] &= \mathrm{E}[\boldsymbol{A}[\boldsymbol{HA}]^H] + \mathrm{E}[\boldsymbol{AN}^H] = \mathrm{E}[\boldsymbol{AA}^H\boldsymbol{H}^H] \\ &= \boldsymbol{C}\,\mathrm{E}[\boldsymbol{BB}^H\boldsymbol{C}^H\boldsymbol{H}^H] = \boldsymbol{H}^H. \end{aligned}$$

Also, $\mathbf{R}_{YY}$, the covariance matrix of $\boldsymbol{Y}$, becomes

$$\mathbf{R}_{YY} = \mathrm{E}\boldsymbol{YY}^H = \boldsymbol{H}\,\mathrm{E}[\boldsymbol{AA}^H]\,\boldsymbol{H}^H + \mathrm{E}\boldsymbol{NN}^H = \boldsymbol{HH}^H + N_0 T_s \boldsymbol{I}_N.$$

That is, the MMSE receiver needs to perform adaptive, real-time matrix inversion. However, in the special case of a channel without any Doppler spreading, thus with $\boldsymbol{H} = T_s \operatorname{diag}(\beta_{0,0}, \dots \beta_{N-1,N-1})$. Then $\boldsymbol{W}$ reduces to a diagonal matrix with elements [8]

$$w_{n,m} = \frac{d_{m.n} b^*_{nn} T_s^{-1}}{b_{nn} b^*_{nn} + \frac{N_0}{T_s}}$$

For mobile channel with Doppler, it appears nonetheless useful to consider a simple receiver in which $\boldsymbol{W}$ is just the above 'automatic gain control'. It needs to estimate only $\beta_{n,n}$, but no $\beta_{m,n}$ with $m \neq n$. Next, we calculate the BER, which involves the statistical analysis of $\beta_{n,n}$ and w_n for Rayleigh channels with

Doppler and delay spread. The decision variable for user bit 0, after combining all subcarrier signals consists of

$$x = x_0 + x_{MUI} + x_{ICI} + x_n$$

where x_0 is the wanted signal, x_{MUI} is the self interference between different user symbols (multi-user interference), x_{ICI} is the intercarrier interference, and x_n is the noise. The terms x_{MUI}, x_{ICI} and x_n are zero-mean Gaussian, when we consider the average over a large collection of Rayleigh channels. We define $M_{ij} = \mathrm{E_{ch}}\, |\beta_{n,n}|^i\, |w_n|^j$, where $\mathrm{E_{ch}}$ denotes the expectation over all channels. The wanted signal is $x_0 = b_0 M_{11} T_s$. The contribution of the multi-user interference is

$$x_{MUI} = T_s \sum_{k=1}^{N-1} b_k \left[\sum_{n=0}^{N-1} b_{n,n} w_{n,n} c_0[n] c_k[n] \right]$$

For orthogonal spreading codes ($\Sigma_n\, c_0[n]c_k[n] = 0$). For a dispersive channel, the orthogonality of spreading codes is eroded. In such case, the variance of the MUI can be evaluated by observing that for any two orthogonal codes $c_j[n]$ and $c_k[n]$ with $j \neq k$, one can partition the set of subcarrier indixes n with n = 0, 1, N–1 into two sets, both with exactly N/2 elements, such that A_{-} = {n: $c_j[n]c_k[n] = -1/N$} and A_{+} = {n: $c_j[n]c_k[n] = +1/N$} [6]. Here $A_{+} \cup A_{-} = A$ ensures that $\Sigma_{A+\cup A-}\; c_j[n]c_k[n] = 0$. Hence

$$x_{MUI} = \frac{T_s}{N} \sum_{k=1}^{N-1} b_k \left[\sum_{n \in A_+} b_{n,n} w_{n,n} - \sum_{n \in A_-} b_{n,n} w_{n,n} \right]$$

Because of independence of user symbols and channel properties, and mutual independence of user signals,

$$s^2_{MUI} = \mathrm{E_{ch}E}\; x_{MUI} x^*_{MUI} = \frac{T_s^2}{N^2} \mathrm{E} \sum_{k=1}^{N-1} b_k^2$$
$$\bullet\, \mathrm{E_{ch}} \left[\sum_{n \in A+} b_{n,n} w_{n,n} - \sum_{n \in A-} b_{n,n} w_{n,n} \right]^2$$

If we may assume that fading of the subcarriers is independent, we can write

$$\mathrm{E_{ch}} \left(\sum_{n \in A+} b_{n,n} w_n \right)^2 = \mathrm{E_{ch}} \sum_{n \in A+} \sum_{m \in A+} b_{n,n} b_{m,m} w_{n,n} w_{m,m}$$
$$= \frac{N}{2} \left(\mathrm{E_{ch}}\, b^2_{n,n} w^2_{n,n} + \left(\frac{N}{2} - 1 \right) \left(\mathrm{E_{ch}}\, b_{n,n} w_{n,n} \right) \right) = \frac{NM_{22}}{2} + \frac{N}{2} \left(\frac{N}{2} - 1 \right) M_{11}^2$$

and since $A_{+} \cap A_{-} = \emptyset$,

$$\mathrm{E_{ch}} \left(\sum_{n \in A+} b_{nn} w_n \right) \left(\sum_{n \in A-} b_{nn} w_n \right) = \left(\frac{N}{2} \right)^2 M_{11}^2$$

Thus,

$$s^2_{MUI} = \frac{N-1}{N} T_s^2 \left(M_{22} - M_{11}^2 \right)$$

The ICI contribution stems from crosstalk between subcarriers. Signal components which are present in $a_n = \Sigma_k\, c_k[n] b_k$ are spilled into $y_m = y_{n+\Delta}$, with strength $\beta_{n+\Delta,n}$. In the receiver, these are weighted by $w_{n+\Delta,n+\Delta}$ and unspread by $c_0[n+\Delta]$.

$$x_{ICI} = T_s \sum_{n=0}^{N-1} a_{\mathrm{n}} \sum_{\Delta \neq 0} b_{\mathrm{n}+\Delta,n} w_{\mathrm{n}+\Delta,\mathrm{n}+\Delta} c_0[n+\Delta]$$

$$x_{ICI} = T_s \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} b_{\mathrm{k}} c_k[n] \sum_{\Delta \neq 0} b_{\mathrm{n}+\Delta,n} w_{\mathrm{n}+\Delta,\mathrm{n}+\Delta} c_0[n+\Delta]$$

Inserting $a_n = \Sigma_k\, c_k[n] b_k$ and interchanging the sequence of the summings

$$x_{ICI} = T_s \sum_{\Delta \neq 0} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} c_k[n] b_k\; b_{\mathrm{n}+\Delta n} w_{\mathrm{n}+\Delta,\mathrm{n}+\Delta}\, c_0[n+\Delta]$$

Thus,

$$s^2_{ICI} = \mathrm{E_{ch}Ex_{ICI}x^*_{ICI}} =$$
$$T_s^2 \mathrm{E_{ch}E} \left[\sum_{m=1}^{N-1} \sum_{k=1}^{N-1} b_k \sum_{n=0}^{N-1} b_{mn} w_n c_0(n) c_k(n-m) \right]^2$$

The square of the triple sum simplifies because of $\mathrm{E}b_k\, b_j^* = \delta_{kj}$ and $\mathrm{E_{ch}}\, \beta_{i,j}\, \beta_{k,l} = \delta_{ij}\delta_{kl}$ and similar properties. Note that each term $c_0[n]c_k[n-m]$ is multiplied by $\beta_{m,n}$ and $w_{n,n}$, which are complex valued with random mutually independent arguments. Thus, even if the code has good autocorrelation properties, the channel delay spread erodes the attenuation of the ICI hoped for. So we can simplify our analysis by attributing no specific ICI-reducing properties to the spreading code, i.e., we take, $\mathrm{E}[c_0[n]\; c_k[n-m]]^2 = N^{-2}$, and

$$s^2_{ICI} = \frac{1}{N} \mathrm{E} \sum_{k=1}^{N-1} b_k^2$$
$$\mathrm{E} \left[\sum_{\Delta \neq 0} \left[c_0(n) c_k(n-m) \right]^2\; \mathrm{E_{ch}} \sum_{n=0}^{N-1} \left| b_{m,n} \right|^2 \mathrm{E_{ch}} \sum_{n=0}^{N-1} \left| w_{n,n} \right|^2 \right]$$

Thus

$$\sigma_{ICI}^2 = \Sigma_{\Delta \neq 0} p_\Delta\;\; M_{02}\, T_s^2.$$

The variance of the noise, collected over all subcarriers weighted by $w_{n,n}$ becomes $s_{noise}^2 = N\, M_{02}\, N_0 T_s$.

Since we consider ensembles of many different channels, x_{MUI}, x_{ICI} and x_{noise} are zero-mean complex Gaussian. So, the local-mean BER for BPSK becomes $B_1 = \frac{1}{2} \mathrm{erfc} \sqrt{\frac{E_N}{N_0}}$, with

$$\frac{E_N}{N_0} = \frac{M_{11}^2}{\left(M_{22} - M_{11}^2 \right) + M_{02} \left[\sum_{\Delta \neq 0} p_\Delta + \frac{N_0}{T_s} \right]}$$

For numerical work, it is useful to realize that

$$M_{11} = T_s^{-2} \left[1 - \frac{c}{p} \exp\left(\frac{c}{p} \right) E_1\left(\frac{c}{p} \right) \right]$$

$$M_{22} - M_{11}^2 = \frac{c}{p} - \frac{c^2}{p^2}\exp\left(\frac{c}{p}\right)E_1\left(\frac{c}{p}\right) - \frac{c^2}{p^2}\exp\left(\frac{2c}{p}\right)\left[E_1\left(\frac{c}{p}\right)\right]^2$$

and

$$M_{02} = \frac{1}{p}\left[\left(1+\frac{c}{p}\right)\exp\left(\frac{c}{p}\right)E_1\left(\frac{c}{p}\right) - 1\right]$$

with $E_1()$ the exponential integral.

4. RESULTS

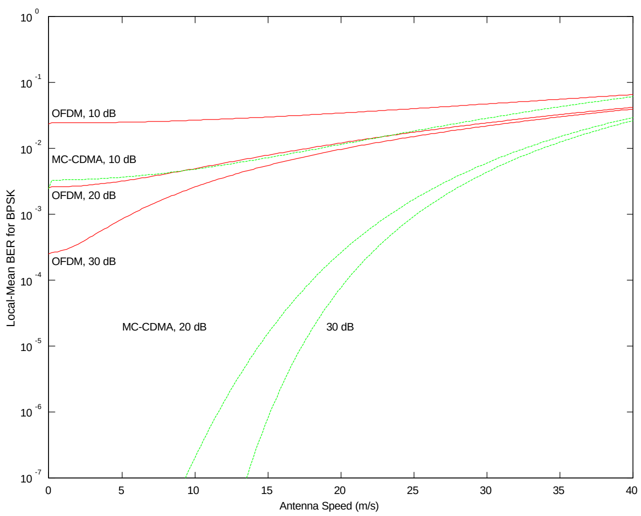

Figure 3: Local-mean BER versus antenna speed for uncoded OFDM and MC-CDMA

Figure 3 gives some results for Doppler spreads at 4 GHz. We inserted typical values for DVB-DTTB but modified it to BPSK MC-CDMA instead of the standardized C-OFDM. The frame duration is 896 microseconds, with an FFT size of 8192. This corresponds to a subcarrier spacing of f_s = 1.17 kHz and a data rate of 9.14 Msymbols/s. Signal to noise ratios are E_b/N_0 = 10, 20 and 30 dB. The BER for BPSK OFDM is plotted for 30 dB E_b/N_0. The curves shows that MC-CDMA outperforms uncoded OFDM. The theoretical analysis is most accurate for a hypotetic system with infinitely many subcarriers. Simulations showed that for the SNR and BER range of interest, systems with more than 64 subcarriers closely follow the theoretical model [7].

5. CONCLUDING DISCUSSION

In MC-CDMA, cancellation of the ICI caused by Doppler spreads is (at least in theory) possible by appropriately weighting subcarriers. However, the weight factors depend on the instantaneous channel amplitudes and the Doppler shifts of relevant reflections. An MMSE receiver presumably is too computationally intensive as it involves an adaptive matrix inversion algorithm. A simplified algorithm is proposed, which involves approximately the same complexity as a conventional MC-CDMA receiver. Meanwhile an improvement of the raw BER is seen for MC-CDMA, relative to OFDM.

MC-CDMA can intuitively be compared with OFDM, in the sense that both MC-CDMA and OFDM use subcarriers and exploit frequency diversity to improve the reliability of reception of individual user bits. While C-OFDM uses error correction coding (with intentionally added redundancy and soft decision information regarding subcarrier amplitudes) to achieve this diversity, MC-CDMA uses orthogonal spreading code sequences (which map N user symbols into N subcarriers, based on a weighted addition). C-OFDM can exploit side information about the subcarrier amplitude, so the error correction can be more effective than for MC-CDMA.

We believe that the results presented here contribute to a better under standing of the merits of OFDM and MC-CDMA systems. From current standardization it appears that OFDM is not only attracting attention for broadcast (DAB and DVB) and wireless LAN (HIPERLAN II) applications, but also for future generation mobile telephone, wireless multimedia and wireless in-home entertainment systems.

6. REFERENCES

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