USING LAPLACE TRANSFORMS TO COMPUTE PERFORMANCE OF MOBILE RADIO LINKS

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ABSTRACT: This paper reviews progress in developing a mathematical framework to evaluate the performance of mobile radio links. It takes into account multi-ray multipath fading with Rician or Nakagami probability density, lognormal shadowing and path loss of wanted and interfering transmitters, Additive White Gaussian, man-made burst noise and discontinuous transmission. The method presented here uses Laplace Transforms of the pdf of the power of each interfering signal. This method is computationally efficient and allows inclusion of a wide range of link aspects. We also show that, in contrast to common belief, it is not realistic to approximate the Rician fading of the wanted signal by a Nakagami model. New results are given for micro-cells along highways.

1 Introduction

The aim of this paper is to review and further develop the techniques for computing and modelling link outage performance. It has been shown in a number of recent papers that Laplace Transforms of the pdf of received signal power, or characteristic functions provide a convenient and computationally efficient way to evaluate the effect of the many factors affecting radio link performance. Link evaluation forms a critical part of any frequency planning tool. The main problem in computing the link performance is the stochastic character of multipath fading and shadowing of the wanted signal and all interferers, the man-made and additive white Gaussian noise and signal self interference due to dispersion. One approach, extensively reported in many papers, is to average the BER or outage probability over the Probability Density Function (PDF) of each of these components. For a hexagonal cellular net with six interferers, this rapidly leads to a 16 fold integral which cannot be solved analytically. Numerical and approximate computations where developed in [1-8] and in numerous other papers. In this paper, we review a method introduced more recently [9-13], based on Laplace Images of the PDFs of received power. Section 2 expresses outage probabilities in terms of Laplace image functions. Section 3 shows how this leads to a McLaurin expansion to obtain approximate results with a computationally very efficient method. Sections 4 - 7 review various interfering signals and present the corresponding characteristic functions of the interference power. Section 8 illustrates the method giving new results for highway cellular networks.

2 Laplace-Transform Expressions

A signal outage is the event that the short-term average power of the wanted signal fails to exceed the joint interference-plus-noise power p_t by at least a factor z. The probability of successful reception (no outage) is

$$\Pr(p_{0,0} > z p_t) = \int_{0-}^{\infty} f_{p_t}(x) \int_{zx}^{\infty} f_{p_{0,0}}(y)\, dy\, dx \quad (1)$$

where the instantaneous power $p_{i,k}$ in the k-th path of the i-th user is denoted as $p_{i,k}$. Its local-mean power is denoted as $\bar{p}_{i,k}$. The total local-mean power from transmitter i accumulated from all resolvable paths is $\bar{p}_i = \Sigma_k \bar{p}_{i,k}$. Depending on the particular delay profile, the mean power in the N_k resolvable paths can written as $\bar{p}_{i,k} = \alpha_{i,k} \bar{p}_i$, where $\alpha_{i,k} \geq 0$ and $\Sigma_k \alpha_{i,k} = 1$. The dominant resolvable path (path 0) of the wanted signal (user 0) has power $p_{0,0}$.

Several models have been proposed to model the stochastic behavior of the path amplitudes. Commonly accepted are Rician, Rayleigh and Nakagami fading. In the event of Rician fading, the instantaneous power $p_{i,k}$ has the pdf

$$f_{p_{i,k}}(p_{i,k} | \bar{p}_{i,k}) = \frac{(1+K)e^{-K}}{\bar{p}_{i,k}} \exp\left\{-\frac{1+K}{\bar{p}_{i,k}} p_{i,k}\right\} I_0\left(\sqrt{4K(1+K)\frac{p_{i,k}}{\bar{p}_{i,k}}}\right) \quad (2)$$

where the Rician K-factor is defined as the ratio of the power in the dominant component and the scattered (multipath) power and $I_0(.)$ denotes the modified Bessel function of the first kind and order zero. In the special case that the dominant component is zero (K = 0), Rayleigh fading occurs. The series expansion

$$I_0(z)=\sum_{n=0}^{\infty}\frac{1}{(n!)^2}\left(\tfrac{1}{4}z^2\right)^n \quad (3)$$

is used to write

$$\begin{aligned}\Pr(p_{0,0}>zp_t|\bar{p}_{0,0}) &= \\ &= \sum_{n=0}^{\infty}\sum_{k=0}^{n}\int_{0-}^{\infty}\exp\left\{-K-zx\frac{K+1}{\bar{p}_{0,0}}\right\}\frac{K^n}{n!}\left(zx\frac{K+1}{\bar{p}_{0,0}}\right)^k f_{p_t}(x)dx \quad (4)\\ &= \sum_{n=0}^{\infty}\frac{K^n}{n!}e^{-K}\sum_{k=0}^{n}\frac{s^k}{k!}\int_{0-}^{\infty}x^k s^{-sx}f_{p_t}(x)dx\end{aligned}$$

where $s = z(K + 1)/\ \bar{p}_{0,0}$ Using the properties of the Laplace Transform, it can be shown that

$$\Pr(p_{0,0}>zp_t|\bar{p}_{0,0}) = e^{-K}\sum_{n=0}^{\infty}\sum_{k=0}^{n} (-1)^k\frac{K^n}{n!}\frac{s^k}{k!}\frac{d^k}{ds^k}\mathcal{L}\{f_{p_t}(x),s\} \quad (5)$$

where $\mathcal{L}\{f,s\}$ denotes the one-sided Laplace transform of the function f at the point s. For m-Nakagami fading, the instantaneous power is gamma distributed [10, 14] with

$$f_{p_{i,k}}(p_{i,k}|\bar{p}_{i,k}=\bar{p})=\frac{1}{\Gamma(m)}\left(\frac{m}{\bar{p}}\right)^m p_{i,k}^{m-1}\exp\left\{-\frac{mp_{i,k}}{\bar{p}}\right\}. \quad (6)$$

where $\Gamma(m)$ is the gamma function, with $\Gamma(m+1) = m!$ for integer m. For a Nakagami-fading wanted signal, probability (1) becomes [12, 13]

$$\begin{aligned}\Pr(p_{0,0}>zp_t|\bar{p}_{0,0}) &= \sum_{l=0}^{m-1}\frac{s^l}{l!}\int_{0-}^{\infty}x^l e^{-sx}f_{p_t}(x)dx \\ &= \sum_{l=0}^{m-1}\frac{(-s)^l}{l!}\frac{d^l}{ds^l}\mathcal{L}\{f_{p_t}(x);s\}\end{aligned} \quad (7)$$

where $s = zm\ /\ \bar{p}_{0,0}$. Thus for a wanted signal with its strongest resolvable part subject to Nakagami or Rician fading, probability (1) can be written as the series

$$\Pr(p_{0,0}>zp_t|\bar{p}_{0,0})=\sum_{l=0}^{\infty}a_l s_0^l\frac{(-1)^l}{l!}\frac{d^l}{ds^l}\mathcal{L}\{f_{p_t};s\}\Big|_{s=s_0} \quad (8)$$

where coefficients a_i and argument s_0 depend on the type of fading. For the case of a Rayleigh-fading signal, $s = z\ /\ \bar{p}_{0,0}$ and the sum reduces to single term with $a_0 = 1$ and all other a_i are zero [9 - 11]. Because of shadowing, the local-mean power $\bar{p}_i$ has the log-normal pdf

$$f_{\bar{p}_i}(\bar{p}\mid\bar{\bar{p}}_i)=\frac{1}{\sqrt{2\pi}\sigma_s\bar{p}}\exp\left\{-\frac{1}{2\sigma_s^2}\ln^2\left(\frac{\bar{p}}{\bar{\bar{p}}_i}\right)\right\}, \quad (9)$$

where σ_s is the logarithmic standard deviation of the shadowing, expressed in natural units. If the wanted signal is subject to shadowing, area-mean probabilities are obtained by averaging the (8) over its lognormal PDF.

3 MacLaurin Expansion

The outage probability at large C/I ratios ($\bar{p}_{0,0}$ >> $\mathrm{E}p_t$) is found from the behavior of the Laplace expression at small values of s. Expanding the Laplace transform into a MacLaurin series gives

$$\mathcal{L}\{f_{p_t},s\} = 1 - s\mathrm{E}p_t + \frac{s^2}{2!}\mathrm{E}p_t^2 - \cdots . \quad (10)$$

Inserting this in (5) and (7) gives the outage probabilities for m-Nakagami fading

$$\begin{aligned}\Pr(\text{out}) &= 1-\Pr(p_{0,0}>zp_t|\bar{p}_{0,0}) \\ &= \frac{1}{m!}\frac{\mathrm{E}\,p_t^m}{\bar{p}_{0,0}^m}+O\left(\frac{\mathrm{E}\,p_t^{m+1}}{\bar{p}_{0,0}^{m+1}}\right)\end{aligned} \quad (11)$$

with m = 2, 3, ..., and, for Rician fading,

$$\begin{aligned}\Pr(\text{out}) &= se^{-K}\mathrm{E}p_t+(1-K)e^{-K}\frac{s^2}{2}\mathrm{E}p_t^2 \\ &+(1-2K+\frac{K^2}{2})e^{-K}\frac{s^3}{2}\mathrm{E}p_t^3+O\left(\frac{\mathrm{E}\,p_t^4}{\bar{p}_{0,0}^4}\right)\end{aligned} \quad (12)$$

where $s = z(K + 1)/\ \bar{p}_{0,0}$ and $\mathrm{E}p_i^m = m!\ \bar{\bar{p}}_i^{\ m}\exp\{m^2\sigma_s^2/2\}$. The series appear most accurate for severe fading and become less accurate for $K > 4$ .. 10 in the range of outage probabilities of practical interest. The effect of shadowing can be included easily into the series [10], giving fairly accurate results for small K ($K < 4$) and less than 6 dB of shadowing.

From (11) and (12) we conclude that approximating the pdf of a Rician-fading *wanted* signal by a Nakagami pdf is inaccurate as the results differ even in first-order. The Nakagami PDF with $m > 1$ does not model deep fades, so it predicts much better performance than Rician fading. Nonetheless it appears accurate to approximate the fading of *interfering* signals by a Nakagami PDF [15].

4 Narrowband Interference

The Laplace image of the instantaneous power received from the i-th Nakagami fading and shadowed interferer with area-mean power can be shown to be of the form

$$\mathcal{L}\{f_{p_i|\bar{p}_i};s\} = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-x^2)dx}{\left(1+\frac{s}{m}\bar{\bar{p}}_i\exp(\sqrt{2}x\sigma_s)\right)^m} \quad (13)$$

The effect of discontinuous transmission can be included using $\mathcal{L}\{f,s\} = (1 - \Pr(i_{ON})) + \Pr(i_{ON})\ \mathcal{L}\{f,s\,|\,\mathrm{ON}\}$ with $\mathcal{L}\{f,s\,|\,\mathrm{ON}\}$ the conditional Laplace Image of the PDF, given that the i-th transmitter is active and $\Pr(i_{ON})$ the probability that the terminal transmitter is on.

5 Wideband Interference

Assume the signals in N_k resolvable paths from user i to be Rayleigh fading and to add Incoherently. So, for a given set of local-mean powers, the pdf of the instantaneous power is found from the N_k-fold convolution

$$\mathcal{L}\{f_{p_i}(p\,|\,\bar{p}_i,i_{ON});s\} = \prod_{k=0}^{N_k-1}\frac{1}{1+s\alpha_{i,k}\bar{p}_i} \quad (14)$$

Taking into account shadowing and $\Pr(i_{ON})$, the Laplace image is found as

$$\mathcal{L}\{f_{p_i};s\} = 1-\Pr(i_{ON})+\frac{\Pr(i_{ON})}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{e^{-x^2}dx}{D(s)} \quad (15)$$

where the denominator function $D(s)$ is a polynomium in s and a function of x. For the case of a three-ray fading channel ($N_k = 3$) one can show that [16]

$$\begin{aligned} D(s) \triangleq\ & 1+s\bar{\bar{p}}_i e^{\sqrt{2}x\sigma_s} \\ & +s^2(\alpha_{i,0}\alpha_{i,1}+\alpha_{i,0}\alpha_{i,2}+\alpha_{i,1}\alpha_{i,2})\bar{\bar{p}}_i^2 e^{2\sqrt{2}x\sigma_s} \\ & +s^3\alpha_{i,0}\alpha_{i,1}\alpha_{i,2}\bar{\bar{p}}_i^3 e^{3\sqrt{2}x\sigma_s} \end{aligned} \quad (16)$$

Hence, the pdf of the power of a narrowband Nakagami and lognormally fading signal is found to be of the same form as that of a wideband signal with shadowing and independent Rayleigh-fading paths.

6 Multipath Self-Interference in a Wideband Channel

While the first resolvable path often contains a dominant component, such as a direct line of sight, which leads to Rician fading, paths arriving later typically experience Rayleigh (or more generally Nakagami) fading. In a N_k-ray Rayleigh-fading channel, N_k - 1 excessively delayed reflections interfere with the wanted signal. Their effect can be described by the pdf and the corresponding image of the interference power is

$$\mathcal{L}\{f,s\} = \prod_{k=1}^{N_k-1}\frac{1}{1+s\alpha_{i,k}\bar{p}_i} \quad (17)$$

7 Noise

We distinguish wideband and narrowband (burst) noise. The bandwidth of the Additive White Gaussian Noise (AWGN) noise is limited by receive filters. Since outage events are defined for a minimum averaging window of duration significantly longer than the inverse of receiver with RF-bandwidth B_T, we may consider the short-term average noise power to be constant [10]. The total received noise power is $\bar{p}_n = N_0/B_T$, where N_0 is the one-sided spectral power density of the AWGN at the receiver input. This noise power has the pdf

$$f_{p_n}(x) = \delta(x-\bar{p}_n)\ , \quad (18)$$

and Laplace image

$$\mathcal{L}\{f_{p_n},s\} = \exp\{-s\bar{p}_n\}\ . \quad (19)$$

In addition to AWGN, the terminal may receive narrowband man-made noise. Middleton [17] defines 'Class A' man-made noise having a bandwidth which is substantially smaller than the RF bandwidth of the receiver. The pdf of the momentary man-made noise voltage ζ was described as an infinite sum of Gaussian pdf's, namely

$$f_Z(\zeta) = e^{-A}\sum_{m=0}^{\infty}\frac{A^m}{m!\sqrt{2\pi\frac{m\bar{p}_m}{A}}}\exp\left\{-\frac{\zeta A^2}{2m\bar{p}_m}\right\}, \quad (20)$$

where A is the impulsive index, and $\bar{p}_m$ the mean man-made noise power. Because of the narrowband nature of the noise, the amplitude of pdf becomes a sum of Rayleigh pdf's. The corresponding pdf of instantaneous power is

$$f_{p_m}(p) = e^{-A}\left[\delta(p)+\sum_{m=1}^{\infty}\frac{A^m}{m!}\frac{A}{m\bar{p}_m}\exp\left\{-\frac{Ap}{m\bar{p}_m}\right\}\right] \quad (21)$$

Typical values for the impulsive index are $0.01 < A < 0.5$ and for the mean power $0.001 < \bar{p}_n/\bar{p}_m < 0.1$, i.e., the man-made noise is 10 to 30 dB above the AWGN noise floor. Since AWGN and man-made noise are independent and add incoherently, the Laplace image of the pdf of the total (AWG plus burst) noise power becomes

$$\mathcal{L}\{f_{p_s} \otimes f_{p_n}, s\} = e^{-\bar{\varphi}_s - A}\left[1 + \sum_{m=1}^{\infty} \frac{A^m}{m!} \frac{1}{1 + s\frac{m\bar{\varphi}_m}{A}}\right]. \quad (22)$$

8 Numerical Results for Highway network

For planning and deployment of networks, many propagation models exist for estimating the area-mean power from a topographical data base. For generic system design, the area-mean power is mostly estimated from the distance r_i between the i-th terminal and the base station. A generally accepted (normalized) model is

$$\bar{\bar{p}}_i = r_i^{-\beta_1}\left(1 + \frac{r_i}{r_g}\right)^{-\beta_2} \quad (23)$$

where typically $\beta_1 \approx 2$ and $\beta_1 \approx 2$. The normalized turnover distance r_g depends on the carrier frequency and the antenna heights.

We illustrate our method for a shadowed Rician-fading wanted signal, experiencing multiple co-channel shadowed Rayleigh-fading interference. The assumption that interference is Rayleigh fading is reasonable as their line-of-sight propagation path is more likely to be obstructed. Conditioning of local-mean powers we get

$$\Pr(p_0 > z p_t | \bar{p}_0 \cdots \bar{p}_N) = \sum_{l=0}^{\infty}\left[1 - e^{-K}\sum_{n=0}^{l-1}\frac{K^n}{n!}\right] \cdot \frac{(-s)^l}{l!} \frac{d^l}{ds^l}\prod_{k=1}^{N}\frac{1}{1+s\bar{p}_k}\Bigg|_{s=(K+1)\frac{z}{\bar{p}_0}} \quad (24)$$

Numerical evaluation of the derivatives can be implemented efficiently by the partial fraction expansion into

$$\prod_{k=1}^{N}\frac{1}{1+s\bar{\bar{p}}_k} = \sum_{k=1}^{N}\frac{B_k}{1+s\bar{\bar{p}}_k} \quad (25)$$

Since the constants B_k depend only on the local-mean powers of the N interfering signals but not on s, the derivatives are found as simple analytical expressions. For $K = 4$, higher order terms vanish relatively fast, and the 20th derivative gave a relative contribution of less than 10^{-6}. The Hermite polynomial method was applied to include the effect of log-normal shadowing for the wanted and all interfering signals. Figure 1 illustrates a simplified highway cellular network with 4 interfering base stations. The turnover distance r_g, normalized to the cell size, was taken 0.1 of the distance between two base station. The cell size is thus [-0.5, 0.5]. Table 1 and Figure 2 give the signal outage probability for $K = 4$ (6 dB) and various degrees of shadowing. A cell layout with $C = 3$ different frequencies ensures acceptable outage probability for 3 dB of shadowing. Figure 3 compares exact results with a truncated MacLaurin series.

Table 1: Outage probability at cell boundary for various reuse patterns for 0, 3 and 6 dB of shadow fading. $K = 4$.

Cluster Size	Shadowing 0 dB	3 dB	6 dB
C = 1	1.0	9.3e-1	8.4e-1
C = 2	4.1e-2	5.6e-2	2.3e-1
C = 3	3.2e-3	3.5e-3	5.6e-2
C = 4	8.0e-4	8.2e-4	1.6e-3
C = 5	3.0e-4	3.1e-4	5.4e-3

9 Conclusions

We reviewed, generalized and extended a Laplace method to calculate outage probabilities in mobile radio networks. We addressed Rician and Nakagami fading, additive white Gaussian and man-made burst noise in a K-ray Rayleigh-fading channel, shadowing and path loss. In contrast to common approximations made in the literature, the method allows numerically tractable investigation of networks with non-i.i.d. interference. Particularly in micro-cellular networks along highways, it may not be realistic to assume that all co-channel interfering signals have the same mean power. Our work confirms that one may approximate the Rician pdf of interference power by a Nakagami pdf. However, approximation of the Rician fading of a wanted signal by a Nakagami model gives misleadingly optimistic results. New results have been presented for highway micro-cells. In practical cell planning, often coarse fade margins are used, to avoid the need for time consuming computations. We developed MacLaurin expansions that also allow rapid link evaluation, without the inaccuracies typically associated with fixed fade margins.

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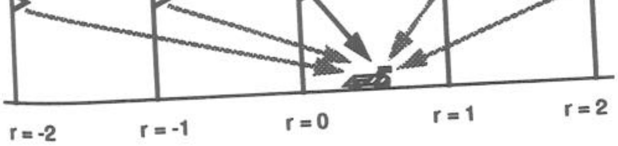

Figure 1 One-dimensional frequency reuse layout for highway micro-cellular communication.

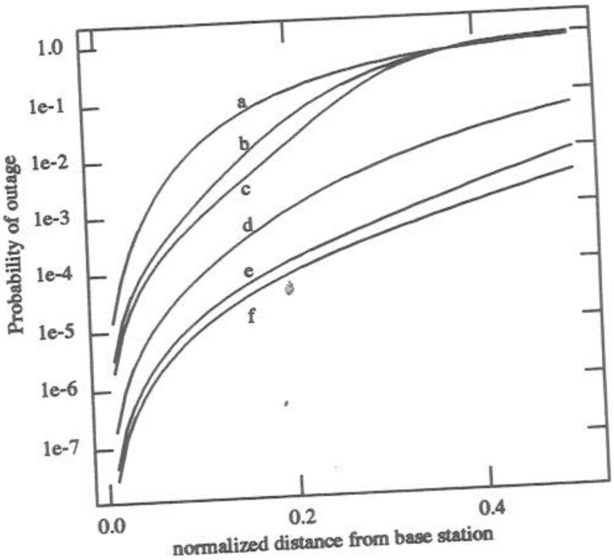

Figure 2 Outage probability versus location, K= 4 (6 dB). a) C= 1, 6 dB shadowing, b) C= 1, 3 dB shadowing, c) C= 1, shadowing, d) C= 3, 6 dB shadowing, e) C= 3, 3 dB shadowing, f) C= 3, no shadowing. Path Loss β_1=2 β_2=2. r_g= 0.1

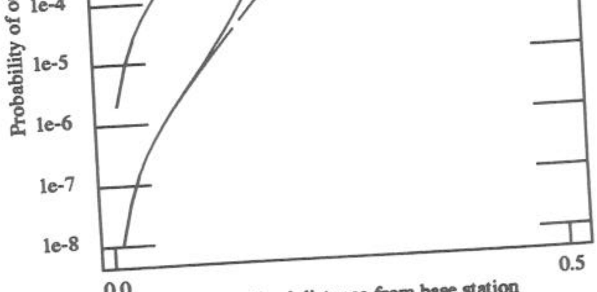

Figure 3 Comparison of exact result with two-term MacLaurin series. a) K = 4, MacLaurin b) K = 4, exact, c) K = 10, MacLaurin, d) K = 10, exact. $\beta_1 = 2$ $\beta_2 = 2$ $r_g = 0.1$, $C = 1$.