X. Wang and J.P.M.G. Linnartz, “Average delay in asynchronous Lightwave ALOHA network”, joint WIC/IEEE SP Symposium on Information Theory and Signal Processing in the Benelux, Brussels, Belgium, May 10-11, 2011

# Average Delay in Asynchronous Visual Light ALOHA Network

Xin Wang, Jean-Paul M.G. Linnartz,
Signal Processing Systems, Dept. of Electrical Engineering
Eindhoven University of Technology
The Netherlands

**Abstract**

In recent years, LED technology emerged as a prime candidate for the future illumination light source, due to high energy efficiency and long life time. Moreover, LEDs offer a superior flexibility to designers in terms of achievable colors and shapes. In addition, LEDs can illuminate and simultaneously transmit information embedded in their light output. Since there are many LEDs in a typical installation, two kinds of message collisions can cause time delays in data exchange, namely collisions among data messages and collisions between data and switching caused by typical LED driver electronics. Based on the two cases, we calculate the average delay time. A suitable partitioning of the message into data packets is proposed for Pulse Width Modulation LED drivers with known and sufficiently stable clock frequency.

## 1 Introduction

Light emitting diodes (LEDs) will play an increasingly important role in lighting in the future [1] [2]. These solid state luminaires can also be used to emit data messages that are embedded in the light output. Hence LEDs can form a sensor network in which throughput, number of nodes and delay become key performance indicators. This paper addresses the problem of how to estimate the average delay time due to the collisions, taking into account not only multiple luminaires that emit messages, but also the effect of Pulse Width Modulation (PWM) [3] [4] [5] to control the light intensity of each LED.

The system designer must make a trade off for the format of transmitting a data message. If all bits are transmitted sequentially at once, this has advantages and disadvantages: the chances that a packet is lost due to clock drifts are small. However, the data spurt causes a temporary increase in light intensity which can be perceived as a flash, particularly when the LED illumination output is small at that moment. Moreover, if other luminaires apply PWM, it is very likely that such a LED driver causes harmful interference to the data transmission.

On the other hand, if the transmission strategy is to attach only one bit (or even one CDMA chip [4]) to a PWM illumination pulse, the duration to transmit a full message becomes so large that clock drifts start to become non-negligible. This paper studies the trade-off between the above two extremes.

This paper is organized as follows. First, Section 2 formulates the system. Subsequently, Section 3 describes the reason why the collision happens, and analyzes the average delay time due to the collision. The performance of the proposed system is also illustrated with numerical results in this section. Finally, Section 4 concludes this paper.

# 2 System Description

Our installation contains $N$ luminaires, indexed by $n = 1, \ldots, N$. These may not have a common time (phase) reference, but we assume that their internal clocks are sufficiently stable and identical to allow detection without frequency recovery, thus with phase recovery only. Luminaires use "on-off" keying to allow dimming (by adapting the average duty cycle) and to carry messages

Our system has been inspired by [4] [5], but relieves the need for a phase lock among luminaires. Each luminaire uses a separate clock with period $T_1$. During any $T_1$ interval the illumination of the LED is either on or off. A typical choice is $T_1 = 1\mu s$. To model PWM, an interval of duration $T_2 = N_1 T_1$ is called a $T_2$ block where $N_1$ is the number of dimming steps, typically $N_1 = 1024$ and $T_2 = 1.024ms$. During the $T_2$ block the light source is 'on' during a fraction of the $N_1$ clock cycles. It is common practice in PWM dimming to choose the duty cycle of an LED according the required illumination. There lies $N_L$ sequential clock cycles are used for illumination. In our system we further reserve $N_b$ ($N_b << N_1$) to allow communication. We assume that the illumination ensures that the total light output, including the data-modulated part provide the required illumination.

$N_2 T_2$ blocks make up one $T_3$ frame. During a $T_3$ frame, a message of $N_b$ bits can be transmitted, encoded into $N_h N_v T_1$ clock cycles, where $N_h$ denotes the number of successive $T_1$ pulses used within one block, and $N_v$ denotes the number of blocks used to compose one message. The signal covers an on-off sequence of $N_h$ clock cycles in each block, in $N_v$ consecutive blocks. This data part is followed by an illumination part. The illumination is an "always on" sequence of $N_L$ clock cycles in each block and in $N_2$ consecutive blocks. These two parts (data and illumination) start at different positions from each other.

This illumination pattern is illustrated in Fig. 1, which row-by-row maps sequential samples of the illumination sequence as seen by a sensor into a matrix. Samples taken at rate $T_1$ fill the matrix row by row. The width of the matrix equals to the length of one block. A message covers a rectangle of $N_h$ by $N_v$, with surface area $N = N_h N_v$. Due to a lack of mutual synchronization, all luminaires have a different starting moment for their $T_3$ frames. Within a frame, a luminaire chooses a random starting position for its message, but within one frame the start position in each block remains fixed. The starting position is random and independently chosen for every successive frame. In this way, we avoid repeated collisions. In practice the $T_1$ timing also differs from luminaire to luminaire. Hence the modulated signal does not exactly match the $T_1$ grid of the receiver. In our analysis we will ignore this effect since it only has a negligible effect on the probability of a collision of messages.

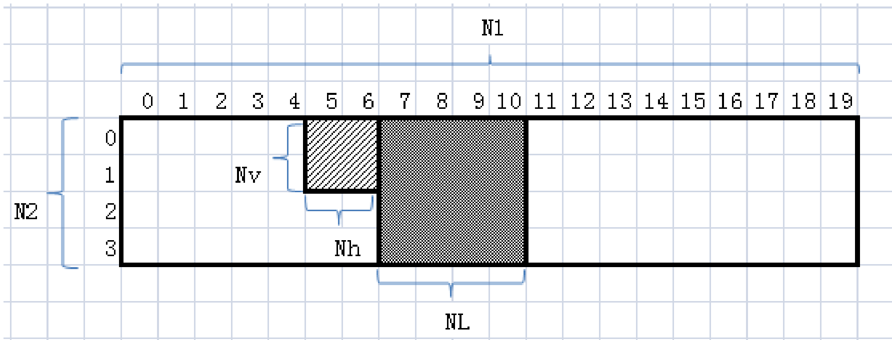


Figure 1: Simplified example of illumination sequence for one LED, containing $N_v N_h$ message bits and $N_1 N_L$ illumination bits. Here $N_v = 2, N_h = 2, N_L = 4, N_1 = 20, N_2 = 4, N_v = 2$

Table 1: Simulation Parameters

| Parameters | Values |
|---|---|
| Number of $T_1$ slots contained in one $T_2$ block, $N_1$ | 1024 |
| Number of $T_2$ blocks contained in one $T_3$ frame, $N_2$ | 1024 |
| Number of bits of a data message in on $T_3$ frame, $N_b$ | 144 |
| Number of $T_1$ slots contained in one $T_2$ block's illumination part, $N_L$ | 500 |
| Number of luminaires / LEDs in the system, $N$ | 100 |

# 3 ALOHA without ACK

Our analysis is based on the slotted ALOHA random access scheme with a finite population of transmitters. Each transmitter only sends at most a single message in every cycle. In contrast to the traditional ALOHA scenario we do not assume that acknowledgements are returned after any successful reception of message. This relieves us from the notorious instability of ALOHA networks, and if the population of LED luminaires is large enough, allows us to model the message arrivals as a stationary stochastic process that closely follow the Poisson distribution.

Our performance measure is *Average delay time*, defined as average time from a message is send to the moment that it is successfully received. That is, the average delay is one $T_3$ frame if the message is successfully delivered at first attempt; the average delay is two $T_3$ frames if the message delivery is failed at first attempt and successful at second attempt, etc.

## 3.1 Overlap between data and data

The data transmitted by an LED could be interfered (overlapped) by another LED's data part or illumination part. A data message covers a rectangle of $N_v$ by $N_h$, so it has a footprint of surface area $N = N_v N_h$ in Fig. 1. The vulnerability region, i.e., the area in which any other LED starting a message transmission will cause harmful interference is a rectangle of height $2N_v - 1$ and width $2N_h - 1$. It has a surface area of $N_{vuln} = (2N_h - 1)(2N_v - 1)$. It is required in the system that $N_h N_v$ is much less than $N_1 N_2$.

Evidently, from a data-to-data collision performance perspective, it is not favorable to transmit "square messages" ($N_h \approx N_v \approx \sqrt{N_b}$), because for a fixed data payload $N_b$ this would maximize the surface area of the vulnerability rectangle. To reduce the surface area of the vulnerability rectangle, preferably fat (small $N_v$) or thin (small $N_h$) messages are send. In the limiting case of very fat messages, all bits are transmitted in the same block ($N_v = 1$, a single row in Fig. 1).

Before we analyze the average delay time, we calculate the average collision probability. Since there are $N$ lamps in the system, then for one LED, there are $N - 1$ interference sources. Then we can derive probability of successful transmission $P_n$ for different numbers of interference sources $n$: when there is only one LED works as interference source, the probability of successful transmission is

$$P_1 = 1 - \frac{(2N_h - 1)(2N_v - 1)}{N_1 N_2} \tag{1}$$

When there are $n$ LEDs which work independently as interference sources, the probability of successful transmission is

$$P_n = P_1^n = \left[1 - \frac{(2N_h - 1)(2N_v - 1)}{N_1 N_2}\right]^n \tag{2}$$

Thus the average delay time for one LED with $n$ interference sources is

$$D_n = \sum_{k=1}^{\infty} P_1^n (1 - P_1^n)^{k-1} N_2 N_1 T_1 k = \frac{N_2 N_1 T_1}{P_1^n} \tag{3}$$

For installations in which all $N$ LEDs continuously emit status updates, this provided a useful performance measure, if we insert $n = N$. From

$$\frac{\partial P_n}{\partial N_h} = \frac{2n(1 - \frac{N_b}{N_h^2})}{N_1 N_2} \left[1 - \frac{(2N_h - 1)(2\frac{N_b}{N_h} - 1)}{N_1 N_2}\right]^{n-1} = 0 \tag{4}$$

we learn that a minimum occurs at $N_h = \sqrt{N_b}$, irrespective of $n$. This result matches our intuition. The "square message" is the worst case to avoid data collisions.

For a sensor network scenario, in which luminaires act as sensing nodes that occasionally have a data message to transmit, the arrival data messages is a random process. If the probability that one LED has data to transmit is $p$. For large $N$ and small $p$, this message traffic may be approximated by a Poisson distribution with arrival rate $\lambda = pN$. The probability that $n$ luminaires transmit in same frame is $\frac{\lambda^n e^{-\lambda}}{n!}$. Then the probability that data message is transmitted successfully is

$$P_{Successful} = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda} P_1^n}{n!} = e^{-(1-P_1)\lambda} \tag{5}$$

We assume that the luminaire is always repeating the transmission, irrespective of whether the message is received. Thus the average delay time for a given LED is

$$\begin{aligned} D_d &= \sum_{k=1}^{\infty} P_{Successful}(1 - P_{Successful})^{k-1} N_2 N_1 T_1 k \\ &= \frac{N_2 N_1 T_1}{P_{Successful}} \\ &= N_2 N_1 T_1 e^{\lambda \frac{4N_b - 2N_h - 2\frac{N_b}{N_h} + 1}{N_1 N_2}} \end{aligned} \tag{6}$$

Taking the derivative and setting this to zero, we find a maximum delay at $N_h = \sqrt{N_b}$.

In the above section we have analyzed an ALOHA network, in which the main new contribution was the introduction of a "two-dimensional" vulnerability area around the message footprint. Next we will also introduce an important cause of data loss in visual light communication, namely interference from the PWM needed for dimming.

### 3.2 Partial overlap between data and illumination

A test data packets can not only be lost due to a collision from a data packet from another LED (as addressed above) but also by the fringes of the illumination part of a another LED, which we will address in this section. If, however, the given LED's data part is fully overlapped by another LED's illumination part, this is experienced as a fixed background lighting condition, which causes in the photo-detector a DC offset that does not harm proper reception, except possibly for some additional shot-noise [4]. One can easily recover the desired data by using differential coding or by subtracting the intensity of constant illumination from the received data. So in this section, we will only calculate the possibility of partial overlap. We assume that the illumination part also randomizes its position in every $T_3$ frame. Since every LED has its own timing reference, and $T_3$ frames do not perfectly line up, both "vertical" and "horizontal" fringes of the illumination harmfully interfere.

The calculation is similar to previous section; the only difference is that in this case the probability of partial overlap $P_1$, needs refinement. The probability of any overlap between data part and illumination part (both fully and partially) is

$$P_O = \frac{(N_h + N_L - 1)(N_v + N_2 - 1)}{N_1 N_2} \tag{7}$$

The probability of a full overlap is

$$P_F = \frac{(N_L - N_h + 1)(N_2 - N_v + 1)}{N_1 N_2} \tag{8}$$

Thus the probability of a partial overlap is

$$P_P = P_O - P_F = \frac{(N_L - N_h + 1)(N_2 - N_v + 1)}{N_1 N_2} \tag{9}$$

We combine this with the probability of overlap between data and data (1), taking into account that $P_P$ and $P_D$ are not mutually exclusive

$$P_{D\cap P} = \frac{(N_h - 1)(2N_v - 1)}{N_1 N_2} \tag{10}$$

We combine these two results; when there is only one LED works as interference source. Using that $N_b = N_v N_h$, the probability of successful transmission is,

$$P_1 = 1 - (P_P + P_D - P_{D\cap P}) = 1 - \frac{2N_h N_2 + 2N_L N_v + 2N_b - 2N_2 - 2N_L - N_h}{N_1 N_2} \tag{11}$$

In the scenario that only $n$ ($n = 0, 1, \ldots, N$) luminaires embed data, the probability of a successful reception is, if we assume that all luminaires randomize their illumination burst including the ones that refrain from data transmission (See Fig. 2),

$$P_n = P_1^{n-1}(1 - P_P)^{N-n} \tag{12}$$

It appears that this success probability hardly depends on $n$, so $P_n \approx P_1^{N-1}$. Hence, $D \approx N_2 N_1 T_1 P_1^{-N+1}$.

A more likely situation may be that luminaires that are not involved in the data transmission maintain a fixed position of their illumination burst, and if $N_h < N_L$. In this case $n$ has a pronounced influence on the success probability

$$P_n = P_1^{n-1}\left[1 - \frac{2(N_h - 1)}{N_1}\right]^{N-n} \tag{13}$$

This results in a delay of

$$D_n = \sum_{k=1}^{\infty} P_n (1 - P_n)^{k-1} N_2 N_1 T_1 k = \frac{N_2 N_1 T_1}{P_n} \tag{14}$$

We now consider a sensor scenario, in which data messages arrive with probability $p$. For large $N$ we arrive at a closed form expression if we approximate the binomial distribution by a Poisson distribution

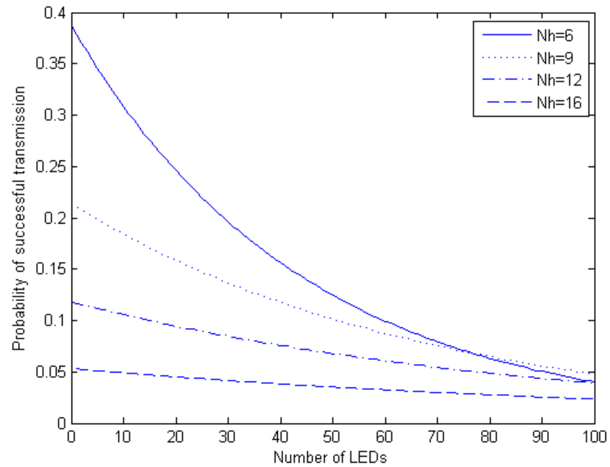


Figure 2: Probability $P_n$ of a successful transmission versus the number of Luminaires (LEDs) that actively transmit $N_b = 144$ bits of data for various data formats $N_h$ in a system with $N = 100$ luminaires

$$D_d \approx N_2 N_1 T_1 \Big[1 - \frac{2(N_h - 1)}{N_1}\Big] e^{-\lambda(1-\frac{N_1-2N_h+2}{N_1 P_1})} \tag{15}$$

Fig. 3 depicts the average delay time in number of $T_1$ slot as a function of $N_h$. To ensure short delay time small values of $N_h$ are preferred, say around $N_{h,opt} = 5$ to 8 bits. Interestingly, our analysis that takes PWM into account, particularly with randomized illumination pulse positions, does not favor extremely thin messages ($N_h = 1$). In fact such messages are too often lost in collision with interfering LEDs that change the position of their illumination pulse during the packet transmission. The best performance occurred at $N_h = 5$ and $N_2 = 270$, with a delay of $1.9 \times 10^6 T_1$ slots, say 1.9 sec. if $T_1 = 1\mu sec.$

# 4 Discussions and Conclusions

In this paper, we analyzed an approach to simultaneously illuminate and transmit information from individual illumination LEDs, in which we do not require a phase lock between different luminaires. The transmission is based on slotted ALOHA scheme. Since there are many luminaires, collisions affect performance. We mainly take two cases into consideration: collisions caused by another LED's data part and collisions by another LED's illumination part. Both effects lead to a time delay in the data transmission; however the former one exerts negligible effect on the average delay time compared to the latter one.

Based on this model, we derive the expression for optimum $N_h$, which will minimize the average delay time. It appeared very disadvantageous to transmit all bits in a single burst of data: such a message is very vulnerable to interference from PWM modulated LEDs. A more favorable transmission method is to transmit only a few a bits at a time,

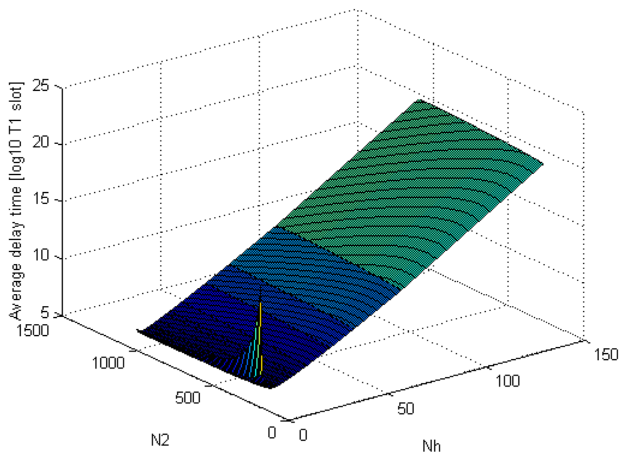


Figure 3: Relationship between average delay time and $N_2$ and $N_h$, for 10 bit dimming ($N_1 = 1024$), messages of $N_b = 144$ bits, consisting of $N = 100$ LEDs, each using an illumination period of duration $N_L = 500$.

and repetitively use the same transmission window in multiple PWM cycles. Further research in recovering data despite partial overlaps with PWM illumination is seen as an important next step to improve performance.

# References

[1] S. Muthu, F. J. P. Schuurmans, and M. D. Pashley, "Red, green, and blue LEDs for white light illumination," *IEEE J. Select. Topics Quantum Electron.*, vol. 8, no. 2, pp. 333 - 338, Mar./Apr. 2002.

[2] E. F. Schubert and J. K. Kim, "Solid-state light sources getting smar," *Science*, vol. 308, pp. 1274 - 1278, May 2005.

[3] Jean-Paul M.G. Linnartz, Tim C.W. Schenk, Lorenzo Feri, Hongming Yang, "On-off switching patterns for smart LED lighting," in *proc. 30-th Symposium on Information Theory in the Benelux*, May 2009, pp. 233-242.

[4] Jean-Paul M.G. Linnartz, Lorenzo Feri, Hongming Yang, Sel B. Colak, Tim C.W. Schenk, "Code Division-based Sensing of Illumination Contributions in Solid-State Lighting Systems," *IEEE Transactions on Signal Processing*, Oct. 2009, Volume: 57 Issue:10, pp. 3984 - 3998

[5] Jean-Paul M.G. Linnartz, Lorenzo Feri, Hongming Yang, Sel B. Colak and Tim C. W. Schenk, "Communications and Sensing of Illumination Contributions in a Power LED Lighting System," *Communications, 2008. ICC '08. IEEE International Conference on*, May. 2008, pp. 5396 - 5400