ONE DIMENSIONAL CELLULAR NETWORK WITH "SPATIAL-ALOHA" PROTOCOL

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1 Abstract

We address the transmission of data packets from base stations to mobile terminals in a one-dimensional cellular network. The probability that a data packet successfully arrives at the destined mobile terminal highly depends on the activity of nearby co-channel base stations. Motivated by favorable results from earlier studies we consider a network in which all base stations use the same radio channel. Packets lost because of mutual interference are retransmitted with a random backoff mechanism. Recursive arguments are applied for the calculation the mean time necessary for a base station to deliver the packet, provided that in an initial state all stations have packet to transmit. The average delivery time, maximized over all base station locations, is also analyzed. The relations obtained give some further insight into behavior of a (highway) cellular data network.

2 Introduction

In this paper, we address the cellular data networks. We consider a high-way environment with a one-dimensional (linear) cell lay-out. Base stations transmit data packets of fixed length to users. Typically such networks are designed with a certain frequency reuse pattern, such that adjacent cells do not share the same frequency. Although this avoids mutual interference among cells, frequency reuse reduces the bandwidth available in each individual cell. Earlier investigations [1, 2] revealed that it is preferable to use the same (full) bandwidth in each cell. This allows base stations to transmit their messages at fast bit rates, thus leaving the channel idle during large periods of time. Other cells can use the bandwidth interference-free during these periods. However, due to channel fading, harmful collisions inevitably occur, both in networks with all cells using the same channel and in networks with large frequency reuse factors. This motivated us to develop methods that can dynamically assign spectrum resources (time slots) to cells, depending on instantaneous needs.

Particularly if dense reuse is applied, interference among cells may cause instability and avalanches of retransmisions [1]. Interference causes packets to be lost. This leads to retransmissions, which causes more interference and new packet losses. Transmitting at the full bandwidth in every cell hence requires measures to resolve 'collisions' in the spatial domain, as opposed to the conventional collision resolution that operates in a single cell only.

Previous work, e.g. [2], proposed 'spatial resolution schemes' to resolve collisions among packet transmitted in adjacent cells. In one such scheme, all base stations share the same transmit channel which has frames of a certain fixed number of time slots. During normal operation, each station may transmit during any time slot. With some probability a harmful collision occurs, and the interfering power is too strong to allow successful receiver capture. In this case the odd-numbered base stations retransmit in odd slots while all even-numbered stations are silenced [2]. Similarly, even-numbered terminals retransmit in even time slots. The coordination (such as switching from free access to collision-resolution mode) is performed by sending instructions through the fixed backbone infrastructure that connects all base stations. Such algorithms solve the interference problem dynamically, so during periods of low load, it will be possible for all stations to transmit at the full bandwidth, thus with minimal delays. Moreover during heavy load, spectrum resources are used efficiently because base stations are only silenced when needed.

To further exploit the benefits of dense frequency reuse under heavy but non-uniform traffic loads, Walrand and Litjens [3] proposed a packet-switched scheme transmitting packets from roadside base stations to vehicles. Arrival processes in the cells were assumed to be homogeneous Poisson processes but with non-identical arrival rates. The proposed queueing and scheduling algorithms assign permissions to transmit to the base stations in a conflict-free manner: when a station is ac-

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tivated, its neighbors are silenced, in order to solve the problem of collisions.

In this paper, we address a system in which base stations do not exchange information about their activity to neighbors. That is, packets lost because of interference have to be retransmitted by trial and error. Our intention is to provide basic results on how fast collisions between base station can be resolved. We call our scheme "spatial slotted ALOHA" because similar to the ALOHA protocol[4], transmitters are allowed to use any time slot, regardless of activity of other transmitters. However, if interference occurs, only transmitters in spatially nearby cells suffer, so the collision has to be resolved only locally.

Our approach thus significantly differs from conventional narrowband cellular networks, which mostly use a fixed cellular frequency reuse pattern, and within each cell a random-access scheme, e.g. ALOHA, is used independently of the traffic characteristics in other cells.

We consider a one-dimensional (linear) cellular network as for instance encountered in Intelligent Vehicle Highway Systems. Cells are likely to carry equal amounts of teletraffic. A base station (BS) transmits packets to mobile users within its cell. The carrier frequency is the same for all base stations.

A user terminal receives transmissions of its own cell and interference signals from other BSs. If a terminal successfully detects a packet from its BS, it sends an acknowledgment to the BS. Otherwise, no acknowledgment is sent. The uplink channel is assumed to be perfect so that acknowledgments are always received by the BS.

BSs are synchronized such that packet transmissions are initiated at time instants $t = 0, 1, \ldots$. A time interval $[t,\ t+1)$ is called a slot t. We assume that the packet transmission time is equal to the slot duration. Cells are numbered sequentially from left to right, the left most cell has number 1. Let M be the number of cells in the system (or briefly, the system size). Cells $i-1, i+1$ are called adjacent to cell i, $1 < i < M$. We only take into account interference from nearest neighboring stations. For example, transmissions by BS i interfere with these of BSs $i-1$ and $i+1$.

A mobile terminal successfully receives the transmission of its BS with probability c_n, where n denotes the number of simultaneously transmitting BSs in the adjacent cells, $n = 0, 1, 2$. Typical values of c_n depend on channel properties, modulation methods etc. [1, 2]. Taking into account both packet collisions and packet corruption by channel noise, we suggest that c_0 is close to one, c_1 is about 0.5, and c_2 about 0.3.

3 Collision resolution

Suppose each BS has only one packet to transmit at time $t = 0$ (full load initial condition). This (pessimistic) full-load initial condition is assumed throughout this paper. Each BS transmits its packet with probability p in slots $0,\ 1, \ldots$ until the packet is successfully received. BSs transmit independently of one another. All BSs have the same retransmission probability; the system is homogeneous.

The output time of the given BS is the time from the initial moment $t = 0$ to the eventual successful transmission of its packet. By $d_M(i)$ we denote the mean output time of the i-th BS for the M cell system.

A BS that has successfully transmitted its packet is silent in subsequent slots. The silent BS makes the part of the system to the left of it to be independent of the part which is to the right of it. Let α be an integer constant, $0 \leq \alpha \leq M-1$. We shall refer to a group of BSs with numbers $\alpha+1, \ldots, \alpha+m$ as a fragment of length m if they all have packets to transmit and BSs $\alpha,\ \alpha+m+1$ are silent. For the purpose of analysis we suppose that there exist two subsidiary cells with numbers 0 and $M+1$ respectively. The first one is the left adjacent to cell 1 and the second is the right adjacent to cell M. The subsidiary BSs are always silent. Now we can refer to the whole system as a fragment of length M.

To derive an equation for $d_M(i)$, we generalize the output time to a fragment of the given length. Suppose BSs $\alpha+1, \ldots, \alpha+n$ form the n cell fragment at time moment t; then by $\tau_n(i)$, $1 \leq i \leq n$ we denote the time from moment t to the end of successful transmission of BS $\alpha+i$. Random variable $\tau_n(i)$, which is called the output time of BS i for n cell fragment, is independent of neither α nor t. Similarly to $d_M(i)$, we denote by $d_n(i)$ the mean output time of BS i for the n cell fragment.

If we assign numbers to fragment cells from right to left such that the rightmost BS has number 1, then we get the fragment identical to the original one. From this it follows that

$$d_n(i) = d_n(n-i+1). \qquad (1)$$

We say that a station is in state "1" in a slot t if it transmits in the slot t. Otherwise, the station is in state "0". Consider a group of sequential BSs with numbers $\alpha,\ \alpha+1, \ldots, \alpha+n+1$. It is assumed that all these BSs have a packet to transmit at moment t. Let i, j, k, l be the states of the BSs $\alpha,\ \alpha+1,\ \alpha+n,\ \alpha+n+1$, respectively.

Suppose A_t is the event {no one of BSs $\alpha+1, \ldots, \alpha+n$ gets successful transmission during the slot t}, B_t is the event {BSs $\alpha+1,\ \alpha+n$ are in states j, k, respectively during the slot t}, and D_t is the event {BSs $\alpha,\ \alpha+n+1$

are in states i, l, respectively during the slot t}. By $f_n(i,j,k,l)$ we denote the joint conditional probability $\Pr\{A_t \bigcap B_t \mid D_t\}$, which is independent of the time t. From identity of two possible cell enumerations it follows that

$$f_n(i,j,k,l) = f_n(l,k,j,i). \quad (2)$$

In the boundary case $n = 1$, we put

$$f_1(i,j,k,l) = 0,\ j \neq k.$$

In what follows $\overline{p} = 1 - p$; $\overline{c}_i = 1 - c_i$; the set $\{0, 1\}$ is denoted by I_2.

Proposition 1 *The function $f_n(i,j,k,l)$, $n > 0$, satisfies the boundary conditions*

$$f_1(i,0,0,l) = \overline{p},\ i \in I_2,\ l \in I_2,$$

$$f_1(i,1,1,l) = p\overline{c}_{i+l},\ i \in I_2,\ l \in I_2,$$

and the recursion relations

$$f_n(i,0,k,l) = \overline{p}(f_{n-1}(0,1,k,l) + f_{n-1}(0,0,k,l)), \quad (3)$$

$$f_n(i,1,k,l) = p(\overline{c}_i f_{n-1}(1,0,k,l) + \overline{c}_{1+i} f_{n-1}(1,1,k,l)), \quad (4)$$

$$i \in I_2,\ k \in I_2, l \in I_2.$$

Proof is omitted.

Proposition 2 *The mean output time $d_M(i)$ of BS i is given by the recurrence relation*

$$d_m(i) = 1 + \sum_{l=i+1}^{m+1} \sum_{k=0}^{i-1} a_{l,k} d_{l-k-1}(i-k), m = 1,2,...,M, \quad (5)$$

where

$$a_{l,k} = \sum_{\alpha=0}^{1} \sum_{\beta=0}^{1} p^{\theta_k}(c_{\alpha+1} r_{k-1} + c_\alpha \overline{r}_{k-1})^{\theta_k}$$

$$\cdot f_{l-k-1}(\theta_k, \alpha, \beta, \theta_l) p^{\theta_l}(c_{\beta+1} r_{l+1} + c_\beta \overline{r}_{l+1})^{\theta_l},$$

$$r_i = p\theta_i,\ \overline{r}_i = 1 - r_i,$$

$$\theta_i = \begin{cases} 1, & 0 < i < m+1, \\ 0, & i \leq 0 \ or\ i \geq m+1. \end{cases}$$

Proof is omitted.

The behavior of $d_M(i)$ as a function of station number i is illustrated in Fig.1 for $p = 0.5$, $c_0 = 1$, $c_1 = 0$, $c_2 = 0$ and different system sizes M.

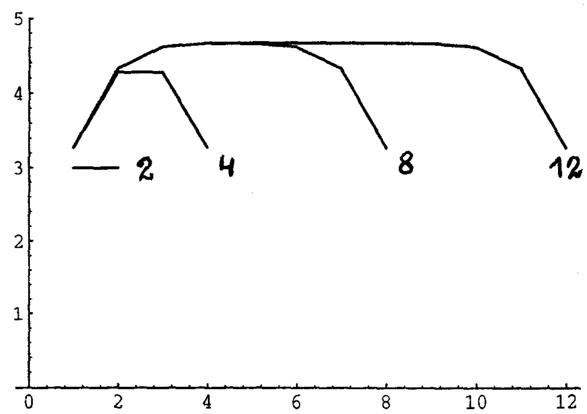

Figure 1: Mean output time as a function of BS number for $M = 2, 4, 8, 12$.

As one might expect, the output time is minimal for edge cells $i = 1$ and $i = M$. These cells have only one adjacent neighbor that produces interference signals. For M greater than 10 only 4 - 5 stations on both edges of the system have preferable positions in the sense that they spend less time for successful transmission of packet.

To investigate the asymptotic behavior of $d_M(i)$ for large M, we consider a linear cellular system with infinite number of cells. The mean output time of a BS under the full load initial condition is independent of its number and denoted by d_∞.

Proposition 3 *The mean output time d_∞ is written in the form of series*

$$d_\infty = 1 + \sum_{i=1}^{\infty}[a'_{2i} d_{2i-1}(i) + 2a'_{2i+1} d_{2i}(i)$$

$$+2\sum_{k=1}^{i-1}(a'_{2i} d_{2i-1}(k) + a'_{2i+1} d_{2i}(k))], \quad (6)$$

where

$$a'_i = \sum_{\alpha=0}^{1} \sum_{\beta=0}^{1} p^2(c_{\alpha+1} p + c_\alpha \overline{p})$$

$$\cdot f_{i-1}(1, \alpha, \beta, 1)(c_{\beta+1} p + c_\beta \overline{p}).$$

Proof is omitted.

The series (6) is found to have quite slow convergence. For the partial sum of series (6) to reach the value of $d_8(4)$ we had to take into account $d_m(i)$ with m up to 65. The value of d_∞ calculated with a precision of three significant digits coincides with the mean output time of centrally located BSs for $M = 12$. It means that for centrally located BSs $d_m(i)$ is in fact independent of M for $M \geq 12$.

The behavior of the mean output time as a function of the retransmission probability p is determined by the capture parameters c_0, c_1 and c_2. To show this we consider two systems: the first has the noiseless channel without capture $c_0 = 1$, $c_1 = c_2 = 0$ and the second has high capture probabilities $c_0 = 0.95$, $c_1 = 0.9$, $c_2 = 0.8$. In both cases, the system size M is equal to 9 and the station under consideration has number 5.

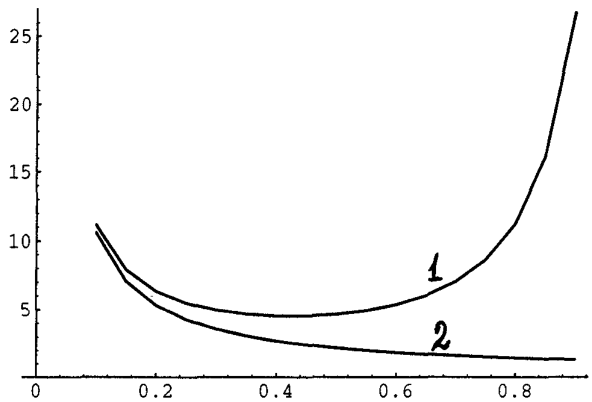

Figure 2: Mean output time as a function of p.

In the case of high capture, the optimal retransmission probability is equal to one. It is not true for the system with low capture. Both small p and p close to unity give long collision resolution periods. There exists a value of p for which the output time is minimal. This value, denoted by p_{opt}, is equal to 0.425 in case one.

The probability p_{opt} is higher for edge stations and lower for central stations. Thus, in homogeneous system there does not exist a value of p, for which all stations have minimal output time. The rigorous choice of optimal probability can be done on the basis of a performance measure of the whole system.

The mean output time makes possible the evaluation of BS throughput. Since $d_M(i)$ is the mean time for station i to transmit the packet successfully, the throughput of BS i is approximately equal to $\frac{1}{d_M(i)}$. A more exact result for the BS throughput is obtained in the next section.

4 The mean session length

The mean session length T_M is defined by the formula

$$T_M = E \max_{1 \leq i \leq M} \tau_M(i),$$

where E is the sign of mathematical expectation and $\tau_M(i)$ is the random output time of BS i for the M cell system. A peculiarity of the session length is that it characterizes the behavior of the whole system in contrast to the output time, which characterizes the given station.

To derive T_M, we generalize the notion of mean session length. Let T_m, $1 \leq m \leq M$, be the mean session length of the m cell fragment, i.e., the mean time from a slot in which this fragment exists to the end of slot in which the last BS of the fragment successfully transmits its packet. Let $Q_{m,n}$, $m \geq 1$, $0 \leq n \leq m$ be the probability of the event that a m cell fragment which exists at time t will split by the time $t+1$ into a set of fragments with maximal length equaled to n.

Proposition 4 *The mean session length T_M is given by the recurrence relation*

$$T_m = 1 + \sum_{n=1}^{m} Q_{m,n} T_n, \ 1 \leq m \leq M. \qquad (7)$$

Proof is omitted.

To find $Q_{m,n}$, we consider a group of $m+2$ sequential BSs with numbers α, $\alpha+1, ..., \alpha+m+1$, where $\alpha \geq 0$, $\alpha+m+1 \leq M$. BSs $\alpha+1, ..., \alpha+m$ have packets at a time t. Let i, j, k, l be the states of the BSs α, $\alpha+1$, $\alpha+m$, $\alpha+m+1$, respectively, in slot t. Suppose G_t is the event {the length of the greatest fragment by the moment $t+1$ is equal to n }. As in section 2, B_t is the event {BSs $\alpha+1$, $\alpha+m$ are in states j, k, respectively during the slot t} and D_t is the event {BSs α, $\alpha+m+1$ are in states i, l, respectively during the slot t}. By $g_{m,n}(i,j,k,l)$ we denote the joint conditional probability $\Pr\{G_t \bigcap B_t \mid D_t\}$, which is independent of the time t and constant α. It is obvious that

$$Q_{m,n} = \sum_{j=0}^{1} \sum_{k=0}^{1} g_{m,n}(0,j,k,0).$$

Proposition 5 *The probability $g_{m,n}(i,j,k,l)$, $m \geq 1$, $0 \leq n \leq m$, satisfies the boundary conditions*

$$g_{m,0}(i,j,k,l) = 0 \ \textit{if}\ jk = 0, \ i \in I_2, \ l \in I_2,$$

$$m \geq 1,$$

$$g_{1,0}(i,1,1,l) = pc_{i+l}, \ i \in I_2, \ l \in I_2,$$

$$g_{m,0}(i,1,1,l) = p^m c_{i+1} c_2^{m-2} c_{1+l}, \ i \in I_2, \ l \in I_2,$$

$$m \geq 2,$$

$$g_{m,m}(i,j,k,l) = f_m(i,j,k,l), \ i \in I_2, \ j \in I_2, \ k \in I_2, \ l \in I_2, \ m \geq 1,$$

and the recursion relations

$$g_{m,m-1}(i,j,k,l) = \delta_{j,1} p \sum_{u=0}^{1} c_{i+u} f_{m-1}(1,u,k,l)$$

$$+\delta_{k,1} p \sum_{w=0}^{1} f_{m-1}(i,j,w,1) c_{w+l},$$

$$i \in I_2, \ j \in I_2, \ k \in I_2, \ l \in I_2, \ m \geq 2,$$

where $\delta_{i,j}$ *is the Kronecker delta; for all other possible pairs* (m,n)

$$g_{m,n}(i,j,k,l) = \delta_{j,1} p \sum_{u=0}^{1} c_{i+u} g_{m-1,n}(1,u,k,l)$$

$$+ \sum_{v=1}^{\min(n-1,m-n-1)} \sum_{w=0}^{1} f_v(i,j,w,1) p \sum_{u=0}^{1} c_{w+u}$$

$$\cdot g_{m-v-1,n}(1,u,k,l) + \sum_{w=0}^{1} \sum_{u=0}^{1} \sum_{s=0}^{\min(n,m-n-1)} c_{w+u}$$

$$\cdot f_n(i,j,w,1) p g_{m-n-1,s}(1,u,k,l), \quad (8)$$

$$i \in I_2, \ j \in I_2, \ k \in I_2, \ l \in I_2.$$

Proof is omitted.

The relations obtained enable us to calculate the mean session length. The mean session length is an example of criteria which can be used for finding the optimal retransmission probability p. Since the session length is a majorant for a BS output time, the inverse $\frac{1}{T_M}$ gives us a lower bound on a BS throughput. It means that the network is stable if the input traffic per each station is less than $\frac{1}{T_M}$.

5 Conclusions

The behavior of linear cellular networks with the SPATIAL ALOHA protocol has been evaluated in terms of the mean output time for a base station with given location in a one-dimensional cell lay-out, and the mean session length. It is found that only 4-5 edge stations at both ends of a linear network have privileged positions in the sense that they spend less time to transmit a packet successfully than the centrally located stations.

In the case of signals that are prone to any interference from adjacent cells ($c_0 = 1, c_1 = c_2 = 0$), there is an optimal value of retransmission probability which maximizes the throughput of the system. The optimal retransmission probability and the corresponding mean output time can be evaluated from recurrence relations obtained. The latter is about 4 packet lengths. This is greater than the mean output time for a linear network with the alternating protocol according to which permission to transmit is given to all stations with even numbers and then to all stations with odd numbers and so on. Lower mean output time for the alternating protocol is obtained at the expense of total network synchronization.

With better receiver capture performance, significantly less time needed to resolve a 'global collision' involving all base stations. For the system with $c_0 = 0.95$, $c_1 = 0.9$, $c_2 = 0.8$ this time can be made as small as 1.2 by choosing the optimal value of protocol parameter p. This is less than two slots, which would be need in a fixed 2-cell reuse pattern.

Moreover, the system addressed by our calculations has more bandwidth available if one cell has a (temporarily) large traffic load while neighboring cells are idle.

Further investigations of linear cellular networks can incorporate nonhomogeneous cells with the protocol parameter p dependent on the base station number, interaction between more distant base stations and more realistic arrival models. The expressions derived here for collision resolution times is a useful basis for such investigation.

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