Controlled Equalization of Multi-Carrier CDMA in an Indoor Rician Fading Channel

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Abstract

In this paper, a novel digital modulation technique called Multi-Carrier Code Division Multiple Access (MC-CDMA) is analyzed. With MC-CDMA, each data symbol is transmitted at multiple subcarriers with each subcarrier modulated by "1" or "-1" based on a spreading code. Analytical results are presented on the performance of this modulation/multiple access scheme in the downlink of an indoor wireless Rician fading channel. The performance of a controlled equalization technique that attempts to restore the orthogonality between users is evaluated.

1. Introduction

This paper will extend on the results of a previous paper [1] involving the analysis of a new spread spectrum transmission method called MC-CDMA. MC-CDMA [1,2,3] addresses the issue of how to spread the signal bandwidth without increasing the adverse effects of the delay spread, which is a measure of the length of the channel impulse response. With MC-CDMA, a data symbol is transmitted over N narrowband subcarriers where each subcarrier is modulated by "1" or "-1" based on a spreading code. Different users transmit over the same set of subcarriers but with a spreading code that is orthogonal to the codes of other users. If the number of and spacing between subcarriers is appropriately chosen, it is unlikely that all of the subcarriers will be located in a deep fade and consequently frequency diversity is achieved. As an MC-CDMA signal is composed of N narrowband subcarrier signals each with a symbol duration much larger than the delay spread, an MC-CDMA signal will not experience significant degradation from inter-chip interference and inter-symbol interference (ISI) [4,5].

In a previous paper [1], MC-CDMA was analyzed in a Rayleigh fading channel. In contrast to the uplink, numerical results indicated that MC-CDMA performed better in the downlink where the ease of phase correction of the interference allows for the partial restoration of orthogonality between users. It was also noted that while Maximum Ratio Combining (MRC) performed better in a noise-limited channel, Equal Gain Combining (EGC) performed better in an interference-limited channel.

In this paper, the performance of MC-CDMA in the downlink of a Rician fading channel will be analyzed. The performance of a controlled equalization scheme applied to this modulation technique will be compared with the performance of EGC and MRC.

2. Basic Principles

The input data symbols, $a_m[k]$, are assumed to be binary antipodal where k denotes the kth bit interval and m denotes the mth user. In the analysis, it is assumed that $a_m[k]$ takes on values of -1 and 1 with equal probability. The generation of an MC-CDMA signal can be described as follows. As shown in Fig. 1, a single data symbol is replicated into N parallel copies. Each branch of the parallel stream is multiplied by a chip from a spreading code of length N. Each copy is then binary phase-shift keying (BPSK) modulated to a subcarrier spaced apart from its neighboring subcarriers by F/T_b Hz where F is an integer number. An MC-CDMA signal consists of the sum of the outputs of these branches. This process yields a multi-carrier signal with the subcarriers containing the orthogonally coded data symbol.

As illustrated in Fig. 1, the transmitted signal corresponding to the kth data bit of the mth user is

$$s_m(t) = \sum_{i=0}^{N-1} c_m[i]\, a_m[k] \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t\right) \times \times p_{T_b}(t - kT_b) \qquad c_m[i] \in \{-1, 1\} \tag{1}$$

where $c_m[0]$, $c_m[1]$, ... , $c_m[N-1]$ represents the spreading code of the mth user and $p_{T_b}(t)$ is defined to be an unit amplitude pulse that is non-zero in the interval of $[0, T_b]$.

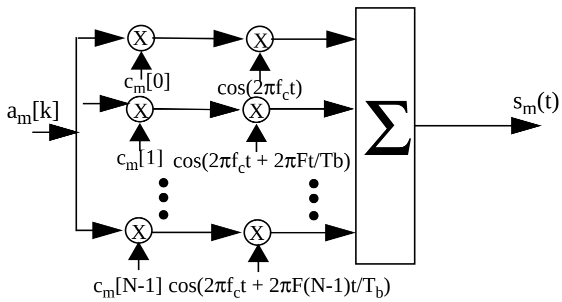

Fig. 1 Transmitter Model

3. Channel Model: Dispersive Rician Fading

In this paper, we will focus on a frequency-selective channel with $1/T_b << BW_c << F/T_b$. This model implies that each modulated subcarrier with transmission bandwidth of $1/T_b$ does not experience significant dispersion ($T_b >> T_d$). It is also assumed that the amplitude and phase remain constant over a symbol duration, T_b, (i.e., Doppler shifts due to the motion of terminals is negligible). This assumption agrees with indoor measurements of Doppler shifts, which tend to be very small and typically in the range of 0.3-6.1 Hz [6].

For downlink transmissions, i.e., from the base station to the terminals, a terminal receives interfering signals designated for other users ($m = 1, 2, ... , M-1$) through the same channel as the wanted signal ($m = 0$). Thus, the transfer function of the continuous-time fading channel for all transmissions from the base station to user $m = 0$ can be represented as

$$H\left(f_c + i\frac{F}{T_b}\right) = \rho_i e^{j\theta_i} \tag{2}$$

where the random amplitude, ρ_i, and phase, θ_i, of the channel at frequency $f_c+i(F/T_b)$ are independent of m. The phase shifts, θ_i for $i = 0, 1, ... N-1$, introduced by the channel are assumed to be independent and identically distributed (iid) random variables uniform on the interval of $[-\pi, \pi]$ for all subcarriers.

As there is often a line-of-sight (LOS) component in an indoor environment, the channel amplitudes, ρ_i for $i = 0, 1, ..., N-1$, are assumed to have the following Rician distribution

$$f_{\rho_i}(\rho_i) = \frac{\rho_i}{\sigma_i^2} e^{-\frac{\rho_i^2 + b_0^2}{2\sigma_i^2}} I_0\left(\frac{b_0\rho_i}{\sigma_i^2}\right) \tag{3}$$

where σ_i^2 represents the power of the scattered component, b_0 is the LOS component and $I_0(\rho)$ is the zeroth order modified bessel function. As the notation suggests, the dominant LOS component b_0 is assumed to be equal for all subcarriers. The Rician distribution is often characterized by the K-factor which is defined as the ratio of the power of the LOS component to the power of the scattered component

$$K = \frac{\frac{1}{2}b_0^2}{\sigma_i^2}. \tag{4}$$

4. Receiver Model

For M active transmitters, the received signal is

$$\begin{aligned} r(t) &= \sum_{m=0}^{M-1}\sum_{i=0}^{N-1} \rho_i c_m[i]\, a_m[k] \\ &\times \cos\left(2\pi f_c t + 2\pi i\frac{F}{T_b}t + \theta_i\right) + n(t) \end{aligned} \tag{5}$$

where $n(t)$ is additive white Gaussian noise (AWGN). The local-mean power at the ith subcarrier is defined to be

$$\begin{aligned} \bar{p}_i &= E\left[\rho_i \cos\left(2\pi f_c t + 2\pi\frac{i}{T_b}t + \theta_i\right)\right]^2 \\ &= \frac{1}{2}E\rho_i^2 \end{aligned} \tag{6}$$

where it is assumed that local-mean powers of the subcarriers are equal. Thus, the total local-mean power of the *mth* user is defined to be $\bar{p} = N\bar{p}_i$. To simplify the analysis, it is assumed that exact synchronization with the desired user ($m = 0$) is possible. As shown in Fig. 2, the first step in

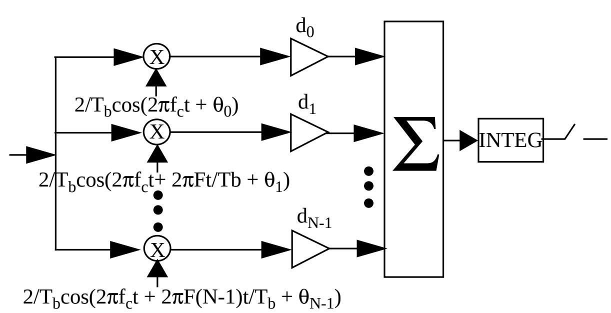

Fig. 2 Receiver Model

obtaining the decision variable involves demodulating each of subcarriers of the received signal, which includes applying a phase correction, $\hat{\theta}_i$, and multiplying the *ith* subcarrier signal by a gain correction, d_i. In the analysis, it is assumed that perfect phase correction can be obtained, i.e., $\hat{\theta}_i = \theta_i$. After adding the subcarrier signals together, the combined signal is then integrated and sampled to yield the decision variable, ν_0. For the *kth* bit, the decision variable is

$$\upsilon_o = \sum_{m=0}^{M-1}\sum_{i=0}^{N-1} \rho_i c_m[i]\, d_i a_m[k]\,\frac{2}{T_b} \times \int_{kT_b}^{(k+1)T_b} \cos\left(2\pi f_c t + 2\pi F\frac{i}{T_b}t + \theta_i\right) \times \cos\left(2\pi f_c t + 2\pi F\frac{i}{T_b}t + \hat{\theta}_i\right) dt + \eta \tag{7}$$

where the corresponding AWGN term, η, is given as

$$\eta = \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} n(t)\,\frac{2}{T_b} d_i \times \cos\left(2\pi f_c t + 2\pi F\frac{i}{T_b}t + \hat{\theta}_i\right) dt \; . \tag{8}$$

In this paper, we will considered three frequency equalization techniques: Equal Gain Combining (EGC), Maximum Ratio Combining (MRC), and Controlled Equalization (CE). EGC and MRC are discussed in [1]. With EGC, the gain correction factor is

$$d_i = c_0[i] \, . \tag{9}$$

With MRC, the gain correction factor is

$$d_i = \rho_i c_0[i] \, . \tag{10}$$

4.1 Controlled Equalization (CE)

While EGC may be desirable for simplicity and MRC for combating noise, neither of these techniques significantly exploit the coding of the subcarriers. With Controlled Equalization, an attempt at restoring the orthogonality between users is made by normalizing the amplitudes of the subcarriers. As the orthogonality of the users is encoded in the phase of the subcarriers, this method is primarily beneficial in the downlink where phase distortion for all users may be corrected. For CE, the gain factor for the ith subcarrier is

$$d_i = c_o[i]\,\frac{1}{\rho_i} u(\rho_i - \rho_{thresh}) \tag{11}$$

where $u(\rho_i)$ is the unit step function. Thus, only subcarriers above a certain threshold will be equalized and retained. This constraint is added to prevent the amplification of subcarriers with small amplitudes that may be dominated by a noise component.

5. Performance Analysis

- **EGC**

Using EGC as the equalization technique results in the following decision variable

$$\nu_0 = a_0[k] \sum_{i=0}^{N-1} \rho_i + \sum_{m=1}^{M-1} a_m[k] \times \sum_{i=0}^{N-1} c_m[i]\, c_0[i]\, \rho_i + \eta \tag{12}$$

where the AWGN term, η, has a variance of NN_0/T_b. Because of the orthogonality of the codes, the interference term may be rewritten as

$$\beta_{int} = \sum_{m=1}^{M-1} a_m[k] \left(\sum_{j=1}^{N/2} \rho_{a_j} - \sum_{j=1}^{N/2} \rho_{b_j}\right) \tag{13}$$

where

$$c_m[a_j]\, c_0[a_j] = 1 \qquad c_m[b_j]\, c_0[b_j] = -1$$
$$\sum_{j=1}^{N/2} \{a_j\} \cup \sum_{j=1}^{N/2} \{b_j\} = \{0, 1, \ldots, N-1\} \tag{14}$$

Applying the Central Limit Theorem (CLT) individually to both inner sums, the interference term can be approximated by a zero-mean gaussian r.v. with a variance of

$$\sigma^2_{\beta_{int}} = 2(M-1)\,(1-\gamma)\,\bar{p} \tag{15}$$

where

$$\gamma = \frac{\pi}{4}\left(\frac{e^{-K}}{K+1}\right)\left[(1+K)\, I_0\left(\frac{K}{2}\right) + K \times I_1\left(\frac{K}{2}\right)\right]^2 \tag{16}$$

and $I_1(K)$ represents the first order modified Bessel function. The probability of making a decision error can be written as

$$Pr\left(error \mid \bar{p}, K, \{\rho_i\}_{i=0}^{N-1}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{\frac{1}{2}\left(\sum_{i=0}^{N-1}\rho_i\right)^2}{2(M-1)\,(1-\gamma)\,\bar{p} + \frac{NN_0}{T_b}}}\right). \tag{17}$$

Finding a closed form expression for the sum of *N* iid Rician r.v.'s has been historically a difficult problem. In [1], it was shown that using the CLT to approximate the sum of iid Rayleigh r.v.'s leads to an adequate approximation. Applying the CLT to the sum of iid Rician r.v.'s, the sum may also be approximated by a zero-mean gaussian distribution. Averaging Eq.(17) over this gaussian distribution results in the following average bit error rate (BER)

$$
Pr\,(error|\,\bar{p}, K) \cong
$$

$$
\cong \frac{1}{2} erfc\left(\sqrt{\frac{\gamma\bar{p}T_b}{2\frac{M}{N}\,[1-\gamma]\,\bar{p}T_b + N_0}}\right). \tag{18}
$$

- **MRC**

In a similar manner, the average BER for MRC can be determined to be

$$
Pr\,(error|\,\bar{p}, K) \cong
$$

$$
\cong \frac{1}{2} erfc\left(\sqrt{\frac{\bar{p}T_b}{\frac{M}{N}\left[\frac{4K+2}{(K+1)^2}\right]\bar{p}T_b + N_0}}\right). \tag{19}
$$

- **CE**

The distribution of the number of subcarriers above the threshold, n_0, is given by the following binomial distribution

$$
p_{n_0}(n_0) \;=\; \binom{N}{n_0}\{pr_{on}\}^{n_0}\{1-pr_{on}\}^{N-n_0} \tag{20}
$$

for $n_0 = 0, 1, 2, \ldots, N$ where

$$
pr_{on} = \int_{\rho_{thresh}}^{\infty} (K+1)\,e^{-K}\frac{\rho_{o,\,i}}{\overline{p_{0,\,i}}}e^{-\frac{K+1}{2\overline{p_{0,\,i}}}\rho_{0,\,i}^2} \times \; \times I_0\left(\rho_{0,\,i}\sqrt{\frac{2K(K+1)}{\overline{p_{0,\,i}}}}\right)d\rho_{0,\,i} \tag{21}
$$

Given that there are n_0 subcarriers above the threshold indexed by j, the decision variable for CE is

$$
\upsilon_o|\,n_0 \;=\; a_{0.}[k]\,n_0 + \beta_{int} + \eta \tag{22}
$$

where the interference term, β_{int}, is given as

$$
\beta_{int} \;=\; \sum_{m=1}^{M-1} a_m[k]\sum^{n_0} c_m[j]\,c_0[j]\,. \tag{23}
$$

The distribution of the inner sum,

$$
\Sigma_m \;=\; \sum^{n_0} c_m[j]\,c_0[j]\ \ , \tag{24}
$$

given n_0 is

$$
p_{\Sigma_m|\,n_0}(\Sigma_m|\,n_0) \;=\; \frac{\binom{N/2}{(n_0+\Sigma_m)/2}\binom{N/2}{(n_0-\Sigma_m)/2}}{\binom{N}{n_0}} \tag{25}
$$

where $-min\{n_0, N-n_0\} \le \Sigma_m \le min\{n_0, N-n_0\}$ and Σ_m can only assume even (odd) values if n_0 is even (odd). The exact distribution of the interference term, β_{int}, given n_0 depends on the specific spreading codes that are used. In this analysis, it is assumed that each of the inner sums acts as an independent r.v., and that the pdf of the interference is given as the convolution of the individual pdfs of Σ_m for $m = 1, 2, \ldots, M\text{-}1$.

The noise term is approximately a zero-mean gaussian r.v. with a variance of

$$
\sigma^2_{\eta|\,n_0} = n_0\frac{N_0}{T_b}\frac{1}{2\overline{p_{0,\,i}}}\frac{1}{pr_{on}}\int_{\rho_{thresh}}^{\infty}(K+1)\,e^{-K}\frac{1}{\rho_{o,\,i}\overline{p_{0,\,i}}} \times \; \times e^{-\frac{K+1}{2\overline{p_{0,\,i}}}\rho_{0,\,i}^2} I_0\left(\rho_{0,\,i}\sqrt{\frac{2K(K+1)}{\overline{p_{0,\,i}}}}\right)d\rho_{0,\,i} \tag{26}
$$

Given n_0 and the interference component, β_{int}, the probability of making a decision error is

$$
Pr\,(error|\,n_0, \beta_{int}) \cong \int_{n_0-\beta_{int}}^{\infty}\frac{1}{\sqrt{2\pi\sigma_\eta^2}}e^{-\frac{x^2}{2\sigma_\eta^2}}dx. \tag{27}
$$

Combining the results given above yield the following expression for the average bit error rate

$$
BER \cong \sum_{n_0=0}^{N} p_{n_0}(n_0)\sum_{\beta_{int}} p_{\beta_{int}|\,n_0}(\beta_{int}|\,n_0) \times \frac{1}{2}erfc\left(\frac{n_0-\beta_{int}}{\sqrt{2}\sigma_\eta}\right) \tag{28}
$$

6. Numerical Results

Plots of the average BER for Rician K-factors of 0 and 10 are shown in Fig. 3 for EGC and MRC. Note that for the Rayleigh fading case ($K = 0$) the expressions in Eqs. [18,19] reduce to the results derived in [1] for Rayleigh fading. As in the case of Rayleigh fading, MRC has

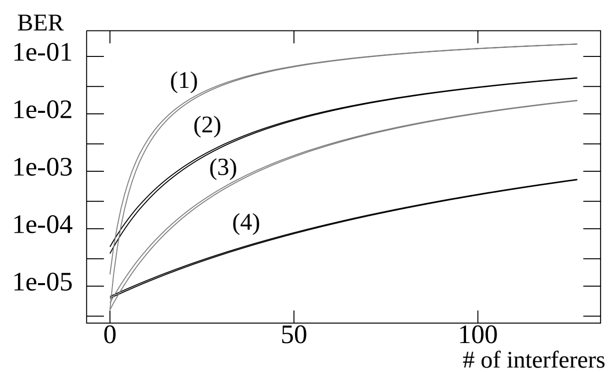

Fig. 3 BER vs. the # of interferers for different Rician K-factors using MRC: K=0 (1) and K=10 (3) and EGC: K=0 (2) and K=10 (4). Curves are shown for both CLT and LLN approximations. The SNR is 10 dB and N = 128.

a better performance for very low number of interferers while EGC outperforms MRC in an interference limited Rician fading channel. This result reflects the observation that MRC distorts the orthogonality further between users and consequently does not perform as well when a large number of interferers are present.

Plots of the BER for CE are shown in Fig. 4 for $K = 5$ and Fig. 5 for $K = 10$. From the curves, it can be seen CE outperforms EGC and MRC in combating interference.

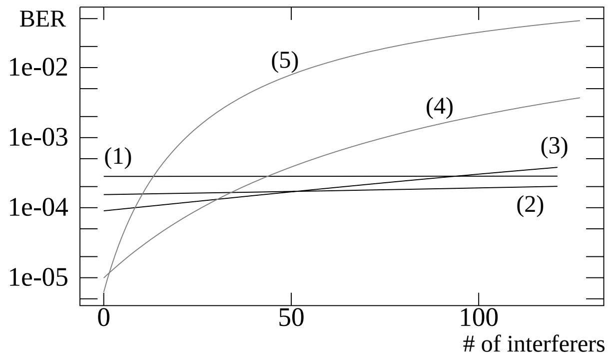

Fig. 4 BER for CE vs. the # of interferers with K=5 for ρ_{thresh}=0.002 (1), ρ_{thresh}=0.008 (2), and ρ_{thresh}=0.014 (3). The SNR is 10 dB, p=0.1, and N=128. Plots for EGC (4) and MRC (5) are included for comparison.

Note that there exists a ρ_{thresh} such that the BER vs. the number of interferers is relatively flat. At this threshold level, there are a sufficient number of subcarriers above the threshold such that orthogonality between users has been significantly restored. As the threshold level is lowered past this point, no benefit occurs since "orthogonality" has already been achieved and only noise amplification results. For higher threshold values, the BER is affected by the number of interferers to a greater extent. For all threshold values, the performance of CE is worse than EGC or MRC for a small number of interferers due to the amplification of noise.

7. Conclusion

In this paper, the performance of MC-CDMA in the downlink of an indoor Rician fading channel was evaluated. Numerical results revealed that the Rician K-factor has a significant effect on the BER. In addition, it was found that a controlled equalization technique that attempts to restore the orthogonality between users outperforms EGC and MRC in combating interference. While these equalization techniques may not be optimal, these detectors do offer advantages over some maximum-likelihood detectors with their relatively low computational complexity.

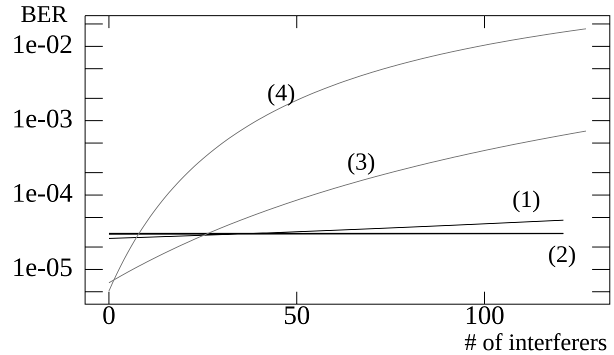

Fig. 5 BER for CE vs. the # of interferers with K=10 for ρ_{thresh}=0.008 (2) and ρ_{thresh}=0.016 (1). The SNR is 10 dB, p=0.1, and N=128. Plots for EGC (3) and MRC (4) are included for comparison.

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