BER of Multi-Carrier CDMA in an Indoor Rician Fading Channel

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Abstract

This paper will analyze a novel digital modulation technique called Multi-Carrier Code Division Multiple Access (MC-CDMA) in which data symbols are transmitted at multiple subcarriers where each subcarrier is modulated by a 1 or -1 based on a spreading code. Analytical results will be presented on the performance of this modulation scheme in the downlink of an indoor wireless Rician Fading channel. In addition, the performance of a controlled equalization technique that attempts to restore the orthogonality between users will be evaluated.

1. Introduction

Recently, there has been a growing interest in indoor wireless radio communication. This paper will extend the analysis presented in a previous paper [1] of the performance of a new spread spectrum transmission method called "MC-CDMA" in an indoor environment.

MC-CDMA [1,2,3] addresses the issue of how to spread the signal bandwidth without increasing the adverse effects of the delay spread, which is a measure of the length of the impulse response of the channel. With MC-CDMA, a data symbol is transmitted over N narrowband subcarriers with each subcarrier being encoded with a 0 or π phase offset based on a spreading code. Different users transmit over the same set of subcarriers but with a spreading code that is orthogonal to the codes of other users. The resulting signal has an orthogonal code structure in the frequency domain. If the number of and spacing between subcarriers is appropriately chosen, it is unlikely that all of the subcarriers will be located in a deep fade and consequently frequency diversity is achieved. As a MC-CDMA signal is composed of N narrowband subcarrier signals each of which has a symbol duration much larger than the delay spread, a MC-CDMA signal will not experience significant degradation from inter-chip interference and inter-symbol interference (ISI) [4,5].

In a previous paper [1], MC-CDMA was analyzed in a Rayleigh fading channel. Numerical results revealed that MC-CDMA's benefits may be exploited to a greater extent in the downlink where phase correction of the interference may be performed to partially restore orthogonality between users. It was also noted that while Maximum Ratio Combining (MRC) performed better in a noise-limited channel, Equal Gain Combining (EGC) performed better in an interference-limited channel.

In this paper, the performance of MC-CDMA in the downlink of a Rician fading channel will be analyzed. The effect of a controlled equalization scheme on this modulation scheme will be compared to the performance of EGC and MRC.

2. Basic Principles

Shown in Fig. 1 is the model of the transmitter. The input data symbols, $a_m[k]$, are assumed to be binary antipodal where k denotes the *kth* bit interval and m denotes the *mth* user. In the analysis, it is assumed that $a_m[k]$ takes on values of -1 and 1 with equal probability. The generation of an MC-CDMA signal can be described as follows. A single data symbol is replicated into N parallel copies. Each branch of the parallel stream is multiplied by a chip from a pseudo-random (PN) or some other code of length N and then BPSK modulated to a different subcarrier spaced apart from its neighboring subcarriers by F/T_b where F is an integer number. The transmitted signal consists of the sum of the outputs of these branches. This process yields a multi-carrier signal with the subcarriers containing the pn-coded data symbol.

As illustrated in Fig. 1, the transmitted signal corresponding to the *kth* data bit of the mth user is

$$s_m(t) = \sum_{i=0}^{N-1} c_m[i]\, a_m[k] \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t\right) \times \times p_{T_b}(t - kT_b) \qquad c_m[i] \in \{-1, 1\} \tag{1}$$

where $c_m[0], c_m[1], \ldots, c_m[N-1]$ represents the spreading code of the *mth* user and $p_{T_b}(t)$ is defined to be an unit amplitude pulse that is non-zero in the interval of $[0, T_b]$.

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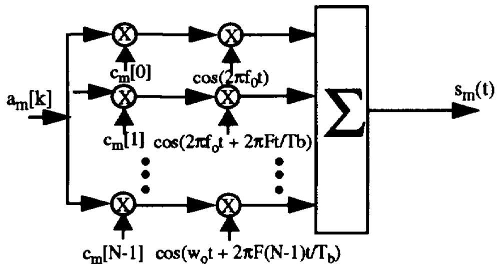

Fig. 1 Transmitter Model

3. Channel Model: Dispersive Rician Fading

In this paper, we will focus on a frequency-selective channel with $1/T_b << BW_c << F/T_b$. This model implies that each modulated subcarrier with transmission bandwidth of $1/T_b$ does not experience significant dispersion ($T_b >> T_d$). It is also assumed that the amplitude and phase remain constant over a symbol duration, T_b, (i.e., Doppler shifts due to the motion of terminals is negligible). This assumption agrees with indoor measurements of the Doppler shifts, which tend to be very small and typically in the range of 0.3-6.1 Hz [6].

For transmissions in the downlink, i.e., from the base station to the terminals, a terminal receives interfering signals designated for other users ($m = 1, 2, \ldots, M-1$) through the same channel as the wanted signal ($m = 0$). Thus, the transfer function of the continuous-time fading channel for all transmissions from the base station to user $m = 0$ can be represented as

$$H\left(f_c + i\frac{F}{T_b}\right) = \rho_i e^{j\theta_i} \quad (2)$$

where the random amplitude, ρ_i, and phase, θ_i, effects of the channel at frequency $f_c + i(F/T_b)$ are independent of m.

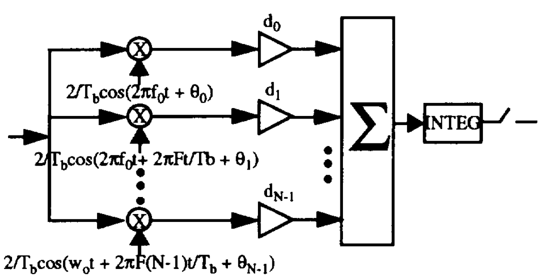

Fig. 2 Receiver Model

The phase effects, θ_i for $i = 0, 1, \ldots N-1$, introduced by the channel are assumed to be independent and identically distributed (iid) random variables uniform on the interval $[-\pi, \pi]$ for all subcarriers.

As there is often a line-of-sight (LOS) component in an indoor environment, the amplitude scaling factors, ρ_i for $i = 0, 1, \ldots, N-1$, are assumed to have the following Rician distribution

$$f_{\rho_i}(\rho_i) = \frac{\rho_i}{\sigma_i^2} e^{-\frac{\rho_i^2 + b_0^2}{2\sigma_i^2}} I_0\left(\frac{b_0 \rho_i}{\sigma_i^2}\right) \quad (3)$$

where σ_i^2 represents the power of the scattered component, b_0 is the LOS component and $I_0(\rho)$ is the zeroth ordered modified bessel function. As the notation suggests, the dominant LOS component b_0 is assumed to be equal for all subcarriers. The Rician distribution is often characterized by the K-factor which is defined as the ratio of the power of the LOS component to the power of the scattered component

$$K = \frac{\frac{1}{2} b_0^2}{\sigma_i^2} \quad (4)$$

A statistical quantity that is of interest is the mean of the Rician distribution which can be found to be

$$E\rho_i = e^{-K/2} \sqrt{\frac{\pi}{2(K+1)} \bar{p}_i} \times \left[(1+K) I_0\left(\frac{K}{2}\right) + K I_1\left(\frac{K}{2}\right) \right] \quad (5)$$

where $I_1(K)$ represents the first ordered modified bessel function.

4. Receiver Model

For M active transmitters, the received signal is

$$r(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \rho_i c_m[i] a_m[k] \times \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_i\right) + n(t) \quad (6)$$

where $n(t)$ is additive white Gaussian noise (AWGN). The local-mean power at the ith subcarrier is defined to be

$$\bar{p}_i = E\left[\rho_i \cos\left(2\pi f_c t + 2\pi \frac{i}{T_b} t + \theta_i\right)\right]^2 = \frac{1}{2} E\rho_i^2 \quad (7)$$

where it is assumed that local-mean powers of the subcarriers are equal. Thus, the total local-mean power of the mth user is defined to be $\bar{p} = N\bar{p}_i$. Shown in Fig. 2 is the model of the receiver. To simplify the analysis, it is assumed that exact synchronization with the desired user ($m = 0$) is possible. The first step in obtaining the decision variable involves demodulating each of subcarriers of the received signal, which includes applying a phase correction, $\hat{\theta}_i$, and multiplying the *ith* subcarrier signal by a gain correction, d_i. In the analysis, it is assumed that perfect phase correction can be obtained, i.e., $\hat{\theta}_i = \theta_i$. After adding the subcarrier signals together, the combined signal is then integrated and sampled to yield the decision variable, ν_0. For the *kth* bit, the decision variable is

$$\nu_o = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \rho_i c_m [i] d_i a_m [k] \frac{2}{T_b} \times \int_{kT_b}^{(k+1)T_b} \cos (2\pi f_c t + 2\pi F \frac{i}{T_b} t + \theta_i) \times \cos (2\pi f_c t + 2\pi F \frac{i}{T_b} t + \hat{\theta}_i)\, dt + \eta \quad (8)$$

where the corresponding AWGN term, η, is given as

$$\eta = \sum_{i=0}^{N-1} \int_{kT_b}^{(k+1)T_b} n(t) \frac{2}{T_b} d_i \times \cos (2\pi f_c t + 2\pi F \frac{i}{T_b} t + \hat{\theta}_i)\, dt \quad (9)$$

The value of the gain correction depends on the diversity method chosen. We will considered three methods: Equal Gain Combining (EGC), Maximum Ratio Combining (MRC), and Controlled Equalization (CE). EGC and MRC are discussed in [1]. With EGC, the gain correction factor is

$$d_i = c_0 [i] \quad (10)$$

With MRC, the gain correction factor is

$$d_i = \rho_i c_0 [i] \quad (11)$$

4.1 Controlled Equalization (CE)

While EGC may be desirable for simplicity and MRC for combating noise, neither of these techniques significantly exploit the coding of the subcarriers. With Controlled Equalization, an attempt at restoring the orthogonality between users is made by normalizing the amplitudes of the subcarriers. As the orthogonality of the users is encoded in the phase of the subcarriers, this method is primarily beneficial in the downlink where phase distortion for all users may be corrected. For CE, the gain factor for the ith subcarrier is

$$d_i = c_o [i] \frac{1}{\rho_i} u (\rho_i - \rho_{thresh}) \quad (12)$$

where $u(\rho_i)$ is the unit step function. Thus, only subcarriers above a certain threshold will be equalized and retained. This constraint is added to prevent the amplification of subcarriers with small amplitudes that may be dominated by a noise component.

5. Bit Error Rates (BER)

• EGC

Using EGC as the equalization technique results in the following decision variable

$$\nu_0 = a_0 [k] \sum_{i=0}^{N-1} \rho_i + \sum_{m=1}^{M-1} a_m [k] \times \sum_{i=0}^{N-1} c_m [i] c_0 [i] \rho_i + \eta \quad (13)$$

where the AWGN term, η, has a variance of NN_0/T_b. Because of the orthogonality of the codes, the interference term may be rewritten as

$$\beta_{int} = \sum_{m=1}^{M-1} a_m [k] \left(\sum_{j=1}^{N/2} \rho_{a_j} - \sum_{j=1}^{N/2} \rho_{b_j} \right) \quad (14)$$

where

$$c_m [a_j] c_0 [a_j] = 1 \qquad c_m [b_j] c_0 [b_j] = -1$$
$$\{a_j\} \cup \{b_j\} = \{0, 1, \ldots, N-1\} \quad (15)$$

Applying the Central Limit Theorem (CLT) individually to both inner sums, the interference term can be approximated by a zero-mean gaussian r.v. with a variance of

$$\sigma^2_{\beta_{int}} = 2(M-1)(1-\gamma)\bar{p} \quad (16)$$

where

$$\gamma = \frac{\pi}{4} (\frac{e^{-K}}{K+1}) \left[(1+K) I_0 (\frac{K}{2}) + K \times I_1 (\frac{K}{2}) \right]^2 \quad (17)$$

The probability of making a decision error can be written as

$$Pr(error | \bar{p}, K, \{\rho_i\}_{i=0}^{N-1}) = \frac{1}{2} erfc \left(\sqrt{\frac{\frac{1}{2} \left(\sum_{i=0}^{N-1} \rho_i \right)^2}{2(M-1)(1-\gamma)\bar{p} + \frac{NN_0}{T_b}}} \right) \quad (18)$$

Finding a closed form expression for the sum of N iid Rician r.v.'s has been historically a difficult problem. In [1], it was shown that using the LLN or CLT to approximate the sum of iid r.v.'s leads to approximately the same numerical results. Thus, in the rest of this paper, analytical results will be given only for the CLT approximations.

Applying the CLT, the sum may be approximated by a zero-mean gaussian distribution. Averaging Eq.(18) over the gaussian distribution results in the following BER

$$Pr(error|\bar{p}, K) \cong$$

$$\cong \frac{1}{2} erfc\left(\sqrt{\frac{\gamma\bar{p}T_b}{2\frac{M}{N}[1-\gamma]\bar{p}T_b + N_0}}\right). \quad (19)$$

- **MRC**

Following an analysis similar to the analysis above, the BER for MRC can be determined to be

$$Pr(error|\bar{p}, K) \cong$$

$$\cong \frac{1}{2} erfc\left(\sqrt{\frac{\bar{p}T_b}{\frac{M}{N}\left[\frac{4K+2}{(K+1)^2}\right]\bar{p}T_b + N_0}}\right). \quad (20)$$

- **CE**

Given that there are n_0 subcarriers above the threshold indexed by j, applying CE transforms the decision variable in Eq.(8) to

$$\upsilon_o| n_0 = a_0 [k] n_0 + \beta_{int} + \sum_{j=0}^{n_0-1} \frac{\eta_j}{\rho_j} \quad (21)$$

where the interference term, β_{int}, is given as

$$\beta_{int} = \sum_{m=1}^{M-1} a_m[k] \sum_{j=0}^{n_0-1} c_m[j] c_0[j] \quad (22)$$

As the noise term is the sum of n_0 independent random variables (r.v.), where n_0 will be large with a high probability for the values of ρ_{thresh} of interest, the noise can be approximated by the CLT to be a zero-mean gaussian r.v. with a variance of

$$\sigma_\eta^2 = n_0 \frac{N_0}{T_b} \frac{1}{2\bar{p}_i} E_1\left(\frac{\rho_{min}^2}{2\bar{p}_i}\right) \quad (23)$$

where $E_1(\rho)$ is the exponential integral defined as

$$E_1(\rho) = \int_\rho^\infty \frac{e^{-t}}{t} dt. \quad (24)$$

The probability of making a decision error given n_0 and the interference component, β_{int}, is

$$Pr(error| n_0, \beta_{int}) \cong \int_{n_0-\beta_{int}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_\eta^2}} e^{-\frac{x^2}{2\sigma_\eta^2}} dx \quad (25)$$

The distribution of the number of subcarriers above the threshold, n_0, is described by the following binomial distribution

$$p_{n_0}(n_0) = \binom{N}{n_0}\left(e^{-\frac{\rho_{thresh}^2}{2\bar{\rho}_i}}\right)^{n_0}\left(1 - e^{-\frac{\rho_{thresh}^2}{2\bar{\rho}_i}}\right)^{N-n_0} \quad (26)$$

for $n_0 = 0, 1, 2, \ldots, N$.

Note that the orthogonality of the codes, given by the following condition

$$\sum_{i=0}^{N-1} c_m[i] c_0[i] = N\delta_{m,0}, \quad (27)$$

implies that half of the inner products, $c_m[i] c_0[i]$, are positive while the other half are negative. Using this observation, the distribution of the inner sum,

$$\Sigma_m = \sum_{j=0}^{n_0-1} c_m[j] c_0[j] , \quad (28)$$

given n_0 is

$$p_{\Sigma_m| n_0}(\Sigma_m| n_0) = \frac{\binom{N/2}{(n_0+\Sigma_m)/2}\binom{N/2}{(n_0-\Sigma_m)/2}}{\binom{N}{n_0}} \quad (29)$$

where $-min\{n_0, N-n_0\} \le \Sigma_m \le min\{n_0, N-n_0\}$ and Σ_m can only assume even (odd) values if n_0 is even (odd). The exact distribution of the interference term, β_{int}, given n_0 depends on the spreading code, $\{c_m[i]\}$ for $i = 0, 1, \ldots, N-1$, that is used. In this analysis, it is assumed that each of the inner sums act as an independent r.v., and that the pdf of the interference is given as the convolution of the pdfs of Σ_m for $m = 1, 2, \ldots, M-1$. Combining the results given above yield the following expression for the bit error rate

$$BER \cong \sum_{n_0=0}^{N} p_{n_0}(n_0) \sum_{\beta_{int}} p_{\beta_{int}| n_0}(\beta_{int}| n_0) \times \frac{1}{2} erfc\left(\frac{n_0 - \beta_{int}}{\sqrt{2}\sigma_\eta}\right) \quad (30)$$

6. Numerical Results

Plots of the BER for Rician K-factors of 0 and 10 are shown in Fig. 3 for EGC and MRC. Note that for the Rayleigh fading case ($K = 0$) the expressions in Eqs. [19,20] reduce to the results derived in [1] for Rayleigh fading. As in the case of Rayleigh fading, MRC has a better performance for very low number of interferers while EGC outperforms MRC in an interference limited Rician fading channel. This result reflects the observation that MRC distorts the orthogonality further between users and consequently does not perform as well when a large number of interferers are present.

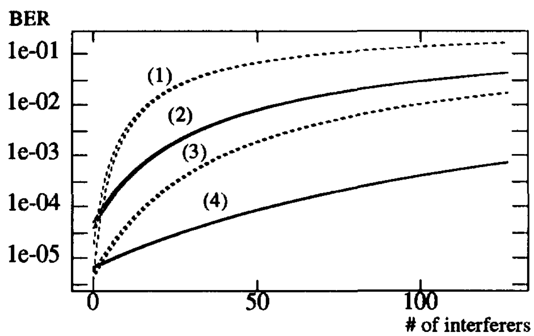

Fig. 3 BER vs. the # of interferers for different Rician K-factors using MRC: K=0 (1) and K=10 (3) and EGC: K=0 (2) and K=10 (4). Curves are shown for both CLT and LLN approximations. The SNR is 10 dB and N = 128.

Plots of the BER for CE are shown in Fig. 4 for $K = 5$ and Fig. 5 for $K = 10$. From the curves, it can be seen CE outperforms EGC and MRC in combating interference. Note that there exists a ρ_{thresh} such that the BER vs. the number of interferers is relatively flat. At this threshold level, there are a sufficiently number of subcarriers above the threshold such that orthogonality between users has been significantly restored. As the threshold level is lowered past this point, no benefit occurs since "orthogonality" has already been achieved and only noise amplification results. For higher threshold values, the BER is affected by the number of interferers to a greater extent. However, for all threshold values, the performance of CE is worse than EGC or MRC for a small number of interferers due to the amplification of noise.

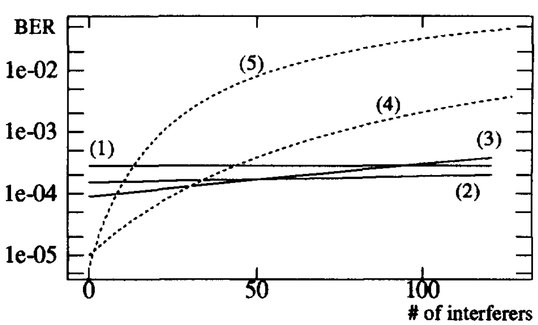

Fig. 4 BER for CE vs. the # of interferers with K=5 for ρ_{thresh}=0.002 (1), ρ_{thresh}=0.008 (2), and ρ_{thresh}=0.014 (3). The SNR is 10 dB, p=0.1, and N=128. Plots for EGC (4) and MRC (5) are included for comparison.

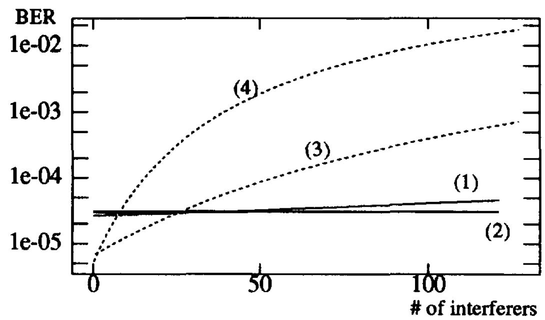

Fig. 5 BER for CE vs. the # of interferers with K=10 for ρ_{thresh}=0.008 (2) and ρ_{thresh}=0.016 (1). The SNR is 10 dB, p=0.1, and N=128. Plots for EGC (3) and MRC (4) are included for comparison.

7. Conclusion

In this paper, the performance of MC-CDMA in the downlink of an indoor Rician fading channel was evaluated. Numerical results revealed that the Rician K-factor has a significant effect on the BER. In addition, it was found that a controlled equalization technique (CE) that attempts to restore the orthogonality between users outperforms EGC and MRC in combating interference.

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