the widely differing values of penetration factors obtained, and extend the work to cover data traffic, but will also hopefully thereby provide some insight into future international telephony and data traffic in Europe.

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## Optimising delay and throughput in packetswitched CDMA network with collision resolution using stack algorithm

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Indexing terms: Code division multiple access, Packet radio networks

The authors address the effect of code division multiple access (CDMA) on the performance of wireless packet-switched networks. Although CDMA can enhance throughput, the delay at small traffic loads suffers from bandspreading. For imperfect signal separation at the receiver, performance benefits from CDMA may be lost. The capacity of the particular stack algorithm addressed for collision resolution can be enhanced from 0.32 to ~0.40 if a 10dB spreading gain with perfect signal separation is employed.

Introduction: In multiuser wireless networks, substantial effort is being made to use the scarce radio spectrum in the most efficient way. Approaches to enhance the performance of packet-switched networks with bursty traffic can employ advanced transmission techniques such as code division multiple access (CDMA) [1] and advanced medium access protocols, such as the tree or stack algorithms for collision resolution [2–5]. This Letter addresses networks involving both CDMA transmission and collision resolution schemes.

Stack algorithm: We address a slotted multiaccess channel with an infinite number of terminals sending packets to a central receiver. New packet arrivals form a Poisson process with mean  $\lambda$  packets per second. The message length is L bits and the total system bandwidth is  $B_C$ . For a system with modulation compactness  $\eta$  chip/s/Hz and spreading gain C, the packet duration and slot time is  $T_L = C_L/(\eta B_C)$ . The message arrival rate, expressed in packets per time slot becomes  $\mu = \lambda C_L/(\eta B_L)$ . The slot beginning at  $t = nT_L$  is called slot n. Let K denote the number of packets transmitted in a slot, then with probability  $q_{k,K}$ , only k of these packets are received successfully. CDMA with multisignal detection ideally achieves k = K if  $K \leq C$ , while k = 0 if K > C. Under such conditions, CDMA enhances the throughput of ALOHA [1]. However, even ideal CDMA with finite C cannot avoid that, with probability 1, the backlog and delay of slotted ALOHA eventually grows beyond any finite bound. Such instability can be overcome by col-

lision resolution algorithms, such as the stack algorithm [3–5]. Stack algorithms differ from ALOHA in that unsuccessful packets use feedback information about whether the last slot was *idle* (K = 0), a *capture* (k > 0), or a *conflict* (k = 0; K > 0). Associated with the message buffer is a stack counter  $l_n$  which changes from slot to slot according to the following rules [5]:

(i) a packet transmitted in slot *n* for the first time has  $l_n = 0$ 

(ii) when  $l_n = 0$  this packet will be transmitted in slot *n*; when  $l_n > 0$ , the packet is not transmitted

(iii) when  $l_n = 0$  and a conflict is reported in slot *n*, then  $l_{n+1} = 1$  with probability 1/2 and  $l_{n-1} = 0$  with probability 1/2

(iv) when  $l_n \ge 0$  and a capture is reported, then  $l_{n-1} = l_n$ 

(v) when  $l_n > 0$  and slot *n* is reported idle, then  $l_{n+1} = l_n - 1$ 

(vi) when  $l_n > 0$  and a conflict is reported, then  $l_{n+1} = l_n + 1$ 

We compare perfect CDMA with a receiver that sees multiuser interference. We assume that the radio channel is linear, time-invariant, frequency nonselective with signal to (additive white Gaussian) noise ratio  $\gamma$ . The simplification is often made to model interference as Gaussian noise attenuated by the spread factor C; then the bit error rate given K - 1 competing packets of equal power is

$$P_{s|K} = \frac{1}{2} \operatorname{erfc} \sqrt{\frac{\gamma}{\frac{K-1}{C}\gamma + 1}}$$
(1)

From this conditional BER we find the probability  $P_{s|K}$  that a packet of L bits is received with not more than t bit errors. Using  $P_{s|K}$  as the success probability for an individual packet, we derive  $q_{k,K}$ .

The delay in stack is taken here as the time elapsed between the first transmission attempt of a packet and the start of its successful transmission. We define a session as the time interval between two successive moments  $n_1 < n_2$  such that for some packet  $l_{n1} = 1$ and  $l_{n2} = 0$  for the first time. The expected length of a session with K packets transmitted in slot  $n_1$  is denoted as  $h_K$ . The unconditional expected length h of a session starting from an empty system is  $h = \sum p_K h_K$  with  $p_K = \mu^K \exp(-\mu)/K!$ . The expected number of new packets arriving during a session is  $\mu h$ . Let  $d_K$  denote the expected sum of the delays of all packets transmitted in a session of multiplicity K, then the expected delay of a packet expressed in slots is

$$d = \sum_{j=1}^{\infty} \frac{p_j d_j}{\mu h} \tag{2}$$

The values of  $d_{K}$  satisfy a system of linear equations, namely

$$d_{K} = \sum_{k=1}^{\infty} q_{k,K} \left( K - k + \sum_{i=0}^{\infty} p_{i} d_{K-k+i} \right) + q_{0,K} \left( K + \sum_{i=0}^{\infty} \sum_{j=0}^{K} \binom{K}{l} \right) \times 2^{K} [p_{i} \{ d_{l+i} + h_{l+i} (K-l) \} + p_{j} d_{K-l+j} ]$$
(3)

Here, the first term accounts for the case where k packets capture the receiver. K - k messages are retransmitted with *i* newcomers after spending one slot in the stack. The next term accounts for conflicts when no packet leaves the system. K packets split and spend at least one slot in the system. With binomial probability, *l* packets are retransmitted with *i* new arrivals. The remaining K - lmessages stay in the stack during the time of a session of multiplicity l + i, and are then retransmitted with *j* new arrivals. The expected lengths of sessions  $h_K$  are obtained similarly. In Fig. 1 we take  $L/(\eta B_C) = 1$  and assume packets of 256 bits.

In Fig. 1 we take  $L/(\eta B_C) = 1$  and assume packets of 256 bits. Including packet transmission times, the total delay is [3/2 + d]C. The capture models discussed here give almost identical results for C = 1, while for C = 10 results highly depend on the performance of multisignal detection.

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Fig. 1 Delay against message arrival rate with and without 10 dB spreading

 $\gamma = 100 (20 \text{ dB})$ ; performance of imperfect CDMA depends on error correction distance t

Conclusion: We have shown that in a lightly loaded network, the access delay incurred for transmission and collision resolution increases if spread-spectrum transmission is applied. At large traffic loads, however, spreading may enhance performance, provided that good multisignal detection methods or interference cancellation schemes are used. The throughput gain over unspread transmission is of the order of 20–30%. An optimum solution would be to adjust the spreading factor in a wireless network dynamically, depending on the instantaneous demand for low-delay or high-throughput services.

## Statistically tailored binary sequence for improved error rate measurements in 256 QAM digital radio

W.D. Grover and B.D. Venables

Indexing terms: Binary sequences, Bit error rate, Error statistics, Amplitude modulation, Digital radio systems

The authors address considerations of repeatability and accuracy in the measurement of BER on a 256 QAM digital radio system. When viewed in terms of state occupancies and vector transition magnitudes, pseudorandom binary sequences (PRBSs) only weakly exercise the sequences of state occupancies and transition magnitudes that statistically characterise an infinite random QAM payload. This is further aggravated if the mapping of bits into QAM symbols is static in each PRBS repetition. We have synthesised and tested a binary sequence which better imitates the statistics of an infinite random sequence in the vector symbol space. In simulation trials, the new test sequence more closely predicts the BER obtained with a much longer nonrepeating random pattern.

Background: During development of a 256 QAM digital radio system, difficulties were encountered in obtaining repeatable BER measurements in laboratory testing of back-to-back transmitreceive bays with flat loss attenuators and additive noise. By disconnecting and reapplying the DS3-rate PRBS input, the BER measurement could change up to an order of magnitude, without other changes in system conditions. Some simple considerations can account for this and suggest an idea for the design of binary patterns for testing vector modulated systems with high-order constellations: A PRBS of maximal length 2n-1 bits exhausts all possible n-bit sequences [1]. This is excellent for binary channels because intersymbol interference (ISI) is well characterised if  $n \ge$ 15. In contrast, consider that  $256^4 = 2^{32} = 4.29 \times 10^9$  symbols would be needed only to exercise all four-symbol sequences in 256 QAM. Therefore an n = 21 PRBS will exercise only about 1 in 2000 of the four-symbol sequences. But the QAM system is usually tightly bandlimited, with ISI tails that are important for well over four symbol times. In the best case, pattern coverage may be greater than the estimate above, by up to eight times, if the Acknowledgment: This work was supported by the Dutch Science Foundation N.W.O., under a project supervised by B.Tsybakov and J.C. Arnbak.

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number of internal overhead bits that the radio terminal adds, per PRBS word length  $\varepsilon$ , is a multiple of 8. Then, repetitions of the  $2^{n}-1$  PRBS word advance through all eight mappings of bits into QAM symbols before repeating. Conversely, pattern coverage and repeatability will be at their worst when  $\varepsilon \mod 8 = 1$  (as in our system), because PRBS repetitions then map into exactly the same repeated set of QAM symbols. This can explain a BER repeatabilty problem because an arbitrary (and then unchanging) mapping of the PRBS word into QAM symbols is adopted each time the binary input is applied. Eight distinct QAM sequences can then arise from one PRBS, each of which individually represents a small sample of possible trajectories in state space. Different worst-case ISI effects, hence differing BERs, can therefore be expected. In contrast, BER will be repeatable (whether accurate or not) when  $\varepsilon \mod 8 \neq 1$  because regardless of the initial mapping phase the same set of other QAM symbol mapping phases is revisited cyclically as the PRBS word is repeated.

Hypothesis and theory: This led us to the hypothesis that, even when repeatable, the absolute accuracy of 256 QAM BER testing may be relatively poor and could be improved if the binary test sequence were specifically designed to imitate the state occupancy and vector transition statistics of an infinite length random OAM sequence. Repeatability would also be assured if these properties were satisfied in each binary to QAM mapping phase. The principal idea is that in practical systems the most BER-affecting instantaneous ISI effects follow the largest swings across the signal state. Hence, a test pattern should have the statistically correct number of signal swings of all sizes. To pursue this, we characterised the ideal transition magnitude statistics of an infinite random QAM sequence in its vector space. A first statistic is the probability of occupancy of each state  $X_0$  in the constellation. A perfect random sequence will visit all states with equal likelihood, so  $p(X_0) = 1/256$ for 256 QAM. The next statistic is the Euclidean distance  $X_1$ between successive QAM symbols, i.e.  $X_1 = \sqrt{[(i_n - i_{n-1})^2 + (q_n - q_{n-1})^2]}$ where  $i_n$ ,  $q_n$  are the *n*th *i* and *q* symbol components. We obtained  $p(X_1)$  for 256 QAM by visiting every state of one quadrant and, from there, recording the distance of jumps to all other equally likely states, in all quadrants. Higher-order statistics give the distributions of total path distance travelled in any k-symbol trajectory around the state space. These are obtainable by repeated convolution of  $p(X_1)$  with itself. Specifically,  $p(X_2)$  is the probability distribution of the sum of two independent instances of the  $X_1$ 

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