Wiener Filtering of Multi-Carrier CDMA in Rayleigh Fading Channel

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Abstract

In this paper, we will analyze the application of Wiener filtering to the detection of Multi-Carrier Code Division Multiple Access (MC-CDMA) signals. Simulation results are included to test the validity and accuracy of the analytical results.

1. Introduction

In an earlier paper, the authors discussed the utilization of MC-CDMA as a digital modulation and multiple access technique in an indoor wireless environment [1]. Due to its special signal structure, MC-CDMA signals will experience little linear distortion in fading channels where the symbol duration, T_b, is much larger than the delay spread, T_d.

With MC-CDMA [2], each data symbol is simultaneously transmitted at N binary phase shift-keying (BPSK) narrowband subcarriers, each seperated by F/T_b Hz where F is an integer. As shown in the transmitter model of Fig. 1, each of the N subcarrier waveforms is modulated (multiplied) by a single chip of a spreading code of length N. Different users transmit at same set of subcarriers but with a different spreading code. Due to its narrowband composition, MC-CDMA signals are not significantly affected by delay spreads as much as Direct-Sequence CDMA (or other wideband) signals. This modulation technique should not be confused with transmitting multiple DS-CDMA signals at different frequencies as done so by Milstein [3].

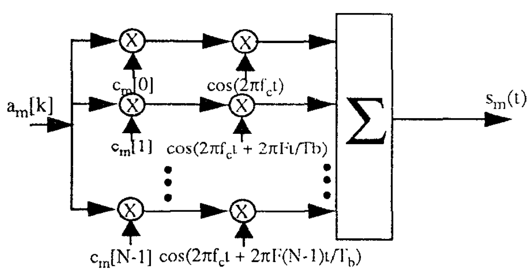

Fig. 1 Transmitter model of the *mth* user

While the general shape of the each individual subcarrier will not be experience significant linear distortion after transmission, the flat fading that each subcarrier will experience results in an amplitude scaling. Unequal scalings at different subcarriers will distort the orthogonality between users. While the conventional detection methods of equal gain combining (EGC) and maximal ratio combining (MRC) may be sufficient in an additive white guassian noise (AWGN) channel, these detection methods do not directly address the issues of orthogonality and of interference cancellation. Consequently, these detection methods do not perform as well in interference limited channels.

In this paper, we will apply Wiener filtering to the detection of MC-CDMA signals. Wiener filtering is optimal in a mean-squared error sense with respect to both the noise and the interference. In most fading channels, the determination of the Wiener coefficients is performed adaptively. In this paper, we are concerned primarily with the theoretical limits and not the implementational aspects. Thus, it is assumed that accurate estimates of the complete channel state information (i.e., the fading at the subcarriers) is available and that adaptive structures are not necessary. Under this assumption, the implementation of this detection method can be performed with low complexity multiplications.

2. Channel Model

Denote the vector of the data symbols by

$$A = \left[a_0\ a_1\ \dots\ a_{N-1}\right]^T \qquad (1)$$

where $a_m \in \{-1, 1\}$ corresponds to the data symbol of the *mth* user. If user m is inactive, then $a_m = 0$. Define the code matrix C to be

$$C = \begin{bmatrix} c_0[0] & c_1[0] & \dots & c_{N-1}[0] \\ c_0[1] & c_1[1] & \dots & c_{N-1}[1] \\ \dots & \dots & \dots & \dots \\ c_0[N-1] & c_1[N-1] & \dots & c_{N-1}[N-1] \end{bmatrix} \qquad (2)$$

where the *mth* column vector of C corresponds to the spreading code, $\{c_m[i] \mid i = 0, 1, \dots, N-1\}$, of the *mth* user. Using the vector and matrix notation described above, an equivalent discrete vector representation of the transmitted signal described in Fig. 1 is

$$S = \left[s_0\ s_1\ \dots\ s_{N-1}\right]^T = CA \tag{3}$$

where s_i represents the signal components of all users at the *ith* subcarrier. The actual continuous-time signal transmitted by the *mth* user is

$$s_m(t) = a_m \sum_{i=0}^{N-1} c_m[i] \cos\{2\pi (f_c + \frac{F}{T_b} i) t\}. \tag{4}$$

In this paper, we will focus on downlink transmissions, i.e., from the base station to the mobiles. Assuming that the delay spread is much smaller than the symbol duration, the effect of the channel at the *ith* subcarrier may be approximated by a amplitude scaling, ρ_i, and a phase offset, θ_i. Applying the received signal to the receiver model shown in Fig. 2, the equivalent discrete representation of the received signal is

$$Y = \left[y_0\ y_1\ \dots\ y_{N-1}\right]^T = HCA + N_1 \tag{5}$$

where y_i represents the component of the received signal at the *ith* subcarrier, the channel matrix H is defined to be

$$H = \begin{bmatrix} \rho_0 & \dots & 0 & 0 \\ \dots & \rho_1 & \dots & 0 \\ 0 & \dots & \dots & \dots \\ 0 & 0 & \dots & \rho_{N-1} \end{bmatrix}, \tag{6}$$

and $N_1 = \left[\eta_0\ \eta_1\ \dots\ \eta_{N-1}\right]^T$ is a vector containing the corresponding AWGN terms with η_i representing the noise term at the *ith* subcarrier with power N_0/T_b. Obviously, the discrete representation of the demodulated signal given in Eq.(5) is an approximation. However, under the condition $T_b \gg T_d$, each subcarrier faces a relatively flat channel and Eq.(5) provides a reasonable representation.

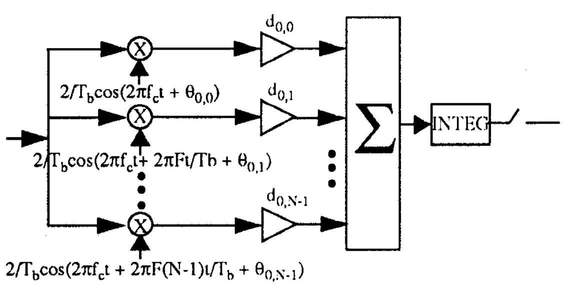

Fig. 2 Receiver Model

The application of Wiener filtering to this received signal involves linearly combining the different subcarrier diversity components to yield the decision variable

$$v_0 = D \bullet Y \tag{7}$$

where $\bullet$ denotesthe inner product between two vectors and the vector $D = \left[d_0\ d_1\ \dots\ d_{N-1}\right]^T$ represents the optimal weighting coefficients. These coefficients may also be viewed as an amplitude equalization to compensate for the fading at the subcarriers. Arbitrarily chosing $m = 0$ as the desired signal, the optimal choice of the equalization vector D in the mean-squared error sense can be determined to be

$$D = R_y^{-1} R_{a_0 Y} \tag{8}$$

where

$$R_Y = E\{YY^T\} = HCE(AA^T)C^T H + \frac{N_0}{T_b} I \tag{9}$$

represents the autocorrelation matrix of the received vector Y and $R_{a_0 Y}$ represents the cross-correlation vector between the desired symbol, a_0, and the received signal vector, Y. In the formulation of this method, it is assumed that estimates of the channel amplitudes, $\{\rho_i | i = 0, 1, \dots, N-1\}$, are available and are treated as deterministic constants.

A closed form solution of Eq.(8) is difficult to obtain except for the case of a full load, i.e., the case when all users are active. The corresponding equalization coefficient at the *ith* subcarrier for a full load is

$$d_i = \frac{\rho_i}{N\rho_i^2 + N_0/T_b} c_0[i]. \tag{10}$$

As expected, Eq.(10) indicates that the linear combination of the subcarrier components should include the inner product of the desired user's spreading code vector with the received signal vector. For small ρ_i, the equalization coefficient will be small to avoid excessive amplification of the noise. For large ρ_i, the correction factor will be proportional to taking the inverse of the channel, $1/\rho_i$, inorder to restore orthogonality between users.

For sufficiently large values of N, the average bit error rate (BER) for the case of a full load may be approximated using the Central Limit Theorem by

$$BER \cong \frac{1}{2} erfc \sqrt{\frac{1}{2} \frac{(NEw_i)^2}{N^2\sigma_{w_i}^2 + N\frac{N_0}{T_b}Ev_i^2}} \quad (11)$$

where $w_i = \dfrac{\rho_i^2}{N\rho_i^2 + N_0/T_b}$ and $v_i = \dfrac{\rho_i}{N\rho_i^2 + N_0/T_b}$.

Simulation and Numerical Results

Due to the lengthy time consumption of the simulations, results for only spreading factors of $N = 8$ and $N = 64$ were obtained. Shown in Fig. 3 is a plot of the average BER versus the signal-to-noise ratio (SNR) in a Rayleigh fading channel for a full load with $N = 8$. Independent fading at the subcarriers is assumed. It is also assumed that all users are received with equal power. Note that the analytical results for Wiener filtering differ significantly from the simulation results for large SNR. This discrepancy is due to the application of the Central Limit Theorem for small spreading factors. Shown in Fig. 4 are the results for $N = 64$. Examining Fig. 3 and 4, it can be seen that Eq.(11) is more accurate for large N. Also included in Fig. 3 are the curves for a single narrowband subcarrier with AWGN and with and without fading. Note that the performance of MC-CDMA is better than a single narrowband transmission due to frequency diversity even with co-channel interference.

Shown in Fig. 5 is a plot of the simulation results for the average BER versus the number of interferers. The SNR was chosen to be 10dB. Because of the time consuming nature of the simulator, a relatively small spreading factor of N=8 was used. Besides the curves for Wiener filtering, the simulation results of conventional EGC and MRC detection are also included [1]. As expected, Wiener filtering outperforms EGC and MRC substantially in combating interference.

Conclusion

In this paper, the application of Wiener filtering was applied to the detection process of MC-CDMA. In contrast to conventional diversity combining techniques, this detection technique directly addresses the effects of the interference on the BER. It was shown that even with a full load, MC-CDMA with Wiener filtering outperforms single narrowband transmissions in a Rayleigh fading channel. It was also shown that Wiener filtering outperforms conventional dectection methods substantially.

References

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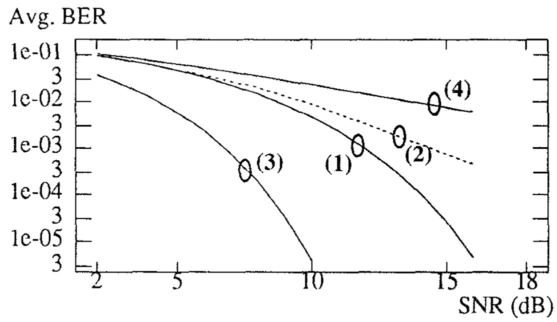

Fig. 3 Average BER vs SNR for N = 8 with Rayleigh fading and a full load for Wiener Filtering: (1) analytical and (2) simulation and single narrowband: (3) with noise only and (4) with noise and fading.

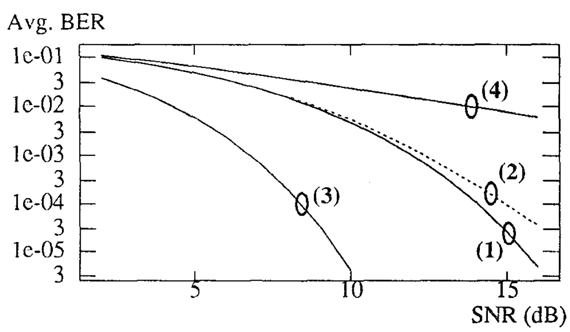

Fig. 4 Average BER vs SNR for N = 64 with Rayleigh fading and a full load for Wiener Filtering: (1) analytical and (2) simulation and single narrowband: (3) with noise only and (4) with noise and fading.

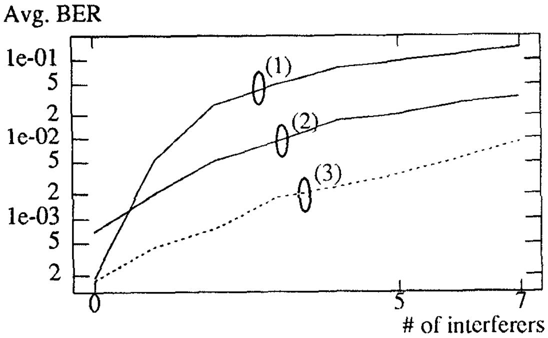

Fig. 5 Average BER vs # of interferers for spreading factor N = 8 with Rayleigh fading and SNR=10dB. Simulation results for (1) Maximal Ratio Combining (2) Equal Gain Combining (3) Wiener filtering.