

# Imperfect Sector Antenna Diversity in Slotted ALOHA Mobile Network

Jean-Paul M. G. Linnartz, *Member, IEEE*

**Abstract**— In a wireless network with path loss and fading channels, receiver capture is known to substantially enhance the performance of the slotted ALOHA random access protocol. The efficiency of narrowband slotted ALOHA radio networks can be enhanced further by using sector antennas, each receiving signals from a particular segment of the network area. This paper investigates the effect of realistic, i.e., partially overlapping antenna patterns and the resulting correlation of received power levels at different receiver branches. A method is derived for computing the joint throughput from two base station receivers with overlapping antennas patterns. The *a posteriori* information provided by the event of one message capturing one antenna is used to find conditional probabilities of capturing the other antennas as well. The paper shows that any overlap in the antenna patterns decreases the throughput, but transmissions from the overlapping area may face a larger probability of capture than signals from directions in which one antenna has maximum gain.

## I. INTRODUCTION

THE ALOHA protocol has been proposed as a possible method to provide random access for many spatially distributed terminals to a common receiving base station. In slotted ALOHA, the time axis is divided into time slots of a fixed duration. The basic idea is that a terminal with a packet ready for transmission can transmit in any time slot. Because of the absence of coordination among terminals, transmissions from terminals may harmfully interfere with each other, which is called a message “collision.” To ensure reliable communication, a terminal waits for an acknowledgment of successful reception from the base station and retransmits after a random time if it fails to receive an acknowledgment. It has been shown in several papers that the ALOHA protocol performs better in fading radio channels with capture than on wired channels where any collision destroys all packets involved, e.g., [1]–[5]. If two or more packets collide in a radio network, it is likely that the power of one signal sufficiently exceeds the total interference power from all other terminals. Such a packet may successfully “capture” the receiver, despite the other colliding messages.

The performance of the radio ALOHA network may be further enhanced by using multiple antennas, e.g., [6]–[10]. Two different approaches have been proposed: one may sectorize the network area using directional antennas, such that each receiver sees only a limited amount of radio traffic [6]–[8],

Paper approved by P. T. Mathiopoulos, the Editor for Wireless Personal Communications of the IEEE Communications Society. Manuscript received February 8, 1994; revised March 8, 1985. This paper was presented in part at GLOBECOM'94, San Francisco, CA, 1994.

The author is with Philips Research, Eindhoven, The Netherlands and the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA 94720 USA.

Publisher Item Identifier S 0090-6778(96)07375-8.

or one may adaptively combine the signals from multiple antennas in order to reduce the interference caused by colliding packets [9], [10]. The case of idealized sector antennas has been addressed by Lau and Leung [6]–[8]. Adaptive beam steering and interference nulling by optimally combining the outputs of multiple antennas was assumed by Ward and Compton [9]–[10]. Such signal processing optimizes receiver capture and message success probabilities, so it also enhances the throughput-delay performance of the network. The capture model in [9] and [10] assumed that an adaptive antenna array can resolve packets arriving with at least a certain (threshold) offset in time and angle of arrival. It was further assumed that receiver fails to null interference if the number of interfering messages exceeds the number of degrees of freedom in the adaptive filters.

In a typical mobile radio environment, the distribution of the terminals over the area also plays a significant role in the performance. This paper addresses the aspects of propagation path loss and multipath fading. As our receiver does not perform adaptive filtering, it does not need channel and interference estimation. So, packets do not contain a training sequence, but only a short synchronization preamble. This is particularly relevant to our application, which is in collecting very short telemetric data messages.

Earlier, Abramson [1] and Sousa [4] addressed a network with a very large population of users and messages arriving from random locations. The message generation process was modeled as a spatially uniform Poisson process. Abramson assumed perfect reception of a packet from distance  $r$  if no other interfering packet transmission occurred within a “vulnerability” circle of radius  $cr$ , with  $c$  a system constant. Cumulation of multiple interfering signals was modeled in [2] and [4]. Here, we address the same spatial packet generation process as in [1] and [4]. However, we do not limit ourselves to message arrivals at one particular receiver, but we investigate the related and highly correlated signal powers at two different receivers for imperfect sector antennas with considerably overlapping gain patterns.

Differences in received signal power lead to receiver capture and allow successful packet reception despite interference from other users. Hence, even without adaptive antennas, the throughput of the wireless slotted ALOHA networks can be significantly above the performance known for wired networks. This paper addresses an infinitely large area, in which each base station receives messages from its vicinity. The total traffic offered in the entire area is unbounded. Unlike in ALOHA systems without capture, the system can have nonzero throughput because of receiver capture and a sufficiently steep path loss law.

The application which motivated our analysis was in advanced traffic management and information systems (ATM/IS), in particular the collection of telemetric data from probe vehicles [13]. Probe vehicles are private or commercial vehicles that are equipped with specific positioning and communications equipment that participate in road traffic. These vehicles randomly transmit short reports of a few hundred bits, containing their experienced road travel times. Messages typically include a set of consecutive locations, the travel times between these locations, and a vehicle identification code. For privacy reasons, this identification code (or the message itself) may be encrypted by an asymmetric algorithm that recognizes the probe as a legitimate user, without being able to establish the identity of the vehicle. The function of transmitting probe messages may be incorporated in the navigation and traveller information system onboard future "intelligent" vehicles.

Typically, multiple base stations can be located in the area of interest. An access scheme that allows two-way communication for several coexisting services in ATM/IS has been proposed and described in [19]. In this scheme, only a fraction of the time slots are allocated to probe transmissions, but these slots are the same in the entire system area. Other services use dynamic frequency reuse algorithms optimized for the quality-of-service (QoS) required for that service. Earlier investigations [5] concluded that in a wide-area system with Poisson traffic, covered by many base stations and many "cells," optimum spectrum efficiency can be achieved if cells use the same inbound ALOHA frequency. This also allows extensive use of receiver diversity. All base stations forward received probe messages via a fixed communications infrastructure to a traffic control center (TCC). If a message is received at more than one base station simultaneously, the TCC discards all but one copy of the message. In our application, it is not necessary to retransmit messages lost because of interference. This is in contrast to interactive services where messages lost in interference or fades are retransmitted. Such retransmissions lead to considerably higher traffic loads from remote areas with relatively small capture probabilities [5]. Without retransmissions, and if the area covered by each receiver is large compared to the grid spacing of the road network, a uniform distribution of the offered packet traffic appears a reasonable assumption.

As contiguous frequency reuse avoids the need for handovers to other cell areas, ALOHA random access has advantages over polling vehicles to gather road traffic data.

Interestingly, the ALOHA concept has recently also been proposed for voice and multimedia traffic, particularly if multiple base stations can be involved in the processing of signals from a particular terminal [14], [15]. The analysis discussed here also applies to such systems.

## II. CAPTURE PROBABILITY

We consider one particular signal, called the "test signal", which is denoted with index 0, attempting to reach either of two receivers, denoted  $A$  and  $B$ . The probability of successful reception depends not only on the instantaneous signal-to-interference (C/I) ratio, but also on the type of modulation and coding and the character of the interference [3]. In narrowband radio, the probability of successful reception typically makes

a rapid transition from close to zero to close to one for C/I ratios changing a few dB. Because of UHF path loss, antenna gains, shadowing and fading in the absence of power control, the dynamic range of the received signals is many tens of dBs. If the probability that the C/I ratio is in the transition range is relatively small, it is reasonable to approximate the probability of successful reception by the probability that the C/I ratio is above a certain threshold  $z$ . From [5] and [16], we conclude that for BPSK transmission  $z$  typically is close to four.

In Section III, we will briefly address the effect of direct sequence code-division multiple-access (DS-CDMA).

The event that the C/I-ratio of a test signal from user 0 at receiver  $A$  is above the receiver threshold  $z$  is denoted as  $A_0$ . We assume constant received power during a packet transmission time. The probability of successful reception is

$$P(A_0) = P(p_{A,0} > zp_{A,t}) \\ = \int [1 - F_{p_{A,0}}(zp_{A,t})] dF_{p_{A,t}} \quad (1)$$

where  $p_{A,t}$  is the joint interference power at receiver  $A$  and  $p_{A,0}$  is the instantaneous received power of the wanted signal (with index 0). Here  $F_p$  denotes the distribution function of random variable (rv)  $p$ . In a Rayleigh-fading channel, the instantaneous power of the wanted signal is exponentially distributed. Hence, assuming the local-mean power  $\bar{p}_{A,0}$  to be known, one finds

$$P(p_{A,0} > zp_{A,t} | \bar{p}_{A,0}) = \int e^{-(zp_{A,t}/\bar{p}_{A,0})} dF_{p_{A,t}} \\ \triangleq x\left\{p_{A,t}, \frac{z}{\bar{p}_{A,0}}\right\} \quad (2)$$

where  $x\{p_{A,t}, s\}$  denotes the characteristic function (cf) of the joint interference power  $p_{A,t}$  at the point  $js$  with  $j = \sqrt{-1}$ . It can also be written as the expectation value  $\{p_{A,t}, s\} = E[\exp\{-sp_{A,t}\}]$ . This also implies that  $x\{p_{A,t}, s\}$  is the Laplace transform of the probability density function (pdf) of  $p_{A,t}$  [18]. For incoherent cumulation [2] of signals, thus, if the power of the joint signal equals the sum of the powers from individual interferers, and if signals powers are independent, the pdf of the joint interference power is the  $N$ -fold convolution of the pdf of the individual powers. Transformation results in the multiplication of  $N$  factors, each containing a characteristic function of the received power from an individual interfering component. So

$$P(p_{A,0} > zp_{A,t} | \bar{p}_{A,0}) \\ = x\left\{p_{A,\text{noise}}, \frac{z}{\bar{p}_{A,0}}\right\} \prod_{j=1}^N x\left\{p_{A,i}, \frac{z}{\bar{p}_{A,0}}\right\}. \quad (3)$$

Here,  $N$  denotes the total number of terminals in the system and  $p_{A,\text{noise}}$  denotes the received noise power [5]. In the remainder of the text, we will use  $P_{NA}$  to denote the probability that the test signal is received successfully at  $A$  in an interference-free but noise-limited channel, i.e.,  $P_{NA} = x\{p_{A,\text{noise}}, z/\bar{p}_{A,0}\}$ .

Denoting the probability of the interferer  $i$  being active as  $P(i_{\text{ON}})$ , one finds

$$x\{p_{A,i}, s\} = 1 - P(i_{\text{ON}})[1 - E[e^{-sp_{A,i}} | i_{\text{ON}}]]. \quad (4)$$

It appears useful to denote the conditional probability of unsuccessful reception (not  $A_0$ ) given that (only) interfering terminal  $i$  was active as  $W_{A,i}(0 \leq W_{A,i} \leq 1)$ . We call this factor the "vulnerability weight factor" [5]. Thus, for a finite population of  $N$  terminals, the probability that a "test" packet with known local-mean power captures receiver  $A$  becomes

$$P(A_0|\bar{p}_{A,0}) = P_{NA} \prod_{i=1}^N [1 - W_{A,i}P(i_{ON})]. \quad (5)$$

It can easily be shown that, for a Rayleigh-fading channel without shadowing, thus with known local-mean powers of the wanted signal (0) and interfering signal ( $i$ ), the weight factor becomes [5]

$$W_{A,i} \triangleq 1 - E \left[ \exp \left\{ -\frac{z\bar{p}_{A,i}}{\bar{p}_{A,0}} \right\} \right] = \frac{z\bar{p}_{A,i}}{z\bar{p}_{A,i} + \bar{p}_{A,0}}. \quad (6)$$

The local-mean power  $\bar{p}_{A,i}$  of signal  $i$  at receiver  $A$  is assumed to be

$$\bar{p}_{A,i} = H_A(\phi_i)r_i^{-\beta} \quad (7)$$

where  $H_A(\phi)$  is the antenna gain pattern of receiver  $A$  in direction  $\phi$  and  $r_i$  denotes the distance between mobile transmitter  $i$  and the receiver site.  $\beta$  is the path loss exponent. The locations  $(r_i, \phi)$  of interfering transmitters are regarded as independent and identically distributed (i.i.d.) rv's with pdf  $f(r, \phi)$  and distribution  $F(r, \phi)$ .

### III. THROUGHPUT AT ONE RECEIVER

The capture probability averaged over all possible locations of the  $n$  interferers known to be active, can be expressed if one recognizes that the resulting  $n$ -dimensional integral over the product (5) can be written as a product of integrals. This is allowed because each factor only contains one random interference variable. Thus, for  $n$  terminals known to be active

$$P(A_0|n, r_0, \phi_0) = P_{NA} \left[ \int (1 - W_{A,i}) dF(r_i, \phi_i) \right]^n. \quad (8)$$

In order to study a Poisson field of interferers, we take the limit for  $N \rightarrow \infty$  under the condition that the total offered traffic  $NP(i_{ON}) = G_t$  is a constant. The number of active interferers is a Poisson rv with mean  $G_t$ . Initially we limit the spatial distribution of transmitters to distances  $R_1 < r_i < R_2$  to ensure that  $G_t$  and the moments of the received signal powers are finite. The capture probability becomes

$$P(A_0|r_0, \phi_0) = \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} P(A_0|n, r_0, \phi_0). \quad (9)$$

Following [1], we introduce the spatial distribution of offered traffic  $G(r, \phi)$ , at distance  $r$  from the base station in direction  $\phi$ , where  $G(r, \phi)$  is expressed in packets per slot per unit area (ppspa). So  $f(r, \phi) = 2rG(r, \phi)/G_t$  [2]. One finds

$$\begin{aligned} & A_0|r_0, \phi_0 \\ &= P_{NA} \exp \left\{ - \int_0^{2\pi} \int_{R_1}^{R_2} W_{A,i} G(r, \phi) r dr d\phi \right\} \end{aligned}$$

$$\begin{aligned} &= P_{NA} \exp \left\{ - \int_0^{2\pi} \int_{R_1}^{R_2} \frac{zH_A(\phi)r_0^\beta}{zH_A(\phi)r_0^\beta + H_A(\phi_0)r^\beta} \right. \\ & \quad \left. \cdot G(r, \phi) r dr d\phi \right\}. \quad (10) \end{aligned}$$

For a uniform offered packet traffic with  $G(r, \phi) \equiv G_0$  for all  $r$  and  $\phi(R_1 \rightarrow 0$  and  $R_2 \rightarrow \infty)$  we find

$$\begin{aligned} P(A_0|r_0, \phi_0) &= P_{NA} \exp \left\{ - \frac{\pi G_0 r_0^2}{B \sin \frac{2\pi}{\beta}} \left( \frac{z}{H_A(\phi_0)} \right)^{2/\beta} \right. \\ & \quad \left. \cdot \int_0^{2\pi} H_A^{2/\beta}(\phi) d\phi \right\}. \quad (11) \end{aligned}$$

In a noise-free network ( $P_{NA} = 1$ ), the throughput of this receiver is found after some straightforward algebraic operations, as

$$\begin{aligned} S_t &= \int_0^{2\pi} \int_0^\infty P(A_0|r_0, \phi_0) G_0 r_0 dr_0 d\phi_0 \\ &= \frac{\beta}{2\pi z^{2/\beta}} \sin \frac{2\pi}{\beta} \quad (12) \end{aligned}$$

provided that the propagation attenuation is faster than in free-space ( $\beta > 2$ ). In the event of a single receiver, the throughput with  $\beta = 4$  is  $2/(\pi\sqrt{z})$ , or approximately 0.32 for  $z = 4$ . We conclude that this throughput depends neither on its antenna pattern nor on the offered traffic density. As the offered traffic per unit of area has an arbitrary but constant density per unit of area ( $0 < G_0 < \infty$ ), the total traffic is unbounded ( $G_t \rightarrow \infty$  since we let  $R_2 \rightarrow \infty$ ). For a system without capture ( $z \rightarrow \infty$ ), the throughput would thus be zero.

If DS-CDMA is used, the above expression changes in two ways: the threshold improves, roughly inversely proportional to the spreading gain  $M$ . On the other hand, if one assumes a fixed transmission bandwidth independent of  $M$ , the message duration increases proportionally to  $M$  [17]. Combining these two effects, the normalized throughput per unit of time, becomes roughly

$$S_t \approx \frac{\beta}{2\pi M^{1-2/\beta} z_0^{2/\beta}} \sin \frac{2\pi}{\beta} \quad (13)$$

where  $z_0$  is a threshold that is appropriate if  $M = 1$ , with  $z \approx z_0/M$ . Thus bandspreading ( $M > 1$ ) may not be beneficial, unless more sophisticated interference cancellation is used, that allows the capture performance to improve more rapidly than inversely proportional to  $M$  [17]. It should however be acknowledged that the assumption of a threshold may not accurately model the receiver performance in DS-CDMA networks.

### IV. A POSTERIORI PROBABILITY OF INTERFERENCE

In order to evaluate the joint throughput at two receivers, it appears useful to return to probabilities conditional on known locations of terminals. The probability  $P(k_{ON}|A_0)$  that a terminal  $k$  has transmitted an interfering packet, given that the

wanted packet captures base station  $A$ , is found from Bayes rule

$$P(k_{\text{ON}}|A_0, \{\text{all } r, \phi\}) = \frac{P(A_0|k_{\text{ON}}, \{\text{all } r, \phi\})}{P(A_0|\{\text{all } r, \phi\})} P(k_{\text{ON}}) \quad (14)$$

where  $P(A_0|k_{\text{ON}})$  is the probability that a wanted segment is received successfully, given that interferer  $k$  was active, but unconditional on the activity of any other terminals. Taking account of the noise floor, and possible activity of  $N - 1$  other interferers, each transmitting with probability  $P(i_{\text{ON}})$ , we insert

$$P(A_0|k_{\text{ON}}, \{\text{all } r, \phi\}) = P_{NA} \{1 - W_{A,k}\} \prod_{i=1, i \neq k}^N (1 - W_{A,i} P(i_{\text{ON}})). \quad (15)$$

Many factors in the numerator and denominator of (14) cancel. So, the *a posteriori* probability that interferer  $k$  was active given that our test signal captured receiver  $A$  is equal to

$$P(k_{\text{ON}}|A_0, \{\text{all } r, \phi\}) = \frac{1 - W_{A,k}}{1 - W_{A,k} P(k_{\text{ON}})} P(k_{\text{ON}}) \quad (16)$$

independent of whether other interferers were active.

## V. ANTENNA DIVERSITY

We assume that the antennas of the two receivers  $A$  and  $B$  are separated far enough to ensure independent Rayleigh fading, while the path losses to  $A$  and  $B$  are identical. The conditional probability that the test packet captures receiver  $B$  given that it also captures receiver  $A$  is found as

$$\begin{aligned} P(B_0|A_0, \{\text{all } r, \phi\}) &= P_{NB} \prod_{k=1}^N (1 - W_{B,k} P(k_{\text{ON}}|A_0, \{\text{all } r, \phi\})) \\ &= P_{NB} \prod_{k=1}^N \left(1 - \frac{1 - W_{A,k}}{1 - W_{A,k} P(k_{\text{ON}})} W_{B,k} P(k_{\text{ON}})\right) \end{aligned} \quad (17)$$

where  $P_{NB}$  is the probability that the received signal fails to exceed the noise floor at receiver  $B$ . The probability that a test packet captures at least one of the two base stations (event  $A_0 \cup B_0$ ), given the position of all terminals  $i$  ( $i = 0, 1, 2, \dots, N$ ), equals

$$\begin{aligned} P(B_0|\{\text{all } r, \phi\}) &= P(A_0|\{\text{all } r, \phi\}) \\ &\quad + P(B_0|\{\text{all } r, \phi\}) - P(A_0|\{\text{all } r, \phi\}) \\ &\quad \cdot P(B_0|A_0, \{\text{all } r, \phi\}) \end{aligned} \quad (18)$$

Inserting (5) and (17) in (18) gives, after some algebraic simplifications

$$P(A_j \cap B_j|\{\text{all } r, \phi\}) = P_{NA} P_{NB} \prod_{i=1}^N (1 - W_{A,B,i} P(i_{\text{ON}})) \quad (19)$$

TABLE I  
COMMON PACKET THROUGHPUT (IN PACKETS PER SLOT) FOR TWO RECEIVERS WITH IMPERFECT SECTOR ANTENNAS,  $z = 4$  (6 dB),  $\beta = 4$ ,  $S_A = S_B = 1/\pi$

Rician factor	$K$	$S_{A \cap B}$
$-\infty$ dB	0	0
-30 dB	0.001	0.019
-20 dB	0.01	0.055
-10 dB	0.1	0.136
-0 dB	1	0.212

where the joint weight function  $W_{A,B,i}$  ( $0 \leq W_{A,B,i} \leq 1$ ) is defined as

$$W_{A,B,i} \triangleq W_{A,i} + W_{B,i} - W_{A,i} W_{B,i}. \quad (20)$$

It can be interpreted as a factor weighing the disturbance caused by a interfering packet  $i$  from position  $(r_i, \phi_i)$  to the reception of a data packet from the "test" terminal at the two receivers  $A$  and  $B$  simultaneously. For the interfering traffic being a spatial Poisson process, (18) becomes

$$\begin{aligned} P(A_0 \cup B_0|\{\text{all } r, \phi\}) &= \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} P(A_0 \cup B_0|n, \{\text{all } r, \phi\}). \end{aligned} \quad (21)$$

In terms of  $G(r, \phi)$ , this can be written in the elegantly structured expression

$$\begin{aligned} P(A_0 \cup B_0|r_0, \phi_0) &= P_{NA} \exp \left\{ - \int_0^{2\pi} \int_{R_1}^{R_2} W_{A,i} G(r, \phi) r dr d\phi \right\} \\ &\quad + P_{NB} \exp \left\{ - \int_0^{2\pi} \int_{R_1}^{R_2} W_{B,i} G(r, \phi) r dr d\phi \right\} \\ &\quad - P_{NA} P_{NB} \exp \left\{ - \int_0^{2\pi} \int_{R_1}^{R_2} W_{A,B,i} G(r, \phi) r dr d\phi \right\}. \end{aligned} \quad (22)$$

This expression describes the probability of capturing at least one receiver, given an arbitrary spatial distribution  $G(r, \phi)$  of the offered traffic. The expression contains three terms: the first two terms express the capture probability for the individual receivers  $A$  and  $B$ , respectively; the third term corresponds to successful reception at two receivers simultaneously. For  $R_1 \rightarrow 0$  and  $R_2 \rightarrow \infty$ , the argument of the third term has the form

$$\arg = \int_0^{2\pi} \int_0^{\infty} \left[ W_{A,i} + W_{B,i} - \frac{p}{(p+r^\beta)} \frac{q}{(q+r^\beta)} \right] \cdot G(r, \phi) r dr d\phi \quad (23)$$

where

$$p = z \sqrt{\frac{H_A(\phi)}{H_A(\phi_0)}} r_0^\beta \quad \text{and} \quad q = z \sqrt{\frac{H_B(\phi)}{H_B(\phi_0)}} r_0^\beta. \quad (24)$$

Solving the integral over  $r$  for  $\beta = 4$  and  $G(r, \phi) = G_0$ , gives

$$\begin{aligned} P(A_0 \cap B_0|r_0, \phi_0) &= \exp \left\{ - \frac{\pi}{4} z G_0 r_0^2 \int_0^{2\pi} \sqrt{\frac{H_A(\phi)}{H_A(\phi_0)}} + \sqrt{\frac{H_B(\phi)}{H_B(\phi_0)}} \right. \end{aligned}$$

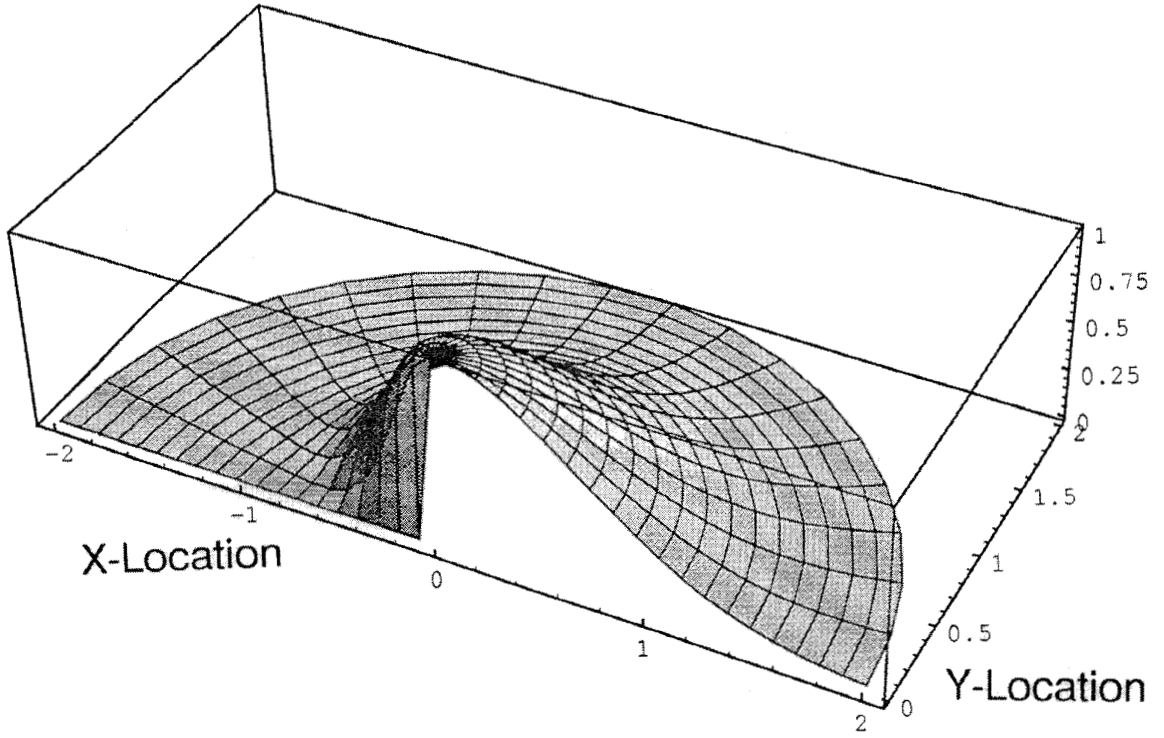


Fig. 1. Probability of capturing a receive system with cardiac antenna, versus transmitter location. UHF groundwave propagation ( $\beta = 4$ ), Receiver threshold of  $z = 4$  (6 dB). Offered traffic  $G_0 = 1$  packet per slot per unit of area (ppspa).

$$\left. - \frac{\sqrt{H_A(\phi)H_B(\phi)}}{\sqrt{H_A(\phi)H_B(\phi_0)} + \sqrt{H_B(\phi)H_A(\phi_0)}} d\phi \right\} \quad (25)$$

The total throughput is obtained by integrating the probability of capture over the cell area. We use

$$S_{A \cup B} = S_A + S_B - S_{A \cap B} \quad (26)$$

with the common throughput

$$\begin{aligned} S_{A \cap B} &= \int_0^{2\pi} \int_0^\infty P(A_0 \cap B_0 | r_0, \phi_0) G_0 r_0 dr_0 d\phi_0 \\ &= \frac{2}{\pi\sqrt{z}} \int_0^{2\pi} \left\{ \int_0^{2\pi} \sqrt{\frac{H_A(\phi)}{H_A(\phi_0)} + \frac{H_B(\phi)}{H_B(\phi_0)}} \right. \\ &\quad \left. - \frac{\sqrt{H_A(\phi)H_B(\phi)}}{\sqrt{H_A(\phi)H_B(\phi_0)} + \sqrt{H_B(\phi)H_A(\phi_0)}} d\phi \right\}^{-1} \\ &\quad \cdot d\phi_0. \end{aligned} \quad (27)$$

## VI. COMPUTATIONAL RESULTS

Next we consider four different antenna patterns; one offering perfect sectorization though with multipath scattering causing crosstalk from other sectors, and three realistic antennas.

### A. Sector Antennas

In the case of two perfect sector antennas, no overlapping traffic occurs, thus  $H_A(\phi_i)$  is a nonzero constant, say  $H_A = 1$ ,

for  $0 < \phi < \pi$  and zero otherwise, and  $H_B(\phi_i) = H_A(\phi_i\pi)$ . The probability of capturing at least one base station reduces to

$$P(A_0 \cup B_0 | r_0, \phi_0) = \exp \left\{ -\frac{\pi}{2} \sqrt{z} r_0^2 G_0 \right\} \quad (28)$$

for a noise-free Rayleigh-fading channel with  $\beta = 4$ . However, if this antenna is mounted at a relatively low position with limited "clearance", Rician scattered waves will erode the perfect sectorization. The ratio of the gain into the other sector over the gain in the main lobe is denoted as  $K$ , so  $H_A(\phi_i)$  is taken unity for  $0 < \phi < \pi$  and  $K$  otherwise. In practice, scattering will not result in such an idealized antenna pattern, but wave interference will cause random gain fluctuations, possibly with multipath nulls in certain directions. In our analysis, these multipath fluctuations are not included in the antenna pattern, but are accounted for in the Rayleigh or Rician pdf of the received signal amplitudes. It can be shown that Rician multipath scattering near the base station and Rayleigh scattering near the mobile terminal result in a Rayleigh pdf of the received envelope [11] which is already accounted for in (1)–(6). Including a wave interference pattern in the antenna diagram would thus be inappropriate.

Inserting the sector gain in expression (27) gives

$$S_{A \cap B} = \frac{4}{\pi\sqrt{z}} K^2 + \frac{\sqrt{K}(K+1)}{\frac{3}{2} K \sqrt{K} + K + \frac{3}{2} \sqrt{K} + 1}. \quad (29)$$

The joint throughput for several  $K$  factors is given in Table I.

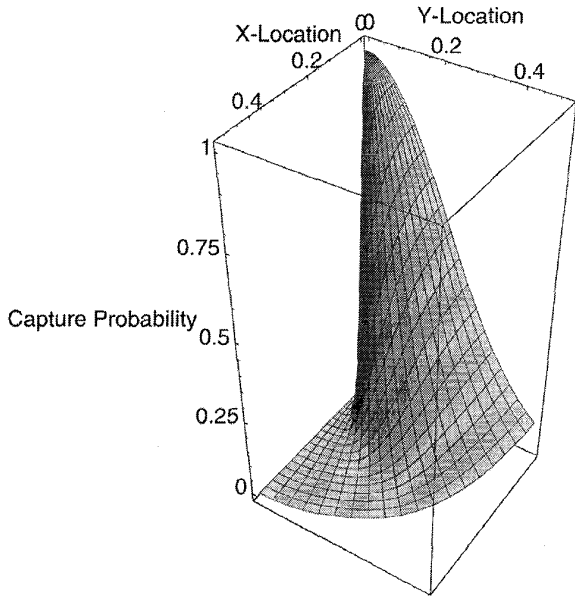


Fig. 2. Probability of capturing a receive system with a vertical array of half wave dipoles, versus transmitter location. Dipoles spacing  $\lambda/2$ . UHF groundwave propagation ( $\beta = 4$ ), Receiver threshold of  $z = 4$  (6 dB). Offered traffic  $G_0 = 1$  ppspa.

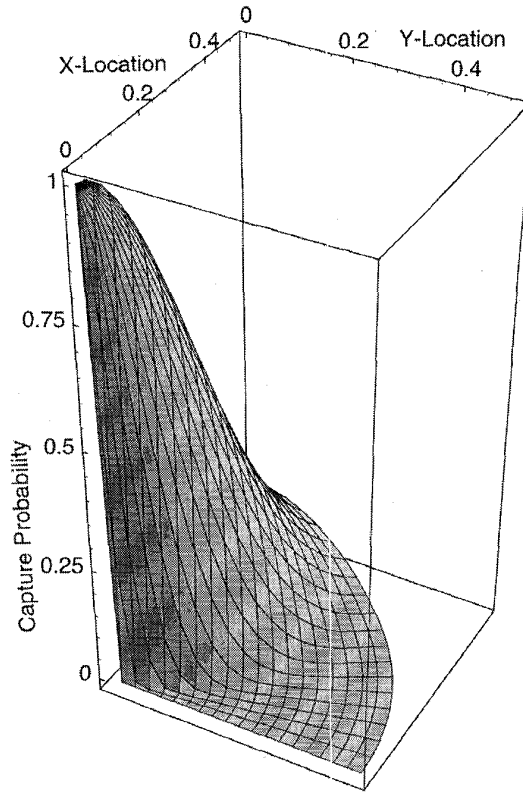


Fig. 3. Probability of capturing both receivers a two-branch diversity receive system versus transmitter location. Half wave dipoles spaced  $\lambda/2$ . UHF groundwave propagation ( $\beta = 4$ ), Receiver threshold of  $z = 4$  (6 dB). Offered traffic  $G_0 = 1$  ppspa.

**B. Horizontal Half-Wave Dipoles**

For horizontal half-wave dipoles, the antenna pattern in the horizontal plane is [12]

$$H_A(\phi) = k_1 \left\{ \frac{\cos \left[ \frac{\pi}{2} \cos \phi \right]}{\sin \phi} \right\}^2$$

with  $k_1 \approx 1.64$  a constant. The antenna gain at the second receiver is taken  $H_B(\phi_i) = H_A(\phi_i - \pi/2)$ . The common traffic  $S_{A \cap B}$  is found to be approximately 0.14 packets per slot.

**C. Cardiac Pattern**

An array of two  $\lambda/2$  vertical dipoles separated by a quarter wavelength and fed by currents equal in magnitude and  $\pi/2$  out of phase, gives the cardiac pattern [12]

$$H_A(\phi) = k_2 \left\{ \cos \left[ \frac{\pi}{4} (\cos \phi - 1) \right] \right\}^2$$

This antenna receives signal from a sector in forward direction ( $\phi = 0$ ) and attenuates signals from a backward direction. Fig. 1 gives the probability of receiving a signal from another vehicle terminal. If used in a diversity scheme with sectorization  $H_B(\phi_i) = H_A(\phi_i - \pi)$ , the common throughput is 0.11 packets per slot.

**D. Array of Vertical Dipoles**

A two-element linear array of vertical half-wave dipoles, with spacing  $d_a$  has the pattern [12]

$$H_A(\phi) = k_3 \cos^2 \left( \frac{\pi d_a}{\lambda} \cos \phi \right)$$

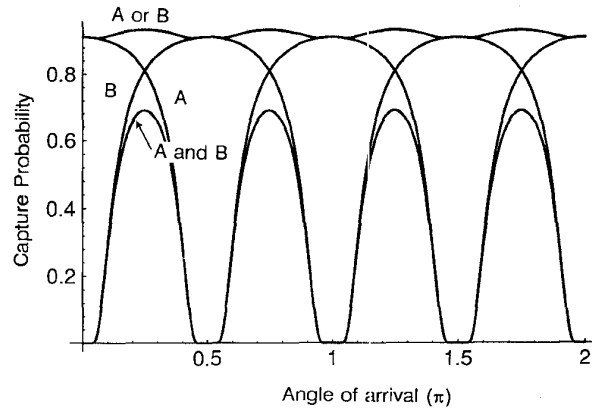


Fig. 4. Probability of capture at receiver A, B, {A or B} and {A and B} at distance  $r = 0.1$ . UHF groundwave propagation ( $\beta = 4$ ). Receiver threshold of  $z = 4$  (6 dB). Offered traffic  $G_0 = 1$  ppspa.

We address  $d_a = \lambda/2$  and  $H_B(\phi_i) = H_A(\phi_i - \pi/2)$ . This gives a common throughput of 0.08 packets per slot. An array of two vertical dipoles thus gives a relatively high joint throughput compared to the other antenna patterns considered here. Figs. 2 and 3 present the throughput of a single receiver and the common throughput, respectively, for UHF groundwave propagation ( $\beta = 4$ ), a receiver threshold of  $z = 4$  (6 dB), an offered traffic of  $G_0$  is 1 packet per slot per

TABLE II  
COMMON AND JOINT THROUGHPUT OF A SLOTTED ALOHA NETWORK  
WITH VARIOUS ANTENNA SYSTEMS. UHF GROUNDWAVE  
PROPAGATION ( $\beta = 4$ ), A RECEIVER THRESHOLD OF  $z = 4$  (6 dB)

Antenna Type	Throughput	
	$S_{A \cap B}$	$S_{A \cup B}$
Ideal Sector	0	0.64
Horizontal Dipole	0.14	0.50
Array Vert. Dipole	0.08	0.56
Cardiac	0.11	0.53

unit area. Fig. 4 gives the probability of capture at receiver  $A, B, \{A \text{ or } B\}$  and  $\{A \text{ and } B\}$  for a packet transmitted from a distance of  $r = 0.1$ . Clearly this system provides fair access probabilities for transmissions from any direction. Signals coming from overlapping areas are slightly favorable in their probability of capture.

## VII. CONCLUSION

The performance of a slotted ALOHA network with antenna diversity has been analyzed (Table II). A method has been developed to quantify the effect of realistic antenna patterns with overlapping gain patterns. To our knowledge, this is the first time these effects have been modeled realistically, while previous investigations considered only idealized patterns. We obtained numerical results for the capture probability and the network throughput, for the case of a uniform spatial distribution of the traffic offered to the network.

For the application of collecting data from probe vehicles participating in the road traffic, the slotted ALOHA random access scheme can be an efficient method to exchange travel times and short status reports. Although some channel capacity is lost because of message collisions, the ALOHA system does not require any overhead. This is in contrast to "polling" schemes, where the transmission sequence has to be reconfigured continuously according to vehicle activity. Our results showed that the performance of a system for collecting data from probe vehicles can be enhanced significantly by sector receive antennas, even if their gain pattern is far from perfect.

It turned out that a receive system using an array of two half-wave dipoles in each diversity branch gave better performance than other simple, inexpensive antenna designs. Larger throughput would be obtained for perfectly disjoint antenna coverage patterns. Any overlap reduces the throughput.

## REFERENCES

- [1] N. Abramson, "The throughput of packet broadcasting channels," *IEEE Trans. Commun.*, vol. COM-25, no. 1, pp. 117-128, Jan. 1977.
- [2] J. C. Arnbak and W. van Blitterswijk, "Capacity of slotted ALOHA in Rayleigh fading channel," *IEEE J. Select. Areas Commun.*, vol. JSAC-5, no. 2, pp. 261-269, Feb. 1987.
- [3] K. Zhang and K. Pahlavan, "A new approach for the analysis of the slotted ALOHA local radio networks," in *Proc. Int. Conf. Comm. ICC*, Atlanta, Apr. 1990, pp. 1231-1235.
- [4] E. S. Sousa, "Performance of a spread spectrum packet radio network link in a Poisson field of interferers," *IEEE Trans. Inform. Theory*, vol. IT-38, no. 6, pp. 1743-1754, Nov. 1992.
- [5] J. P. M. G. Linnartz, *Narrowband Landmobile Radio Networks*. Norwood, MA: Artech House, 1993.
- [6] C. T. Lau and C. Leung, "Throughput of a slotted ALOHA channel with multiple antennas and receivers," *Electron. Lett.*, vol. 24, no. 25, pp. 1540-1542, Dec. 1988.
- [7] ———, "A slotted ALOHA packet radio system with multiple antennas and receivers," in *Proc. IEEE Veh. Technol. Conf.*, Philadelphia, PA, June 1988, pp. 402-406.
- [8] ———, "Antenna selection in multi-sector packet radio system" *IEEE Trans. Commun.*, vol. 39, no. 11, pp. 1709-1718, Nov. 1991.
- [9] J. Ward and R. T. Compton, Jr., "Improving throughput and delay performance of slotted aloha packet radio network using adaptive arrays," *IEEE Trans. Commun.*, vol. 40, no. 2, pp. 292-300, Feb. 1992.
- [10] ———, "High throughput slotted ALOHA packet radio network with adaptive arrays" *IEEE Trans Commun.*, vol. 41, no. 3, pp. 460-461, Mar. 1993.
- [11] T. Tank and J. P. M. G. Linnartz, "Vehicle-to-vehicle channel modeling," *Int. J. Wireless Inform. Networks*, vol. 2, no. 1, 1995.
- [12] S. Ramo, J. R. Whinnery, and T. van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1985.
- [13] J. P. M. G. Linnartz and M. Westerman, "Mobile radio data system for collecting road traffic data," *IEICE Trans. Commun.*, Japan, vol. E77-B, no. 7, pp. 934-938, July 1994.
- [14] M. Zorzi and S. Pupolin, "Slotted ALOHA for high-capacity voice cellular Communications," *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, pp. 1011-1021, Nov. 1994.
- [15] H. J. Kim and J. P. M. G. Linnartz, "Virtual cellular network: a new communications architecture with multiple access ports," in *44th IEEE Veh. Technol. Conf.*, June 1994, Stockholm, pp. 1055-1059.
- [16] J. P. M. G. Linnartz, R. Hekmat, and R.-J. Venema, "Near-far effects in random access networks with narrowband Rayleigh fading," *IEEE Trans. Veh. Technol.*, vol. 41, no. 1, pp. 77-90, Feb. 1992.
- [17] J. P. M. G. Linnartz and N. Vvedenskaya, "Optimizing throughput and delay in CDMA network with stack algorithm for collision resolution," *IEE Electron. Lett.*, vol. 30, no. 18, pp. 1470-1471, Sept. 1994.
- [18] R. F. Diesta and J. P. M. G. Linnartz, "Using Laplace transforms to compute performance of mobile radio links," in *IEEE Int. Conf. Personal Indoor Mobile Radio Commun. (PIMRC)*, The Hague, Sept. 19-23, 1994, vol. 1, pp. 301-305.
- [19] J. P. M. G. Linnartz, "Packet-switched cellular communications architecture for IVHS using a single radio channel," in *IEEE Conf. Personal Indoor Mobile Radio Commun. (PIMRC)*, The Hague, Sept. 19-23, 1994, vol. 4, pp. 1222-1225. Also: PATH report UCB-ITS-PWP-94-13, Sept. 1994, ISSN 1055-1417.



**Jean-Paul M. G. Linnartz** (S'85-M'87) was born in Heerlen, The Netherlands, in 1961. He received the M.Sc.E.E. degree in electrical engineering (cum laude) from Eindhoven University of Technology, The Netherlands, in 1986, and the Ph.D. degree in multi-user mobile radio nets (cum laude) from the Delft University of Technology, The Netherlands, December 1991.

During 1987-1988, he worked with the Physics and Electronics Laboratory (F.E.L.T.N.O., The Hague) of the Netherlands Organization for Applied Scientific Research on frequency planning and UHF propagation. From 1988-1991, he was Assistant Professor at Delft University of Technology, The Netherlands. Since January 1992, he has been an Assistant Professor in the Department of E.E.C.S. at the University of California at Berkeley, U.S.A. In 1994, he returned to Delft University of Technology in The Netherlands, as an Associate Professor. Currently he is with Philips Natuurkundig Laboratorium, Eindhoven, The Netherlands. His main research interests are in (wireless) multimedia communications, conditional access and information security, electronic watermarks, Intelligent Vehicle Highway Systems, random access to fading channels, and multicarrier CDMA (combining OFDM and CDMA). He is editor of an interactive multimedia CD ROM on Wireless Communications. In 1993, he published the book "Narrowband Land-Mobile Radio Networks." Dr. Linnartz received the Dutch "Veder Prize" in 1991 for his research on tele-traffic aspects in mobile radio networks.