Effect of CDMA Transmission on Performance of Wireless Networks with Stack Algorithm for Collision Resolution [1]

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***Abstract* - This paper evaluates the performance of a packet radio network employing Code Division Multiple Access and a Stack Algorithm to resolve message collisions. We investigate the effects of bandspreading assuming a fixed total system bandwidth. Although bandspreading with perfectly orthogonal signals enhances system capacity, it does not resolve the instability of ALOHA random access unless special measures are taken to control retransmission traffic. Using a Stack Algorithm, at small traffic loads the packet delay is minimized if no spreading is applied. At large traffic loads, perfect CDMA enhances performance, but for imperfect signal separation at the receiver, advantages of CDMA are lost. The capacity of the Stack Algorithm addressed here can be enhanced from 0.32 to at least 0.40 if a large spreading factor with perfect signal separation is employed**

Introduction

In modern multi-user wireless networks, substantial effort is being made to use scarce radio spectrum resources in the most efficient way [1]. Approaches to enhance the performance of packet-switched networks with bursty traffic typically employ advanced transmission techniques such as Code Division Multiple Access (CDMA) [2] or advanced medium access protocols, such as the tree or stack algorithms for collision resolution [3 - 11]. This paper addresses the effect of CDMA transmission on the performance of random access schemes including slotted ALOHA and the stack algorithm.

Medium Access Protocol

We address a multi-access channel with a large (possibly infinite) number of distributed terminals that send data packets to a central receiver. Packets arrive according to a temporal Poisson process with mean arrival rate λ packets per second. The fixed message length is L bits and the total system bandwidth is B_C. For a system without spreading and modulation compactness of η chips/s/Hz or bit/s/Hz, the packet duration T_L is $T_L = L / (\eta\ B_C)$. Ignoring propagation delays and power-up (guard) times, we assume that the slot time is identical to T_L. The system is slotted, i.e., the time axis is divided into slots and any packet transmission starts at the beginning of a slot and finishes at the end of the same slot. We compare this system with CDMA transmission with spreading factor C. For a fair comparison we maintain the same total system bandwidth. This implies that the slot duration and packet transmission time increases by a factor C so it becomes $T_L = C\,L / (\eta\ B_C)$. The message arrival rate, expressed in packets per time slot becomes $\mu = \lambda\ T_L = \lambda\ C\,L / (\eta\ B_C)$. The slot beginning at $t = n\,T_L$ is called slot n. We normalize the unit of time taking $L / (\eta\ B_C) = 1$.

Let random variable K ($K = 0, 1, ..$) denote the number of packets transmitted in a slot, then with probability $q_{k,K}$, k of these packets achieve successful transmission *(capture)*, while the remaining $K - k$ packets are unsuccessful. In ideal CDMA, one often assumes

$$q_{k,K} = \begin{cases} \delta_{k,K} & K = 0,\ 1,\ ...,\ C \\ \delta_{k,0} & K = C+1,\ C+2,\ ... \end{cases} \qquad (1)$$

Thus, in a narrowband system ($C = 1$), successful transmission only occurs if exactly one packet is present in that slot. If the arrival process of packets were truly Poissonic in slotted ALOHA, the throughput per unit of time becomes [2]

$$S = G\ e^{-CG} \sum_{k=0}^{C-1} \frac{CG^{\,k}}{k!} \qquad (2)$$

where S denotes the expected number of successful packets per slot and G denotes the number of attempted transmissions per slot. This tends to $S \to G$ ($S \to C\lambda$, $G \to C\lambda$) for $\lambda < 1$ and $C \to \infty$. This suggests that with increasing C, the ALOHA random access system 'looses' its contention character and its properties become closer to that of a fixed assignment scheme.

Despite these seemingly beneficial effects on throughput, the transmission time per packet increases proportional with C and even perfect CDMA with $C < \infty$ cannot repair the instability [3, 12] of the ALOHA system with a fixed retransmission back-off probability. Even though the

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probability on a destructive collision involving more than C new packets rapidly becomes small with increasing C, retransmission traffic spoils the stability of network. The proof is based on the observation that for any finite C and fixed retransmission probability p, their exists an N such that if more than N terminals are in backlog, the backlog is expected to grow without bounds. As the probability of having N terminals in backlog is nonzero, the system will eventually experience ever-increasing backlog. In order to ensure reliable operation, CDMA wireless packet-switched networks thus need a control mechanism that regulates the number of packets transmitted in each time slot.

Efficient collision resolution algorithms have been proposed, such as the Stack Algorithm [7 - 11]. Similar to ALOHA, a terminal that generates a new user packet during a slot, transmits in the next slot. A packet capturing the receiver in slot n leaves the system at $t = n + 1$. All terminals with packets that fail to capture the receiver use a stack algorithm. For that purpose, each user terminal that still has a back-logged packet for transmission listens to the return channel at the end of a slot to decide what has to be done in the next slot. The stack algorithm addressed here [11] uses ternary feedback: Each terminal can a posteriori distinguish perfectly between an *idle* slot ($K = 0$), a slot with *capture* ($k > 0$), and a slot with *conflict* ($k = 0$; $K > 0$).

Each packet transmitted in a slot without capturing the receiver is either retained in the terminal buffer (with probability r) or retransmitted in the next slot (with probability $1 - r$). Associated with the message buffer is a stack counter l_n which changes from slot to slot according to a set of rules. Generally, the stack counter increases when a conflict is observed in a slot and decreases when a slot is idle. Any packet that captures the receiver leaves the system. We address an algorithm [11] in which

1) A packet transmitted in slot n for the first time (i.e., this packet was generated in slot $n - 1$) has $l_n = 0$.
2) When $l_n = 0$, this packet will be transmitted in slot n. When $l_n > 0$, the packet is not transmitted in slot n.
3) When $l_n = 0$ and a conflict is reported in slot n, then $l_{n+1} = 1$ with probability r and $l_{n+1} = 0$ with probability $1 - r$.
4) When $l_n \geq 0$ and a capture is reported in slot n, then $l_{n+1} = l_n$.
5) When $l_n > 0$ for a packet and slot n is reported idle, then $l_{n+1} = l_n - 1$.
6) When $l_n > 0$ for a packet and a conflict is reported in slot n, then $l_{n+1} = l_n + 1$.

Here we address the near-optimum case [11], $r = 1/2$.

Channel and capture model

We will compare the model of perfect CDMA (1) with a receiver that attenuates interference by a factor C. We assume that all signals are transmitted over a Linear Time-Invariant (LTI) frequency non-selective Additive White Gaussian Noise (AWGN) channel. The energy per bit is taken identical and constant for all users and it is denoted as E_b. The noise spectral power density is N_0. The signal to noise ratio is $\gamma = E_b/N_0$. A commonly used model for the bit error rate given K interferers is

$$P_{e|K} = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\gamma}{\frac{K-1}{C}\gamma + 1}} \tag{3}$$

If an error correcting code can accepts packet of L bits with not more than t bit errors, the probability that one particular packet is successful becomes

$$\mathrm{P}(\mathrm{succ}\,|\,K) = \sum_{i=0}^{t} \binom{L}{i} P_{e|K}^{\,i} \left(1 - P_{e|K}\right)^{L-i} \tag{4}$$

The probability that k out of K colliding packets capture the receiver becomes

$$q_{k,K} = \binom{K}{k} \mathrm{P}(\mathrm{succ}\,|\,K)^{k} \left(1 - \mathrm{P}(\mathrm{succ}\,|\,K)\right)^{K-k} \tag{5}$$

Collision Resolution Delay

The definition of expected delay in stack applies to an arbitrary member of the ensemble of packets offered to the system. It is taken here as the expectation of the time interval elapsed between (the beginning of) the first transmission attempt of a packet and (the beginning of) its successful transmission. The stack algorithm has similarities with last-in/first-out systems and branching processes: a packet colliding in slot n that moves into the stack at slot $n + 1$ can be retransmitted (successfully or not) only after all other packets that stayed in the channel ($l_{n+1} = l_n = 0$) have left the system, together with all new packets arriving during the period of their transmission [13 - 14]. One defines a *session* as the time interval between two successive moments n_1 and n_2 ($n_1 < n_2$) such that for some packet $l_{n1} = 1$ and $l_{n2} = 0$ for the first time. In Figure 1, this packet is denoted by o. The expected length of a session depends on the number K of packets transmitted in slot n_1 and is denoted as h_K. Such a session is called a session of multiplicity K.

If at the end of slot $n - 1$ there are no packets in the system, the number of packets K transmitted in slot n is Poissonic with mean μ, so $p_K = \mu^K \exp(-\mu)/K!$ is the probability of K new packets arrivals in a slot. The expected length h of a session starting from an empty system is

$$h = \sum_{K=0}^{\infty} p_K h_K \tag{6}$$

The expected number of new packets arriving and transmitted during a session is μh. Let d_K denote the expected sum of the delays of all packets transmitted in a session of multiplicity K. The expected delay of a packet is

$$d = \sum_{j=1}^{\infty} \frac{p_j d_j}{\mu \bar{h}} \tag{7}$$

The values of d_K are obtained recursively as they satisfy the following system of linear equations

$$\begin{aligned} d_K = & \sum_{k=1}^{\infty} q_{k,K} \left(K - k + \sum_{i=0}^{\infty} p_i d_{K-k+i} \right) \\ & + q_{0,K} \left(K + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{l=0}^{K} \binom{K}{l} r^{K-l} (1-r)^l \cdot \right. \\ & \left. \cdot \left[p_i \left(d_{l+i} + h_{l+i}(K-l) \right) + p_j d_{K-l+j} \right] \right) \end{aligned} \tag{8}$$

Here, the first term on the right-hand side accounts for the case that with probability $q_{k,K}$, k ($k = 1, 2, .. K$) packets capture the receiver. All terminals receive the 'capture' feedback information and $K - k$ messages spend one slot in stack. These unsuccessful packets are retransmitted with i newcomers. The next term accounts for the event that a conflict ($k = 0$) is reported. No packet leaves the system, the K packets split, and spend at least one time slot in the system. With binomial probability, l packets are retransmitted with i new arrivals. The remaining $K - l$ messages stay in the stack during the time of a session of multiplicity $l + i$, and are then retransmitted with j new arrivals. While evaluating this, the values of the expected lengths of sessions h_K are obtained from a similar system of equations that differs from the above expression only in free terms. In order to obtain numerical results, the values of d_K are well approximated by solving the truncated system of only a finite number of equations. For the example of $C = 1$ with (1), one may ignore the probability of sessions of multiplicity higher than 10.

Computational Results

We normalize by taking $L / (\eta B_C) = 1$, i.e., medium access delays are expressed in a time slots for a system with no spreading ($C = 1$) and $\mu = C\lambda$. We assume packets of 256 bits without error correction coding ($t = 0$) and with a two-bit error correcting code ($t = 2$). The arrival rate λ corresponds to the expected number of new packets per slot in an non-spreading system. The message delay expressed in seconds is $[3/2 + d]\, C$, so it includes one packet time and a half for the random waiting time till the beginning of the next slot and the message transmission duration. Figure 2 depicts the message delay versus the arrival rate λ for spreading factors $C = 1$ and 10. Results for $C = 1$ appeared not to be sensitive to the choice of capture model. This is in contrast to the investigation in [11] addressing different received power for different users. The curves show that $C = 1$ gives a significantly smaller delay than a larger spreading factor. For large λ, however, spreading gives better performance. The model of Eqns (3) - (5) (Curve C-D) is more pessimistic than the perfect capture model (curves B). However, even if CDMA allows perfect capture (1), for $\lambda < 0.3$ the delay is minimized if $C = 1$.

The number of packets in the stack can be shown to be ergodic for any packet arrival rate less than a particular λ_{cr}, called the maximum throughput, in which case the packet delay is finite.

Spread Factor	Perfect CDMA	Imperfect CDMA	
		$t = 0$	$t = 2$
	λ_{cr}	λ_{cr}	λ_{cr}
C = 1	0.32	0.32	0.32
C = 2	0.34	0.16	0.18
C = 5	0.37	0.13	0.17
C = 10	0.40	0.12	0.15
C = 15	0.40	0.11	0.15
C = 20	0.40	0.10	0.14

Table: Maximum message arrival rate to ensure finite delay, for various spreading factors. Perfect and imperfect multi-signal detection.

Discussion and Conclusion

We have shown that in a lightly loaded network, the packet delay of a wireless random access scheme increases if one applies spread spectrum transmission. This is explained by the fact that without intentional spreading, messages can have very short duration, which substantially reduces the transmission time. On the other hand, at large traffic loads, spreading may enhance performance, provided that near-perfect detection methods are used. Presumably interference-cancellation techniques and multi-signal detection are needed to reliably separate up to C signals in a CDMA system with spreading factor C. As such filtering techniques inevitably enhance noise, good signal-to-noise ratios are required at the receiver. Another noise constraint is the fact that the receiver has to distinguish between an idle slot and a conflict. As collision resolution relies on estimating the number of backlogged users, the performance degrades if errors occur frequently in this decision due to background noise or interference from other networks.

To some extent, our results disagree with favorable results for CDMA transmission in cellular telephony systems. In circuit-switched telephony the main performance criterion is a guaranteed maximum outage probability for the entire duration of a call. The 'averaging' the effect of interference over time and bandwidth may then favorably affect the probability that the interference occasionally is too large. In efficient packet-switched networks however, the interference can be prohibitively large for a substantial portion of time. In such networks, capture probabilities benefit if the interference power level fluctuates from slot to slot. Moreover, the delay penalty from longer packet transmission times due to spreading appears to be significant. These results motivate a further study of efficient Medium Access Schemes for wireless channels in relation to the current discussion on standards for future universal personal wireless communication systems, focussing heavily on circuit-switched transmission.

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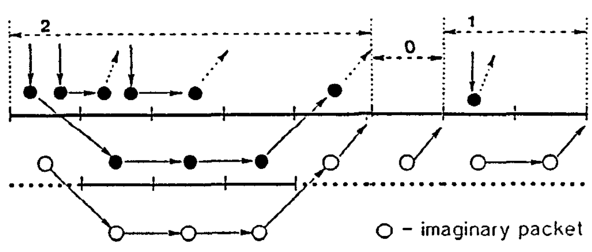

Fig. 1. Packet behavior in stack-algorithm (above) and in the stack memory (below). Packets entering the system are shown with a vertical arrow. Packets leaving the system due to capture are shown with dotted arrows. Definition of sessions in stack-algorithm. Session multiplicities are indicated by the number in the first slot.

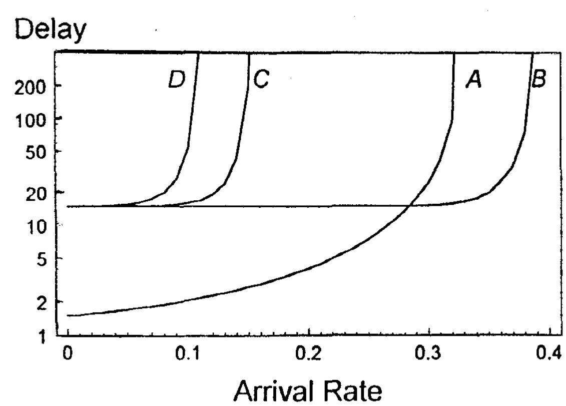

Fig. 2 Expected total delay (including transmission time) versus normalized message arrival rate. Signal-to-noise ratio: $\gamma = 100$ (20 dB). A: no spreading. Curve; B: spread factor $C = 10$, perfect multi-signal detection; C: spread factor $C = 10$, 2-bit error correction; D: spread factor $C = 10$, no error correction; E: $C = 1$ but new packets randomly choose one out of 10 parallel channels.