Single-Frequency Packet Network Using Stack Algorithm and Multiple Base Stations

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This paper evaluates a new method to combine contiguous frequency reuse with random access in wireless packet-switched networks. We consider a radio network with two base stations receiving packets transmitted by a large population of mobile users. Both stations share the same channel and therefore transmissions to one base station in one cell interfere with transmissions to the other base station in the other cell. A simulation of a two-cell system is performed, to study the interaction of retransmission traffic in two cells. To consider the performance of such system analytically, we model a one-cell system with time-varying channel properties: if only one station is receiving messages from its cell, the channel is assumed to be in "good state"; if terminals in both cells are busy the channels are assumed to be in "bad state". For conflict resolution, the stack-algorithm is used. The packet delay is calculated. Our results confirm that in wireless packet networks, it is advantageous to allow neighbouring cells to share the same channel.

Keywords : wireless networks, packet radio, multiple access, stack algorithm, receiver capture, packet delay, markov communication channels, cellular radio networks.

I Introduction

Many of the solutions for sharing communication resources among multiple users and services that have been developed for wireline networks become inefficient or require modifications if they are used for radio resource management in wireless networks. The common goal of most multiplexing, switching and multiple access schemes to is dynamically assign bandwidth during certain periods of time. If such techniques are used in radio data or multimedia networks, the radio spectrum needs to be reused spatially in an efficient (and presumably dynamic) way. However, this issue has hitherto mostly been addressed separately from allowing multiple users to share the same bandwidth–time resources within one cell. This is for instance illustrated by the fact that most existing mobile data networks use a cellular frequency reuse pattern, and within each cell a random-access scheme is used independently of the traffic characteristics in other cells.

This paper shows that if the stack-algorithm is used for random access in one cell (i.e., to handle intracell interference), it can also mitigate the effect of bursty (intercell) interference between neighbouring co-channel cells. Hence, the stack-algorithm allows substantially denser frequency reuse than currently used in radio telephony. In particular, we consider the performance of a wireless network with two base stations receiving packets

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transmitted by a large population of mobile users. The users that are in the cell area of one of the base station transmit their packets to this base station. To do that they compete for (random) access, according to the free-access stack-algorithm with feedback from the base station [1 - 3]. If both base stations share the same channel, transmissions in one cell interfere with transmissions in the other cell. The stack algorithm retransmission scheme not only avoids instability within one cell but also migitates the effect of interference between cells. We simulate the interaction of retransmissions in two interfering cells, each with its own stack algorithm. To compute the performance of a network with two base stations, we model the performance of one-station system with time-varying channel properties. Whenever only one base station is busy (or both are silent), the base stations are assumed to have a relatively reliable channel ("good state"). When both stations are busy, the stations are assumed to have an imperfect ("noisy") channel ("bad state"), due to the mutual interference. To address system performance mathematically, we approximate transitions from one state to the other as an autonomous Markov on/off process. State transitions are assumed to be independent of transmissions in the cell suffering its interference.

The performance of a two-cell system with a common channel is compared with the performance of two-cell system with different channels in each cell. We do not consider any CDMA spreading, as we have found in [4] that such modulation techniques may not be advantageous in our case of packet data transmission. To avoid interference in conventional cellular systems, two different channels would be needed, each with half the total available bandwidth. Hence, the transmission time increases by a factor two. Moreover the arrival rate expressed in (new) packets per time slot increases by factor of two. Particularly under large traffic loads, this appears to lead to a significantly larger delay than our proposal of allowing neighbouring cells to share bandwidth, thus accepting mutual interference.

II Model of Access Scheme

The uplink random access channel is considered to be time slotted and synchronized at time slot level. The slots start at $t = 1, 2, 3, \ldots$. A slot that starts at $t = n$ is called slot n. A full packet is always transmitted within one slot time. A new packet is transmitted for the first time in the first slot following its arrival (free access). The input flow of packets in a cell is modelled as a spatially and temporally uniform Poisson process with flow rate λ packets per a slot. [‡]

Any packet that captures the receiver leaves the system. Terminals are informed about packets successfully arriving at the base station through an instantaneous feedback channel. For unsuccessfully transmitted packets, i.e., for those packet that are not acknowledged, each terminal has a buffer to keep one packet for retransmission. Each packet transmitted in a slot without capturing the receiver is either retained in the user's buffer (with probability 1/2) or is retransmitted in the next slot (with probability 1/2). The main idea here is to split the backlogged terminals into different small groups after a collision. The probability of having a message collision in one of the groups rapidly vanishes after several splits. To ensure that each terminal keeps track of the group

[‡]More sophisticated traffic models may better reflect the burstyness of multi-media traffic, but we believe that such refinements would not fundamentally change the principal conclusions of our investigation.

to which its packet belongs, a stack counter l_n is associated with the user's buffer. The counter contents change from slot to slot according to the stack-algorithm rules, following the feedback information. Terminals transmit (only) when $l_n = 0$. Generally speaking, the stack counter increases when a conflict is reported in a slot and decreases when a slot is idle. The idea is that after a collision, all backlogged groups of packets have to wait before the current group has resolved its collision. As the current group splits, a new level is "inserted", and all existing groups increase their stack counter. Intuitively, the stack algorithm protocol is a "randomised first come - last served" protocol.

We address the particular stack algorithm that uses a ternary "idle slot/success/conflict" feedback [3]. Moreover, in case of capture, the feedback channel reports which packet captured the base station. The protocol rules of the Stack Algorithm addressed here are:

1. A packet transmitted in slot n for the first time (i.e. the packet generated in slot $n-1$) has $l_n = 0$.

2. If $l_n = 0$ for a packet, the packet is transmitted in slot n. If $l_n > 0$, the packet is not transmitted in slot n.

3. If $l_n = 0$ for a packet and a capture is reported in slot n, for that packet, then the successful packet, leaves the system. For other transmitting packets (if any) $l_{n+1} = l_n = 0$.

4. If $l_n = 0$ for a packet and a conflict is reported in slot n, then $l_{n+1} = 1$ with probability 1/2 and $l_{n+1} = 0$ with probability 1/2.

5. If $l_n > 0$ for a packet and a conflict is reported in slot n, then $l_{n+1} = l_n + 1$.

6. If $l_n > 0$ for a packet and slot n is reported idle, then $l_{n+1} = l_n - 1$.

7. If $l_n > 0$ for a packet and a capture is reported in slot n, then $l_{n+1} = l_n$.

The delay of a packet is defined as the length of a time interval between the start of the packet's first transmission and the start of its successful transmission. Note, that the time a packet spends in the system includes the random waiting time till the beginning of the first transmission, the delay, and one slot of successful transmission. The mean duration T a packet spends in the system is equal to the mean delay D plus one and a half slot duration.

The value of λ_{cr} is called the throughput of the system, if there exists such λ_{cr} that for any input flow rate λ, $\lambda < \lambda_{cr}$, all packets are transmitted with finite delay with probability 1, and the mean delay D is finite, while for $\lambda > \lambda_{cr}$, delays grow beyond any finite bound. λ_{cr} depends on channel properties.

For one-cell system and for given channel probabilities, the mean delay D can be expressed in terms of the solutions of the linear algebraic equations [1 - 4].

For an ALOHA system [7 - 11] with an infinite population of user terminals without capture, it has been shown that no such $\lambda_{cr} > 0$ exists. Only for ALOHA systems with capture, the non-zero value $\lambda_{cr} = \lim \inf_{k \to \infty} \pi_k$ is found [10], with π_k the probability of capture if k signals are competing. Moreover, it has been shown that for mobile ALOHA, the capture probability in a case of an infinite number of competing terminals is non-zero only under unrealistic assumptions about the spatial distribution of terminals [5, 11]. Therefore we limit our results to the stack algorithm, which is an improved retransmission strategy of the basic ALOHA protocol. Examples of other methods to stabilize the ALOHA multiple access protocols are presented and discussed in [12 - 13].

III Simulation of Two-Cell System

In our single-frequency two-cell system, both base stations use the same stack-algorithm rules, but without mutual coordination. The cell areas overlap. The performance of this two-station system depends on the probabilities described in following paragraphs.

Let in cell $i, i = 1, 2$, slot n be idle. If in cell $j, j \neq i$, slot n is idle too, then in cell i this slot is reported idle. If in cell j slot n is not idle, and the total received power $S^{(i)}$ is below a detection threshold Q_{th}, then in cell i slot n is reported idle. Otherwise (if $S^{(i)} > Q_{th}$), the receiver in i will attempt to detect a packet, even though all available packets arrive from cell j. In cell i capture is reported with probability $p_{0s} = p_{0s}^{(i)}$ and a conflict is reported with probability $p_{0c} = p_{0c}^{(i)}$. The values of $p_{0s}^{(i)}$ and $p_{0c}^{(i)}$ depend, among other things, on the propagation distances between the base station i and the active terminals. In our simulation we use known location of terminals, which remain fixed from the first arrival of a packet until the successful reception of the packet. However, new packets arrive in terminals with randomly chosen locations.

Let one terminal transmit in cell $i, i = 1, 2$, during slot n. If in cell $j, j \neq i$, slot n is idle, then in cell i capture occurs so the packet leaves the system, with probability $p_{1s} = p_{1s}^{(i)}$, and a conflict is reported with probability $1 - p_{1s}^{(i)}$. In our simulations, we compute $p_{1s}^{(i)}$ depending on the distance between the base station and the terminal, and on noise levels. If slot n is not idle in cell j, then in cell i capture occurs (and is correctly reported over the feedback channel) with probability $p_{1s}^{(i)}$ and a conflict is reported with probability $1 - p_{1s}^{(i)}$. In this case the value of $p_{1s}^{(i)}$ is computed taking into account the distances between the base station i and all active terminals, which now are in cells i and j.

Let several, say k_i, ($k_i = 2, 3, ...$) terminals transmit in cell i and k_j, ($k_j = 0, 1, ...$) terminals transmit in cell j. Then, in this cell i capture is reported with probability $p_{cs}^{(i)}$ and a conflict is reported with probability $1 - p_{cs}^{(i)}$. The value of $p_{cs}^{(i)}$ depends on the distances between the base station i and the active terminals in cells i and j, and on the system noise floor.

Note, that the feedback information in cell i does not affect the behavior of terminals in cell j. In our model, transmissions within a cell either result in a capture or a conflict, but are never reported as an idle slot by the intended base station.

We adopt a generally accepted model for (outdoor) macrocellular propagation considering a Rayleigh-fading channel with "40 log d" plane-earth path loss [6]. That is, signal from the m-th ($m = 1, 2, ..., k$) transmitting terminal with distance r_{im} is received at base station i with local-mean power $S_m^{(i)} = r_{im}^{-4}$. Because of Rayleigh fading, the instantaneous power is an exponential r.v. with mean $S_m^{(i)}$ [6].

For k, $k = 1, 2, ..$ transmitting terminals, with the distances r_{im}, $m = 1, ..., k$, the total (local mean) received power $S^{(i)}$ at base station i is $S^{(i)} = \sum_{m=1}^{k} S_m^{(i)} = \sum_{m=1}^{k} r_{im}^{-4}$. In this paper, this variable is used to determine whether the slot is idle or occupied. A refined computation may consider instantaneous, rather than local-mean powers. We assume that the received (instantaneous) power is constant during packet reception ("slow fading" [5]). For most modern mobile communication systems, including systems with GSM, IS54 or DECT parameters, this appears to be a reasonable assumption.

If several terminals (say, $k, k = 1, 2, ..$ terminals) transmit in a slot (may be in different cells), we assume that a packet captures the base station if the instantaneous signal-to-noise-plus-interference ratio exceeds a threshold z. By solving the integrals for the probability density functions of received power, i.e., considering path loss and Rayleigh fading, the capture probability for terminal 1 ('tr. 1') at distance r_{i1} from base station i can be expressed as [5]

$$p_{cs,1}^{(i)} = \Pr\{capture\ by\ tr.1\} = \exp(-zQ_n r_{i1}^4) \prod_{m=2}^{k} r_{im}^4/(zr_{i1}^4 + r_{im}^4).$$

where z is a receiver parameter, typically $z = 4$ and Q_n is the noise floor of the system. A realistic value could be on the order of a 10 dB (local-mean) signal to noise ratio at the cell boundary, where $r_{i1} = 1$, so the local-mean power is $S_m^{(i)} = 1$. So with $Q_n = 0.1$ (-10 dB), the local-mean ratio becomes $S_m^{(i)}/Q_n = 10$. For this value, packets that do not experience interference are only lost occasionally because of noise and fading. The probability that any one packet (out of k packets) captures base station i is $\Pr\{capture\ in\ cell\ i\} = \sum_{m=1}^{k} p_{cs,m}^{(i)}$, which in practice is almost identical to $\max_{m=1,\ldots,k} p_{cs,m}^{(i)}$, i.e., only the packet from the nearest terminal is likely to capture. If only a single terminal (say, the one with index 1) transmits without interference, the probability of capturing the receiver reduces to $p_{cs,1}^{(i)} = p_{1s}^{(i)} = \exp(-zQ_n r_{i1}^4)$.

IV Approximate Markovian Model for Interference

The above (detailed) model was used for simulation, but it makes analysis tedious. To compute results for a two-cell system, we model it as a one-cell system with a two-state channel model. To reflect the idle and busy periods of sessions of transmissions in the other cell, the channel of the "victim" cell system has two states ("good" or "bad"). Markovian transitions from one-state to the other are considered: if during slot n the system is in state $i, i = 1, 2$, then it will stay in this state during slot $n + 1$ with probability q_i, and will be in different state with probability $(1 - q_i)$. The mean duration of being in state i is equal to

$$L_i = 1/(1 - q_i), \quad i = 1, 2.$$

Autonomous Markovian transitions as modelled above are only an approximation of the changes from an idle to a busy periods in the interfering cell. As we explored in our simulation, retransmissions in the two cells interfere, so busy periods interact in a very complicated manner. In the analysis, we simplify the interaction as an autonomous mechanism, however, the average values of busy and idle periods are obtained recursively, considering both intercell and intracell interference.

Let the system be in state i during a slot n. We model the effect of interference as follows: If slot n is idle, then a capture is reported in this slot with probability $\pi_{0s}^{(i)}$, and a conflict is reported with probability $\pi_{0c}^{(i)}$, the slot is reported idle with probability $(1 - \pi_{0s}^{(i)} - \pi_{0c}^{(i)})$. Here event "0s" models excessively strong signals from the other cell, to the extent that an "alian" packet captures the base station. We assume that the base does not immediately recognize that the capturing packet comes from the other cell, so it reports the capture. Event "0c" models interference power from the other cell that is strong enough to obscure the emptiness of the slot, but it does not result in any packet capturing the base station.

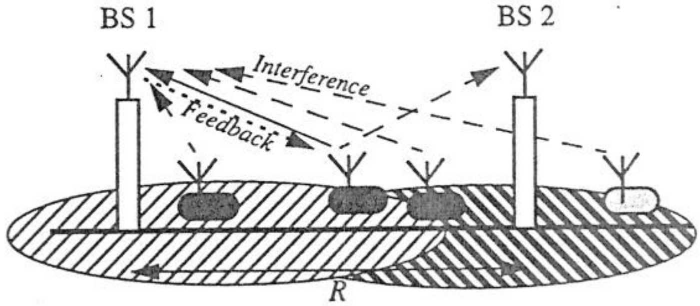

Figure 1: Multiple-access radio system with two base stations and overlapping cells

If in slot n one packet from that cell is transmitted, then a capture occurs (and is reported) with probability $\pi_{1s}^{(i)}$, and a conflict is reported with probability $(1-\pi_{1s}^{(i)})$. Here event 1 models interference from the other cell that destroys the packet, or fading and noise that impair successful transmission.

If in slot n $k, k>1$, packets are transmitted, then one packet captures the receiver with probability $\pi_{ks}^{(i)}$, and a conflict is reported with probability $1-\pi_{ks}^{(i)}$. Here event "ks" models not only interference from within the cell but also interference from the other cell, and noise on the random-access channel.

We assume that in the state "1" the base station can make much better observations on transmissions in its cell, and provides almost perfect feedback channel, $\pi_{0s}^{(1)}=\pi_{0c}^{(1)}=0$. In state "2" however, the observations are obscured due to interference from the other cell, so the base station provides imperfect feedback, $\pi_{0s}^{(2)}>0$, $\pi_{0c}^{(2)}>0$, $\pi_{ks}^{(2)}>0$, and $\pi_{ks}^{(2)}<\pi_{ks}^{(1)}$. We averaged the probabilities $p_{0s}, p_{0c}, p_{1s}, p_{ks}$ over (simulated) terminal locations to obtain the values for $\pi_{0s}, \pi_{0c}, \pi_{1s}, \pi_{ks}$, respectively. Our simulation used uniformly distributed terminals, but their location is kept fixed between generation and successful (re-) transmission of a packet. In this case, the retransmission traffic becomes non-uniform, as most retransmissions are made by remote terminals (large r_{im}). Rayleigh fading was considered to be independent from transmission attempt to attempt. Note that the assumption that terminal distances are kept fixed can substantially affect the performance [11], but its effect is ignored in many other (purely analytical) investigations.

V Results and Discussion

Simulations have been performed for a one dimensional two-cell system with a common radio channel as illustrated in Figure 1: The terminals and the base stations are located on a line ("a highway model"). The base stations are in the centers of their cells.

The locations of the terminals are assumed to be uniformly distributed in the cell area, and independent. The distance between the two base station is taken to be $R, 1<R<2$. We assume that the terminal only considers feedback from one base station. A terminal m can be in the cell of base station i, i.e., it can rely on its feedback if the distance r_{im} is less then 1.

The results for the one-cell system with autonomous interference have been computed analytically. To calculate the performance of a one-station system, we investigate "busy

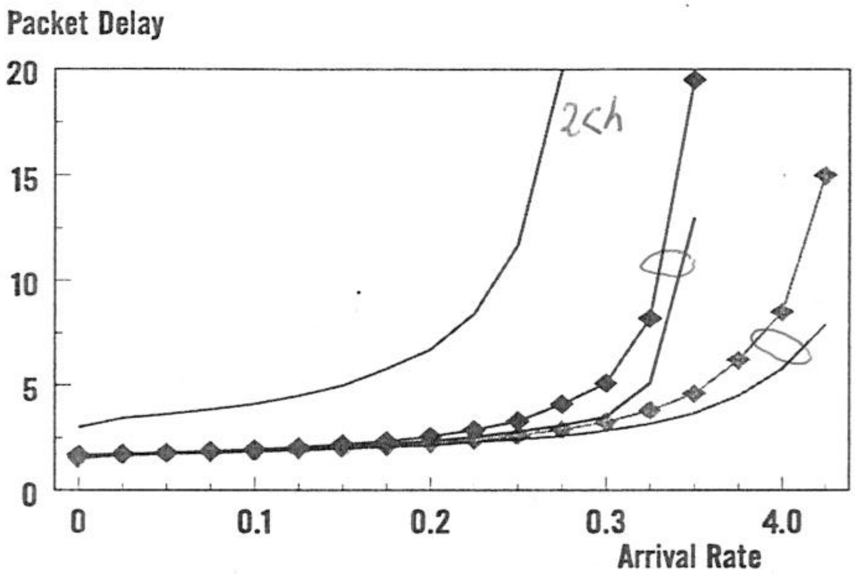

Figure 2: Delay versus normalized message arrival rate. Single frequency net: Simulation (blocks) and compuation (base station separation $R = 1.5$ and 1.9), compared with two separate channels

sessions". Roughly speaking a busy session is a time interval during which there are some packets being transmitted or waiting to be retransmitted in the stack of some terminal. A session ends as soon as all terminals have an empty stack and the channel is idle. To analyse a two-cell system with equal message arrival rates, we consider such a one-cell, two-state system. The mean length of busy session L_b is equal to the mean length L_2 of the state "2" ("bad state"). The mean duration L_1 of the system being in state "1" ("good state") is taken equal to the mean length of the slots without new packets, $L_1 = 1/(1 - e^{-\lambda})$. Therefore $q_1 = e^{-\lambda}$. For given channel properties the value of L_b can be expressed in terms of the solutions of the linear equations (similar to the computation of the delay value D); q_2 is found by iteration, to satisfy the equality $L_2 = L_b$.

The results of computations for one-cell, two-state system follow the results of simulations with 3% accuracy for λ close to .1 and with 10% accuracy for λ close to .2. Note, that our simulations accurately model the interaction between the two stack-algorithms in both cells at slot and packet level, rather than simplifying the interaction as an autonomous Markovian interference process. Apparently, for large λ this effect becomes significant, so the results from computation and similation diverge near λ_{cr}.

The results of simulations for a one-channel two-cell system with input flow rate λ in both cells ($C = 1$,) are compared with the results for single-cell, two-state model ($C = 1$), and with the results for two-cell system with separate channels but with the same propagation and noise and capture parameters. A fair comparison requires us to accommodate the same total bandwidth to each system. As in the case with $C = 2$ reuse, each channel only has half the bandwidth, time slots need to be twice as long. Effectively, for a given arrival rate per second, this corresponds to a flow rate of 2λ new packets per slot. This increase in traffic load and also the fact that time slots are twice as long as in the case $C = 1$ severely affect the delay. However, using different channels ($C = 2$) has the advantage that the cell interference or "feedback errors" do not occur.

From Figure 2, we conclude that it is not advisable to split the radio spectrum resources into two separate channels. We have not yet optimized the choice for the threshold Q_{th}, but experimemnts revealed that the results are not so sensitive to small changes in Q_{th}. Evidently Q_{th} can not be below the noise floor Q_n. A reasonable choice would be to take Q_{th} approximately equal to the expected signal power if packets from two remote terminals in the cell collide (each with $r_{im} = 1$, so $Q_{im} = 1$). In practice, a low threshold would result in many errors of interpreting an idle channel as a collision, because of signals blown over from the other cell. On the other hand, too high a threshold leads to errors of interpreting a collision of weak, faded signals as an idle slot. This however, was not modelled in our present work. Note however that unlike some other collision resolution algorithms, the stack algorithm is fairly robust against occasional observation or feedback errors.

We could in theory obtain the values of $\pi_{0s}, \pi_{0c}, \pi_{1s}, \pi_{ks}$ by integrating over a certain spatial distribution of transmission attempts and a certain distribution of the number of active terminals. We preferred to use values from a Monte Carlo Simulation. This had not only the advantage the capture probabilities more accurately reflect that most (re) transmissions arrive from areas relatively far away from the base stations (as previously argued), but also it more accurately considers that the number of packets in a slot not exactly follows a Poisson distribution.

For the parameter values in Figure 2, i.e., for a receiver capture threshold $z = 4$, a base station separation $R = 1.9$, noise floor $Q_n = 0.1$, and idle/busy threshold of $Q_{th} = 2$, the probability that a single packet is not lost (in the absence of intercell and intracell interference) is $\pi_{1s}^{(1)} = 0.96$. So, with probability 0.04, the base station sees this transmission as a collision (it sees some radio energy, but can not detect the signal without bit errors). If multiple packets ($k = 2, 3, ..$) are transmitted in the cell, but no interference is seen from the other cell, the capture probability is found as $\pi_{kc}^{(1)} = 0.6$. In the "bad state", i.e., if the other cell causes interference, an empty slot is seen as a collision with probability $\pi_{0c}^{(2)} = 0.05$. Moreover, with probability $\pi_{01}^{(2)} = 0.01$, a packet from other cell is strong enough to capture the base station in the reference cell. Interference from other cell is rarely strong enough to destroy the transmission of a single packet in the reference cell: $\pi_{1s}^{(2)} = 0.88$, which is only marginally less than $\pi_{1s}^{(1)}$. Similarly in case of a collision in the reference cell, the capture probability is not significantly affected if intercell interference is also present: $\pi_k^{(2)} = 0.53$.

Our computations show that if the flow rate λ is sufficiently low, and the noise is sufficiently low (as is the case in the given examples), then the delay is less for the single-frequency system (with $C = 1$), both for the simulated model and for the simplified model of a one-cell, two-state system. For larger rates, it is still beneficial to share the same channel. However, the results from the simulation diverge from the computed results.

This suggestion to adopt a single-frequency concept can also be understood intuitively from considering a comparable two-cell ALOHA system, which for sake of simplicity we assume to be stable [11]. If one allows the two base station to share a common channel the delay will be small as collisions are rare. If on the other hand separate channels are used, the slot duration is doubled, which in the limiting case of very light traffic appears the only relevant effect on delay. At larger traffic load increases it furthermore becomes relevanthow heavily the channels are loaded. In the single-frequency collisions occur with probability $\exp(-2\lambda)$ because of packets from two cells. In the separaate channel system, collisions also occur with probability $\exp(-2\lambda)$, but now because the slot length is twice as long while we only see traffic from one cell. However, as capture occurs in radio networks, the former system suffers less from the collisions, as one half of all interfereing packets come from the other cell, and may be too weak to cause harmful interference.[§]

VI Conclusion

Our results show that in a narrowband two-cell system with the stack algorithm used for conflict resolution, the same radio channel can be used in both cells. At low traffic loads, its performance can be approximated by one-cell system with two-states of the system, and with autonomous Markovian transitions from one state to the other.

Our results suggest that in wireless networks with bursty traffic, it may be advantageous to allow nearby cells to use the same channels. The free-access algorithm not only efficiently controls retransmissions needed after a collision within the cell, but also makes the system robust against high levels of interference from co-channel cells. This is in contrast to conventional cellular frequency reuse used for mobile-telephones, where adjacent cells need to use different frequencies.

VII Appendix

Here we present the equations used in calculations of the packet mean delay in a one-cell, two-state system. Let us begin with the rigorous definition of a session and define a session that starts at slot n. We introduce at slot n an additional packet to the system with $l_n = 1$ for this packet. Let $l_{n_1} = 0$ for the first time, $n_1 > n$. The time interval $[n, n_1)$ is called a session. A session is called a k-session if k packets are transmitted at it's first slot n. A busy session is a session with $k > 0$. Denote by $h_i(k)$ the mean length of a k-session if it starts in state i. Denote by $d_(k)$ the mean sum of delays of all packets, that are successfully transmitted in a k-session, if this session starts in the state i. The mean packet delay D is defined as

$$D = \frac{\sum_{i=1,2}\sum_{k=0}^{\infty} p_k Q_i d_i(k)}{\lambda \sum_{i=1,2}\sum_{k=0}^{\infty} p_k Q_i h_i(k)}; \quad p_k = \exp(-\lambda)\lambda^k/k!.$$

The values of $h_i(k)$ and $d_i(k)$ are found as solutions of linear equation systems that differ in free terms only. We present here the equation for $h_i(k)$. In the equations for $d_i(k)$ the free terms depend linearly on $h_i(k)$.

$$h_i(0) = 1 + \pi_{01} \sum_{m=0}^{\infty} p_m[q_i h_i(m) + (1 - q_i) h_j(m)] + \pi_{0c} F_{i,0,0},$$

[§]An interesting corollary is that packet-switched systems require different frequency reuse methods than used in telephony: One base station covering the entire service area gives smaller delays (by a factor of two) than a (more expensive) system with two base stations, each with only half the bandwidth available. Both systems have the same message arrival rate per time slot, but the former has shorter time slots.

$$h_i(1) = 1 + (1-\pi_1)\sum_{m=0}^{\infty} p_m[q_i h_i(m) + (1-q_i)h_j(m)] + \pi_1(F_{i,1,0} + F_{i,0,1})/2,$$

$$h_i(k) = 1 + \sum_{l=0}^{k} 2^{-k} C_k^l (F_{i,l,k-l} + F_{i,k-l,l}),$$

where

$$F_{i,l,k} = \sum_{m=0}^{\infty} p_m\{q_i[h_i(l+m) + s_{i,i}(l+m)h_i(k-l+m) + s_{i,j}(l+m)h_j(k-l,m)] +$$

$$+(1-q_i)[h_j(l+m) + s_{j,i}(l+m)h_i(l+m) + s_{j,j}(l+m)h_j(k-l+m)]\},$$

$$C_k^l = k!/l!(k-l)!, \quad i = 1,2, \quad i \neq j, \quad k > 1.$$

Here $s_{i,j}(k)$ is the probability that a slot is in a state j if it follows a session of multiplicity k, that starts in a state $i, i, j = 1, 2$. To calculate $s_{i,j}(k)$ we first calculate the probability, that a k-session, that starts in state i is of length l. The recursion on l permits to find these probabilities. The Markov chain transition from one state to the other permits to get $s_{i,j}(k)$.

To estimate the value of D with 0.1% accuracy it is sufficient to solve a finite system of equations for systems for $h_i(k), d_i(k), k < 10 \div 12$. The value of λ_{cr} is estimated with 0.01% accuracy by the value of λ, for which $D > 500$.

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