NEAR-FAR EFFECT ON SLOTTED ALOHA CHANNELS WITH SHADOWING AND CAPTURE

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Abstract

Signal statistics are modelled in an environment of shadowing and ground wave path loss. This propagation model is used to investigate channel performance of a slotted ALOHA network. The results are presented as receiver capture probability versus packet propagation distance, with the log-normal variance and the offered traffic as parameters.

1 Introduction

Mobile communication faces the problem of severe and fast signal fluctuations, which appear to reduce the reliability of continuous digital transmission. However, multiple-access studies show that dynamic fading and receiver capture always assist packet throughput in a collision channel. The performance of mobile ALOHA radio channels with near-far effect, shadowing and multipath fading, is therefore superior in terms of throughput, delay and stability, to all wired channels without fluctuating signal power.

The initial interest in the ALOHA channel was based on network simplicity [1]: many users transmit their packets to a common receiver, without mutual regulation. Consequently, access conflicts may occur, so packets lost due to interference have to be retransmitted. This paper addresses slotted ALOHA, where all data packets have equal duration and must be transmitted in predefined (but not user-assigned) time slots.

In a radio channel, a typical receiver may be able to correctly decode (the strongest) one of more colliding packets if its power p_s exceeds the joint interference power p_n of the other packets by at least a factor z, known as the receiver threshold.

In recent papers [3-12] the model for capture and propagation has been developed step by step. Abramson [3] presented the influence of the propagation distance on packet throughput in ALOHA channels with receiver capture. Namislo [12] introduced incoherent cumulation of power when more than two packets collide. Rayleigh fading on channels with coherent or incoherent cumulation was studied by Arnbak and Van Blitterswijk [4]; a specific spatial spread of users over the coverage area was also modelled. The near-far effect originating from a spatial distribution of terminals has been generalised in [5-6]. Alternative methods describe shadow fading [7] combined with multipath fading for coherent [8] and incoherent [9] signals. The radio channel with both near-far effect and shadowing has not yet been considered. This paper deals with this problem and the resulting throughput of ALOHA channels, which has not been modelled before.

It is shown in section 2 that, for not too heavy traffic loads, the calculated throughput of an ALOHA channel can be quite accurately determined using an approximate probability density function (pdf) of signal power. General spatial distributions can be estimated by a log-normal distribution of the area mean power (sections 3 and 4). Capture probability is presented as a function of distance in section 5.

Comparison of numerical results in literature [3-12] leads to the conclusion that channel throughput for high traffic loads critically depends on the spatial distribution of traffic close to the receiver. The log-normal subscriber density, presented in section 3 and 4, is compared with uniform distributions of traffic.

2 Distribution of signal powers

The (normalised) area mean power p_a, received from a mobile transmitter at a (normalised) distance r from the central receiver, is

$$p_a = r^{-\beta} \qquad (1)$$

with the power attenuation exponent β in the order of 3 to 4. In mobile radio channels, different types of propagation behaviour take place, depending on terrain features. Multiple obstacle diffraction, reflection, ground wave propagation and scattering are the main phenomena in mobile UHF propagation, resulting in an average loss (1). In many empirical propagation models, e.g. [13], the local path loss is expressed as the addition of losses (in dB) due to individual isolated terms. This is likely to result in log-normally fluctuating path loss. The power p_l received from a mobile terminal moving over an area of a few tens of meters is empirically found to approximate a log-normal distribution about the area-mean p_a [14]. The probability density function (pdf) for p_l, given p_a, is

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thus

$$f_{pl}(p_l|p_a) = \frac{1}{\sqrt{2\pi}\,\sigma_s p_l} \exp\left(\frac{-(\ln p_l - \ln p_a)^2}{2\sigma_s^2}\right), \tag{2}$$

with σ_s the logaritmic standard deviation due to shadowing, expressed in natural units. The logaritmic standard deviation expressed in dB-values σ_{dB} ($\sigma_{dB} = 4.34\ \sigma_s$) is in the range 4 to 12dB. The normally distributed shadow attenuation (in dB) can be modelled by assuming many cascaded (multiplicative) attenuators on the channel, varying in a more or less uncorrelated way.

It is generally accepted that, if mobile multipath fading is superimposed on the above mechanisms of shadowing, the microscopic distribution of the signal amplitude, given the local mean power p_l, has a Rayleigh distribution. Finding the exact distribution of instantaneous received power, resulting from combined shadowing and multipath fading, is a complicated mathematical exercise. However, in the model of many cascaded attenuators, the assumption of a log-normal approximation can be retained after spreading by one extra mechanism, i.e., multipath reception. This is in agreement with the observation that combined Rayleigh fading and shadowing have approximately a log-normal distribution, especially if $\sigma_{dB} > 6$ dB. In [7], this approximation was applied for mobile ALOHA channels, with the resulting logaritmic mean and variance of the approximating log-normal distribution

$$m_t \stackrel{\Delta}{=} \ln p_t = \ln p_a + \tfrac{1}{2} \ln 2 \tag{3}$$

$$\sigma_t^2 = \sigma_s^2 + \ln 2. \tag{4}$$

It has been verified [10] by the weight-function method [6], that results obtained with the modified moments (3) and (4) agree closely with exact solutions. This suggests that an approximate pdf of area-mean received power may also yield correct throughput. In analogy with the above reasoning for multipath statistics, in this paper we shall consider the spatial distribution of terminals and resulting propagation losses to produce a log-normal distribution of the received power.

3 Near-far effect

The (stationary) spatial distribution of the generated traffic G(r) is defined as the offered traffic per unit of area at a distance r from the common receiver and is expressed in packets per slot (pps) per unit of area. Using (1), the pdf of area-mean power becomes [4]

$$f_{pa}(p_a) = \frac{2\pi}{\beta} r^{\beta+2} \frac{G(r)}{G_t}. \tag{5}$$

The total traffic G_t offered to the channel, as determined by polar integration of G(r), ensures that (5) is normalised. Now, the unconditional pdf of the local mean power p_l can be found by (2) and (5)

$$f_{pl}(p_l) = \int_0^\infty \frac{f_{pa}(p_a)}{\sqrt{2\pi}\,\sigma_s p_l} \cdot \exp\left(-\frac{(\ln p_l - \ln p_a)^2}{2\sigma_s^2}\right) dp_a. \tag{6}$$

In general, the resulting pdf can not easily be described in a simple analytical form. However, if (5) is of a log-normal form, the result (6) becomes log-normal. A log-normal distribution of the area-mean power is obtained for spatial distributions of the traffic of the form [10]

$$G(r) = \frac{G_t\ \beta}{(2\pi)^{3/2}\ r^2 \sigma_d} \exp\left(\frac{-(-\beta \ln r)^2}{2\sigma_d^2}\right). \tag{7}$$

Since the ALOHA channel is essentially contention-limited, we can normalise the overall mean power in (7) to unity. Traffic distributions for various σ_d in the range corresponding to 2 to 12 dB are shown in Figure 1, for $\beta = 4$. Using (2), (5) and (7), (6) becomes [15, eq(7.4.32)]

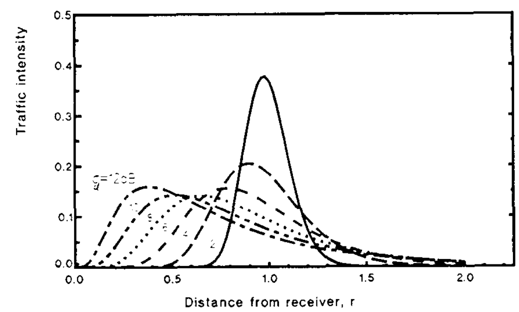

Fig. 1: Spatial distribution of traffic (7) for a total traffic of 1 packet per time slot offered to the channel. The spatial spread σ_d corresponds to 2, 4, .. 12 dB, and β=4.

$$f_{pl}(p_l) = \frac{1}{\sqrt{2\pi}\,\sigma_t p_l} \exp\left(-\frac{\ln^2 p_l}{2\sigma_t^2}\right) \tag{8}$$

where

$$\sigma_t^2 = \sigma_s^2 + \sigma_d^2. \tag{8a}$$

This result is intuitively appealing: The near-far effect and shadowing are multiplicative, so after logaritmic operation they become additive Gaussian processes. Consequently, the cumulated effect is found by summing the logaritmic variances.

4 Approximation for arbitrary spatial distributions

As the near-far effect is studied using the log-normal distribution, shadowing can be replaced by an additional spreading mechanism in the domain of the propagation distance.

An equivalent (fictitious) traffic $G^*(\rho)$, incorporating the extra spread σ_s compensating for shadowing, is defined as

$$G^*(\rho) \stackrel{\blacktriangle}{=} \frac{\beta G_t}{2\pi\ \rho^{\beta+2}}\ f_{p1}(\rho^{-\beta}) \tag{9}$$

where the resulting path attenuation for the fictituous traffic is expressed by $p_1=\rho^{-\beta}$. The equivalent traffic thus summarises statistics of the signal power p_1 due to shadowing and the near-far effect. Using (5), (6) and (9), $G^*(\rho)$ becomes

$$G^*(\rho) = \int_0^\infty \frac{r\ \beta}{\sigma_s \rho^2\sqrt{2\pi}}\ G(r)\ \exp\left(\frac{-(\beta\ln\rho-\beta\ln r)^2}{2\ \sigma_s^2}\right)\ dr \tag{10}$$

where G(r) now represents any spatial distribution, including (7). Only if G(r) happens to be on the form (7), the equivalent traffic (10) is again of the log-normal form (7). For other spatial distributions, however, the equivalent traffic can be approximated by (7), with logaritmic standard deviation σ_t obtained by matching the first and second order logaritmic moments from (10). This is equivalent to selecting the new variance

$$\sigma_t^2 = \sigma_s^2 + \sigma_d^2 = \sigma_s^2 + \mu_2 - \mu_1^2, \tag{11}$$

with the (logaritmic) k-th moment μ_k of G(r) defined as

$$\mu_k \stackrel{\blacktriangle}{=} \int_0^\infty [-\beta\ln(r)]^k\ 2\pi r\ \frac{G(r)}{G_t}\ dr. \tag{12}$$

Figure 2 shows an (original) distribution of traffic, constant for 0.5 < r < 1.5, its equivalent $G^*(\rho)$, compensated for shadowing, and the approximation by (7) and (11).

The approximation simplifies the analysis of packet radio channels with near-far effect and shadowing, within acceptable limits of accuracy. It is seen from Figure 3 that the method looses accuracy if the non-vanishing part of the subscriber density is extended closer to the receiver. This problem, which is frequently encountered in literature, e.g. [5] [12], is fundamental: (quasi-) uniform distributions with G(r) > 0 for $r \to 0$ have unbounded linear moments. Nevertheless, analytical results can be obtained from (12) for the uniform distribution

$$G(r) \equiv G_t\ /\ \pi \quad \text{for } r < 1, \tag{13}$$

since the logaritmic moments exist, viz.

$$\mu_k = (-\beta)^k \int_0^1 \ln^k r\ 2r dr = (-\beta)^k \frac{k!}{2^k}. \tag{14}$$

For β=4, the logaritmic standard deviation σ_d equals 2 (or 8.68dB) for the uniform distribution (13).

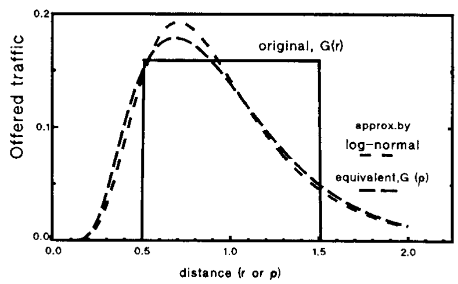

Fig. 2: Equivalent traffic density $G^*(\rho)$ for a uniform subscriber density in the range 0.5 < r < 1.5, 6 dB of shadowing and the log-normal approximation (12).

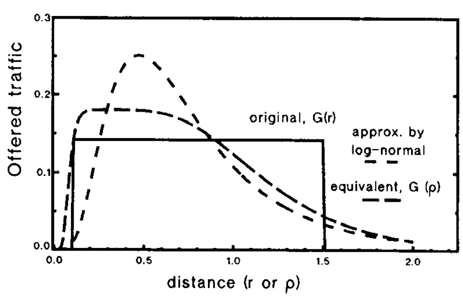

Fig. 3: Equivalent traffic density $G^*(\rho)$ for a uniform subscriber density in the range 0.1 < r < 1.5, 6 dB of shadowing and the log-normal approximation (12).

5 Access probability versus propagation distance

Since data communication sessions are generally short relative to the time it takes to cross the coverage area, network (ensemble) averages do not equal (time) averages for individual terminals. Therefore, we concentrate on capture probabilities conditional on terminal position. The conditional capture probability for packets transmitted at a distance r from the central receiver is derived analogous to eq. (10) in [7] and is given by

Prob{capture|r} =

$$1 - \sum_{n=1}^{\infty} \frac{G^n}{n!} \exp(-G)\, Q(l_n), \qquad (15)$$

with $Q(l_n)$ the cumulated normal probability function [7] [12, eq. 26.2.3] with argument

$$l_n \triangleq \frac{-\beta \ln r - m_u - \ln z}{\sqrt{\sigma_s^2+\sigma_u^2}}$$

where m_u and σ_u are the logaritmic mean and standard deviation of the cumulative interference signal, due to the composite near-far effect and shadowing. m_u and σ_u are obtained using Schwartz and Yeh's method [16,17], with the standard deviation σ_t from (8a) of individual signals. It is assumed that σ_s (the portion of the standard deviation due to shadowing) is equal for the desired signal and all interferers, which implies that terrain features are assumed uniformly spread over the coverage area. Since the capture probability (15) is conditional on propagation distance of the desired signal, no variance due to near-far effect (σ_d) is included for the desired signal.

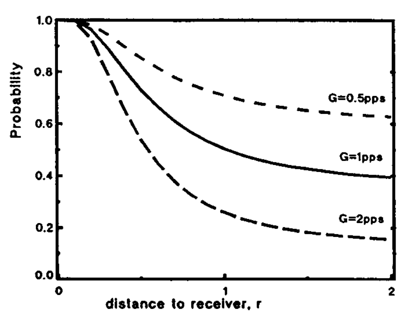

Fig. 4: Capture probability versus propagation distance for a receiver with threshold of 6dB and 6 dB of shadowing. The total offered traffic is 0.5, 1 and 2 packets per time slot.

Figure 4 portrays the capture probabilty versus propagation distance r for a total offered traffic G_t of 0.5, 1 and 2 packets per time slot. The receiver threshold z is 4 (6dB). Signal power fluctuations due to shadowing of σ_{dB} = 6dB and a (approximately uniform) spatial spread of σ_d = 2 (8.68dB) about the mean at r=1. Fig. 5 gives the capture probability as a function of distance, for G_t=1 and σ_s corresponding to 0, 6 and 12 dB. As long as the total

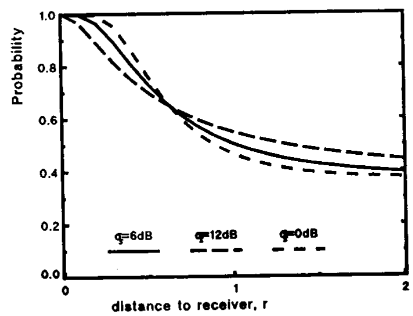

Fig. 5: Capture probability versus propagation distance for a receiver with threshold of 6dB and 0, 6 and 12 dB of shadowing. The total offered traffic is 1 packet per time slot.

traffic is not too high (G_t<2), mutual differences for various σ_s are relatively small. At heavy traffic (G_t>1), total throughput [7] is mainly determined by the near-far effect, rather than by shadowing, if σ_{dB} < 8.68dB.

6 Conclusions

The paper compares log-normal spatial distributions and uniform distributions of users over the coverage area. The former facilitate the analysis of mobile packet radio channels with near-far effect and shadowing and can often approximate the latter type of distribution.

Channel throughput for high traffic loads ($G_t \to \infty$) critically depends on the distribution of traffic close to the receiver. Unfortunately, the uniform distribution causes inaccurate limits for high traffic loads. As the total traffic increases, only the packets generated very close to the common receiver have a chance to capture. In the limit for very high traffic loads ($G_t \to \infty$), only packets from terminals arbitrarily close to the receiver capture it, since they are received with (arbitrarily) high power. If an exactly uniform spatial distribution (12) is assumed, throughput is known to be non-zero, though $G_t \to \infty$ [3,4,5,11,12]. Frequently, the correctness of a non-zero limit has been discussed, e.g. [5,12]. From this point of view, the log-normal distribution (7) may be preferred above (13). Even for the perfect capture receiver (z=1), throughput tends to zero for $G_t \to \infty$ with log-normal distributions [7-9].

Consequently, stability and throughput cannot be realistically studied by (quasi-) uniform distributions. The log-normal subscriber density with a spatial spread of 8.68 dB, as presented here, gives a viable alternative to the study of the heavily loaded collision-type multiple access channel. Further, uniform distributions with heavy traffic over very short ranges are not always providing the best approximations to practical situations. Many mobile networks use antennas in the fixed stations with relatively high clearance, with all mobile users necessarily separated some distance from these antennas. Even in wireless office systems, the nearest terminal can only induce limited power in the network receiver. In militairy packet radio systems, the probability of radio units in the same net being very close together may not be likely either, if only for tactical reasons.

The model described does not explicitly consider Rayleigh fading. If packets are short with respect to the average non-fade duration, the effect of Rayleigh fading can be considered in the choice of σ_S. For long packets, the receiver threshold z has to be modelled to allow occasional fades in the signal.

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