AVERAGE BIT ERROR RATE FOR COHERENT DETECTION IN MACRO AND MICRO CELLULAR RADIO

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Abstract

The probability of bit error is investigated, taking account of propagation effects in macro and micro cellular networks, noise and multiple interferers. Coherent detection of BPSK, QPSK and BFSK is considered with combined log-normal shadowing and Rayleigh fading for macro cellular radio and combined log-normal shadowing and Rician fading for micro cellular radio. The effect of interference with bit intervals randomly aligned with the wanted signal is studied for the case of BPSK.

1 INTRODUCTION

Cellular networks increasingly use digital rather than analogue transmission. Much attention has been paid in the past to assess the system performance by evaluating the probability that the signal-to-interference ratio drops below a required threshold, e.g. [1,2]. In digital nets, error probabilities also are a relevant measure of the performance of a radio link [3]. Although a number of computer simulations [3] have been performed, analytical models to investigate the effect of co-channel interference in nets with fading channels have not yet fully been developed. Most results of studies on the average bit error rate in a fading channel documented so far, e.g. in [4], consider only additive white Gaussian noise (AWGN) without interference while other studies consider only co-channel interference in non-fading channels. Recently, Wong and Steele [5] studied the BER for an MSK signal in mobile radio networks with both Rayleigh fading and multiple interfering signals. Models as proposed in [5] are rather complicated so in this paper a mathematically much simpler, but acceptably model to study the performance of digital communication systems is considered. This paper presents the probability of a bit error focussing on propagation effects in macro and micro cellular networks, considering a number of symplifying assumptions. This paper presents the effect of the Rician *K*-factor (micro cellular nets), and the logarithmic standard deviation σ_s of the shadowing (micro and macro cellular networks), and compares the link performance in macro and micro cellular nets.
A comparison of BPSK, QPSK and BFSK is given, altough the model considered here may be somewhat coarse to draw definite conclusions on the relative performance from the comparison made in this paper.

2 PROPAGATION MODEL

In a typical frequency non-selective Rayleigh fading channel, a received carrier from the *j*-th transmitter is on the form of

$$v_j(t) = \zeta_j \cos\omega_c t + \xi_j \sin\omega_c t \tag{1}$$

The in-phase component and quadrature components ζ_j and ξ_j, respectively, consist of many reflections and are independently Gaussian distributed random variables with identical pdf's, on the form of $N(0,\bar{p}_j)$, i.e., with zero mean and with a variance equal to the local-mean power $\bar{p}_j$. If (1) is expressed in terms of the amplitude ρ_j and the phase θ_j, the amplitude is found to have a Rayleigh pdf and the corresponding total instantaneous power p_j ($p_j = ½\rho_j^2 = ½\zeta_j^2 + ½\xi_j^2$) is exponentially distributed with mean $\bar{p}_j$. We assume that, at least during one bit, the phase θ_j and the amplitude ρ_j of each signal, whether wanted ($j=0$) or interfering ($1 \le j \le n$), remains constant. In *macro* cellular nets, log-normal shadow attenuation co-exists with Rayleigh fading [6]. In *micro* cellular nets, for the wanted signal, shadowing is mostly absent and a dominant line-of-sight component C_0 is present. The instantaneous amplitude ρ_0 of the desired signal has the Rician pdf [4]

$$f_{\rho_0}(\rho_0) = \frac{\rho_0}{\sigma_{r,0}^2}\exp\left(-\frac{\rho_0^2+C_0^2}{2\sigma_{r,0}^2}\right) I_0\left(\frac{\rho_0 C_0}{\sigma_{r,0}^2}\right), \tag{2}$$

with local-mean power $\bar{p}_0 = ½C_0^2 + \sigma_{r,0}^2$ and specular-to-scattered ratio $K = C_0^2/2\sigma_{r,0}^2$. The local-mean signal-to-interference ratio is defined as $\bar{p}_0/\bar{p}_t$. Because of the larger propagation distances, interference signals exhibit Rayleigh fading ($C_j=0$ for $j>0$) and shadowing. Shadowing can be described by a log-normal distribution of the local-mean power (expressed in a logarithmic value such as dB) about the area-mean power. This corresponds to the log-normal pdf [7]

$$f_{\bar{p}_j}(\bar{p}_j|\bar{\bar{p}}_j) = \frac{1}{\sqrt{2\pi}\,\sigma_s \bar{p}_j}\exp\left(-\frac{\left(\ln(\bar{p}_j)-m_j\right)^2}{2\sigma_s^2}\right) \tag{3}$$

where σ_s and $\bar{\bar{p}}_j = exp(m_j)$ are the logarithmic standard deviation in natural units and area mean respectively. The standard deviation in dB is $s_s = 4.34\sigma_s$, with s_s in the range 4-12 dB. The area-mean signal-to-interference ratio is defined as $\bar{\bar{p}}_0/\bar{\bar{p}}_t$.
A number of assumptions are considered in this paper. Narrowband systems with frequency non-selective fading channels are considered, so delay spread of the wanted signal and interfering signals is ignored. The time constants of the fading are assumed large compared to the bit duration. Receivers which perfectly synchronise to the wanted signal are considered here so any synchronisation impairment of the receiver is neglected.

3 BINARY PHASE SHIFT KEYING

Initially, for each of the interfering signals, exactly overlapping bit periods are assumed, i.e., none of the interfering carriers exhibits a phase reversal during the integration over the bit duration of the wanted signal. Interfering signals are assumed to have exactly the same carrier frequency as the wanted signal: phase fluctuations, e.g. caused by frequency drifts or Doppler shifts, are considered neglible during a bit duration. The received joint signal $v(t)$ is on the form [8]

$$v(t) = \rho_0\kappa_0\cos(\omega_c t + \theta_0) + \sum_{j=1}^{n}\rho_j\kappa_j\cos(\omega_c t + \theta_j) + n(t), \tag{4}$$

where κ_j ($\kappa_j = \pm 1$) represents the binary phase modulation of the j-th carrier and $n(t)$ is the additive white Gaussian noise. The received energy per bit is $E_b = p_0 T_b = \frac{1}{2}\rho_0^2 T_b$. In the detector, $v(t)$ is multiplied by a locally generated cosine $(2\cos\omega_c t)$ and integrated over the entire bit duration T_b. The decision variable ν for synchronous bit extraction from a desired BPSK signal (with index 0) in the presence of n interferers with random phase relative to the local oscillator is

$$\begin{aligned}\nu &= \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} v(t)\cos(\omega_c t)dt \\ &= \rho_0\kappa_0\cos(\theta_0) + \sum_{j=1}^{n}\zeta_j\kappa_j + n_I\end{aligned} \qquad (5)$$

with n_I the sample of the in-phase Gaussian noise. The variance of this noise sample is

$$E\left[n_I^2\right] = \frac{N_0}{T_b} \qquad (6)$$

where N_0 is the spectral power density of the additive white Gaussian noise (AWGN). In a Rayleigh fading channel, the in-phase components ζ_j ($\zeta_j = \rho_j\cos\theta_j$) of the n interferers are all independent Gaussian variables. Phase reversals caused by modulation of κ_j do not affect the Gaussian distribution of the n interference samples, provided that the interfering bits are exactly aligned to the bits of the wanted signal. The variance of the j-th interference sample is

$$E\left[\zeta_j^2\right] = \bar{p}_j \qquad (7)$$

Thus, AWGN as well as the Rayleigh fading interference have a Gaussian pdf, but the correlation between samples during successive bit intervals differs substantially. With AWGN, successive samples are evidently uncorrelated, whereas for a slow-fading signal successive samples have (almost) identical amplitude ζ_j. Interference more or less behaves like burst noise since each interference carrier has a Doppler bandwidth that is substantially smaller than the bandwidth of the bandpass noise. The aspect of time statistics [9] of bit errors is not elaborated here but (long-term) averages are addressed. The Gaussian distributed in-phase samples $\{\zeta_j\}$ originating from the interference signals, added to the Gaussian noise produce a new Gaussian variable. The corresponding conditional bit error probability for a receiver locked to the wanted signal with BPSK is thus, for the event that $\kappa_0 = -1$,

$$P_b(e|\theta_0 = 0, p_0, \bar{p}_t, \kappa_0 = -1) = Pr\left(\sum_{j=1}^{n}\zeta_j\kappa_j + n_i > \rho_0\right) \qquad (8)$$

Because of symmetry, this BER also holds if wanted signal carries a "1", thus if $\kappa_0 = 1$. The joint interference-plus-noise sample has the distribution $N(0, \bar{p}_t + N_0/T_b)$, with $\bar{p}_t = \Sigma\bar{p}_j$. The conditional bit error probability for a receiver locked to the wanted signal is thus

$$\begin{aligned}P_b(e|p_0, \bar{p}_t) &= \int_{\rho_0}^{\infty}\frac{1}{\sqrt{2\pi}}\left(A\bar{p}_t + \frac{BN_0}{T_b}\right)^{-\frac{1}{2}}\exp\left\{-\frac{\frac{1}{2}\lambda^2 T_b}{BN_0 + A\bar{p}_t T_b}\right\}d\lambda \\ &= \frac{1}{2}erfc\left(\sqrt{\frac{p_0 T_b}{A\bar{p}_t T_b + BN_0}}\right)\end{aligned} \qquad (9)$$

with $A = 1$ and $B = 1$. The average error probability is found by taking account of the Rayleigh pdf of the amplitude of the wanted signal.

4 QUADRATURE PHASE MODULATION (QPSK)

In a QPSK-transmitter, the bitstream is split into two bit streams, each with a bit rate $\frac{1}{2}r_b$. The i-th signal $s_i(t)$ transmitted on the radio channel is composed by two BPSK signals in quadrature phase, viz.,

$$s_i(t) = \frac{1}{2}\sqrt{2}\,\kappa_{I,i}\cos(\omega_c t) + \frac{1}{2}\sqrt{2}\,\kappa_{Q,i}\sin(\omega_c t) \qquad (10)$$

where $\kappa_{I,j}$ and $\kappa_{Q,j}$ represent the binary phase modulation of the in-phase and quadrature-phase components, respectively. The received signal $v_i(t)$ is on the form

$$\begin{aligned}v_i(t) &= \frac{1}{2}\sqrt{2}\rho_i\kappa_{I,i}\cos(\omega_c t + \theta_i) + \frac{1}{2}\sqrt{2}\rho_i\kappa_{Q,i}\sin(\omega_c t + \theta_i) \\ &= \frac{1}{2}\sqrt{2}\kappa_{I,i}\left\{\zeta_i\cos(\omega_c t) + \xi_i\sin(\omega_c t)\right\} \\ &\quad + \frac{1}{2}\sqrt{2}\kappa_{Q,i}\left\{-\xi_i\cos(\omega_c t) + \zeta_i\sin(\omega_c t)\right\}\end{aligned} \qquad (11)$$

In the receiver, this signal is multiplied by $2\cos\omega_c t$ and integrated over the interval $2T_b$. The variance of the received interference sample then becomes

$$E\left[\left(\frac{1}{2}\sqrt{2}\right)^2\left(\kappa_{I,j}\zeta_i - \kappa_{Q,j}\xi_j\right)^2\right] = E\left[\zeta_j^2\right] = \bar{p}_j \qquad (12)$$

After integration (and normalisation) over $2T_b$, the variance of the noise sample is

$$E\left[n_i^2\right] = \frac{N_0}{2T_b} \qquad (13)$$

The bit error probability is on the form of (8) and (9) with $A = 2$ and $B = 1$.

In agreement with well-known results, we observe that AWGN has the same effect on BPSK as on QPSK. However, it is seen that BPSK is 3dB more robust against co-channel interference than QPSK. This result is understood as follows: If, in a multi-user network with BPSK signals, the bit rate of all signals is reduced by one half and simultaneously the amplitude is reduced by a factor of $\frac{1}{2}\sqrt{2}$, the BER of the wanted signal remains unchanged. If these modifications are made all transmitters change from BPSK modulation to QPSK modulation. However, with QPSK, the in-phase bit stream of the wanted signal experiences interference from the in-phase and from the quadrature-phase modulation of each interfering signal. Hence, the interference experienced in QPSK is twice as large as with BPSK.

5 BINARY FREQUENCY SHIFT KEYING (BFSK)

With synchronous detection of orthogonal FSK, each interfering signal introduces a Rayleigh phasor (and a corresponding Gaussian in-phase component) in only one of the two branches of the detector, whereas AWGN introduces a Rayleigh phasor in both branches simultaneously. It is assumed that the interfering terminals $\{1,..,n_0\}$ transmit a "0", whereas the terminals $\{n_0+1,..,n\}$ transmit a "1", with n_0 a binomial random variable ($0 \le n_0 \le n$). The desired signal is assumed to contain a "1", but results for a "0" are evidently identical. For exactly aligned bits, the decision variable consists of two terms, ν_0 and ν_1, with

$$\begin{aligned}\nu_0 &= \sum_{j=1}^{n_0}\zeta_j + n_{i,f_0} \\ \nu_1 &= \rho_0\cos(\theta_0) + \sum_{j=n_0}^{n}\zeta_j + n_{i,f_1}\end{aligned} \qquad (14)$$

where $n_{i,f0}$ and $n_{i,f1}$ are samples of the Gaussian noise at the orthogonal frequencies f_0 and f_1, respectively. The integrated in-phase samples of signals in the two branches are subtracted, and evaluated at the sampling instant, i.e., the decision variable is taken $\nu=\nu_0-\nu_1$. Hence, the joint interference sample has the variance

$$E\left[\left(\sum_{i=1}^{n}\zeta_j\right)^2\right] = \sum_{i=1}^{n}\bar{p}_j = \bar{p}_t \tag{15}$$

and the noise has the variance

$$E\left[\left(n_{i,f_0}-n_{i,f_1}\right)^2\right] = 2\frac{N_0}{T_b} \tag{16}$$

The probability of a bit error for coherent detection of BFSK is found from (8) and (9), though with N_0 replaced by $2N_0$, thus $A=1$ and $B=2$. This agrees with the well-known observation that antipodal signal structures, such as BPSK, are 3dB more robust against AWGN than orthogonal structures, such as BFSK. However equations (6) and (16) suggest that binary PSK and FSK are equally sensitive to co-channel interference.

6 BER FOR COHERENT DETECTION IN MICRO CELLULAR NETS

In micro cellular nets the wanted signal has a Rician pdf (2) and the interfering signals have a combined log-normal shadowing and Rayleigh pdf. If only Rayleigh fading is considered for the interfering signals, the conditional bit error probability for a receiver locked to the wanted signal becomes after integrating (9) over the Rician pdf (2):

$$P_b(e|\bar{p}_0,\bar{p}_t) = \frac{1}{2}\int_0^{\infty} erfc\left(\rho_0\sqrt{\frac{1}{2}\frac{T_b}{A\bar{p}_tT_b+BN_0}}\right)\frac{\rho_0}{\sigma_{r,0}^2}\exp\left(-\frac{\rho_0^2+C_0^2}{2\sigma_{r,0}^2}\right)I_0\left(\frac{\rho_0C_0}{\sigma_{r,0}^2}\right)d\rho_0 \tag{17}$$

with $A=1$, $B=1$ for BPSK, $A=1$, $B=2$ for BFSK and $A=2$, $B=1$ for QPSK. The average error probability is found by taking account of the pdf (2) of the amplitude of the wanted signal. If the wanted signal is Rayleigh fading ($C_0=0$), a closed form expression can be found

$$P_b(e|\bar{p}_0,\bar{p}_t) = \frac{1}{2}-\frac{1}{2}\sqrt{\frac{\bar{p}_0T_b}{\bar{p}_0T_b+A\bar{p}_tT_b+BN_0}} \tag{18}$$

The first-order asymptotic approximation for the BER (17) becomes in the event of high signal-to-interference ratios ($\bar{p}_0>>\bar{p}_t$):

$$P_b(e|\bar{p}_0,\bar{p}_t) \approx \exp(-K)\left[\frac{1}{2}-\frac{1}{2}\sqrt{\frac{\bar{p}_0T_b}{\bar{p}_0T_b+(K+1)(A\bar{p}_tT_b+BN_0)}}\right] \tag{19}$$

This equation asymptotically tends to

$$P_b(e|\bar{p}_0,\bar{p}_t) \approx (K+1)\exp(-K)\frac{A\bar{p}_t}{4\bar{p}_0} \tag{20}$$

In the same way the first-order asymptotic approximation for high signal-to-interference ratios in a Rayleigh fading channel ($C_0=0$) can be derived from (18), viz.,

$$P_b \approx \frac{A\bar{p}_t}{4\bar{p}_0} \tag{21}$$

The interfering signals are now considered to be exposed to combined Rayleigh fading and shadowing, with logarithmic standard deviation σ_s. This is a reasonable assumption if interfering signals arrive from relatively remote transmitters with obstructed propagation paths. For the wanted signal line-of-sight propagation is considered, i.e. Rician fading without shadows attenuation. In good approximation, the local-mean joint interference power caused by the power sum of a number of n log-normally distributed signals also has a log-normal distribution [10]. If combined log-normal shadowing and Rayleigh fading is considered for the interfering signals, the BER (17) has to be integrated over the log-normal pdf (3). After substituting the integration variable into

$$y^2 = \frac{1}{2\sigma_s^2}\ln^2\left(\frac{\bar{p}_t}{\bar{\bar{p}}_t}\right) \tag{22}$$

the conditional bit error probability becomes

$$P_b(e|\bar{\bar{p}}_0,\bar{\bar{p}}_t) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\int_0^{\infty} erfc\left(\sqrt{\frac{\frac{1}{2}\rho_0^2T_b}{A\bar{\bar{p}}_tT_b\exp(\sqrt{2}y\sigma_s)+BN_0}}\right)\frac{\rho_0}{\sigma_{r,0}^2}\exp\left(-y^2-\frac{\rho_0^2+C_0^2}{2\sigma_{r,0}^2}\right)I_0\left(\frac{\rho_0C_0}{\sigma_{r,0}^2}\right)d\rho_0\,dy \tag{23}$$

with $A=1$, $B=1$ for BPSK, $A=1$, $B=2$ for BFSK and $A=2$, $B=1$ for QPSK.

In the same way as for (17), the first-order asymptotic approximation for high signal-to-interference ratios ($\bar{p}_0>>\bar{p}_t>>N_0$) can be derived

$$P_b(e|\bar{\bar{p}}_0,\bar{\bar{p}}_t) \approx (K+1)\exp(-K)\frac{A\bar{\bar{p}}_t}{4\sqrt{\pi}\bar{\bar{p}}_0}\int_{-\infty}^{\infty}\exp\left(-y^2+\sqrt{2}y\sigma_s\right)dy \tag{24}$$

This equation can be further simplified into

$$P_b(e|\bar{\bar{p}}_0,\bar{\bar{p}}_t) \approx (K+1)\exp(-K)\frac{A\bar{\bar{p}}_t}{4\bar{\bar{p}}_0}\exp\left(\frac{\sigma_s^2}{2}\right) \tag{25}$$

A formal proof that this limit may be computed in a two step manner is omitted here, but (25) is compared with exact results. To obtain numerical results for the BER's (17) and (23), numerical integration methods as the Hermite polynomial method [11] are used. For the BER (23), only 6 interfering signals are considered because hexagonal cells are considered. The area-mean power and logarithmic standard deviation then become, according to Schwartz and Yeh [12], $\bar{p}_t=11.16$ and $\sigma_s=0.70$ respectively for $s_s=6$dB and $\bar{p}_t=47.19$ and $\sigma_s=1.55$ for $s_s=12$dB. The numerical results can be found in figs. 1, 2 and 3.

7 BER FOR COHERENT DETECTION IN MACRO CELLULAR NETS

In macro cellular nets, wanted and interfering signals are exposed to combined (but independent) Rayleigh fading and shadowing, with logarithmic standard deviation σ_j. For a joint interference signal with the area-mean power $\bar{p}_t$ and the logarithmic variance σ_t, the BER for synchronous detection of a BPSK signal becomes

$$P_b = \frac{1}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy \exp(-x^2-y^2) \sqrt{\frac{\bar{\bar{p}}_0 T_b \exp(\sqrt{2}x\sigma_s)}{\bar{\bar{p}}_0 T_b \exp(\sqrt{2}x\sigma_s) + A\bar{\bar{p}}_t T_b \exp(\sqrt{2}y\sigma_t) + BN_0}} \quad (26)$$

with $A=1$, $B=1$ for BPSK, $A=1$, $B=2$ for BFSK and $A=2$, $B=1$ for QPSK. To calculate BER (26), the Hermite polynomial method [11] was used twice. Values for $\bar{p}_t$ and σ_t have been discussed in the previous section. Fig. 4 gives the BER versus the signal-to-interference ratio $\gamma=\bar{p}_0/\bar{p}_t$. In the event of negligible noise and high signal-to-interference ratios ($\bar{p}_0 >> \bar{p}_t >> N_0$), (21) goes into the first-order asymptotic approximation

$$P_b(e|\bar{\bar{p}}_0, \bar{\bar{p}}_t) \approx \frac{A\bar{\bar{p}}_t}{4\bar{\bar{p}}_0} \exp\left\{\frac{\sigma_s^2+\sigma_t^2}{2}\right\} \quad (27)$$

8 EFFECT OF NON-SYNCHRONOUS INTERFERENCE ON BPSK

In practical communication nets, signals arriving from different transmitters may not contain exactly synchronised bit streams. In this section it is assumed that the bit duration is exactly identical for each transmitter, but the starting time of the bits may differ from terminal to terminal. The receiver is assumed to be in perfect bit synchronisation with the wanted signal, but not necessarily with interfering signals. If, in an interfering BPSK signal, a carrier reversal occurs at the instant $(k+\alpha_j)T_b$ with α_j the bit synchronisation offset ($0<\alpha_j<1$), the decision variable becomes

$$v = \rho_0\kappa_0\cos(\theta_0) + \sum_{j=1}^{n} \zeta_j\{\kappa_{j-}\,\alpha_j + \kappa_{j-}(1-\alpha_j)\} + n_i \quad (28)$$

with κ_{j-} and κ_{j+} the binary digit of the j-th interference signal during the interval $\langle(k-1+\alpha_j)T_b, (k+\alpha_j)T_b\rangle$ and $\langle(k+\alpha_j)T_b, (k+1+\alpha_j)T_b\rangle$, respectively (see fig. 5). If the transmitters are independent, the bit synchronisation offset α_j is likely to be uniformly distributed. Because of symmetry of the case $0<\alpha_j<½$ with $½<\alpha_j<1$, it is assumed that

$$f_{\alpha_j}(\alpha_j) = \begin{cases} 2 & \text{for } 0<\alpha_j<\frac{1}{2} \\ 0 & \text{elsewhere} \end{cases} \quad (29)$$

If a phase reversal of the j-th interfering signal occurs at the instant $k+\alpha_j$, thus if $\kappa_{j-}=-\kappa_{j+}$, signal energy in the period $\langle kT_b,(k+\alpha_j)T_b\rangle$ (a in fig. 5) cancels with the phase-reversed signal during the interval $\langle(k+\alpha_j)T_b,(k+2\alpha_j)T_b\rangle$ (b in fig. 5). In this case, the variance of the interference sample is effectively reduced to $\bar{p}_t(1-2\alpha_j)^2$.

Single interferer

If "0"'s and "1"'s are transmitted with equal probability, and have independent probability of occurrence, the probability of a phase reversal is one half. Thus, if the interference consists of one single signal ($n=1$) with a bit synchronisation offset of $\alpha_j T_b$, the bit error probability goes into

$$P_b(e|\theta_0=0,\bar{p}_0,\bar{p}_1,\alpha_1) = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}} - \frac{1}{4}\sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b (1-2\alpha_1)^2 + N_0}} \quad (30)$$

Integrating over the pdf (29) of α_I, the BER becomes

$$P_b(e|\theta_0=0,\bar{p}_0,\bar{p}_1,\alpha_1) = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0}} - \frac{1}{4}\sqrt{\frac{\bar{p}_0}{\bar{p}_1}} \ln\left(\frac{\sqrt{\bar{p}_0 T_b + \bar{p}_1 T_b + N_0} - \sqrt{\bar{p}_1 T_b}}{\sqrt{\bar{p}_0 T_b + N_0}}\right) \quad (31)$$

For $\alpha_I=0$, i.e., in the case of perfect bit synchronisation, (30) becomes identical to (18). Curves b and a in fig. 6 respectively show the numerical results.

Many interfering signals

If the interference is caused by the sum of many weak signals, the joint interference term in (6) becomes a Gaussian distributed random variable. Since ζ_j and α_j are stochastically independent, the variance of a sample of an interfering signal that exhibits a phase reversal is

$$\begin{aligned} E\left[\zeta_j^2(1-2\alpha_j)^2\right] &= E\left[\zeta_j^2\right] E\left[(1-2\alpha_j)^2\right] \\ &= \bar{p}_j \; 2\int_0^{\frac{1}{2}} 1-4\alpha_j+4\alpha_j^2 \, d\alpha_j \\ &= \bar{p}_j \; \cdot \frac{1}{3} \end{aligned} \quad (32)$$

Alternatively, if no phase reversal occurs, the variance is found from (6). If a phase reversal occurs with probability one half, and if the bit alignment offset is random, the experienced total interference sample has the variance $½(1+{}^1/_3)\bar{p}_t$. This corresponds to a reduction of 1.8 dB of the effective interference power. Thus, for random bit offsets, and many interfering signals, the average BER for coherent detection of a wanted BPSK signal becomes

$$P_b(e|\theta_0=0,\bar{p}_0,\bar{p}_t,n\to\infty) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{p}_0 T_b}{\bar{p}_0 T_b + \frac{2}{3}\bar{p}_t T_b + N_0}} \quad (33)$$

The numerical results are represented by curve c in fig. 6.

9 COMPARISON WITH RESULTS PRESENTED EARLIER

The results presented in this paper will now be compared with experimental and theoretical results presented earlier. Fig. 1A of [13] shows the BER for M-ary FSK signals with differential phase detection as a function of the signal-to-noise ratio in a very slowly fading channel. The shapes of the curves for a Rician factor $K=1$ (0dB) and $K=10$ (10dB) are similar to the curves in fig. 1 in this paper. For $1<K<\infty$, two bends occur in the curve. If a channel without interference is considered ($\bar{p}_t=0$), equation (20) becomes

$$P_b(\epsilon|\bar{p}_0,\bar{p}_t) \approx (K+1)\exp(-K)\frac{BN_0}{4\bar{p}_0} \qquad (34)$$

For BFSK ($M=2$), our results (with $B=2$) suggest $P_e \approx 4.10^{-7}$ for $K=10$ and $\bar{p}_0/N_0=28$dB. This is in good agreement with [13].

Previously a model was proposed with

$$P_e = \begin{cases} 0.5 & \text{if } p_0 < \frac{N_0}{T_b} + \bar{p}_t \\ 0 & \text{if } p_0 > \frac{N_0}{T_b} + \bar{p}_t \end{cases} \qquad (35)$$

Thus perfect reception occurs if the signal is stronger than the joint interference plus noise, whereas bit errors occur with probability one half if the signal is weaker. This allows computation of BER's for outage probabilities [14,6]. Our results in fig. 4 are in good agreement with results for outage probabilities by French (fig. 3 in [14]) and by Hansen and Meno (fig. 2 in [6]) . In [6], an asymptotic value for the BER for a wanted signal with combined Rayleigh fading and shadowing has been derived for a channel without interference. Similar to (27) of this paper, the asymptotic BER appeared to depend linearly on C/N and exponentially on the standard deviation of the shadowing.

Experimental results for the BER in macrocellular networks are reported and compared with (35) in the CCIR document [15]. Fig. 28 of [15] shows a difference of about 2 dB between the measured BER for direct FM and the theoretically calculated BER. The difference between these BER's was explained [15] by correlation of the fading of wanted and interfering signals but, to our impression, may have been caused by the effect of non-synchronous interference. For the case of BPSK, we found that this effect is in the order of 1.8dB.

10 CONCLUSION

The long-term average BER has been expressed for the case of a wanted signal with Rician fading for micro cellular nets and Rayleigh fading and shadowing in the presence of multiple fading interfering signals for macro cellular nets. As seen from figs. 1 and 2, the Rician K-factor has a substantial effect on the BER. The standard deviation s_s of the shadowing influences the BER for low area-mean signal-to-interference ratios in the case of a Rician fading channel (see fig. 3) and for high area-mean signal-to-interference ratios when a Rayleigh fading channel is considered (see fig. 4). It has been found that the BER for the case of a wanted signal signal with Rician fading is much lower than in the case of Rayleigh fading. Nonetheless, the BER for a Rician K-factor of 12 dB is even much higher than for the case when there is no fading at all. Asymptotic expressions have been derived for the limiting case of negligible noise and high area-mean signal-to-interference ratios.

It is well known that in classical time-invariant AWGN channels without interference, BPSK and QPSK are 3 dB more immune against noise than BFSK. Interestingly, our results suggest that coherently received BPSK and BFSK are *equally* resistant against Rayleigh fading co-channel interference, and are 3dB *more robust* than QPSK. However, it should be noted that these observations originate from a simplified model. It is not known whether this conclusion still holds if interfering bits have random bit timing with respect to the wanted signal.

For the case of BPSK, the effect of interference with bit intervals that are not synchronised to the desired signal has been reported for the special case of BPSK. It was seen that if the bit intervals of interfering signals are not aligned with the bit intervals of the wanted signal, the bit error rate is slightly lower than in the case interference with exactly coinciding bit intervals. As seen from fig. 6, the latter result for $n \to \infty$ closely approximates the BER for one single interferer ($n=1$). Only a small effect occurs for local-mean signal-to-interference ratios below 0dB ($C/I<1$). The assumption of perfectly bit-synchronised interference is pessimistic and overestimates the effect of co-channel interference by about 1.8 dB and overestimates bit error rates.

Investigation of a refined model, e.g. taking random bit alignment into consideration, is recommended. Another recommendation is further verification of the model by means of simulation.

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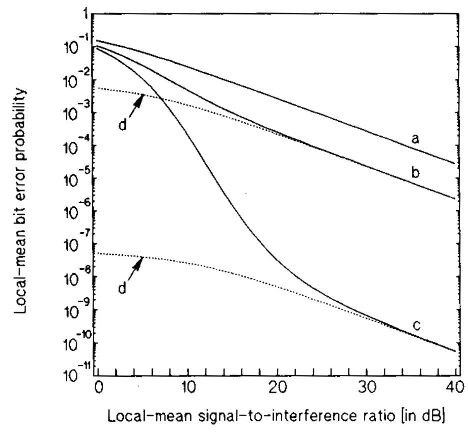

Fig.1: BER in a Rician fading channel versus local-mean signal-to-interference ratio, with specular-to-scattered ratio K=0 (Rayleigh fading - *a*), K=6dB (*b*), K=12dB (*c*) and asymptotes (*d*).

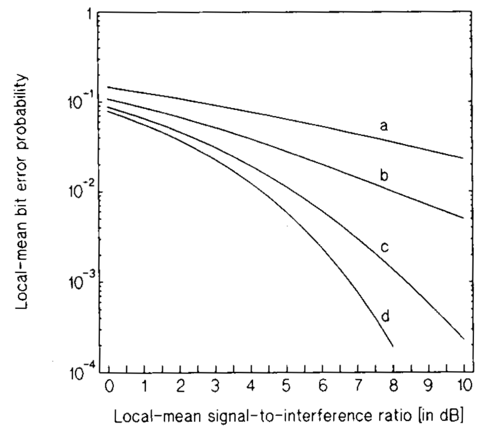

Fig.2 BER in a Rician fading channel for small local-mean signal-to-interference ratios, with specular-to-scattered ratio K=0 (Rayleigh fading - *a*), K=6dB (*b*), K=12dB (*c*) and $K=\infty$ (non fading - *d*).

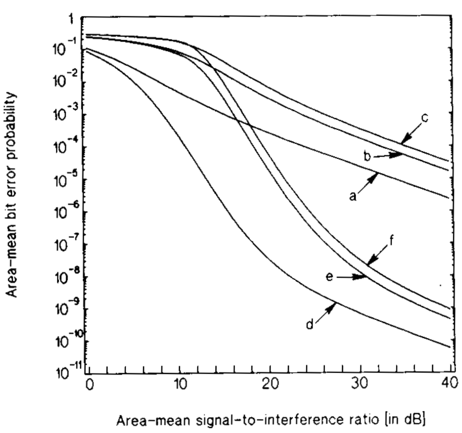

Fig.3: BER in a shadowed-Rician fading channel versus area-mean signal-to-interference ratio for a specular-to-scattered ratio K=6dB (*a,b,c*) and K=12dB (*d,e,f*) with standard deviation s_s=6dB and s_s=12dB respectively.

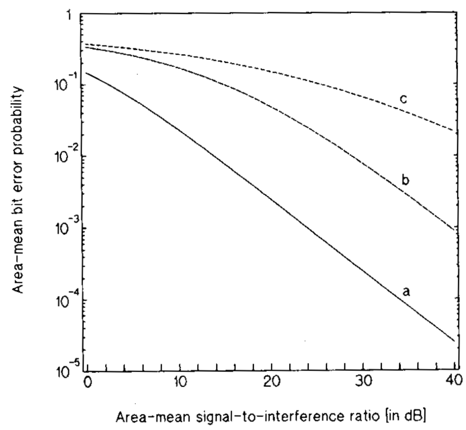

Fig.4: BER in a shadowed-Rayleigh fading channel versus area-mean signal-to-interference ratio, with no shadowing (*a*), standard deviation s_s=6dB (*b*) and s_s=12dB (*c*).

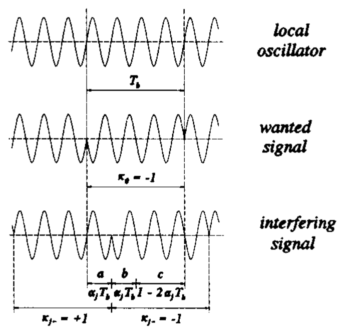

Fig.5: BPSK signal for the local oscillator, the wanted signal with perfect bit alignment and the interfering signal with random bit alignment.

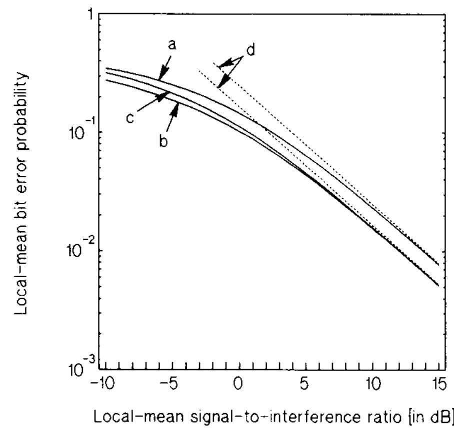

Fig.6: BER with BPSK in a Rayleigh fading channel versus local-mean signal-to-interference ratio for $n=1$ and ∞ interferers, with (*a*) perfect bit alignment, (*b,c*) random bit alignment and (*d*) asymptotes.