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Statistical Characterization of Rician Multipath Effects in a Mobile-to-Mobile Communication Channel

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A statistical model is developed for a narrowband mobile-to-mobile channel taking into consideration Rician scattering near receiving and transmitting antennas both individually and concomitantly. From the proposed channel model we obtain the probability density function of the received signal envelope, the time correlation function and RF spectrum of the received signal, and level crossing rates and average fade durations. We briefly discuss the impact of these parameters on communication networks supporting an intelligent vehicle highway system (IVHS).

KEY WORDS: Rician multipath effects; mobile communication channel; intelligent vehicle highway system.

1. INTRODUCTION

Advanced communication technology may mitigate some of the existing ground transportation problems, such as traffic congestion. Projects such as Road Automobile Communication Systems (RACS) in Japan [1], PROMETHEUS in Europe [2], and Partners for Advanced Transit and Highway (PATH) in the United States are currently engaged in the design of such systems called intelligent vehicle highway systems (IVHS) in Japan and the United States and road transport informatics (RTI) in Europe. Many see these projects as a means of improving safety and efficiency of the highway system, which in turn would lead to an increase in the productivity of commuters as well as alleviate pollution [3]. Mobile-to-mobile communication is of critical importance to such IVHS projects, especially in automated vehicle control systems (AVCS) employing platoons [4]. This study was motivated by research on AVCS, in which vehicles periodically exchange telemetric data on their speed and acceleration. This allows smooth control of vehicles' speeds without the effects of small acceleration errors in the control loop propagating backwards and being amplified in the reactions of following vehicles [4]. Communication could be achieved through roadside base stations, but the frequency reuse can be denser and the system may be more economical if direct car-to-car links are used. Infrared is considered as an option for communication between cars within a platoon, while (leaders of) platoons always communicate over longer ranges where radio may be preferred. Although communication occurs only over relatively short range, on the order of tens of meters, the communication links have to be extremely reliable. Studies [4] showed that reliable operation of such systems requires updates to arrive at least once every 50 ms for communication within a platoon and somewhat more lenient requirements for communication between platoons. Communication channel fades exceeding this duration would thus threaten system reliability.

The application of microwave data links in a land-to-mobile environment has been shown to suffer from multipath fading, shadowing, and Doppler phase shifts. These effects limit the performance of the system. It is thus desirable to have a model of the channel and its limiting effects. This paper presents a statistical model for the effects of multipath fading in a mobile-to-mobile environment, extending the statistical model for Rayleigh fading by Clarke [5] and Jakes [6] for mobile-to-

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land and by Akki and Haber [7] for mobile-to-mobile communication.

In a mobile channel, energy arrives at the receiver by scattering and diffraction over and/or around the surrounding environment. A short-range mobile-to-mobile channel in a highway environment will also contain a much stronger direct line-of-sight component, possibly also with a strong ground-reflected wave. These components combine vectorially at the receiver and give rise to a resultant signal that varies greatly, depending on the distribution of the phases of the various components. These short-term variations in the received signal are called multipath fading. Long-term variations in the signal, such as shadowing or path loss, are also present. The relative motion of the vehicles will give rise to a Doppler shift in the signal. Thus, the mobile radio signal varies rapidly over short distances (fading), with a local mean power that is constant over a small area but varies slowly as the receiver moves. We will concentrate on the short-term effects for narrowband channels. In contrast to Refs. 5–7, we include a dominant component, resulting in Rician fading.

2. PROBABILITY DENSITY FUNCTION OF RECEIVED SIGNAL

In deriving the probability density function of the received envelope, we will follow Clarke's two-dimensional scattering model [5]. Work has been done by Aulin [8] to extend this to a three-dimensional model [9], but we will adopt Clarke's model, which assumes that the field incident on the mobile antenna is comprised of horizontally traveling plane waves of random phase. Also all reflections occur in a plane and both mobile are at the same height. We will augment Clarke's model by considering a dominant (e.g., line-of-sight) component as well as reflections at both transmitter and receiver. On a highway, reflections may also occur against vehicles in between the transmitter and receiver. However, their signal strength is often much smaller, as the path contains two relatively long segments, each with substantial free-space loss. This is similar to the situation in land-to-mobile channels.

At every receiving point we assume the signal to be comprised of many plane waves, as shown in Fig. 1. Here N_T waves experience reflections at the transmitter only, N_R waves experience reflections at the receiver only, and $N_T N_R$ waves experience reflections at both transmitter and receiver. We denote waves by an index i indicating the path and reflection near the transmitter and an index k denoting the path and reflections near the receiver. The (i, k)th incoming wave has a phase shift $\phi_{i,k}$, a spatial angle of arrival α_{Rk}, and a spatial angle of departure α_{Ti} with respect to the velocity of the receiver. We use $i = 0$ and $k = 0$ for the dominant component that is not subject to random scattering. The (i, k)th wave has a real amplitude given by $E_{i,k}$, depending on the reflections and additional path loss that the wave undergoes. In practice, the amplitudes $E_{i,k}$ may be difficult to estimate. We model this as $E_{i,k} = E_0 C_{i,k} D_{i,k}$, where $C_{i,k}$ accounts for scattering near the transmitter and $D_{i,k}$ describes scattering near the transmitter. Here $E_0 C_{0,0} D_{0,0}$ is the deterministic amplitude of the dominant component, which in case it consists only of the line-of-sight wave is found from free space loss. The parameters $\phi_{i,k}$, α_{Rk}, $\alpha_{Ti} C_{i,k}$, and $D_{i,k}$ are all assumed to be random and statistically independent. Maffett [10] has shown that the radar cross section, which is analogous to the dimensionless parameters $C_{i,k}$ and $D_{i,k}$ is a function of polarization and area of incidence. Since the transmitted waves were assumed to be vertically polarized, the area of incidence is the important factor in modeling these parameters. If the separation distance between the two mobiles is sufficiently greater than the distance between mobile and scattering object, the process of scattering at the transmitter and at the receiver may be assumed to be statistically independent. This is particularly the case if the receiver and transmitter are separated sufficiently far to approximate the sum of the waves traveling di-

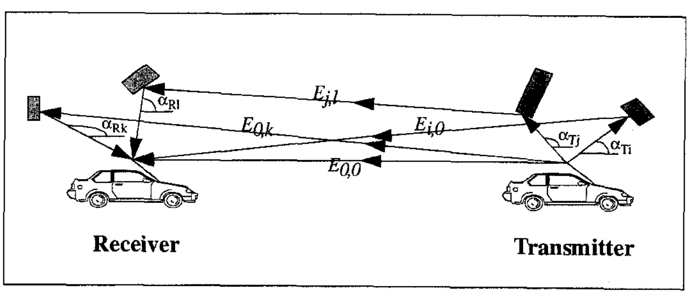

Fig. 1. Mobile-to-mobile propagation channel with scatters near both antennas.

rectly or via one or two reflections as a plane transversal electromagnetic wave through some plane perpendicular to the transmitter-receiver line of sight. This suggests $C_{i,k} = C_i$ and $D_{i,k} = D_k$. In the following analysis we will consider this to be a special case. Then E_{ik} becomes equal to $C_i D_k E_0$, while $E_{i,0}$ and $E_{0,k}$ tend to $C_i D_0 E_0$ and $C_0 D_k E_0$, respectively.

More in general, for reflections at both transmitter and receiver, the signal consists of a double sum over both reflections. If an unmodulated carrier is transmitted, the resulting electric field can be expressed as

$$\begin{aligned} E(t) &= E_{0,0} \cos [(\omega_c + \omega_d)t + \phi_{0,0}] \\ &+ \sum_{i=1}^{N_T} E_{i,0} \cos [(\omega_c + \omega_{Ti})t + \phi_{i,0}] \\ &+ \sum_{k=1}^{N_R} E_{0,k} \cos [(\omega_c + \omega_{Rk})t + \phi_{0,k}] \\ &+ \sum_{k=1}^{N_R} \sum_{i=1}^{N_T} E_{i,k} \cos [(\omega_c + \hat{\omega}_{Rk} - \hat{\omega}_{Ti})t + \phi_{i,k}] \end{aligned} \quad (1)$$

This field consists of a dominant component, which is treated deterministically, along with components that take into account reflections at the receiver, transmitter, and both receiver and transmitter. Measurements [11] indicated that at short range both the line-of-sight and the ground reflection are substantially stronger than the sum of weak scattered waves. Reflections off metal surfaces of the vehicle can also be strong. Hence, one may wish to model the channel using deterministic assumptions about strong paths, resulting in $E_{0,0}$, and using a statistical approach for $E_{i,0}$, $E_{0,k}$, and $E_{i,k}$. In [12] we addressed the Rician channel model developed here, where we considered the dominant component $E_{0,0}$ to consist of the phasor sum of the line-of-sight and a ground reflection.

The motion of the transmitter and receiver is evident in a Doppler shift in each wave component. Our model differs from the single-reflection (Rayleigh fading) model by Akki and Haber [7]. However, for certain simplifying approximations, and if the Rician K-factor of our model is chosen appropriately, both models lead to the same result. From the geometry of Fig. 1, these Doppler shifts are found as follows:

$$\begin{aligned} \omega_d &= \frac{2\pi}{\lambda} (V_R \cos \gamma_R - V_T \cos \gamma_T) \\ \omega_{Ri} &= \frac{2\pi}{\lambda} V_R \cos (\gamma_R - \alpha_{Ri}) \\ \omega_{Ti} &= \frac{2\pi}{\lambda} V_T \cos (\gamma_T - \alpha_{Ti}) \\ \hat{\omega}_{Ri} &= \frac{2\pi}{\lambda} V_R \cos (\gamma_R - \beta_{Ri}) \\ \hat{\omega}_{Ti} &= \frac{2\pi}{\lambda} V_T \cos (\gamma_T - \beta_{Ti}). \end{aligned} \quad (2)$$

Here V_T and V_R are the velocities of the transmitter and receiver, respectively, and γ_T and γ_R are the angles that the motion of transmitter and receiver make with the x axis. In a typical IVHS environment vehicles are following each other; thus $\gamma_T = \gamma_R = 0$. The received field can now be expressed as

$$\begin{aligned} E(t) &= I(t) \cos \omega_c t - Q(t) \sin \omega_c t \\ &+ E_{0,0} \cos [(\omega_c + \omega_d)t + \phi_{0,0}], \end{aligned} \quad (3)$$

where

$$\begin{aligned} I(t) &= \sum_{i=1}^{N_T} E_{i,0} \cos (\omega_{Ti} t + \phi_{i,0}) \\ &+ \sum_{k=1}^{N_R} E_{0,k} \cos (\omega_{Rk} t + \phi_{0,k}) \\ &+ \sum_{k=1}^{N_R} \sum_{i=1}^{N_T} E_{i,k} \cos [\omega_{Rk} t - \omega_{Ti} t + \phi_{i,k}], \end{aligned} \quad (4)$$

and

$$\begin{aligned} Q(t) &= \sum_{i=1}^{N_T} E_{0,0} \sin (\omega_{Ti} t + \phi_{i,0}) \\ &+ \sum_{k=1}^{N_R} E_{0,k} \sin \omega_{Rk} t + \phi_0, \\ &+ \sum_{k=1}^{N_R} \sum_{i=1}^{N_T} E_{i,k} \sin [\omega_{Rk} t - \omega_{Ti} t + \phi_{i,k}]. \end{aligned} \quad (5)$$

If N_T and N_R are sufficiently large, in theory infinite (in practice Bennet [13] has shown that greater than eight multipaths will suffice), the central limit theorem implies that both $I(t)$ and $Q(t)$ are jointly Gaussian random variables for a particular time t and the probability density of the angle of arrivals and departures is uniform between $(-\pi, \pi]$. If we assume that the separation distance between the two mobiles is much larger than the distance between the mobile and scattering objects, both $I(t)$ and $Q(t)$ are uncorrelated and thus independent [5]. The mean values of $I(t)$ and $Q(t)$ are both zero, the variance of $I(t)$ and $Q(t)$, or local-mean scattered power, is given by

$$\sigma^2 = E\left[\sum_{i=1}^{N_T} \frac{E_{i,0}^2}{2} + \sum_{k=1}^{N_R} \frac{E_{0,k}^2}{2} + \sum_{i=1}^{N_T} \sum_{k=1}^{N_R} \frac{E_{i,k}^2}{2} \right] \quad (6)$$

and they are jointly Rayleigh distributed. Following Rice [14] we find that the joint probability density function of the received amplitude, $r(t)$, and phase, $\theta(t)$, is

$$f_{r,\theta}(r, \theta) = \frac{r}{2\pi\sigma^2} \cdot \exp\left(\frac{-(r^2 - 2rE_{0,0}\cos(\theta - \omega_d t) + E_{0,0}^2)}{2\sigma^2}\right), \quad (7)$$

and thus the probability density function of the amplitude is given by

$$f_r(r) = \frac{r}{\sigma^2}\exp\left(\frac{-(r^2 + E_{0,0}^2)}{2\sigma^2}\right) I_0\left(\frac{rE_{0,0}}{\sigma^2}\right), \quad (8)$$

where $I_0(\cdot)$ is defined as the modified zero-order Bessel function of the first kind. We further define the Rician K-factor as the ratio of the power in the direct line-of-sight component to the local mean scattered power:

$$K = \frac{E_{0,0}^2}{2\sigma^2} \quad (9)$$

We define the local mean power as

$$\bar{p} = \frac{1}{2}E_{0,0}^2 + \sigma^2 = P_D + P_R + P_T + P_B \quad (10)$$

where P_D, P_T, P_R, and P_B are the portions of the local mean power in the dominant path, the waves that are scattered only near the transmitter, those scattered only near the receiver, and those scattered twice, respectively. The pdf of the signal envelope r can be expressed as

$$f_r(r) = \frac{r}{\bar{p}}(1 + K)\exp\left(\frac{-K(1 + K^2)r^2}{2\bar{p}}\right) \cdot I_0\left(r\sqrt{\frac{2K(1 + K)}{\bar{p}}}\right) \quad (11)$$

For the special case of sufficiently large antenna separation, we may further define Rician K-factors at the receiver and transmitter as the ratio of the power in the direct line-of-sight wave and the local mean scattered power at the receiver and transmitter respectively, with

$$K_R = \frac{D_0^2}{E\sum_{i=0}^{N_R} D_i^2} \quad (12)$$

and

$$K_T = \frac{C_0^2}{E\sum_{i=0}^{N_T} C_i^2} \quad (13)$$

The pdf of the signal envelope can be expressed in terms of these new Rician K-factors by making the following substitution of variables:

$$K = \frac{K_R K_T}{K_T + K_R + 1 + K_R K_T} \quad (14)$$

We note that the resulting fading is Rician, which is similar to the case of a line-of-sight component with reflections occurring only at one of the antennas. Reflections at both transmitter and receiver are subject to two Doppler shifts. This results in a larger variance in both the in-phase and quadrature field components, which is tantamount to an increase in the scattered man power.

3. RF SPECTRUM

The transmitted signal will be subject to Doppler shifts in the various paths. These Doppler shifts will tend to spread the bandwidth of the transmitted signal, which will be evident in the RF spectrum. The RF spectrum can be found by taking the Fourier transform of the temporal autocorrelation function of the electric field, the latter defined as

$$E[E(t)E(t + \tau)] \quad (15)$$

If we let

$$a(\tau) = E[I(t)I(t + \tau)] = E[Q(t)Q(t + \tau)] \quad (16)$$

and

$$c(\tau) = E[I(t)Q(t + \tau)] = -E[Q(t)I(t + \tau)] \quad (17)$$

then the autocorrelation can be expressed as [5]

$$E[E(t)E(t + \tau)] = a(\tau)\cos\omega_c\tau - c(\tau)\sin\omega_c\tau + E_{0,0}^2\cos(\omega_c + \omega_d)\tau \quad (18)$$

The following simplification can be made:

$$a(\tau) = P_R E[\cos\omega_T\tau] + P_T E[\cos\omega_T\tau] + P_B E[\cos(\omega_T\tau - \omega_R\tau)] \quad (19)$$

$$c(\tau) = P_R E[\sin\omega_R\tau] + P_T E[\sin\omega_T\tau] + P_B E[\sin(\omega_T\tau - \omega_R\tau)] \quad (20)$$

A critical assumption in [5] is that waves are modeled to arrive (or depart) from all angles in the azimuth plane with uniform probability density. For short-range vehicle-to-vehicle communication this assumption is less obvious than for macrocellular propagation environments.

If, for ease of analysis, the probability density functions for α_T and α_R are nonetheless modeled by independent uniform distributions in $(-\pi, \pi]$, we can now evaluate the above expectations as

$$a(\tau) = P_R J_0[2\pi f_{MR}\tau] + P_T J_0[2\pi f_{MT}\tau] + P_B J_0[2\pi f_{MT}\tau] J_0[2\pi f_{MR}\tau] \quad (21)$$

$$c(\tau) = 0 \quad (22)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind and f_{MR} and f_{MT} are the maximum Doppler shifts at the transmitter and receiver, respectively:

$$f_{MT} = \frac{V_T}{\lambda} \quad (23)$$

$$f_{MR} = \frac{V_R}{\lambda} \quad (24)$$

The fact that $c(\tau)$ is zero is a mathematical consequence of sin $(\cdot)$ being an odd function. Physically this result can be related to the fact that the RF spectrum is symmetric about f_c. In order to calculate the power spectral density of $I(t)$ and $Q(t)$, we must first find the Fourier transform of $a(\tau)$. The Fourier transform of the first two terms can be found as [15, p. 707]

$$F[P_R J_0[2\pi f_{MR}\tau] + P_T J_0[2\pi f_{MT}\tau]] = \frac{P_R}{2\pi\sqrt{f_{MR}^2 - f^2}}\Pi\left(\frac{f-f_c}{2f_{MR}}\right) + \frac{P_T}{2\pi\sqrt{f_{MT}^2 - f^2}}\Pi\left(\frac{f-f_c}{2f_{MT}}\right) \quad (25)$$

where $\Pi(f/x)$ is the rectangular pulse function centered at $f = 0$ with a width x and unity amplitude. The transform of the third term can be found from [15, p. 709] as

$$F[P_B J_0[2\pi f_{MR}\tau] J_0[2\pi f_{MT}\tau]] = \frac{P_B}{2\pi^2\sqrt{f_{MT}f_{MR}}} Q_{-1/2}\left(\frac{f_{MR}^2 + f_{MT}^2 - f^2}{2f_M Z_R f_{MT}}\right) \cdot \Pi\left(\frac{f-f_c}{2f_{MR} + =2f_{MT}}\right) \quad (26)$$

where $Q_{-1/2}(\cdot)$ is the Legendre function of the second kind. By using the identity [15]

$$Q_{-1/2}(x) = K\left(\sqrt{\frac{1+x}{2}}\right) \quad (27)$$

the above transformation can be written in terms of the complete elliptical integral of the first kind $K(\cdot)$ as

$$F[P_B J_0[2\pi f_{MR}\tau] J_0[2\pi f_{MT}\tau]] = \frac{P_B}{2\pi^2\sqrt{f_{MT}f_{MR}}} K\left(\sqrt{\frac{(f_{MR}+f_{MT})^2 - f^2}{4f_{MR}f_{MT}}}\right) \cdot \Pi\left(\frac{f-f_c}{2f_{MR} + 2f_{MT}}\right) \quad (28)$$

Setting $V_T = 0$, we get an expression analogous to that in [5, p. 969] for the baseband output spectrum from a square law detector. This output spectrum appears to be the convolution of the input spectrum with itself. This argument can be applied to our result. Namely the spectral contribution to the RF spectrum of the waves that undergo reflections at both receiver and transmitter can be viewed as the convolution of the spectral components that undergo reflections only at the receiver with the spectral components that undergo reflection only at the transmitter. Stated mathematically,

$$F[P_B J_0[2\pi f_{MR}\tau] J_0[2\pi f_{MT}\tau]] = P_B\{F(J_0[2\pi f_{MR}\tau]) \otimes F(J_0[2\pi f_{MT}\tau])\} \quad (29)$$

The RF spectrum can now be found by noting that $a(\tau)$ is modulated by cos $\omega_c\tau$, thus shifting the spectrum of $a(\tau)$ by the carrier frequency, and the direct line-of-sight wave will give rise to a delta function since this wave will only undergo a deterministic Doppler shift. Thus, the RF spectra can be written as

$$S_{RF}(f) = F\{a(\tau)\cos\omega_c\tau + E_{0,0}^2[\cos(\omega_c + \omega_d)\tau]\} \quad (30)$$

or

$$S_{RF}(f) = \frac{P_R}{2\pi\sqrt{f_{MR}^2 - (f-f_c)^2}}\Pi\left(\frac{f-f_c}{2f_{MR}}\right) + \frac{P_T}{2\pi\sqrt{f_{MT}^2 - (f-f_c)^2}}\Pi\left(\frac{f-f_c}{2f_{MT}}\right) + \frac{P_B}{2\pi^2\sqrt{f_{MT}f_{MR}}} K\left(\sqrt{\frac{(f_{MR}+f_{MT})^2 - (f-f_c)^2}{4f_{MR}f_{MT}}}\right)\Pi\left(\frac{f-f_c}{2f_{MR}+2f_{MT}}\right) + P_D 2\pi\delta(f - f_c - f_{MD}) \quad (31)$$

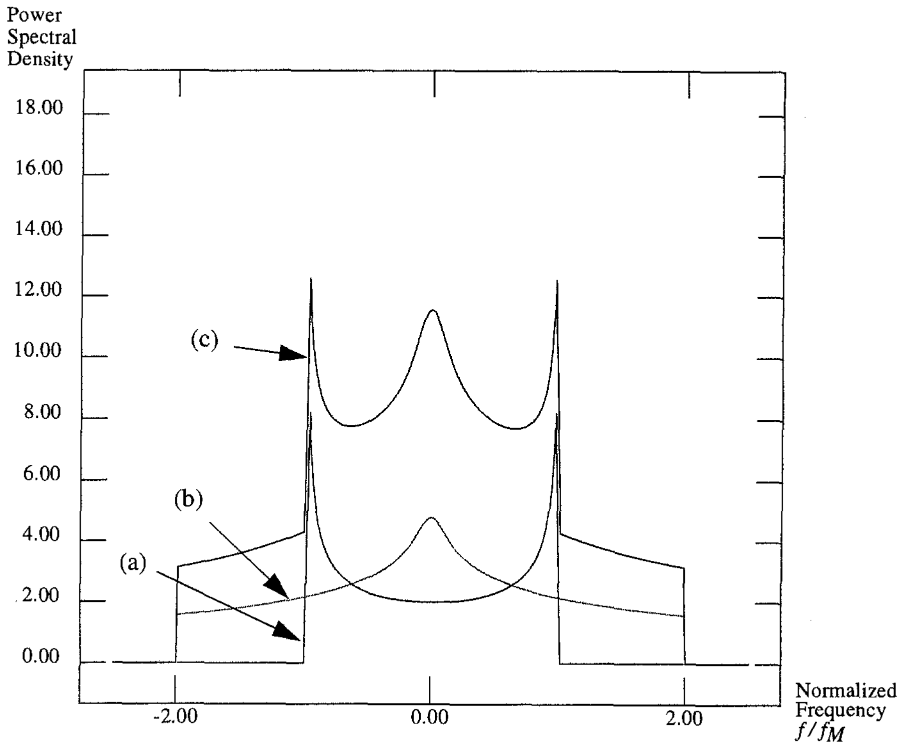

Fig. 2. RF spectra of mobile-to-mobile radio channel with equal transmitter and receiver velocities. (a) Reflections solely at either receiver or transmitter. (b) Reflections at both receiver and transmitter. (c) Summation of (a) and (b).

where the Doppler shift of the direct line-of-sight component is

$$f_{MD} = f_{MR} \cos \gamma_R + f_{MT} \cos \gamma_T \tag{32}$$

In Fig. 2, we see that the RF spectrum is centered around the carrier frequency and bandlimited to $2(f_{MT} + f_{MR})$, which is a direct consequence of the Doppler shift incurred by the movement of transmitter and receiver. The probability densities of α_R and α_T affect the shape of the spectrum inside this band. If we set $V_T = 0$, we do not obtain Clarke's spectrum for a mobile receiver and stationary transmitter. This is due to the fact that Clarke's model assumes no scattering at the transmitter. However, if we set the Rician factors K_T and K_R to zero, we obtain a spectrum analogous to that of Akki and Haber [7] for a Rayleigh fading channel with scatters at transmitter and receiver only.

4. MOMENTS OF POWER SPECTRAL DENSITY

The correlation functions $a(\tau)$ and $c(\tau)$ defined earlier can be expressed as inverse Fourier transforms of the power spectral density without the line-of-sight component as

$$a(\tau) = \int_{f_c-(f_{MT}+f_{MR})}^{f_c+(f_{MT}+f_{MR})} S_i(f) \cos [2\pi(f - f_c)\tau] \, df \tag{33}$$

$$c(\tau) = \int_{f_c-(f_{MT}+f_{MR})}^{f_c+(f_{MT}+f_{MR})} S_i(f) \sin [2\pi(f - f_c)\tau] \, df \tag{34}$$

where

$$S_i(f) = S_{RF}(f) - E_{0,0}^2 \pi\delta(f - f_c - f_{MD}). \tag{35}$$

We saw earlier that $c(t)$ was zero for all t. This can further be explained from the above equation since the RF spectrum is an even function, and sin $(\cdot)$ is odd. These autocorrelations evaluated at zero will give expressions for the moments of the power spectrum. Following Jakes [6],

$$\begin{aligned} E[I^2(t)] &= E[Q^2(t)] = a(0) = b_0 = P_B + P_T + P_R \\ E[I(t)Q(t)] &= c(0) = 0 \\ E[I(t)\dot{I}(t)] &= E[Q(t)\dot{Q}(t)] = \dot{a}(0) = 0 \\ E[I(t)\dot{Q}(t)] &= -E[\dot{I}(t)Q(t)] = \dot{c}(0) = b_1 = 0 \\ E[\dot{I}^2(t)] &= E[\dot{Q}^2(t)] = -\ddot{a}(0) = b_2 \\ &= \tfrac{1}{2}(P_T \omega_{MT}^2 + P_R \omega_{MR}^2 + P_B(\omega_{MT} + \omega_{MR})^2) \end{aligned} \tag{36}$$

where dots represent differentiation with respect to time. Thus $b_n = 0$ for all odd n, again due to the symmetric nature of $S_{RF}(f)$. The moments of the power spectrum for n even can be generalized as

$$b_n = \left(\frac{1 \cdot 3 \cdot 5 \ldots (n-1)}{2 \cdot 4 \cdot 6 \ldots n}\right) \cdot (P_T \omega_{MT}^n + P_R \omega_{MR}^n + (P_B(\omega_{MT} + \omega_{MR}))^n. \quad (37)$$

Using the results of Eq. (37) we will now investigate the pdf's of derivatives of the in-phase and quadrature components. The in-phase and quadrature components and their derivatives are zero mean jointly Gaussian. The covariance matix can be expressed as

$$V = \begin{bmatrix} a(0) & c(0) & \dot{a}(0) & \dot{c}(0) \\ c(0) & a(0) & -\dot{c}(0) & \dot{a}(0) \\ \dot{a}(0) & -\dot{c}(0) & -\ddot{a}(0) & \ddot{c}(0) \\ \dot{c}(0) & \dot{a}(0) & \ddot{c}(0) & -\ddot{a}(0) \end{bmatrix} = \begin{bmatrix} b_0 & 0 & 0 & 0 \\ 0 & b_0 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_2 \end{bmatrix} \quad (38)$$

so

$$V^{-1} = \left(\frac{1}{b_0^2 b_2^2}\right) \begin{bmatrix} b_0 b_2^2 & 0 & 0 & 0 \\ 0 & b_0 b_2^2 & 0 & 0 \\ 0 & 0 & b_0^2 b_2 & 0 \\ 0 & 0 & 0 & b_0^2 b_2 \end{bmatrix} \quad (39)$$

The joint pdf can be written as [14, 16]

$$f_{I,Q,\dot{I},\dot{Q}}(I, Q, \dot{I}, \dot{Q}) = \frac{1}{4\pi^2 b_0 b_2} \exp\left[\frac{-1}{2b_0 b_2}[b_2(I^2 + Q^2) + b_0(\dot{I}^2 + \dot{Q}^2)]\right]. \quad (40)$$

The in-phase and quadrature components can be expressed in terms of an amplitude r and phase θ, as follows

$$\begin{aligned} I(t) &= r \cos \theta - E_{0,0} \cos (\omega_d t + \phi) \\ Q(t) &= r \sin \theta - E_{0,0} \sin (\omega_d t + \phi) \\ \dot{I}(t) &= \dot{r} \cos \theta - r\dot{\theta} \sin \theta \\ \dot{Q}(t) &= \dot{r} \sin \theta - r\dot{\theta} \cos \theta \end{aligned} \quad (41)$$

The joint pdf of the amplitude, phase, and derivatives can be expressed as

$$f_{r,\theta,\dot{r},\dot{\theta}}(r, \theta, \dot{r}, \dot{\theta}) = \frac{r^2}{4\pi^2 b_0 b_2} \exp\left[\frac{-1}{2b_0 b_2}[b_2 r^2 - 2rE_{0,0} \cdot \cos (\omega_d t + \theta) + E_{0,0}^2] + b_0(\dot{r}^2 + r^2\dot{\theta}^2)\right] \quad (42)$$

If we uncondition this expression over the phase and both derivatives, we obtain the same expression for the pdf of the signal envelope derived earlier (with $b_0 = \sigma^2$).

5. LEVEL CROSSING RATE AND AVERAGE FADE DURATION

The fading of the signal envelope was evident in the derivation of the probability density function of the envelope. From this pdf we can obtain an expression for the overall percentage of time that the envelope lies below a certain level and on average how long these fades last. We are also interested in finding the rate at which the envelope crosses a particular level R. These expressions would thus provide parameters in selecting transmission bit rates, word lengths, and coding schemes. The level crossing rate N_R is defined as the expected number of times per second that the envelope crosses R in the positive direction. Rice [14] gives this value as

$$N_R = \int_0^\infty \dot{r} f_{r,\dot{r}}(R, \dot{r})\, d\dot{r} \quad (43)$$

Thus, we must first find the joint pdf of the envelope and its derivative. This can be derived by integrating the phase and its derivative over the joint pdf derived earlier.

$$\begin{aligned} f_{r,\dot{r}}(r, \dot{r}) &= \int_{-\infty}^{\infty} \int_0^{2\pi} f_{r,\theta,\dot{r},\dot{\theta}}(r, \theta, \dot{r}, \dot{\theta})\, d\theta\, d\dot{\theta} \\ &= \frac{r}{b_0} I_0\left(\frac{rE_{0,0}}{b_0}\right) \exp\left[\frac{-(r^2 + E_{0,0}^2)}{2b_0}\right] \\ &\times \frac{1}{\sqrt{2\pi b_2}} \exp\left(\frac{-\dot{r}^2}{2b_2}\right) \end{aligned} \quad (44)$$

We see that since both the envelope and its derivative are independent and thus uncorrelated, their joint pdf can be expressed as the product of individual pdf's. Thus, the derivative of the envelope is zero mean

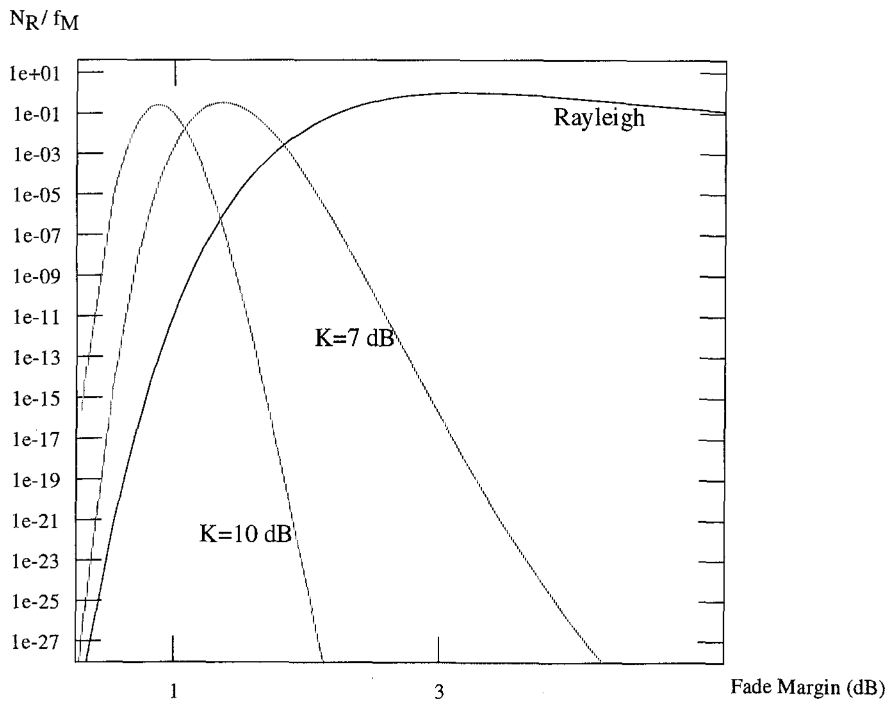

Fig. 3. Normalized level crossing rate vs. fade margin for various Rician K-factors. Equal transmitter and receiver velocities. Equal Rician K-factors K_R and K_T. Maximum Doppler shift for both antennas, f_M. $K = -30$ dB approximates Rayleigh fading.

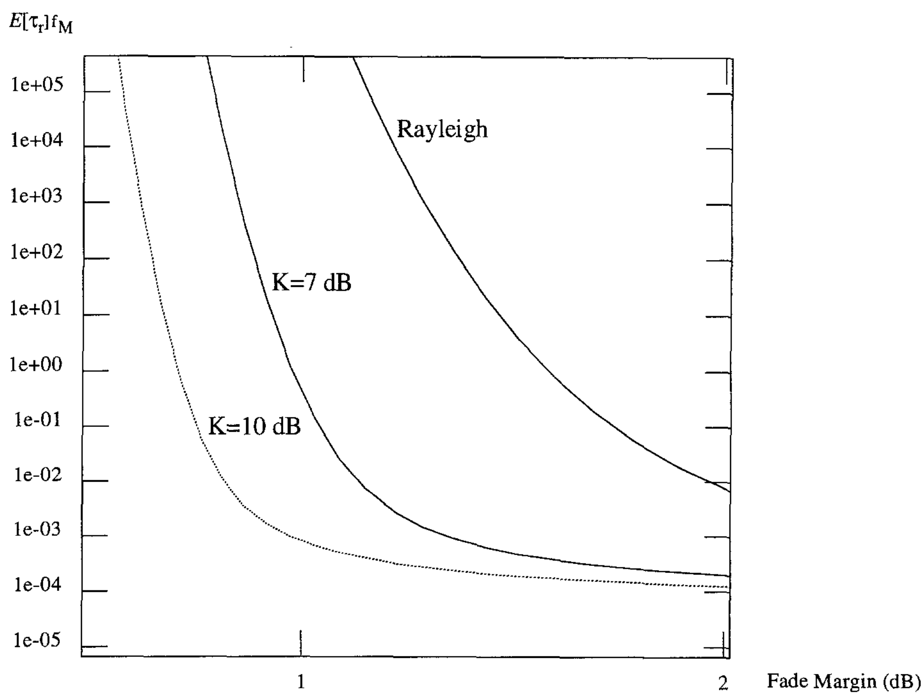

Fig. 4. Average fade duration vs. fade margin for various K-factors. Equal transmitter and receiver velocities. Equal Rician K-factors K_R and K_T. Maximum Doppler shift for both antennas, f_M.

Gaussian with a variance of b_2 and the pdf of the envelope is the same as before. The level crossing rate can now be expressed as

$$N_R = \frac{R}{b_0}\sqrt{\frac{b_2}{2\pi}}\, I_0\left(\frac{RE_{0,0}}{b_0}\right) \exp\left[\frac{-(R^2 + E_{0,0}^2)}{2b_0}\right] \quad (45)$$

We further define the fade margin as the ratio of the mean signal power to the specified level, R. Figure 3 plots the normalized level crossing rate, N_R/f_M or level crossings per wavelength, for various Rician K-factors, where we approximated Rayleigh fading by $K = 0.001$ (-30 dB).

Another important statistical measure of the envelope is the average fade duration. The fade duration, τ, below a specified level R, is defined as the period of fade below this level. The overall fraction of time for which the signal is below a specified level R is given by the cumulative distribution function, $F_r(R)$, of the received signal envelope. This function is obtained by integrating over the pdf the envelope.

$$F_r(R) = \int_0^R f_r(r)\, dr \quad (46)$$

The average fade duration can now be expressed as [17]

$$E[\tau_r] = \frac{F_r(R)}{N_R} \quad (47)$$

Figure 4 plots normalized average fade durations for various Rician K-factors.

6. CONCLUSIONS

A statistical model for a mobile-to-mobile channel has been presented that extends the work on Rayleigh fading channels, without a line-of-sight component, by Clarke [5], Jakes [6], and Akki and Haber [7]. The channel model examines multipath fading and Doppler shifts for a narrow-band signal taking into consideration scatters at both receiving and transmitting antennas both individually and concomitantly, as well as a strong line-of-sight component between antennas. This is in contrast to the model by Akki and Haber [7], which focused on single reflections, ignoring line-of-sight or double reflections. A proposed simplified model loses some of its accuracy when the transmitting and receiving antennas become too close. This suggests that the special case addressed here for independent scattering at the transmitter and receiver may be better acceptable for platoon-to-platoon communications, but needs further verification or it may need refinements for communication within a platoon.

Even with scatters at both transmitter and receiver, the line-of-sight component causes the fading to be Rician, with a new Rician K-factor that is a function of the K-factors at the transmitter K_T and receiver K_R. As K_T and K_R approach zero, we obtain results analogous to Akki and Haber [7] for Rayleigh fading. Also Doppler spreads, for vehicles traveling at roughly the same speed, are twice as large as those reported in textbooks like [6]. Thus, the fading has components that are twice as fast. We interpret from our curves that fade durations longer than 50 ms diminish rapidly with fade margin. Shladover *et al.* [4] have shown that this is of particular importance to AVCS.

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