Slotted ALOHA land-mobile radio networks with site diversity

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Abstract: An analytical method for investigating slotted ALOHA land-mobile networks is presented. The probability of successful reception and throughput is assessed for a single base station, taking account of contending transmissions and receiver noise. Receiver capture is assumed to occur if the received signal power exceeds the joint interference power by a certain margin, called the receiver threshold. Rayleigh fading and UHF groundwave propagation are considered. Results are extended for a cellular network considering the interference from packet transmissions in other (co-channel) cells. It is seen that in packet-switched cellular nets, frequency re-use distances may be substantially smaller than in circuit-switched (CW) telephony networks, where each cell has to be safeguarded continuously from co-channel interference. Moreover, a technique to assess the throughput of ALOHA networks with multiple, geographically-separated base stations is presented and numerical results are given for uniform Poisson-distributed packet transmissions in the service area.

1 Introduction

In present mobile radio networks, high spectrum efficiency is achieved by extensive frequency re-use. In the late 70s, cellular engineering [1] was developed to accommodate the rapid growth of mobile telephony. In continuous wave (CW) communication, as used in circuit-switched mobile telephony, optimum cell repetition patterns have been determined, and their spatial spectrum efficiency has been assessed. However, in numerous mobile communication systems, packet switching (often of a routine type and with short data messages [2]) can provide more efficient use of the available bandwidth. In this case, a large number of terminals can operate within one cell and messages are sent to the base station over a common radio channel, according to an appropriate random-access scheme. Cellular packet networks applying a multiple-access protocol based on slotted ALOHA, such as the public packet-switched network 'Mobitex', are in public or private operation in an increasing number of countries. In this case, the access to the radio channels has a 'bursty' character, and packets lost due to interference are automatically retransmitted [3]. In these networks, therefore, the need to continuously safeguard the traffic in one cell from excessive interference co-channel transmissions in other cells is less demanding than in mobile CW telephony. Thus, frequency re-use distances and cell-cluster sizes may be smaller. Moreover, a packet-switched cellular radio network might exploit the diversity resulting from packet reception by base stations outside the particular cell in which the terminal is located. Most of the recent studies of the performance of mobile ALOHA networks address the case of a single base station, whereas cellular frequency re-use and site diversity has received relatively little attention. A better understanding of multi-cell multiple-access networks appears important to the design of future mobile and personal digital communications networks.

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2 Random access to wide-area network

Throughout this paper, random access for mobile terminals by (slotted) ALOHA is considered. In pure (unslotted) ALOHA networks, transmissions from mobile subscribers to fixed base stations occur in an uncoordinated manner, so packets may be lost owing to mutual interference between participating terminals. In slotted ALOHA, the only regulation is that packets must be contained in predefined, but not user-assigned, time-slots. After receiving a packet, the base station sends an acknowledgment to the mobile terminal. If the mobile terminal does not receive an acknowledgment, it will assume that the packet is lost and will retransmit it after a random time. It is well known that in wired channels in which a collision destroys all packets involved, the maximum throughput is about 18% ($1/2e$) for unslotted ALOHA and about 36% ($1/e$) for slotted ALOHA [4]. Furthermore, the ALOHA network with an infinite number of users has a tendency towards instability [5]. In mobile radio channels, receiver capture may occur if the received signal powers of the colliding packets differ sufficiently. This enhances the throughput [6–15] and mitigates instability [13].

In a wide-area network, a single receiver may not be sufficient to support the offered packet traffic. Two scenarios for increasing the system capacity are considered in this paper: cellular frequency re-use and site diversity. In a cellular ALOHA network, adjacent cells use different in-bound frequencies, so inter-cell interference arrives only from distant terminals. It will be illustrated that relatively short frequency re-use distances can be tolerated without substantial sacrifices to the throughput of each cell. In an extreme case, adjacent cells use the same frequency and slot synchronisation, so that packets near the boundary of the two cells can be received successfully at more than one base station. This offers 'site diversity'.

If the in-bound radio channel is common to all base stations in the network, a wider bandwidth is available for this channel, as the available spectrum is not divided to provide for a cluster of cell frequencies. The corresponding shorter packet duration results in fewer colliding packets per time-slot, which may improve the performance of the network.

In the analysis, it is assumed that time-slot synchronisation is perfect throughout the entire network and known to all terminals; propagation delays and technical difficulties of implementation are ignored. This assumption might be reasonable in networks of limited size employing a relatively low bit rate, for instance 1200 bit/s [3]. If a higher bit rate is chosen, as is increasingly the case in mobile data nets, propagation delays can become of the same order of magnitude as the duration of a data packet. For instance, a call request by a mobile station in the GSM system ($r_b \approx 270$ kbit/s) is made by means of slotted ALOHA. During the telephone call, in TDMA operation, timing advance and retard is possible for each mobile to ensure slotted arrival of blocks of bits. In contrast to this, in random-access networks, isolated data packets are offered to the channel and the provision of feedback information on variable propagation delays is not feasible. In this case, the effective throughput of the random-access channel is severely impaired by guard times to ensure that, despite random propagation delays, received packets fit within prescribed time-slots. Efficiency is further reduced if guard times have to be large enough to ensure slotted arrival of packets at base stations outside the cell. In the Appendix, it is shown that the advantages of slotted ALOHA over pure (unslotted) ALOHA vanish if required guard times exceed the duration of a packet. The Appendix also discusses how the results derived in this paper for slotted ALOHA can be adapted to (pessimistically) approximate the throughput for pure ALOHA by considering interfering traffic with doubled intensity.

Despite the guard times required in practical networks, all data packets are assumed to be of uniform duration, equal to the slot length, which is taken as the normalised unit of time. The duration of a packet in seconds is T_s. If a packet is received at more than one base station simultaneously, the network is assumed to recognise multiple reception and to ignore all but one version of the packet. The feedback channel for acknowledgments of correctly-received packets is assumed to be perfect. This implies that transmissions by the base stations are co-ordinated to guarantee that messages to a mobile terminal experience no harmful interference. The network protocol requires the presence of a suitable control network interconnecting all base stations. Here, we do not address the design of this (wired) network and of the supporting protocols.

3 Channel and capture model

The normalised mean power, received from a mobile terminal i at a normalised distance a_i from base station A is taken to have the form [13, 6]

$$\bar{p}_{A_i} = \alpha a_i^{-\beta} \tag{1}$$

where α is taken to be unity and power and range are normalised so that $0 < a_i < 1$ for terminals within the service area (cell) of the receiver A. The empirical attenuation exponent β is in the range 2 to 4 for UHF propagation. Owing to Rayleigh fading in a narrowband channel, the instantaneous power p_{A_i} received from the ith mobile terminal is exponentially distributed about the local mean power $\bar{p}_{A_i}$ [13], viz.

$$f_{p_{A(i)}}(p_{A_i} \mid \bar{p}_{A_i}) = \frac{1}{\bar{p}_{A_i}} \exp\left(-\frac{p_{A_i}}{\bar{p}_{A_i}}\right) \tag{2}$$

The received packet power from two successive transmissions of the terminal is assumed to be entirely uncorrelated, because of Rayleigh fading. This assumes that the terminal is moving. We will distinguish the case where, during a retransmission waiting time, the mobile terminal moves sufficiently far to assume uncorrelated distances a_i during each retransmission and, alternatively, the case where for each packet, the distance a_i remains constant until eventually a (re-)transmission attempt is successful (see Section 4.3). Shadowing [8, 13] is ignored.

If two or more packets are transmitted from different locations in the service area, their received powers can differ significantly. This effect gives rise to receiver capture: the packet with the strongest signal may be received correctly despite the presence of other contending packet signals. In 1977, the influence of path loss (1) and receiver capture on the throughput of the ALOHA channel was assessed by Abramson [6]. The additional effect of multipath fading was studied in Reference 7. A number of papers also included the effect of different modulation techniques [9–11, 14] and shadowing [8, 13] on receiver capture. It was shown in References 9 and 10 that, for Gaussian distributed interference with mean power $\bar{p}_t$, the bit error probability for a BPSK signal in a Rayleigh fading channel is

$$P_{be} = \tfrac{1}{2} \operatorname{erfc}\left\{\sqrt{\left(\frac{p_{A_j} T_b}{N_0 + \bar{p}_t T_b}\right)}\right\} \tag{3}$$

where N_0 is the spectral density of the additive white Gaussian noise in W/Hz and T_b denotes the bit duration in seconds.* If an error correction code that can correct up to t errors is used, the probability of correct reception of a data packet of L bits from terminal j at receiver A (denoted as event A_j), given the instantaneous received signal power, is

$$\Pr(A_j \mid p_{A_j}) = \sum_{m=0}^{t} \binom{L}{m} (1 - P_{be})^{L-m} P_{be}^{m} \tag{4}$$

where we assume that bit error probabilities are independent from bit to bit [9, 10].

Fig. 1*a* plots the packet success probability (4) against the C/I ratio for various values of t. In contrast to the smooth transition seen in Fig. 1*a* for Gaussian interference, the immunity to one single (constant-envelope) interfering signal is likely to be more abrupt, and is mainly determined by the ability of the receiver to lock onto the test signal j [15]. For a perfectly synchronising receiver, a step function near $C/I \approx 0$ dB may be considered [16]. The above refinements to the receiver capture model are not considered here. For ease of analysis, a packet is assumed to capture the receiver in a base station if, and only if, its instantaneous power exceeds the instantaneous joint interference power by a certain margin (factor) z, called the receiver threshold [7]. Although this simplifies the detection process in the receiver, the threshold model may be considered appropriate

* All variables and parameters concerning transmission aspects are expressed in dimensions related to the *second*. This is in contrast to parameters concerning protocol issues, where *packet length* is used as the normalised unit of time

if the probability that the C/I ratio in the transition range is relatively low.

Anticipating the results to be derived in following sections, the throughput of the ALOHA channel against the threshold z is depicted in Fig. 1b. The accuracy of the 'threshold model' depends highly on the probability that

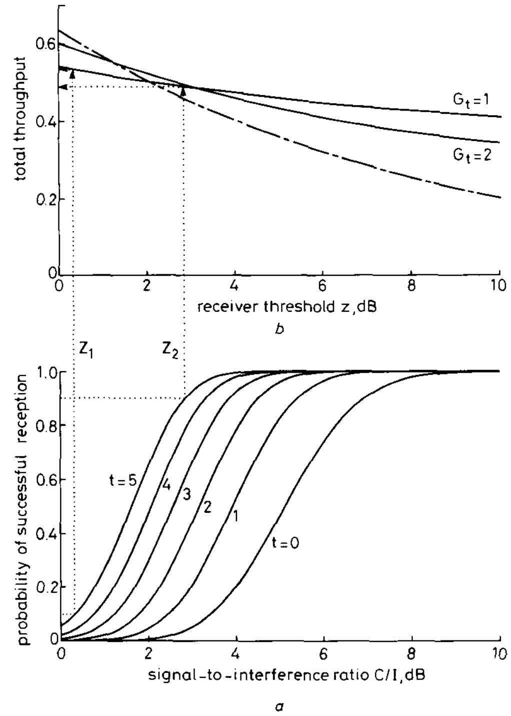


Fig. 1 *Variation of probability of successful reception and throughput with C/I ratio and receiver threshold, respectively*

a Probability of correct reception of a packet of 127 bits with BPSK modulation against C/I ratio with Gaussian interference for various error correction capabilities

b Throughput against receiver threshold z. Uniform offered traffic with $G_t = 1$, 2 pps and infinity

during a certain time-slot the C/I ratio is in the transition range ($z_1 < C/I < z_2$), which is found to be the throughput for z_2 minus the throughput for z_1. Fig. 1b confirms that (under certain conditions to be discussed later) the variations in received signal powers are sufficiently high to ensure that this probability is relatively small. In this event, the threshold model may yield acceptable estimates of the probability of capture and of the channel throughput. This observation is in contrast to conclusions reported from recent investigations of spread spectrum channels, where it was assumed that all signals arrive with almost identical power, e.g. Reference 17. In such cases, the C/I ratio does not vary substantially and the threshold model becomes inappropriate. Moreover, for analogue FM modulation by an FFSK subcarrier [3, 18], the threshold model is believed to be reasonable even if fluctuations in received power are relatively small, because of the FM capture effect. In this case, practical values for z are about 10–20 dB, corresponding to the FM threshold [19].

In this paper numerical results are presented for $z = 4$ (6 dB), which is more pessimistic than $z = 1 \cdots 2$ ($0 \cdots 3$ dB) as suggested by Fig. 1a. This is believed to be a more realistic value for practical receivers for binary-keyed signals in narrowband channels, because imperfect synchronisation may impair the capture performance as compared in eqn. 4. During the reception of a packet, the received signal powers (and thus also the C/I ratio) are assumed to be constant: packets are considered to be short compared with the time-constants of the multipath fading. As reported in the Appendix, the results presented here may be optimistic if, in a practical system, packet durations are not small compared with the time-constants of the fading.

4 Single base station

The event where the C/I ratio of a packet signal from terminal j at receiver A is above the receiver threshold z is denoted as A_j. The probability of capture $\Pr(A_j \mid \bar{p}_{A_j})$, given the local mean power of the desired test packet, can be expressed as

$$\Pr(A_j \mid \bar{p}_{A_j}) = \Pr\left(\frac{p_{A_j}}{p_t} > z \mid \bar{p}_{A_j}\right) \quad (5)$$

where p_t denotes the instantaneous joint interference power of the contending packets. Arnbak and Van Blitterswijk [7] showed by transformation of the stochastic variables p_{A_j} and p_t that, in a Rayleigh fading channel

$$\Pr(A_j \mid \bar{p}_{A_j}) = \int_z^\infty \int_0^\infty \frac{1}{\bar{p}_{A_j}} \exp\left\{-\frac{xy}{\bar{p}_{A_j}}\right\} f_{p_t}(x) x \, dx \, dy \quad (6)$$

This reduces to

$$\Pr(A_j \mid \bar{p}_{A_j}) = \int_0^\infty \exp\left\{-\frac{xz}{\bar{p}_{A_j}}\right\} f_{p_t}(x) \, dx \quad (7)$$

This probability is seen to equal the Laplace image of the PDF $f_{p_t}(\cdot)$ of the joint interference power at the point $z/\bar{p}_{A_j}$ [12, 13]. If the interfering signals accumulate incoherently [7]

$$p_t = \sum_{i=1}^{n} p_{A_i} \quad (8)$$

where n is the number of contaminating signals. In this case, the PDF of p_t is the n-fold convolution of the PDF of the individual signal powers. Laplace transformation of this PDF corresponds to multiplication of n factors, each containing the Laplace image of the PDF of the power received from an individual interferer, so

$$\begin{aligned} \Pr(A_j \mid n, \bar{p}_{A_j}) &= \mathscr{L}\left\{f_{p_t}, \frac{z}{\bar{p}_{A_j}}\right\} \\ &= \mathscr{L}\left\{f_{p_{A_1}} \otimes \cdots \otimes f_{p_{A_n}}, \frac{z}{\bar{p}_{A_j}}\right\} \\ &= \prod_{i=1}^{n} \mathscr{L}\left\{f_{p_{A_i}}, \frac{z}{\bar{p}_{A_j}}\right\} \end{aligned} \quad (9)$$

where $\otimes$ denotes a convolution. Rayleigh fading of the test signal is included in eqn. 9: this probability is conditional on the *mean* power received from the jth mobile transmitter. Rayleigh fading of the interfering signals is to be incorporated in $f_{p_{A_i}}(\cdot)$.

Two cases of contaminating signals are now considered. Expressions are obtained for the Laplace image of interference power caused by a terminal occasionally transmitting a packet, and for a Laplace image embodying the effect of a receiver noise floor. The probability that terminal k offers an interfering packet during the time-slot in which the 'test' packet from terminal j is transmitted is denoted as $\Pr(k_{ON})$. This probability is assumed stationary for each terminal. Also, $\Pr(k_{OFF})$ denotes the probability that terminal k does not transmit, with $\Pr(k_{OFF}) = 1 - \Pr(k_{ON})$. The Laplace transform of the received signal power from the kth terminal is

$$\mathscr{L}\{f_{pA_k}, v\} = \Pr(k_{OFF})\mathscr{L}\{\delta(p_{A_k}), v\} + \Pr(k_{ON})\mathscr{L}\{f_{pA_k}(p_{A_k} | k_{ON}), v\} \quad (10)$$

where v is the variable in the domain of the image, and $\delta(p_{A_k})$ is a delta function at $p_{A_k} = 0$ to account for zero received power at k_{OFF}.

A receiver noise floor may be taken into account by considering a contaminating signal additional to the interfering packet signals. The amplitude of bandlimited noise, is Rayleigh-distributed at any instant during the reception of a data packet. However, here we address the signal-to-noise ratio during the entire reception of a data packet, thus averaged over a duration much longer than the time-constants of fluctuations of the amplitude of the noise. Therefore, we assume a fixed noise power N_A, and a fixed signal-to-noise ratio p_{A_i}/N_A, during a packet reception. The noise power p_n thus has the PDF $\delta(p_n - N_A)$ with Laplace image $\exp\{-vN_A\}$. We denote the probability that the signal from terminal j fails to sufficiently exceed the noise floor at receiver A as P_{NA}, with

$$P_{NA} = \exp\left\{-N_A \frac{z}{\bar{p}_{A_j}}\right\} \quad (11)$$

Because noise and interference add incoherently, P_{NA} may be included as a multiplicative factor in the product in eqn. 9.

4.1 Finite population

We now assume a finite population of N terminals with known positions. If the position of an interfering terminal i is known, the PDF of the received power is given by eqn. 2 with eqn. 1. Since $\mathscr{L}\{\delta(p_{A_i}), v\} \equiv 1$ and $\mathscr{L}\{q^{-1}\exp(-p_{A_i}q^{-1}), v\} = (1 + vq)^{-1}$, the Laplace image eqn. 10 of received power is

$$\mathscr{L}\{f_{pA_i}(p_{A_i} | \bar{p}_{A_i}), v\} = 1 - \Pr(i_{ON}) + \frac{1}{1 + v\bar{p}_{A_i}} \Pr(i_{ON}) \quad (12)$$

The probability $\Pr(A_j | \{a_j\})$ that a 'test' packet transmitted by terminal j captures the receiver in base station A, given all distances a_i ($i = 1, \ldots, N$), is found from inserting the images eqns. 11 and 12 in probability (9). Thus

$$\begin{aligned}\Pr(A_j | \{a_i\}_{i=1}^N) &= P_{NA} \prod_{i=1, i \neq j}^{N} [1 - \Pr(i_{ON}) + \mathscr{L}\{f_{pA_i}(p_{A_i} | \{a_i\}_{i=1}^N, i_{ON}), za_j^\beta\} \Pr(i_{ON})] \\ &= P_{NA} \prod_{i=1, i \neq j}^{N} 1 - \frac{za_j^\beta}{za_j^\beta + a_i^\beta} \Pr(i_{ON}) \end{aligned} \quad (13)$$

For ease of notation, we define a vulnerability weight function $W(a_j, a_i)$ $[0 \leqslant W(.,.) \leqslant 1]$ as

$$W(a_j, a_i) \triangleq \frac{za_j^\beta}{za_j^\beta + a_i^\beta} = 1 - \frac{1}{1 + z\left(\frac{a_j}{a_i}\right)^\beta} \quad (14)$$

Fig. 2 illustrates the vulnerability of a packet transmitted from unity distance ($a_j = 1$) to an interfering packet from distance a_i. The classical results for slotted ALOHA without receiver capture, e.g. eqns. 3 and 4 in Reference 4,

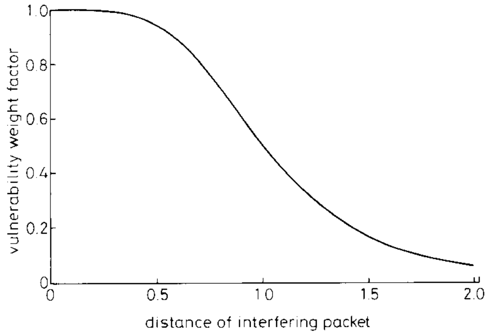


Fig. 2 *Vulnerability weight of a packet from distance $a_j = 1$ to an interfering packet from distance a_i*

are recovered by making the vulnerability weight equal to unity, thus $W(.,.) \equiv 1$. Similarly, for a high receiver threshold ($z \to \infty$) or for a distant terminal ($a_j \gg a_i$), the high vulnerability to any interfering signal is represented by a weight factor close to unity $[W(a_j, a_i) \to 1]$. On the other hand, for test packets from a nearby terminal, interference from remote terminals may be ignored, which is confirmed by the weight factor tending to zero $[W(a_j, a_i) \to 0]$ for $a_j \ll a_i$.

4.2 Infinite population

The probability of receiving signal j during a collision with n interfering signals ($j \notin \{1, \ldots, n\}$) is found from eqn. 9. We now address the situation where only the position of test terminal j is known. After averaging over the unknown positions of the interfering terminals, the conditional probability of capture becomes

$$\begin{aligned}\Pr(A_j | n, a_j, \{i_{ON}\}_{i=1}^n) &= \int_0^\infty \cdots \int_0^\infty P_{NA} \times \prod_{i=1}^n \mathscr{L}\left\{f_{pA_i}(p_{A_i} | \bar{p}_{A_i} = a_i^{-\beta}, i_{ON}), \frac{z}{\bar{p}_{A_j}}\right\} \\ &\quad \times f_{a_1}(a_1), \ldots, f_{a_n}(a_n)\, da_1, \ldots, da_n \\ &= P_{NA} \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^n [1 - W(a_j, a_i)] \\ &\quad \times f_{a_1}(a_1), \ldots, f_{a_n}(a_n)\, da_1, \ldots, da_n \\ &= P_{NA} \left\{\int_0^\infty [1 - W(a_j, a_i)] f_{a_i}(a_i)\, da_i\right\}^n \end{aligned} \quad (15)$$

Here, integration and product may be interchanged because each integration variable a_i occurs in only one factor of the product.

The PDF of range $f_{a_i}(.)$ in eqn. 15 depends on the distribution of the terminals over the service area. Analogous to anlayses in References 6, 7, 9, 12, 13 and 15, we define the spatial density of the offered packet traffic $G(a_i)$ as the average number of packets per time-slot per normalised unit of area (ppsa), transmitted at a distance a_i. The total offered traffic, expressed in packets per time-slot (pps), is obtained by polar integration, namely

$$G_t = \int_0^\infty 2\pi a_i\, G(a_i)\, da_i \quad (16)$$

Also, $G_t = \sum_{i=1}^{N} \Pr(i_{ON})$ with $N \to \infty$. The PDF of the propagation range is found from

$$|f_{a_i}(a_i)\, da_i| = \left|\frac{2\pi a_i G(a_i)}{G_t}\, da_i\right| \quad (17)$$

Inserting eqn. 17 into eqn. 15, the conditional probability of capture becomes

$$\Pr(A_j \mid n, \{i_{ON}\}_{i=1}^{n}, a_j) = P_{NA}\left[\frac{1}{G_t}\int_0^\infty \{1 - W(a_j, a_i)\} G(a_i) 2\pi a_i\, da_i\right]^n \quad (18)$$

Assuming that Poisson-distributed packet traffic is offered by the infinite number of participating terminals ($N \to \infty$), the capture probability at base station A becomes

$$\Pr(A_j \mid a_j) = P_{NA} \sum_{n=0}^{\infty} \frac{G_t^n}{n!} \exp(-G_t) \Pr(A_j \mid a_j, n, \{i_{ON}\}_{i=1}^{n}) = P_{NA} \exp\left\{-\int_0^\infty 2\pi a_i W(a_j, a_i) G(a_i)\, da_i\right\} \quad (19)$$

Because the vulnerability $W(.,.)$ to colliding packets increases with distance a_j, probability 19 is a smoothly-decreasing function of a_j. Limiting cases are $\Pr(A_j \mid a_j) \to 1$ for $a_j \to 0$, $\Pr(A_j \mid a_j) \to 0$ for $a_j \to \infty$ in a noisy channel ($N_A > 0$), and $\Pr(A_j \mid a_j) \to \exp\{-G_t\}$ for $a_j \to \infty$ in a noise-free channel ($N_A = 0$).

The throughput per unit area $S(a_j)$, defined as the average number of successful packets per unit area per time-slot, is obtained from $S(a_j) = G(a_j) \Pr(A_j \mid a_j)$. Analogous to eqn. 16, the total throughput at receiver A, denoted as S_A and expressed in successful packets per slot, is obtained by polar integration of $S(a_j)$.

4.3 Spatial distribution of offered traffic

In a cellular ALOHA network, contending packets arrive from within the cell. If we assume the offered traffic to be uniformly distributed over a cell with unity radius, we may write $G(a_i) = G_0 = G_t/\pi$ inside the cell ($0 < a_i < 1$) and $G(a_i) = 0$ outside the cell ($a_i > 1$). If we ignore interfering traffic from other co-channel cells, eqn. 19 becomes, for $\beta = 4$

$$\Pr(A_j \mid a_j) = P_{NA} \exp\left\{-\sqrt{(z)} a_j^2 G_t \arctan\left[\frac{1}{\sqrt{(z)} a_j^2}\right]\right\} \quad (20)$$

and the total throughput S_A is

$$S_A = G_t \int_0^1 2a_j \Pr(A_j \mid a_j)\, da_j \quad (21)$$

If, on the other hand, the same frequency were to be utilised over an infinitely large area, the offered traffic becomes globally uniform, i.e. $G(a_i) = G_0$ for $0 < a_i < \infty$. For this offered traffic and UHF groundwave propagation ($\beta = 4$), the capture probability eqn. 19 goes into

$$\Pr(A_j \mid a_j) = P_{NA} \exp\left\{-2\pi z G_0 \int_0^\infty \frac{a_j^4 a_k\, da_k}{a_k^4 + z a_j^4}\right\} = \exp\left\{-z N_A a_j^4 - \frac{a_j^2}{2\sigma^2}\right\} \quad (22)$$

where

$$\sigma^2 \triangleq \frac{1}{G_0 \pi^2 \sqrt{(z)}} \quad (23)$$

The total throughput S_A at base station A becomes

$$S_A = \int_0^\infty 2\pi a_j G_0 \Pr(A_j \mid a_j)\, da_j = \begin{cases} \dfrac{\pi}{2} G_0 \sqrt{\left(\dfrac{\pi}{z N_A}\right)} \exp\left\{\dfrac{\pi^4 G_0^2}{16 N_A}\right\} \operatorname{erfc}\left[\dfrac{\pi^2 G_0}{4\sqrt{(N_A)}}\right] & \text{if } N_A > 0 \\ \dfrac{2}{\pi\sqrt{(z)}} & \text{if } N_A = 0 \end{cases} \quad (24)$$

Fig. 3 compares the probability of capture against distance a_j for uniform offered traffic within a cell of finite

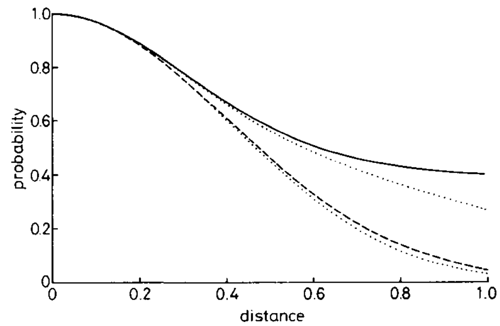


Fig. 3 *Probability of successful reception against distance a_j*
Offered traffic within the unit circle 1 pps. Receiver threshold $z = 4$ (6 dB)
——— uniform offered traffic ($G_t = 1$ pps)
– – – – globally uniform offered traffic
······· corresponding results for noisy channels with $N_A = 0.1$

size and for globally uniform offered traffic, eqns. 20 and 22, respectively. The effect of a receiver noise floor is considered for $N_A = 0.1$, i.e. for an average signal-to-noise ratio of 10 dB for a terminal at the boundary of the cell ($a_j = 1$). Fig. 1*b* shows the total throughput S_A against z for uniform offered traffic within the unit cell for $N_A = 0$, obtained from eqns. 20 and 21. In the limiting case $G_t \to \infty$, the throughput tends to $(2/\pi)/z$. It is seen that, particularly if the offered traffic is reasonably low, the throughput appears not unacceptably sensitive to the receiver threshold, which justifies the use of threshold model.

In the above two examples, it was assumed that the *offered* traffic is constant with range. Such an assumption is reasonable if the packet traffic load is relatively low, ensuring that few packets are lost in collisions. Also, if vehicle speeds are sufficiently large to ensure that the position of a terminal becomes uncorrelated between each (re-)transmission, the assumption that there is a uniform offered packet may be reasonable. Moreover, in certain vehicle location systems or telemetry applications, a (routine) status report lost in a collision may not need to be retransmitted, which may lead to uniform offered traffic if the road traffic is uniformly distributed around the location of the base station.

In contrast to the foregoing cases, in practical random-access networks, most retransmissions can be expected to occur in areas with poor propagation to the base stations [12]. We now assume that retransmissions are always transmitted from the same distance at which the first transmission was attempted. This corresponds to uniform *throughput* per unit of area, i.e. $S(a_j) \equiv S_0$ for $0 < a_j < 1$. This aspect and the effect of hexagonal cellular frequency re-use [1] can be studied if eqn. 19 is rewritten in the form

$$G(a_j) = \frac{S_0}{\Pr(A_j \mid a_j)} = S_0 \exp\left\{+zN_A a_j^\beta + \int_0^\infty 2\pi a_i W(a_j, a_i) \times [G(a_i) + 6G_I(a_i)]\, da_i\right\} \quad (25)$$

where S_0 is the required throughput per unit area $G(a_i) = G(a_j)$ denotes the offered traffic within the cell. In eqn. 25, the effect of interference from six other (co-channel) cells is taken into account by including $G_I(a_i)$ in the integral.

In the analysis of hexagonal cellular networks, it is common practice to normalise the length of the sides of each hexagon (and the radius) to unity (see Fig. 4). In this case, each hexagonal cell has surface area $(3\surd 3)/2$. This corresponds to the surface of a circular cell with radius $3^{3/4}/\surd(2\pi) \approx 0.91$. If C different carrier frequencies are used in each cluster of cells, the frequency re-use distance $R_u = \surd(3C)$, normalised to the radius of a hexagonal cell [1]. If the frequency re-use distance R_u is relatively large ($R_u \gg 1$), all interfering signals from co-channel cells arrive with almost identical mean power. In this case we may write

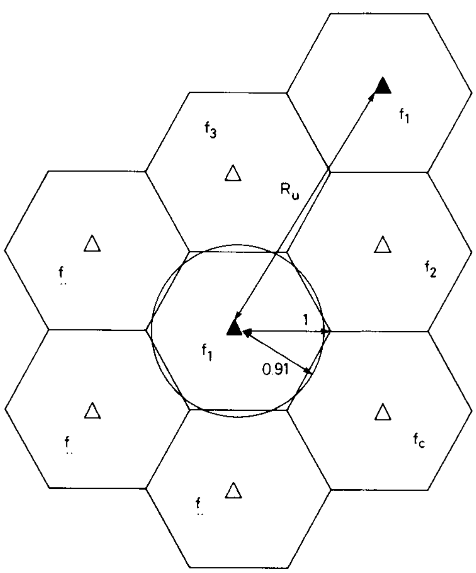


Fig. 4 *Cellular frequency re-use in a mobile ALOHA network. Hexagonal cells are approximated by circular cells of identical surface*

$$G_I(a_i) = \frac{G_t}{2\pi}\,\delta(R_u - a_i) \quad (26)$$

where G_t is the total offered traffic within one cell.

Eqn. 25, with eqn. 26, is solved by means of an iterative computational technique to obtain the traffic $G(a_j)$ to be offered to achieve uniform throughput $S_0 = S_A/(0.91^2\pi)$ within a circular cell $(0 < a_j < 0.91)$. Fig. 5 gives the traffic $G(a_j)$ to be offered to achieve uniform throughput with a total of $S_t = 0.4$ pps for various cluster sizes ($C = 4$, 9 and 100). Table 1 gives the expected number of required (re-)transmission attempts before successful reception occurs, for terminals at the boundary of the cell and for an average position in the cell. Also, the effect of a noise floor $N_A = 0.1$, which gives an average C/N of 11.6 dB for a terminal at $a_j = 0.91$, is considered. For a receiver threshold of 6 dB, a fade margin of 5.6 dB remains at $a_j = 0.91$. Interestingly, the effect of intercellular interference is relatively small, even for $C = 4$, compared with the effect of noise. The observation that noise may substantially diminish the performance in mobile ALOHA channels is in contrast to observations in Reference 10, where a prescribed offered traffic was assumed. It is seen in Fig. 5 that packets from remote terminals are likely to be lost because of noise, which causes a significant increase in the number of (re-)transmission attempts at larger ranges. Poor reception at the fringe of the cell may also threaten the stability of the network [13]. A higher fade margin appears desirable.

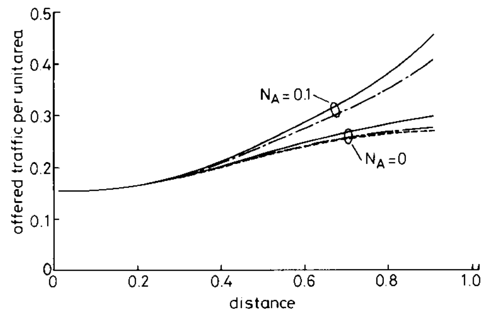


Fig. 5 *Traffic per unit area $G(a_j)$ to be offered to achieve throughput $S_A = 0.4$ pps with uniform distribution*
Receiver threshold $z = 4$ (6 dB)
——— cluster size $C = 4$
— · — cluster size $C = 9$
------ cluster size $C = 100$

Table 1: Expected number of required transmission attempts in a cellular ALOHA network for various cluster sizes and noise levels

C	$R_u = \surd(3C)$	N_A	G_t	Number of attempts	
				Average	Fringe of cell
9	5.20	0	0.62	1.56	1.79
9	5.20	0.1	0.75	1.88	2.68
4	3.46	0	0.64	1.61	1.94
4	3.46	0.1	0.80	2.00	2.97
100	17.3	0	0.62	1.54	1.76

Throughput per cell is $S_A = 0.4$ pps
Receiver threshold $z = 4$ (6 dB)

If slots in other (co-channel) cells are not synchronised, one may (pessimistically) approximate the effect of the interference by multiplying the interfering traffic $G_I(a_j)$ by a factor of 2 (see Appendix).

5 Two-branch site diversity

Although some results are available for antenna (or micro-)diversity at a single base station, e.g. Reference 20, the effect of applying multiple reciever sites has received little attention in the literature. In Reference 21, Chang studied the throughput of a random-access network with multiple receiver sites by extensive evaluation of expected collision events. Here, the vulnerability-weighting technique developed in Section 4 is extended to produce analytical expressions.

To study the throughput of two co-operating base stations, the probability of reception at a single base station (eqn. 19), together with the conditional probability of capture at base station B, given that the packet is not received successfully at base station A, are to be assessed. To this end, we exploit the *a posteriori* information that the occurrence of A_j gives about the activity of other (i.e. interfering) terminals.

5.1 Activity of interfering terminals

The probability $\Pr(k_{ON} \mid A_j)$ that terminal k has transmitted a packet, given the (*a posteriori*) knowledge A_j that a packet from terminal j captures base station A, is found from Bayes' rule

$$\Pr(k_{ON} \mid A_j) = \frac{\Pr(A_j \mid k_{ON})}{\Pr(A_j)} \Pr(k_{ON}) \tag{27}$$

We address the case that the position (and thus also the local mean power) of each participating terminal is known. Inserting capture probabilities of the form of eqn. 13, it follows that

$$\Pr(k_{ON} \mid A_j, \{a_i\}) = \frac{[1 - W(a_j, a_k)\Pr(k_{ON} \mid k_{ON})]P_{NA} \times \prod_{i=1, i\neq j, i \neq k}^{N} [1 - W(a_j, a_i)\Pr(i_{ON} \mid k_{ON})]}{P_{NA} \prod_{i=1, i \neq j}^{N} [1 - W(a_j, a_i)\Pr(i_{ON} \mid k_{ON})]} \times \Pr(k_{ON}) \tag{28}$$

In the numerator, the factor embodying the effect of the kth interferer is not included in the product but is written as a separate factor. Because of the *a priori* knowledge of k_{ON}, and as $\Pr(i_{ON} \mid k_{ON}) = 1$ for $i = k$ and $\Pr(i_{ON} \mid k_{ON}) = \Pr(i_{ON})$ otherwise, eqn. 28 reduces to

$$\Pr(k_{ON} \mid A_j, \{a_i\}) = \frac{1 - W(a_j, a_k)}{1 - W(a_j, a_k)\Pr(k_{ON})} \Pr(k_{ON}) \tag{29}$$

Remarkably, the probability that k was active during a slot in which j captured A, is defined uniquely by $\Pr(k_{ON})$ and the vulnerability weight $W(a_j, a_k)$: only variables concerned with terminal j and k appear in eqn. 29. The *a posteriori* probability appears to be dependent neither on the noise level, nor on further interfering terminals. For $\Pr(k_{ON}) < 1$, the *a posteriori* probability $\Pr(k_{ON} \mid A_j)$ is always less than the *a priori* probability $\Pr(k_{ON})$. Capture of a packet from terminal j at base station A gives the information that during that particular time-slot the number of interfering signals is presumably relatively low. Fig. 6 illustrates the effect of the position of the mobile terminals j and k on the probability of an interfering transmission from terminal k. Probability 29 is now used to assess the conditional probability of capture at receiver B.

5.2 Infinite population

The positions of the N terminals in the two-dimensional service area are denoted as $\boldsymbol{x}_i$. Since terminal i is at (normalised) distance a_i to base station A and at distance b_i to base station B, $|\boldsymbol{x}_j - \boldsymbol{x}_A| = a_j$ and $|\boldsymbol{x}_j - \boldsymbol{x}_B| = b_j$ (see Fig. 7). The normalised distance between the two base stations is R. Similar to eqn. 13, the conditional probability that the packet from terminal j captures base station B, given that it also captures base station A, is

$$\begin{aligned}\Pr(B_j \mid A_j, \{\boldsymbol{x}_i\}_{i=0}^{N}) &= P_{NB} \prod_{k=1, k\neq j}^{N} 1 - W(b_j, b_k)\Pr(k_{ON} \mid A_j, a_j, a_k) \\ &= P_{NB} \prod_{k=1, k \neq j}^{N} 1 - \frac{1 - W(a_j, a_k)}{1 - W(a_j, a_k)\Pr(k_{ON})} \\ &\quad \times W(b_j, b_k)\Pr(k_{ON})\end{aligned} \tag{30}$$

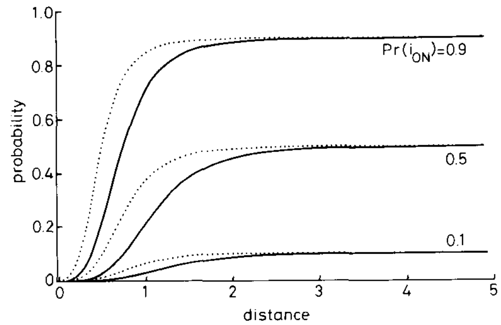


Fig. 6 *A posteriori probability $P(i_{ON} \mid A_j)$ of activity of interferer i against distance of the interferer, given that a packet from terminal j at distance $a_j = 1$ capture receiver A*

A priori probabilities $P(i_{ON}) = 0.1, 0.5$ and 0.9
.... $z = 1$
——— $z = 4$

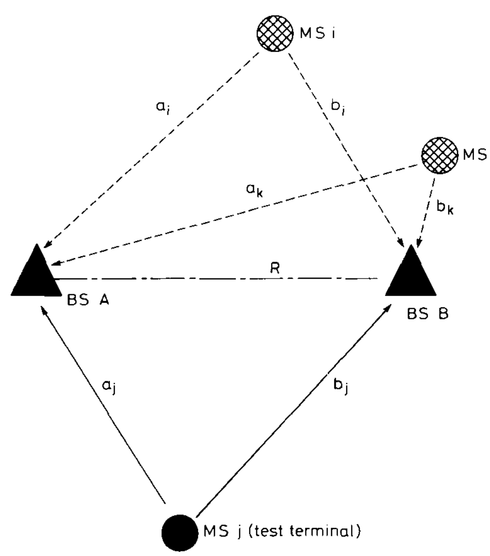


Fig. 7 *Multiple access scenario with mobile terminals and two base stations*

● mobile terminal; ▲ base station

where P_{NB} is the probability that the received signal fails to exceed the noise floor at receiver B. Comparison of eqn. 13 with eqn. 30 shows that $\Pr(B_j \mid A_j) \geq \Pr(B_j)$, i.e. a packet that captures one base station has an enhanced probability of capturing the second base station, also. This can be understood from the fact that, with a relatively high (*a posteriori*) probability, the interference level during that particular time-slot is low, as illustrated in eqn. 29.

The probability that a packet from terminal j will capture at least one of the two base stations (A or B), given the position of each terminal i ($i = 1, 2, \ldots, N$), is

$$\Pr(A_j \vee B_j \mid \{x_i\}_{i=1}^{N}) = \Pr(A_j \mid \{x_i\}) + \Pr(B_j \mid \{x_i\}) - \Pr(A_j \mid \{x_i\}) \Pr(B_j \mid A_j, \{x_i\}) \quad (31)$$

Substituting eqns. 13 and 30 into eqn. 31 gives

$$\Pr(A_j \vee B_j \mid \{x_i\}_{i=1}^{N}) = P_{NA} \prod_{i=1, i \neq j}^{N} 1 - W(a_j, a_i) \Pr(i_{ON}) + P_{NB} \prod_{i=1, i \neq j}^{N} 1 - W(b_j, b_i) \Pr(i_{ON}) - P_{NA} P_{NB} \prod_{i=1, i \neq j}^{N} 1 - W_{AB} \Pr(i_{ON}) \quad (32)$$

including the joint weight function

$$W_{AB} \triangleq W(a_j, a_k) + W(b_j, b_k) - W(a_j, a_k) W(b_j, b_k) = 1 - \frac{a_k^{\beta} b_k^{\beta}}{(z a_j^{\beta} + a_k^{\beta})(z b_j^{\beta} + b_k^{\beta})} = \frac{1}{\left\{1 + z\left(\frac{a_j}{a_k}\right)^{\beta}\right\}\left\{1 + z\left(\frac{b_j}{b_k}\right)^{\beta}\right\}} \quad (33)$$

Here, the factor W_{AB} weights the disturbance caused by an interfering packet signal from position x_k to a reception of a data packet by terminal j at the two base stations A and B simultaneously. For a large receiver threshold ($z \to \infty$) or for an interferer relatively close to either of the base stations ($a_k \ll a_j$ or $b_k \ll b_j$), the weight factor tends to unity. This represents the fact that the interference signal is very likely to disturb duplicated reception of the test packet from terminal j at the two base stations. For a remote interferer (b_k, $a_k \to \infty$), W_{AB} tends to zero: despite interference from k, the test packet is likely to be received correctly at both base stations simultaneously.

5.3 Infinite population of terminals

For a packet transmitted from a position x_j in the presence of n interfering packets, the probability of capture at at least one of the two base stations is found by rewriting eqn. 32 for a population of n interfering terminals known to transmit, and averaging over all possible position of the n interferers. So

$$\Pr(A_j \vee B_j \mid x_j, n, \{i_{on}\}) = \iint_{area} \cdots \iint_{area} \Pr(A_j \vee B_j \mid x_j, \{x_i, i_{ON}\}_{i=1}^{n}, n) \times f_{x_1}(x_1) \cdots f_{x_n}(x_n)\, dx_1 \cdots dx_n \quad (34)$$

Inserting $\Pr(i_{ON} \mid i_{ON}) = 1$ for i = 1, 2, ..., n in eqn. 32 and eqn. 34, gives

$$\Pr(A_j \vee B_j \mid n, x_j, \{i_{ON}\}) = \iint_{area} \cdots \iint_{area} \left\{ P_{NA} \prod_{i=1}^{n} [1 - W(a_j, a_i)] + P_{NB} \prod_{i=1}^{n} [1 - W(b_j, b_i)] - P_{NA} P_{NB} \prod_{i=1}^{n} [1 - W_{AB}] \right\} \times f_{x_1}(x_1) \cdots f_{x_n}(x_n)\, dx_1 \cdots dx_n \quad (35)$$

Since each of the integration variables x_i occurs only in one factor of each of the products, one may interchange product and integration. Hence,

$$\Pr(A_j \vee B_j \mid n, x_j, \{i_{ON}\}) = P_{NA} \left[\iint_{area} \{1 - W(a_j, a_i)\} f_{x_i}(x_i)\, dx_i \right]^n + P_{NB} \left[\iint_{area} \{1 - W(b_j, b_i)\} f_{x_i}(x_i)\, dx_i \right]^n - P_{NA} P_{NB} \left[\iint_{area} \{1 - W_{AB}\} f_{x_i}(x_i)\, dx_i \right]^n \quad (36)$$

For n Poisson distributed, this becomes a sum of exponential functions, namely

$$\Pr(A_j \vee B_j \mid x_j) = \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} \Pr(A_j \vee B_j \mid n, x_j \{i_{ON}\}) = P_{NA} \exp\left\{ - \iint_{area} W(a_j, a_i) G(x_i)\, dx_i \right\} + P_{NB} \exp\left\{ - \iint_{area} W(b_j, b_i) G(x_i)\, dx_i \right\} - P_{NA} P_{NB} \exp\left\{ - \iint_{area} W_{AB}\, G(x_i)\, dx_i \right\} \quad (37)$$

with $G(x_i)$ the offered trafic per unit area at terminal location x_i. Eqn. 37 offers a mathematical expression for the probability of capture at at least one receiver, for an arbitrary spatial distribution of the offered traffic. The expression contains three terms: the first and second terms are of the form of eqn. 19 for the individual receivers A and B; the third terms correspond to successful reception at both receivers simultaneously.

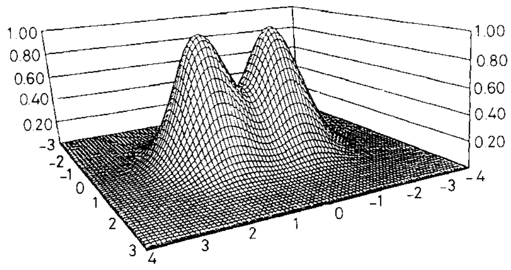
5.4 Globally uniform offered traffic

We assume a uniform offered packet traffic of $G(x) \equiv G_0$ for all x. Using the substitutions $r = a_k$ and $b_k^2 = r^2 + R^2 - 2Rr \cos\phi$, the third term in eqn. 37 equals, for a noise-free system

$$\Pr(A_j \wedge B_j \mid x_j) = \exp\left\langle - \int_0^{2\pi} \int_0^{\infty} \left\{ 1 - \frac{r^4 (r^2 + R^2 - 2rR\cos\phi)^2}{(z a_j^4 + r^4)[z b_j^4 + (r^2 + R^2 - 2rR\cos\phi)^2]} \right\} \times G_0\, r\, dr\, d\phi \right\rangle \quad (38)$$

which has been evaluated numerically. Finally the probability of correct reception for a packet from a known position x_j is found from eqns. 38 and 22.

Figs. 8*a* and *b* give numerical results for the probability of capture at at least one of two geographically



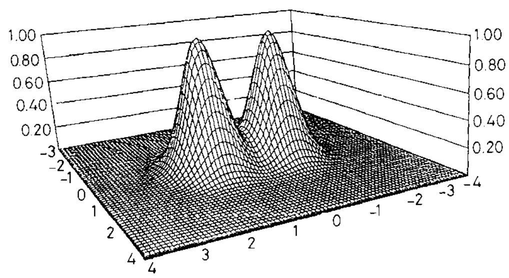


Fig. 8 *Probability of capture Pr $(A_j \vee B_j | x_j)$ as a function of the position of the terminal*

Base stations are located at (1, 0) and (−1, 0). The receiver threshold is 6 dB ($z = 4$)
a Offered traffic per unit area $G_0 = 0.1$ ppsa
b Offered traffic per unit area $G_0 = 0.2$ ppsa

separated base stations as a function of the (two-dimensional) position of the transmitting terminal. Base station A is located at $(-1, 0)$ and B is located at $(+1, 0)$. Fig. 9 presents the cross-section of Fig. 8*a* along the axis through both base stations. The exact result for the probability $\Pr(A_j \vee B_j | x_j)$ of capture at one of the two receivers is compared with the approximation $Q(x_j)$, calculated by

$$Q(x_j) \triangleq \Pr(A_j | x_j) + \Pr(B_j | x_j) - \Pr(A_j | x_j) \Pr(B_j | x_j) \tag{39}$$

Also, the probability $\Pr(A_j \wedge B_j)$ of successful reception at both receivers simultaneously is compared with $\Pr(A_j)$ $\Pr(B_j)$. These approximations correspond to the event where the interference level at the two receivers is uncorrelated at both sites, which underestimates $\Pr(A_j \wedge B_j)$ and overestimates $\Pr(A_j \vee B_j)$. It is seen that for the traffic load studied ($G_0 = 0.1$ ppsa, so $R \approx 3.95\sigma$), probability (38) of reception at both base stations simultaneously is underestimated by about 20% in the worst case, i.e. for a terminal located halfway between the two receivers $[x_j = (0, 0)]$. The effect on the probability of capture $\Pr[A_j \vee B_j | x_j = (0, 0)]$ is less than 5%.

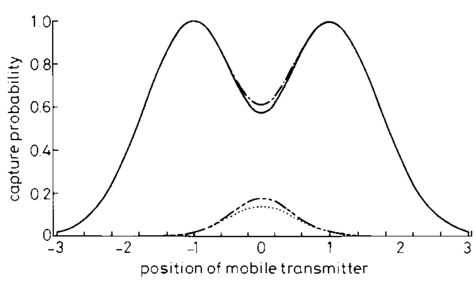


Fig. 9 *Probability of successful reception by at least one receiver Pr $(A_j \vee B_j | x_j)$ and at both receivers simultaneously Pr $(A_j \wedge B_j | x_j)$ for a terminal located on the line through the base stations at (1, 0) and (−1, 0)*

Total offered traffic per unit area is $G_0 = 0.1$ pps. Capture threshold is 6 dB ($z = 4$). Note that Fig. 9 represents a section through Fig. 8*a*
——— reception at one receiver; —·— approximation
–··– reception at both receivers; ········ approximation

6 Total throughput and approximation techniques

The total throughput of the two base stations is given by the integral

$$S_{A \vee B} = \iint_{area} G_0 \Pr(A_j \vee B_j | x_j) \, dx_j \tag{40}$$

which, taking account of the three terms in eqn. 37, may also be written as

$$S_{A \vee B} = S_A + S_B - S_{A \wedge B} \tag{41}$$

The common throughput of the two base stations can be computed from

$$S_{A \wedge B} = S_A \int_0^\infty f_{b_j}(b_j | A_j) \Pr(B_j | b_j, A_j) \, db_j \tag{42}$$

but is now approximated by

$$S'_{A \wedge B} \triangleq S_A \int_0^\infty f_{b_j}(b_j | A_j) \Pr(B_j | b_j) \, db_j \tag{43}$$

The approximation $S'_{A \wedge B}$ ignores the fact that during each slot, the number of interferers and the interference power level at receivers A and B are dependent, and the vulnerabilities of the test packet to these interferers are correlated at the two receivers. This causes B_j and A_j to be mutually correlated, as is considered in eqn. 42. The exact total throughput depicted in Fig. 10 is obtained by

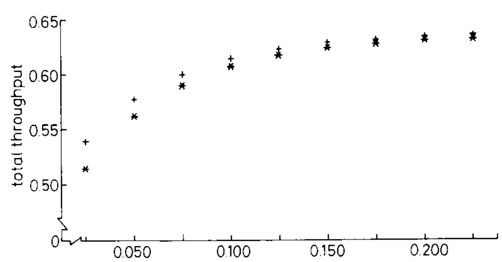


Fig. 10 *Exact total throughput (*) of two base stations against the traffic G_0 per unit of area, and an approximation (+)*

Receiver threshold $z = 4$ (6 dB); receiver separation $R = 2$

4-fold integration, arising from combining eqns. 38 and 40, to take due account of the correlated interference experienced at the two base stations. We now address the approximate and much simpler technique based on eqn. 43 to evaluate the common throughput $S'_{A \wedge B}$.

As seen from eqn. 22, in a noise-free channel, the *a posteriori* PDF of the propagation distance a_j of a correctly-received packet is Rayleigh distributed, with PDF

$$f_{a_j}(a_j \mid A_j) = \frac{2\pi a_j S(a_j)}{S_A} = \frac{2\pi a_j G_0 \Pr(A_j \mid a_j)}{S_A} = \frac{a_j}{\sigma^2} \exp\left\{-\frac{a_j^2}{2\sigma^2}\right\} \quad (44)$$

We now consider distances to base station B, which is a distance R from base station A. Transforming the Rayleigh distributed a_j into the statistical variable of the distance b_j to base station B, one obtains the Rician distribution

$$f_{b_j}(b_j \mid A_j) = \frac{b_j}{\sigma^2} \exp\left\{-\frac{R^2 + b_j^2}{2\sigma^2}\right\} I_0\left(\frac{Rb_j}{\sigma^2}\right) \quad (45)$$

where $I_0(.)$ is the zeroth order modified Bessel function of the first kind. Inserting eqn. 45 and using capture probabilities of the form of eqn. 19 to describe the probability of capture at B, eqn. 43 becomes

$$S'_{A \wedge B} = \frac{2}{\pi\sqrt{z}} \int_0^\infty \exp\left\{-\frac{b_j^2}{2\sigma^2}\right\} \frac{b_j}{\sigma^2} \times \exp\left\{-\frac{R^2 + b_j^2}{2\sigma^2}\right\} I_0\left(\frac{Rb_j}{\sigma^2}\right) db_j \quad (46)$$

The approximation 43 is seen to allow an analytical solution, i.e. by substituting

$$x^2 \triangleq 2b_j^2 \quad \text{and} \quad y^2 \triangleq \tfrac{1}{2}R^2 \quad (47)$$

we find

$$S'_{A \wedge B} = \frac{1}{\pi\sqrt{z}} \exp\left(-\frac{R^2}{4\sigma^2}\right) \quad (48)$$

The approximate total throughput $S'_{A \vee B}$ of the two base stations is found from

$$S'_{A \vee B} = S_A + S_B - S'_{A \wedge B} \quad (49)$$

The above approximate joint throughput (eqn. 49) (indicated by + in Fig. 10) has been compared with the results found from numerical integration of the exact expression eqn. 40 with eqns. 37 and 38 (indicated by * in Fig. 10) and from numerical integration of the approximation 39. The latter method, containing a 4-dimensional numerical integration, agreed with eqns. 49 and 48 up to the 5th decimal place. Comparison with exact results confirmed that the approximation technique underestimates the number of packets that capture both receivers simultaneously; however, it is noted that the accuracy of the approximation is better than a few percent for sufficiently spaced receivers, say $R \gg \sigma$. In this event, the separation between the two receivers is large compared to the expected distance from which received packets arrive. Each receiver will then mainly receive packets from its own vicinity, so that the probability of simultaneous reception of some packet at two base stations becomes low. More generally, the approximation appears relatively accurate if the amount of common packet traffic is not larger than, say, 10% of the total throughput, i.e. if

$$S'_{A \wedge B} < 0.1(S_A + S_B)$$

7 Conclusions

The performance of slotted ALOHA networks with a single base station, or with two base stations, has been investigated using a 'receiver threshold' model. It appears that this method can be acceptable if (i) the differences in received signal power from various terminals are relatively large, as is the case in narrowband mobile radio channels without adaptive power control, and (ii) the offered packet traffic is not excessively large. Systems with short packet durations compared with the time-constants of the fading have been analysed. It has been illustrated that short packets are preferable, to take maximum advantage of the throughput enhancements caused by the receiver capture effect.

The spatial distribution of the packet traffic to be offered to the channel to achieve a uniform throughput has been evaluated. It is seen that the effect of noise can be large: particularly, packets from remote terminals are lost because of noise, so that these terminals are likely to perform many retransmissions, which increases the number of collisions and degrades the total system performance. Results also show that in (packet-switched) ALOHA networks, frequency re-use distances can be substantially smaller than in (circuit-switched) networks for CW radio telephony. This suggests that if frequency re-use patterns are designed for mixed traffic, e.g. to support circuit-switched voice for emergency calls [3], spectrum usage during normal packet-switched operation is far from optimum. Moreover, results may motivate the use of wide-area networks with one contiguous 'cell', i.e. the use of the entire available bandwidth over the entire area rather than splitting the bandwidth into a number of channels, as is conventional in cellular telephony networks. In this case, packets can be received outside the cell where the transmitting terminal is located. This type of site diveristy may enhance the system throughput.

Although a fair comparison of the spectrum efficiency of such a system with a cellular ALOHA net cannot yet be given, a technique is proposed to assess the probability of successful reception of a data packet if site diversity is employed. A two-receiver configuration has been studied analytically. It is shown that a particular packet, when received correctly at one base station, also experiences an enhanced probability of capturing the second base station. This is explained by the observation that, with high *a posteriori* probability, the interference level during that particular time-slot is low. Numerical results have been obtained for the capture probability and the network throughput for the case of a uniform spatial distribution of the total packet traffic offered to the net.

For a cellular ALOHA network with each base station mainly supporting packet traffic from its own cell and receiving packets from transmitters in neighbouring cells relatively rarely, the interference levels experienced at two base station sites may be assumed to be uncorrelated. For this case, relatively simple, approximate expressions are proposed for the joint throughput of two base stations.

Because of propagation delays, the requirements for a fully synchronised random-access network may be too prohibitive to implement wide-area coverage with a 'contiguous cell' using one (common) frequency and exactly aligned time-slots. The results presented here may nonetheless be relevant to assess the enhancement of the throughput if a second receiver is geographically separated from the first receiver.

8 Acknowledgment

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10 Appendix

10.1 Packet duration and non-fade durations

In the literature, the analysis of mobile ALOHA systems mostly addresses two extreme cases: slow fading [7–10, 12, 13 and 15] and fast fading [9, 10 and 15]. In the former case, during reception of a data packet, received powers are assumed constant, whereas in the latter case, successive bits of a data packet are assumed to experience uncorrelated Rayleigh fading, and error correction coding is used to correct erroneously received bits during a signal fade. In this appendix we address the case where the time-constants of the fading are of same order of magnitude as the duration of the packet. In digital cellular (CW) telephony systems, the probability that a block of bits is located in a non-fade interval can be expressed in terms of the Doppler spectrum of the fading and the fade margin [22]. In a packet-switched net with contending signals, the fade margin becomes a stochiastic variable, which will be studied here.

Linnartz and Prasad [23] studied the average duration that the power of wanted signal with fading exceeds the incoherent sum of the powers of n independently Rayleigh fading interfering signals by a factor of at least z. If the interfering signals have an identical mean power, the joint interference signal is Nakagami fading, with the gamma distributed power [7, 23]

$$f_{p_t}(p_t \mid \bar{p}_t) = \frac{1}{\Gamma(m)} \frac{m}{\bar{p}_t} \left(\frac{m p_t}{\bar{p}_t} \right)^{m-1} \exp\left\{ -\frac{m p_t}{\bar{p}_t} \right\} \tag{51}$$

where $\Gamma(m)$ is the gamma function [$\Gamma(n+1) = n!$ for integer n], $\bar{p}_t$ is the mean joint interference power and m is the 'shape factor' with $m = n$. If $n = 1$, Rayleigh fading (eqn. 2) is recovered. The variance of eqn. 51 relative to the mean power $\bar{p}_t$ decreases with n. If the mean powers of individual interference signals are different, eqn. 51 was shown to closely approximate the PDF of received signal power if an appropriate (real) value for m is inserted [23].

For a wanted Rayleigh fading signal with local mean power $\bar{p}_{A_j}$ in the presence of Nakagami fading interference with mean power $\bar{p}_t$, the average non-fade duration (expressed in seconds) was found to be of the form

$$\bar{\tau}_{NF} = \sqrt{\left(\frac{\bar{p}_{A_j}}{z \bar{p}_t} \right)} \frac{1}{\sqrt{(2\pi)} f_m} \frac{\sqrt{(m)}\Gamma(m)}{\Gamma(m + \frac{1}{2})} \tag{52}$$

where f_m is the Doppler shift in hertz, caused by the mobility of the vehicle with speed v, with $f_m = v f_c / c$, where f_c is the carrier frequency and c is the speed of light. Here, fading of each interfering signal was assumed to have the same Doppler spectrum as the wanted signal. For a certain mean joint interference power $\bar{p}_t$, the character of the interfering signal, whether one strong dominant interferer ($n = 1$) or a noise-type signal consisting of large number of weak interferers ($n \to \infty$), produces almost identical average non-fade durations. In fact

$$1 < \frac{\sqrt{(m)}\Gamma(m)}{\Gamma(m+\frac{1}{2})} \leqslant \frac{2}{\sqrt{\pi}} \approx 1.13 \quad (53)$$

for $1 \leqslant m < \infty$ shows that eqn. 52 is relatively independent of m.

The probability that a test packet is located in a non-fade interval can be investigated from eqn. 52 if a number of additional assumptions are made:

(i) Threshold crossings are memory-less, so that non-fade durations are exponentially distributed. It may be argued that non-fade durations corresponding to the maximum Doppler shift prevail in mobile reception [22]. However, if the speed of the vehicle in an urban environment is also seen as a stochastic variable, non-fade intervals tend to be spread over a wider interval and lacking further details, an experimental distribution appears reasonable.

(ii) The joint interference signal behaves as a non-fading signal. This is correct in the limiting case $n \to \infty$, but underestimates the duration of non-fade intervals if the interference is dominated by a single fading signal ($n = 1$) by about 13% (see eqn. 53).

(iii) The offered traffic is quasi-uniformly distributed with

$$G(a_j) = \frac{G_t}{\pi} \exp\left\{-\frac{\pi}{4} a_j^4\right\} \quad (54)$$

This is an approximation of the exactly uniform spatial distribution (for $0 < a_i < 1$) by a smooth analytical function. Arnbak and Van Blitterswijk [7] have shown that in this case the mean joint interference power is of the form

$$f_{\bar{p}_t}(\bar{p}_t \mid n) = \begin{cases} \dfrac{n}{2\sqrt{\pi}} \bar{p}_t^{3/2} \exp\left\{-\dfrac{n^2}{4\bar{p}_t}\right\} & \text{for } n = 1, 2, \ldots \\ \delta(\bar{p}_t) & \text{for } n = 0 \end{cases} \quad (55)$$

Similar to the analysis in References 24 and 25 for noise-limited channels, the probability of successful reception is found from the requirements that C/I is larger than z at the start of the packet and that the packet duration must be shorter than the time towards the next fade. Considering assumptions (i) and (ii), this produces

$$\Pr(A_j \mid \bar{p}_t, \bar{p}_{A_j}) = \exp\left\{-z\frac{(\bar{p}_t + N_A)}{\bar{p}_{A_j}} - \sqrt{(2\pi)} T_s f_m \sqrt{\left[z\frac{(\bar{p}_t + N_A)}{\bar{p}_{A_j}}\right]}\right\} \quad (56)$$

For a Poisson-distributed number of interferers with unknown positions, the probability of capture is

$$\begin{aligned} \Pr(A_j \mid a_j) &= \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} \times \int_0^{\infty} \Pr(A_j \mid \bar{p}_{A_j}, \bar{p}_t) f_{\bar{p}_t}(\bar{p}_t \mid n)\, d\bar{p}_t \\ &= e^{-G_t} \Pr(A_j \mid \bar{p}_{A_j}, p_t = 0) + \sum_{n=1}^{\infty} \frac{G_t^n}{n!} \exp\{-G_t\} \int_0^{\infty} 2nr \\ &\quad \times \exp\left\{-z(r^{-4} + N_A)a_j^4 - \frac{\pi}{4} n^2 r^4 - a_j^2 \sqrt{[2\pi z(r^{-4} + N_A)]} f_m T_s\right\} dr \end{aligned} \quad (57)$$

where we substituted $\bar{p}_t = r^{-4}$. Fig. 11 gives the probability of capture (eqn. 57) against distance a_j for $f_m T_s = 0$, 1 and 10 in a noise-free channel ($N_A = 0$). For comparison, the exact solution for packets of zero duration [15] i.e.

$$\Pr(A_j \mid a_j) = \exp\left\{-\frac{\pi}{2}\sqrt{(z)} G_t a_j^2 \times \operatorname{erfc}\left[\frac{\sqrt{(z\pi)}}{2} a_j^2\right] \exp\left(\frac{\pi}{4} z a_j^4\right)\right\} \quad (58)$$

obtained from inserting eqn. 54 in eqn. 19, is also included. It is seen that eqn. 57 with $f_m T_s = 0$ is pessimistic compared with eqn. 58. A possible explanation is that in the event of very short packet durations in slots with one (or few) interfering signal(s), fades of the test packet may coincide with fades of the interference [23]. In such cases the assumption (ii) of non-fading interference underestimates capture probabilities.

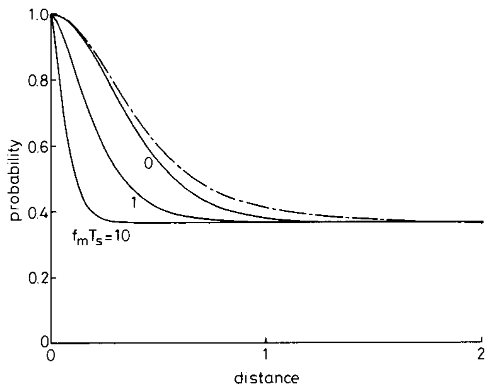

Fig. 11 *Probability of successful reception against distance for packets of normalised duration $f_m T_s = 0$, 1 and 10*
Receiver threshold $z = 4$ (6 dB)
— - — exact solution of eqn. 58 for packets of zero duration

For a carrier frequency of $f_c = 900$ MHz and a vehicle speed of 72 km/h ($v = 20$ m/s), $f_m \approx 50$ Hz and packet durations may not exceed a few milliseconds to justify the assumption of constant received power. The packet duration T_s, of course, depends on the bit rate and on the data format. In the high-level data link control (HDLC) protocol, a frame contains at least $L = 48$ bits to accommodate flags, address, control bits and block error detection. At a bit rate $r_b = 1200$ bits/s this corresponds to $T_s > 0.04$ s ($T_s f_m > 2$). In the first generation of the Mobitex public packet-switched mobile network [3], the bit rate is $r_b = 1200$ Hz. For a data message of minimum duration ($T_s \approx 13$ ms, $L = 16$ bits), one finds $T_s f_m \approx 0.65$. An average packet duration of about $T_s \approx 200$ ms [3] gives $T_s f_m \approx 10$. In systems with higher data rates, $T_s f_m$ is found to be substantially smaller. In the GSM system with a bit rate $r_b \approx 270$ kbit/s, a time-slot has a duration of $T_s \approx 0.577$ ms, so $T_s f_m \approx 0.029$. It should be noted that the GSM radio link may not behave as a narrowband Rayleigh fading channel if delay spreads are large.

This Appendix has revealed that in a random-access channel, the fade margin during a collision can be relatively small. In such cases, the assumption of constant received power appears optimistic and overestimates channel performace if packet durations are of the same order of magnitude as (or larger than) the time-constants of the fading.

10.2 Unslotted ALOHA

We assume that a guard time T_g is required to ensure synchronised arrival of packets. For a network with a maximum roundtrip propagation distance of R meters, $T_g = R/c$. The duration each slot has to be at least $T_s + T_g$, with T_s and T_g expressed in seconds. We consider the situation that the offered packet traffic per *second* is independent of the guard time. Consequently, in a system where guard times are required, the offered traffic G_{tg} expressed in packets per *time-slot*, is larger than the offered traffic per slot G_t in a theoretical network without guard times, i.e.

$$G_{tg} = \frac{T_s + T_g}{T_s} G_t \tag{59}$$

If required guard times become excessively large, say if $R > T_s c$, unslotted ALOHA might be considered. Exact analysis of unslotted ALOHA is complicated by the fact that the number of interfering signals changes during the reception of a packet. We now propose a model that somewhat overestimates the effect of interference in pure ALOHA. A test packet is assumed to capture the receiver if the received power exceeds z times the total power accumulated from *all* signals that are present during at least a part of the duration of reception of the test packet. For an infinite population of terminals, the number of packets overlapping with the test packet is Poisson-distributed with mean $2G_t$, and one may use the expression presented in the main body of this paper if, for the contending traffic, G_t is replaced by $2G_t$, or if $G(x)$ is replaced by $2G(x)$. This also corresponds to the case $G_{tg} = 2G_t$ in eqn. 59. Hence, the probability of capture can be estimated from

$$\begin{aligned} \Pr(A_j \mid n, \bar{p}_{A_j}) &> P_{UA} \\ &\triangleq P_{NA} \prod_{i=1}^{n} \mathscr{L}\left\{f_{P_{A_i}}, \frac{z}{\bar{p}_{A_j}}\right\} \\ &= P_{NA} \prod_{i=1}^{n_1} \mathscr{L}\left\{f_{P_{A_i}}, \frac{z}{\bar{p}_{A_j}}\right\} \prod_{i=n_1+1}^{n} \mathscr{L}\left\{f_{P_{A_i}}, \frac{z}{\bar{p}_{A_j}}\right\} \end{aligned} \tag{60}$$

where $1, \ldots, n_1$ denote the interfering packet present at the instant of arrival of the test packet j, and $n_1 + 1, \ldots, n$ denote packets present at the end of the test packet. One may conclude that unslotted ALOHA becomes favourable if T_g approaches or exceeds T_s.