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CONTIGUOUS FREQUENCY ASSIGNMENT IN WIRELESS PACKET-SWITCHED NETWORKS

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ABSTRACT - **Efficient use of the available radio spectrum resources is of key importance to the performance of mobile radio networks. We show that for packet-switched cellular networks for bursty data traffic a cluster size C = 1, i.e., the same radio channel is reused in all cells, gives the smallest packet delay and highest spectrum efficiency. The analysis addresses the probability of successful reception over a channel with UHF path loss, shadowing and Rayleigh fading, AWGN and man-made burst noise and with intermittent interference from co-channel base stations because of packets arriving in each base station buffer according to a Poisson process. The conclusions are in sharp contrast to commonly used reuse schemes in most existing mobile data nets, where cluster sizes of C = 7 or 9 are typical. "Spatial Collision Resolution" (SCR) is proposed as a novel scheme for further increasing the spectrum efficiency by coordinating packet retransmission attempts in adjacent cells.**

1 Introduction

The user interest in wireless access to communication services is rapidly growing and new networks and systems are developed and implemented at pace. Efficient use of the allocated spectrum resources is essential to accommodate the expected traffic growth. Many researchers have addressed optimum (cellular) frequency re-use strategies for circuit-switched (telephone) networks. This paper studies efficient frequency re-use schemes for packet-switched data networks. In contrast to cellular telephony networks our objective will not be to ensure sufficiently small outage probabilities at the cell boundary per sé. Rather, we wish to minimize the packet queueing delay which occurs if unsuccessful messages have to be retransmitted. We focus on the downlink (base to terminal) as the random-access issues in the uplink have been addressed previously in [1, 2].

The outline of the papers is as follows: Section 2 and 3 present a model for successful data reception in cellular mobile fading channels. In Section 4, the packet queueing delay in the base station is computed as a function of the carried traffic per unit area and per unit of time. Section 5 presents a method for spatial collision resolution.

2 System model

In a (frequency non-selective) narrowband fading channel, the signal amplitude $\rho_i(t)$ and phase $\theta_i(t)$ received from the i-th base station are subject to multipath fading, log-normal shadowing and path loss. Because of multipath fading in a macro-cellular network, the amplitude is Rayleigh distributed and the instantaneous power p is exponentially distributed with mean $\bar{p}_i$, called the 'local-mean power'. Because of shadowing, the received local-mean power $\bar{p}_i$ has the log-normal pdf

$$f_{\bar{p}}(\bar{p} \mid \bar{\bar{p}}_i) = \frac{1}{\sqrt{2\pi}\,\sigma_s \bar{p}} \exp\left\{-\frac{1}{2\sigma_s^2}\ln^2\left(\frac{\bar{p}}{\bar{\bar{p}}_i}\right)\right\}, \quad (1)$$

where σ_s is the logarithmic standard deviation of the shadowing, expressed in natural units, so the local-mean power expressed in logarithmic values, such as dB or neper, has a normal distribution. For given area-mean power $\bar{\bar{p}}_i$, the instantaneous power in each path has the Suzuki pdf

$$f_{p_i}(p \mid \bar{\bar{p}}_i) = \int_0^\infty \frac{1}{\bar{p}_i}\exp\left\{-\frac{p}{\bar{p}_i}\right\}\frac{1}{\sqrt{2\pi}\,\sigma_s \bar{p}_i}\exp\left\{-\frac{\ln^2\frac{\bar{p}_i}{\bar{\bar{p}}_i}}{2\sigma_s^2}\right\}d\bar{p}_i \triangleq \frac{1}{\bar{\bar{p}}_i}\, su_\sigma\!\left(\frac{p}{\bar{\bar{p}}_i}\right). \quad (2)$$

Here we introduced the notation $su_\sigma(\cdot)$ for the normalised pdf for Suzuki fading of a signal with unity area-mean power. Evidently, in the special case of a channel without shadowing ($\sigma_s = 0$), $su_0(\cdot)$ goes into the exponential pdf $su_0(p) = \exp\{-p\}$. In UHF macrocellular radio, the area-mean power typically decreases with distance r_i according to $\bar{\bar{p}}_i = r_i^{-\beta}$ with β approximately 4.

3 Laplace-Transform Expressions

The probability of successful reception depends on the instantaneous signal-to-interference (C/I) ratio, but also on the type of modulation and coding and the character of the interference [3]. In narrowband radio, typically the probability of successful reception is close to zero for C/I ratios less than 4 dB and is close to one for C/I ratios larger than 10 dB. Because of UHF path loss, shadowing and Rayleigh fading, the dynamic range of the received signals are many tens of dBs. The probability that the C/I ratio is in this particular transition range 4 to 10 dB is relatively small and does not critically influence the performance. Therefor one may approximate the probability of successful reception by the probability that the C/I ratio is above a ceratin threshold z with $z \approx 3$ .. 10. This is in contrast to CDMA transmission where power control is required to control the dynamic range of all signals and threshold models become unrealistic. Assuming constant received power during a packet transmission time T_L, the probability of successful reception is

$$\Pr(p_0 > zp_t) = \int_{0-}^{\infty} f_{p_t}(x) \int_{zx}^{\infty} f_{p_0}(y)\, dy\, dx \qquad (3)$$

where p_t is the joint interference power. This expression was interpreted as an integral transform of the pdf of joint *interference* power. In a Rayleigh-fading channel, the conditional pdf f_{p_0} of the instantaneous power of the wanted signal is exponentially distributed. Hence, assuming the local-mean power $\bar{p}_0$ to be known

$$P(p_0 > zp_t|\bar{p}_0) = \int_{0-}^{\infty} e^{-\frac{xz}{\bar{p}_0}} f_{p_t}(x)\, dx \triangleq \mathcal{L}\left\{f_{p_t}, \frac{z}{\bar{p}_0}\right\}. \qquad (4)$$

where $\mathcal{L}\{f,s\}$ denotes the one-sided Laplace transform of the pdf f at the point s. For incoherent cumulation of statistically independent signals, the pdf of the joint interference power is the n-fold convolution of the pdf of the individual powers. Laplace transformation results in the multiplication of n factors, each containing a Laplace image of the pdf of the received power from an individual component. For ease of notation, this conditional probability of successful transmission is denoted as $\bar{Q}(s)$ with $s = z/\bar{p}_0$. Averaging over the shadowing experienced by the desired signal, the area-mean probability becomes

$$Q(r_0) \triangleq \int_0^{\infty} f_{\bar{p}_0}(\bar{p}_0 \,|\, \bar{\bar{p}}_0 = r_0^{-\beta})\, \bar{Q}\left(\frac{z}{\bar{p}_0}\right) d\bar{p}_0 \qquad (5)$$

which depends on the location of the terminal in the cell.

3.1 *Co-channel Interference*

The image of the instantaneous power received from the i-th interferer with area-mean power $\bar{\bar{p}}_i = r_i^{-\beta} = \sqrt{(3C)}$ where C is the cluster size, is denoted as $SU_\sigma(s\bar{\bar{p}}_i)$ with

$$SU_\sigma(s\bar{\bar{p}}_i) \triangleq \mathcal{L}\{f_{\bar{p}_i|\bar{\bar{p}}_i}; s\} = \int_0^{\infty} \frac{1}{1+s\bar{p}_i} \frac{1}{\sqrt{2\pi}\sigma_s \bar{p}_i} e^{-\frac{\ln^2 \frac{\bar{p}_i}{\bar{\bar{p}}_i}}{2\sigma_s^2}}\, d\bar{p}_i\,, \qquad (6)$$

This can be rewritten more conveniently as the transform pair

$$\frac{1}{\bar{\bar{p}}_i} su_\sigma\left(\frac{p}{\bar{\bar{p}}_i}\right) \overset{\mathcal{L}}{\leftrightarrow} SU_\sigma(s\bar{\bar{p}}_i) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-x^2} dx}{1 + s\bar{\bar{p}}_i\, e^{\sqrt{2}x\sigma}} \qquad (7)$$

For Rayleigh-fading channels without shadowing ($\sigma_s = 0$), this image becomes $SU_0(s\bar{p}_i) = (1 + s\bar{p}_i)^{-1}$. If the i-th base station transmits with probability $P(i_{ON})$, $\bar{Q}(s)$ becomes Π_i [1 + $P(i_{ON})$\{$SU_\sigma(s\bar{p}_i)$ −1\}].

3.2 *Noise*

We distinguish two types of noise: wideband and narrowband (burst) noise. The bandwidth of the Additive White Gaussian (AWGN) noise is limited by receive filters optimized to detect the desired signal. Since outage events are defined for a minimum averaging window of duration T significantly longer than the inverse of the receiver RF-bandwidth B_T, ($B_T \approx 1/T_b$ with T_b the bit time), we may consider the short-term average noise power to be constant. The total received noise power is $\bar{p}_n = N_0/T_b$, where N_0 is the one-sided spectral power density of the AWGN at the receiver input. This noise power has the pdf

$$f_{p_n}(x) = \delta(x - \bar{p}_n)\,, \qquad (8)$$

and Laplace image

$$\mathcal{L}\{f_{p_n}, s\} = \exp\{-s\bar{p}_n\}\,. \qquad (9)$$

In addition to the AWGN, the mobile terminal receives narrowband man-made noise. The bandwidth of burst noise is substantially smaller than the RF bandwidth of the receiver. The pdf of this "class A noise" [4] is described as the infinite sum of Gaussian pdf's of momentary noise voltages. Converting Middletons' expressions [4] into amplitudes and and powers we get

$$f_{p_m}(p) = e^{-A}\left[\delta(p) + \sum_{m=1}^{\infty} \frac{A^{m+1}}{(m+1)!\bar{p}_m} e^{-\frac{Ap}{m\bar{p}_m}}\right] \qquad (10)$$

The expected value of the noise power is $\bar{p}_m$. Typical values for the impulsive index are $0.01 < A < 0.5$ and for the mean power $0.001 < \bar{p}_n/\bar{p}_m < 0.1$, i.e., the man-made noise is 10 to 30 dB above the AWGN noise floor. Since AWGN and man-made noise are independent and add incoherently, the image of the pdf of the total (AWG plus burst) noise power now becomes

$$\mathcal{L}\{f_{p_n} \otimes f_{p_m}, s\} = e^{-s\bar{p}_n - A}\left[1 + \sum_{m=1}^{\infty} \frac{A^m/m!}{1 + s\frac{A}{m\bar{p}_m}}\right]. \qquad (11)$$

4 PERFORMANCE AND SPECTRUM EFFICIENCY

The method developed in the previous sections is now applied to generic packet data networks, focussing on the downlink, i.e., from base station to mobile terminal. It will appear that packet switching requires entirely different frequency reuse scheme than circuit-switched telephone networks.

The queueing delay incurred at the base station buffer is taken as the performance measure for the network in Figure 1. It appears reasonable to assume that each base station transmits messages to terminals uniformly distributed over the cell, i.e., the required *throughput* per unit of area is uniform. The successful throughput per base station, expressed in packets per cell per slot is denoted as S_0 ($0 \le S_0 \le 1$). For a cellular network with cluster size C, we define the spectrum efficiency (or the spatial packet throughput density) as

$$SE_D = \eta_r \frac{S_0}{C} \qquad (12)$$

expressed in gross user bits per base station per Hz per second. The bit rate per bandwidth is denoted as η_r. The normalized spectrum efficiency SE_d is found from $\eta_r = 1$ bit/s/Hz.

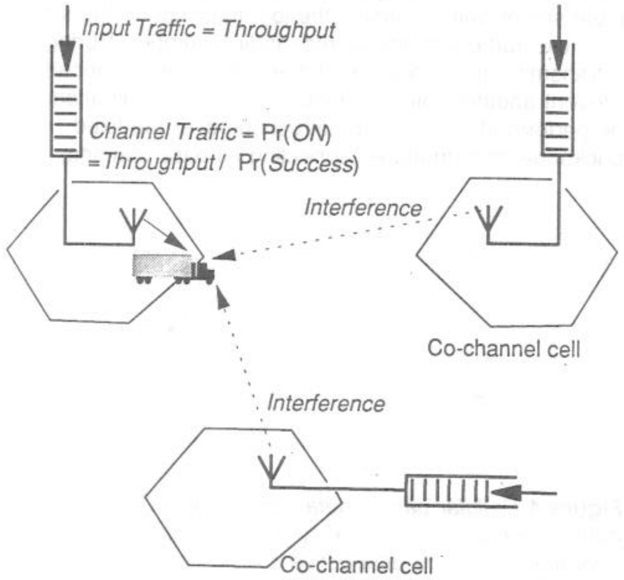

Figure 1 *Cellular packet-switched network configuration.*

Since the probability of successful reception $Q(r)$ decreases with increasing propagation distance r, the expected number of (re-) transmission attempts $M(r)$ increases with increasing r. Figure 2 gives the expected number of required transmission attempts per message in a 1-cell cluster for various propagation conditions. Noise mainly affects the reception at the fringe of the cell. The C/N of more than 20 dB appears sufficient, but poorer noise performance leads to many retransmissions and severely reduces the throughput. Shadowing decreases the overall performance but occasional shadow up-fades help transmission to terminals at the very boundary of the cell.

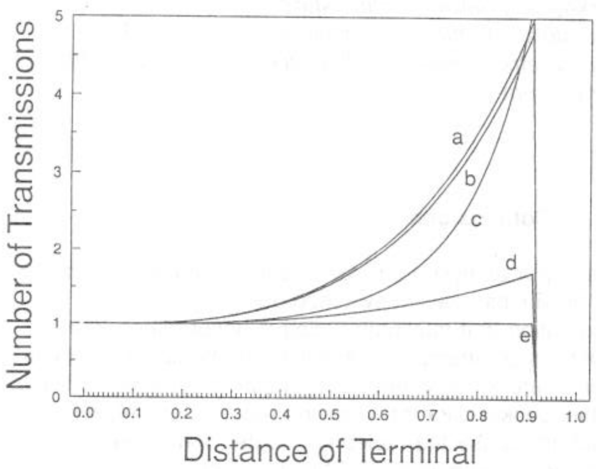

Figure 2 *Expected number of required transmission attempts versus distance for* 6 dB *shadowing. Cluster size* $C = 1$, (a) *with and* (b) *without AWG noise* ($\overline{p}_n = 0.01$), *and* (c) *without shadowing. Comparision with* (d) $C = 3$ *and* (e) $C \to \infty$.

We assume that the Rayleigh fading and shadow attenuation are independent from one transmission attempt to the next, but that the area-mean power remains constant during retransmission delay time. The cell throughput is found from

$$S_0 = P(0_{ON})Q_s = P(0_{ON})\left[\int_0^R \frac{2rdr}{R^2 Q(r)}\right]^{-1} \quad (13)$$

packets per time slot, where 0_{ON} is the event that the (reference) base station transmits a packet and Q_s is cell-average probability of successful reception. The surface area of a unit hexagonal cell corresponds to the surface area of a circular cell with radius $R \approx 0.91$. Since we assume uniform throughput, rather than uniform attempted traffic, we may not compute $Q_s = \int Q(r)\, 2\pi r dr$.

Table 1 Maximum uniform throughput S_0 per base station for various cluster sizes C. Spectrum Efficiency SE_D and expected number of retransmissions at the cell boundary. Modulation technique: $z = 4$ (6 dB) and $\eta_r = 1$ bit/s/Hz. Shadowing 6 dB ($\sigma_s = 1.36$). AWGN −40 dB at $r = 1$, Man-made noise −20 dB at $r = 1$, Impulsive index $A = 0.05$.

C	$S_{0,MAX}$	SE_D	# of transmissions average	fringe (r=0.91)
1	0.40	0.40	2.49	5.00
3	0.78	0.26	1.29	1.70
4	0.84	0.26	1.19	1.47
7	0.91	0.10	1.10	1.22
9	0.93	0.10	1.07	1.16
∞	0.99	0	1.00	1.01

The result in Table 1 that a cluster size $C = 1$ can support a uniform throughput of 0.40 packets per base station clearly shows that increasing the cluster size to 3, 4 or higher cannot increase the spectrum efficiency. The capacity $S_{0,MAX}$, however, can only be achieved if the base station always has a packet ready for transmission, in which case its queue length is unstable and the delay tends to infinity. The delay $D(r)$ is mainly determined by queueing of packets in the base station: packets with nearby destinations may have to wait because of transmission time required for packets to remote destinations. The delay is significantly less sensitive to r than $M(r)$, and may depend on strategy by which the base station selects packets queued for (re-) transmission. Therefore, the following analysis addresses cell-average delays. The number of packets waiting in the base station is modelled as a Discrete Time $M/M/1/\infty$ queue, that is, the service time is approximated as a geometric random variable with mean $1/Q_s$. In steady state, $Q_s = S_0 / P(0_{ON})$. For sake of simplicity we consider a 'slow backbone network', i.e., during a certain time slot, the probability of excatly one packet arrival is S_0, the probability of more than one arrival is zero, and the probability of no arrival is $1 - S_0$. The probability of increasing the number of packets queued is $S_0(1 - Q_s)$ and the probability of decrease is $Q_s(1 - S_0)$. The traffic parameter ρ is found from $\rho = S_0(1 - Q_s) / (Q_s(1 - S_0))$. The probability of m packets in the queue is geometric with $P_m(m) = (1 - \rho)\rho^m$. The average number of packets in the queue is $\rho (1 - \rho)^{-1}$. Using Little's formula, the average delay, expressed in seconds, is

$$D_a = \frac{\rho}{1-\rho}\,\frac{1}{S_0}\,T_L = C\,\frac{1-Q_s}{Q_s-S_0}\,\frac{L}{\eta_r B_N}\,, \quad (14)$$

because the duration of the transmission of a packet transmission with L bits is $T_L = L\,T_b = L\,/\,(\eta_r\,B_T) = L\,C\,/\,(\eta_r\,B_N)$, where B_N is the total bandwidth allocated to the network ($B_N = CB_T$). We normalize by taking $L\,/\,(\eta_r\,B_N) = 1$ second. Figure 3 gives the normalised delay versus the spectrum efficiency SE_d for various cluster sizes (C = 1, 3, 4, 7, 9). These results have been computed by taking $P(i_{ON})$ as an independent parameter. This determines the statistics of interference from other cells and gives the corresponding S_0 and Q_s to be inserted in (12) and (14). The optimum cluster size appears $C = 1$, as it is ensure the smallest delay for any spatial throughput traffic density SE_d.

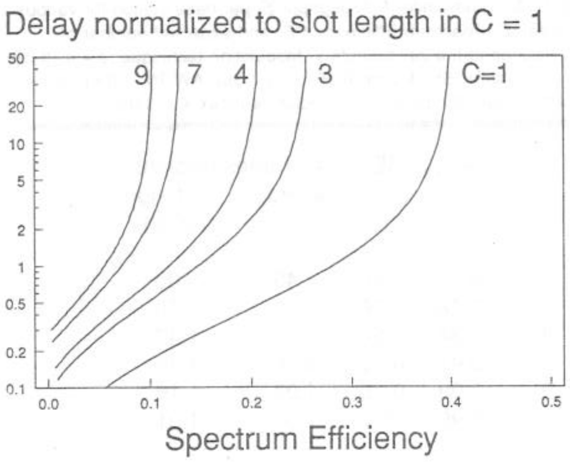

Figure 3 *Delay, normalised to slot duration in network with* $C = 1$, *versus normalised spectrum efficiency for various cluster sizes.* 6 dB *of shadowing. Receiver threshold* $z = 4$ (6 dB). *UHF groundwave propagation* ($\beta = 4$).

5 Spatial Collision resolution

Intuitively one would regard the downlink mobile data channel as a queueing or multiplexing problem rather than a collision-type multiple-access problem: each base station has full control of the traffic flows to any terminal in its range. However, according to the approach in this paper, in high capacity spectrum-efficient packet switched networks, adjacent base stations have to compete for non disjoint (i.e., interfering) spectrum resources. The analysis was confined to the case of independently operating base stations. Resolution of collisions between packet transmission attempts in adjacent cells appears to enhance the spectrum efficiency of radio data networks further.

In particular, we are currently performing a performance analysis of the following 'spatial collision resolution' (SCR) scheme [5]: all base stations share the same transmit channel, which has frames of four time slots. The areas covered by each base station are assigned a sequence number {1, 2 or 3} according to map-coloring scheme which ensures that adjacent areas always have a different number. In normal operation, a base station can transmit in any time slot regardless of its number (Figure 4). If base stations in adjacent areas happen to transmit simultaneously, all signals may nonetheless 'capture' their intended receiver. With some probability however, interference power levels are too strong to allow reliable detection of all messages involved. In the latter case, the base station will retransmit the lost message in the slot of the next frame with the corresponding number. During this retransmission, all adjacent base stations are silenced to prevent another collision (Figure 5). This coordination can be performed by sending instructions over the fixed backbone infrastructure connecting all base stations.

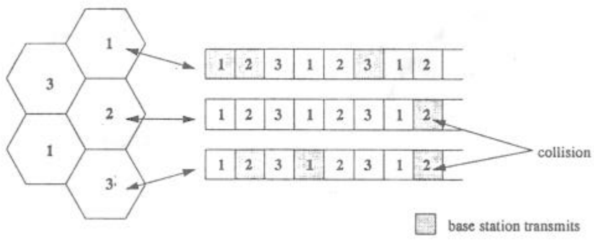

Figure 4 *Cellular packet data network with* $C = 1$ *under normal Contiguous Frequency Assignment (CFA) operation, i.e., if no destructive collision occurs. Any base station can transmit in any time slot, accepting the risk of excessive interference from cochannel transmissions in nearby cells.*

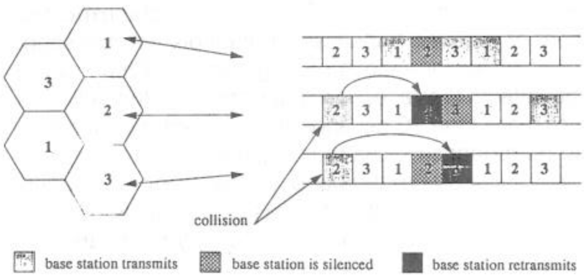

Figure 5 *Spatial Collision Resolution (SCR) in cellular packet data network. Base stations only transmit in the time slots with the corresponding sequence number. During retransmissions, all adjacent base stations must refrain from transmitting.*

6 Conclusions

This paper analysed cellular frequency reuse in the down-link of wireless packet-switched data network. Our assumption that multipath fading is uncorrelated from transmission attempt to transmission attempt is particularly appropriate for slow frequency-hopping networks. Efficient mobile packet data transmission requires entirely different spectrum re-use than telephone nets. Contiguous Frequency Assignment (CFA), i.e., cluster size $C = 1$, can support approximately 0.4 bit/s/Hz/cell and it appeared substantially more efficient than cellular frequency re-use ($C = 3, 4, 7, \ldots$). It also provides the smallest packet delay at a given spatial packet throughput intensity.

Results confirm the general observation that, to ensure minimum delay at maximum user capacity, mobile radio data networks are best designed with relatively dense frequency re-use. The resulting low signal-to-interference ratios and large packet loss probabilities at the link level pose severe constraints on the design of future end-to-end packet-switching protocols. For instance, for fast shadowing we found that the delay and user capacity are optimised if the Medium Access protocol performs an average of up to five transmission attempts per packet for terminal at the fringe of the cell. Even for these remote terminals, increasing the cluster size to $C = 3, 4, ..$ does not decrease the delay.

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