Design of a Frequency Hopping Spread Spectrum Communication System for an Automated Highway System

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Abstract

Frequency hopping spread spectrum communication techniques are investigated in the context of the PATH Automated Highway System project, where reliable communication links are needed from vehicle to vehicle. Multiple access interference and Ricean multipath fading will be considered. Performance measures studied include probability of packet error and the probability that no message is received in a certain time window.

1.0 Introduction

Communications is essential in the implementation of a successful automated highway system to coordinate the vehicles sharing the same road space and to prevent collisions while maintaining a high vehicle throughput. According to a design proposed by researchers at PATH (Partners for Advanced Transit and Highways), cars will travel in small groups termed "platoons" [1]. Cars within the same platoon transmit to each other information that includes vehicle velocity and acceleration.

In this paper, we investigate the relevant issues in the design of a vehicle-to-vehicle radio link using frequency hopping spread spectrum. Two factors in particular combine to degrade performance. The first is multiple access interference. The second is signal fading due to multipath. Taking into account these two factors, we determine the performance of a vehicle-to-vehicle radio link as a function of distance between two vehicles. The optimal rate for a Reed-Solomon code is found, which turns out to be dependent on inter-vehicle distance.

2.0 System description

In the design of a communication system for an automated highway system, a promising option appears to be frequency hopping spread spectrum (FHSS) radio [2]. In addition to its multiple access capability, FHSS radio can be used for vehicle-to-base station communication as well. This paper focuses only on vehicle-to-vehicle communication, where each vehicle transmits information to the following car and receives from the preceding one within the same platoon.

In the analysis to follow, we use typical parameter values of commercially available FHSS radios. We choose a radio which hops over 84 frequencies in the 2.4-2.483 GHz ISM-band, each hop separated by 1 MHz. At each frequency, BFSK modulation is used, and the space and mark are assumed to be orthogonal. We assume that the radio has a transmission rate of 500 kbits/sec, and has a maximum frequency hopping rate of 1000 hops/sec.

3.0 Bit error analysis

3.1 Multiple access interference

We adopt the model and expressions for the probability of a bit error due to multiple access interference as given by Geraniotis [3]. He provides an approximate expression for the probability of error given m user groups of size M_i and interference power P_i, $1 \le i \le m$, where P_o is the power of the wanted received signal. The probability of a bit error due to multiple access interference, P_{ma}, is given by

$$P_{ma} = \sum_{M_{1,f}=0}^{M_1} \sum_{M_{1,p}=0}^{M_1-M_{1,f}} \cdots \sum_{M_{m,f}=0}^{M_m} \sum_{M_{m,p}=0}^{M_m-M_{m,f}} p(M) P_e(M) \quad (1)$$

where $p(M)$ represents the probability of M interferers

$$p(M) = \prod_{i=1}^{m} \binom{M_i}{M_{i,f}} \binom{M_i - M_{i,f}}{M_{i,p}} P_f^{M_{i,f}} P_p^{M_{i,p}} (1-P_h)^{M_i - M_{i,f} - M_{i,p}} \quad (2)$$

and P_e(M) is the corresponding bit error rate.

$$P_e(M) = \frac{1}{2} exp \left\{ -\frac{1}{4} \left[\left(\frac{2T_b P_o}{N_0} \right)^{-1} + \sum_{i=1}^{n} \left[\frac{5}{24} M_{i,f} + \frac{1}{12} M_{i,p} \right] \frac{P_i}{P_o} \right]^{-1} \right\} \quad (3)$$

P_h is the probability of a "hit" (the event that another vehicle transmits at the same frequency during a given bit interval), and P_f and P_p are the probabilities of a "full

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hit" and "partial hit" as defined in [3]. T_b represents the bit duration, and N_o the noise power spectral density.

In order to keep computations tractable, for the computations to follow we consider a deterministic scenario where there are 4 interfering vehicles, each at a distance of 10 meters away.

3.2 Multipath fading

We model the channel probabilistically as a Ricean fading channel based on measurements as reported in [4]. Ricean fading is appropriate because the received signal will likely contain a strong dominant component in addition to the multipath signals if the antennae are placed on the bumpers.

The amplitude A of the received signal, modelled as a random variable comprising of a deterministic and a scattered statistical component, has a density function

$$f_A(a) = \frac{a}{\sigma^2}exp\left\{-\frac{a^2+B^2}{2\sigma^2}\right\} I_o\left(\frac{B}{\sigma^2}a\right) \tag{4}$$

where B is the amplitude of the dominant component and σ^2 represents the expected power of the scattered component, and I_o is the zeroth-order modified Bessel function.

We adopt a two-path model used in [2], where the dominant component consists of two factors: 1) the direct line-of-sight wave and 2) the ground reflected wave. The dominant component is therefore a deterministic value which will be a function of inter-vehicle distance.

Past measurements suggest selecting a path loss exponent of 2 (corresponding to free space loss) and a value of 10 for the Ricean K factor (defined as the ratio of the power of the direct line-of-sight wave to that of the received scattered components) [4]. We choose our transmit power so that the energy per bit (defined as the power of the LOS component multiplied by a bit duration) versus noise power at 10 meters has a value of 100 (20 dB). For practical vehicle spacing of only a few meters, the system will principally be interference limited. From a link budget analysis, a transmitter power of 50 microwatts would be enough.

3.3 Overall bit error probability

The received signal is comprised of a direct LOS wave, ground reflected wave, scattered waves, white additive Gaussian noise, and interference from other vehicles

To determine the probability of a bit error, P_{bit}, we condition P_{ma} given by equation (1) on the amplitude of the received signal and integrate over the probability density function of the signal amplitude, resulting in the following modified set of equations.

$$P_{bit} = \int_0^\infty (P_{bit}|a) f_A(a)\, da \tag{5}$$

$$P_{bit}|a = \sum_{M_{1,f}=0}^{M_1} \sum_{M_{1,p}=0}^{M_1-M_{1,f}} \cdots \sum_{M_{n,f}=0}^{M_n} \sum_{M_{n,p}=0}^{M_n-M_{n,f}} p(M) P_e(M|a) \tag{6}$$

$$P_e(M|a) = \frac{1}{2}exp \left\{-\frac{1}{4}\left[\left(\frac{T_b}{N_0}a^2\right)^{-1} + \sum_{i=1}^{n}\left[\frac{5}{24}M_{i,f} + \frac{1}{12}M_{i,p}\right]\frac{2P_i}{a^2}\right]^{-1}\right\} \tag{7}$$

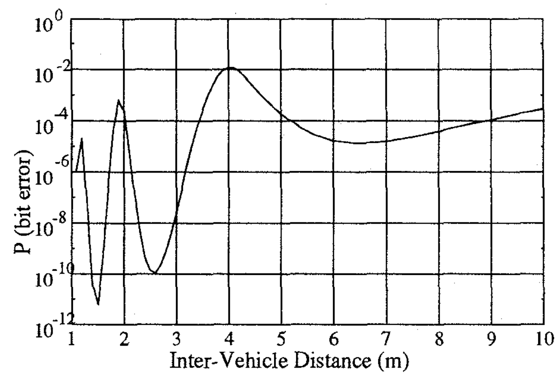

FIGURE 1. Average bit error probability for carrier frequencies 2.4 - 2.483 GHz. Frequency hopping spread spectrum with BFSK modulation over 84 frequencies. 2-path propagation model; Ricean K factor of 10; interference from 4 vehicles 10 meters away; E_b/N_0 = 100 at 10 meters.

The high probability of bit error at inter-vehicle distances of 2 and 4 meters is due to the destructive interference between the line-of-sight and the ground reflected waves.

4.0 Performance analysis

4.1 Packet erasure

The length of a packet is 128 bits, consisting of vehicle data (e.g. - ID, velocity, acceleration, and time stamp) and framing bits. For an FHSS system, bit errors tend to occur in bursts whenever different transmissions coincide in time on the same hopping frequency. In addition, several adjacent bits will be likely to be in error during periods of deep fade. Because of its capability of correcting burst errors, an (n,k) Reed-Solomon code is utilized to encode each packet.

If the elements of GF(2^4) are represented by symbols of length 4 bits (hence n=15), an (n,k) Reed-Solomon code over GF(2^4) can be treated as a $(4n,4k)$ binary code. Because the dimensions in bits for each of the above codes (i.e. - $4k$) are all less than the size of our packet in bits, we break up the packet into $\lceil r \rceil$ segments of k symbols long and code each segment individually, where r =

$128/(4k)$ and $\lceil r \rceil$ denotes the smallest integer greater than or equal to r.

The steps involved in the transmission of a packet are as follows. First groups of 4 bits are formed into symbols. Then segments k symbols long are error coded to become $n = 15$ symbols long. Finally, each segment is transmitted over a different frequency.

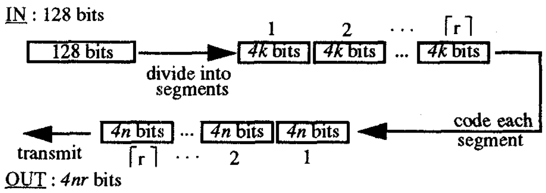

FIGURE 2. Steps involved in the transmission of a packet.

The probability of a symbol error conditioned on the received signal amplitude is given as

$$P_{symbol}|a = 1 - (1 - P_{bit}|a)^4 \tag{8}$$

A segment erasure occurs if the number of symbol errors in the segment exceeds the error correcting capability of the Reed-Solomon code. Since the signal amplitude can be assumed to remain constant during a segment,

$$P_{segment}|a = 1 - \sum_{m=0}^{\frac{n-k}{2}} \binom{n}{m} (P_{symbol}|a)^m (1 - P_{symbol}|a)^{n-m} \tag{9}$$

$P_{segment}$ is obtained by integrating equation (9) over the probability density function of the signal amplitude. Because each segment is transmitted on a different frequency, we assume that segment errors are independent from one another.

A packet erasure results if one or more of the $\lceil r \rceil$ coded segments of length n symbols are received unsuccessfully. The probability of a packet error is equal to

$$P_{packet} = 1 - (1 - P_{segment})^{\lceil r \rceil} \tag{10}$$

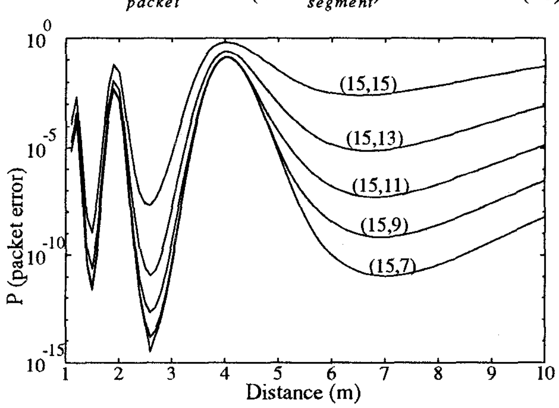

FIGURE 3. Packet error probabilities for various code rates.

From *figure 3*, we see that a lower code rate (defined as k/n) leads to a lower probability of packet error at any given distance.

4.2 Probability of failure

To ensure platoon stability, the control algorithm in each vehicle requires a data packet from the preceding vehicle in the same platoon at least once every 50 ms. Vehicles send multiple packets within the 50 ms. to increase the chance of a successful transmission within the control loop time.

We define the probability of failure as the probability of no successful packet transmissions within 50 ms.

$$P_{failure} = (P_{packet})^s \tag{11}$$

where s is the number of packet transmissions per 50 ms. In our case, s is simply equal to the largest integer less than $50/r$.

Table 1 lists some parameters associated with various codes.

TABLE 1.

(n,k)	correctable symbol errors	number of segments (r)	number of transmissions per 50 ms (s)
(15,15)	0	2.13	23
(15,13)	1	2.46	20
(15,11)	2	2.91	17
(15,9)	3	3.56	14
(15,7)	4	4.57	10

Figure 4 shows the code rate that minimizes $P_{failure}$ at each distance.

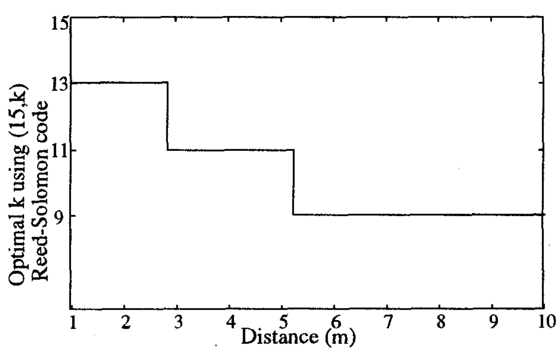

FIGURE 4. Optimal code rate without interleaving.

4.3 Interleaving

In addition to the case as described above, we also consider two schemes of interleaving. In interleaving, the symbols from λ coded segments (each 15 symbols long) are placed in an array with λ rows and 15 columns, where λ is the interleaving depth and the 15 symbols of a segment occupy an entire row. To be consistent with the previous case, we transmit 15 symbols per frequency hop. However, instead of choosing the symbols from the same segment for each hop (i.e. - by rows), we select symbols by columns so that the symbols on each hop are from different segments. The first scheme we consider, which we call "full interleaving", interleaves symbols using a depth of 15. The second one, which we call "partial interleaving", interleaves symbols using a depth of 5.

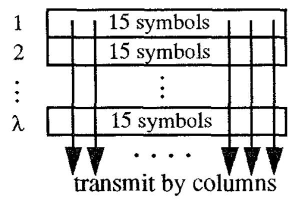

FIGURE 5. Interleaving

In the case of full interleaving, symbol errors within the same segment are independent of one another. Therefore the probability of a segment error is given by

$$P_{segment} = 1 - \sum_{m=0}^{\frac{n-k}{2}} \binom{n}{m} (P_{symbol})^m (1-P_{symbol})^{n-m} \quad (12)$$

where

$$P_{symbol} = 1 - \int_0^\infty (1-P_{bit}|a)^4 f_A(a)\, da \quad (13)$$

and $P_{bit}|a$ is given in equation (6).

However, when interleaving is used, segment errors are no longer independent because segments from the same array are transmitted over the same set of 15 frequencies. Therefore, if a particular segment is in error, then the following segment is also likely to be in error. Likewise, if a segment is received without error, then the following segment is likely to be received without error as well.

If we take into account the positive correlation between the error probabilities of adjacent segments, then expressions (10) and (11) are upper bounds for P_{packet} and $P_{failure}$.

$$P_{packet} \le 1 - [1-P_{segment}]^{\lceil r \rceil} \quad (14)$$

$$P_{failure} \le (P_{packet})^s \quad (15)$$

Although interleaving is simple to implement (it can be performed without any special hardware), this scheme causes a delay at the receiver since the receiver must wait for all 15 segments to arrive before decoding each individual segment. For a real-time application such as this, the delay can be a possible concern.

At the start of each 50 ms. interval, sensors provide measurements which are then arranged in 15 x 15 arrays. Because the contents of each array take 15 ms. to transmit, it is possible to transmit only 3 arrays within the 50 ms. interval. Therefore, the expected number of packet transmissions per 50 ms. becomes

$$s = \left\lfloor \frac{45}{r} \right\rfloor \quad (16)$$

In the case of partial interleaving, 5 hops are required to transmit each segment. Errors of symbols transmitted over different frequencies are assumed to occur independently, although in practical short-range transmission, the channel coherence bandwidth can be several tens of MHz.

The delay of a segment is 5 ms. as opposed to 15 ms. for full interleaving. Because we can transmit exactly ten 5 x 15 arrays in 50 ms., the use of partial interleaving does not force us to transmit fewer packets. Therefore s for partial interleaving is the same for no interleaving.

For an (n,k) Reed-Solomon code, $P_{segment}$ is

$$P_{segment} = 1 - \sum_{i=0}^{\frac{n-k}{2}} prob(i \text{ errors}) \quad (17)$$

When partial interleaving is used,

$$prob(0 \text{ errors}) = p_{0|3}^5 \quad (18)$$

$$prob(1 \text{ error}) = \left[\binom{3}{1} p_{1|3} p_{0|3}^4\right]\binom{5}{1} \quad (19)$$

$$prob(2 \text{ errors}) = \left[\binom{3}{2} p_{2|3} p_{0|3}^4\right]\binom{5}{1} + \left[\binom{3}{1} p_{1|3}^2 p_{0|3}^3\right]\binom{5}{2} \quad (20)$$

$$prob(3 \text{ errors}) = \left[p_{3|3} p_{0|3}^4\right]\binom{5}{1} + \left[9 p_{2|3} p_{1|3} p_{0|3}^3\right] 2\binom{5}{2} + \left[27 p_{1|3}^3 p_{0|3}^2\right]\binom{5}{3} \quad (21)$$

$$prob(4 \text{ errors}) = \left[3 p_{3|3} p_{1|3} p_{0|3}^3\right] 2\binom{5}{2} + \left[9 p_{2|3}^2 p_{0|3}^3\right]\binom{5}{2} + \left[27 p_{2|3} p_{1|3}^2 p_{0|3}^2\right] 5\binom{4}{2} + \left[81 p_{1|3}^4 p_{0|3}\right]\binom{5}{4} \quad (22)$$

$$prob(5 \text{ errors}) = \left[3 p_{3|3} p_{2|3} p_{0|3}^3\right] 20 + \left[9 p_{3|3} p_{1|3}^2 p_{0|3}^2\right] 5\binom{4}{2} + \left[27 p_{2|3}^2 p_{1|3} p_{0|3}^2\right] 5\binom{4}{2} + \left[81 p_{2|3} p_{1|3}^3 p_{0|3}\right] 20 + 243 p_{1|3}^5 \quad (23)$$

where

$$p_{0|3} = \int (1 - P_{symbol}|a)^3 f_A(a)\, da \qquad (24)$$

$$p_{1|3} = \int (P_{symbol}|a)\,(1 - P_{symbol}|a)^2 f_A(a)\, da \qquad (25)$$

$$p_{2|3} = \int (P_{symbol}|a)^2 (1 - P_{symbol}|a) f_A(a)\, da \qquad (26)$$

$$p_{3|3} = \int (P_{symbol}|a)^3 f_A(a)\, da \qquad (27)$$

We use equations (14) and (15) as upper bounds for P_{packet} and $P_{failure}$ as before in our analysis of full interleaving.

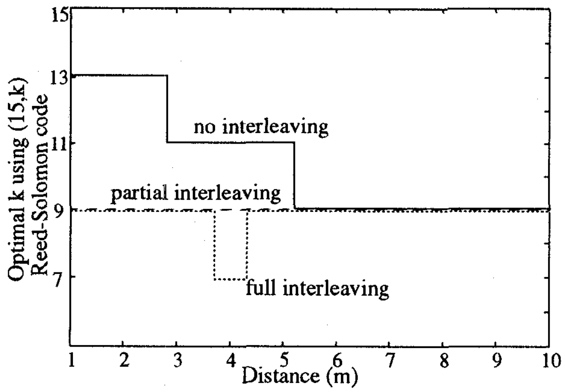

FIGURE 6. Optimal code rate for transmission using no interleaving, partial interleaving, and full interleaving.

Figure 6 shows a comparison of the optimal code rates versus distance for the three schemes that were considered. Of particular interest is the probability of failure at a distance of 4 meters, where the link can be in a deep fade. Nevertheless, our theoretical results show surprisingly small failure rates. At 4 meters, the lowest probability of failure with no interleaving is 10^{-15}, which is achieved with a (15,11) Reed-Solomon code. With full interleaving, a (15,7) Reed-Solomon code results in a value of 10^{-27} at the same distance, and with partial interleaving, a (15,9) Reed-Solomon code produces a failure probability of 10^{-18} at 4 meters.

5.0 Conclusions

In this paper, we computed the probability of failure versus inter-vehicle distance for an FHSS communication system. We analyzed and compared three variations of transmission schemes: no interleaving, partial interleaving, and full interleaving.

Interleaving (especially full interleaving) resulted in a marked reduction in the probability of failure. Because the code that achieves the minimal failure probability is a function of distance, attaining the optimal failure probability for all distances implies changing codes as the inter-vehicle distance changes. This may be impractical from a hardware perspective. Therefore, it might be better to select the code whose performance is optimal at a distance of 4 meters since the error rate is highest at that distance. The scheme using partial interleaving is an exception since with partial interleaving a (15,9) code is the optimal Reed-Solomon code for all distances.

Of particular concern is the high error rate at a distance of 4 meters due to the destructive interference between the line-of-sight and ground reflected waves. Spatial diversity is recommended to avoid prolonged link outages under these conditions.

6.0 Acknowledgments

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7.0 References

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