Effects of Antenna Mutual Coupling on the Performance of MIMO Systems

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Abstract

In this work, we study the effect of mutual coupling between multiple antennas on the outage capacity of a 2x2 multiple-input multiple-output (MIMO) system in flat fading channels. We model the effect of mutual coupling as an extra coupling matrix on top of the correlation matrix due to the propagation environment. The results show that mutual coupling reduces the correlation between the antennas from the propagation environment. On the other hand, mutual coupling also introduces extra power loss when the antennas are placed too close. The combination of these two effects on the outage capacity of MIMO systems is investigated. The simulations show that mutual coupling results in extra degradation of the outage capacity.

1 Introduction

By employing multiple antennas at the transmitter and the receiver, the multiple-input multiple-output (MIMO) system promises great capacity gains in rich scattering wireless environments [1] [2]. MIMO systems also provide diversity gain through using space-time coding techniques [3] [4]. This increases the robustness of the system performance in hostile wireless environments. A common assumption for the study of MIMO systems is that the channels between different transmit and receive antennas are independent and identically distributed (i.i.d.). This is achieved by spacing the antennas sufficiently far apart. In practice, due to constraints on the physical dimensions of devices, the distances between the multiple antennas are usually small. This renders the channel between different antennas correlated. The correlation between the antennas reduces both the capacity and the diversity gains of the MIMO system.The following two factors affect the correlation between different antenna elements.

- Power Azimuth Spectrum (PAS) for the transmit and receive antennas. This factor is determined by the propagation environment.
- Mutual coupling between the antennas. This factor is determined by the polarization, type and physical dimensions of the antennas used.

In [5] and [6], the correlation due to the propagation environment was analyzed for different PAS types and its effect on the capacity of the MIMO systems was studied. The effects of antenna coupling on multiple-antenna systems were studied in [7] and [8]. In [9], it was shown that mutual coupling reduces the correlation between the antennas. On the other hand, mutual coupling also results in extra power penalty when the two antennas are placed too close [10]. In this paper, we study the combined effect of the additional mutual coupling through the outage capacity of the MIMO channels. We show that mutual coupling adds extra degradation in the outage capacity of the MIMO systems. Hence, to guarantee satisfactory performances, the antennas should be spaced further apart compared to the rule of thumb number of $\frac{1}{2}\lambda$.

This paper is organized as follows. In Section 2, we describe the system model. The additional effect of coupling is modeled and studied in Section 3. In Section 4, we present the simulation results in outage capacity and the concluding remarks are given in Section 5.

2 System Model

For a MIMO system with N_t transmit, N_r receive antennas and flat fading channels, the received signal is can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}. \tag{1}$$

where $\mathbf{s}$ is the transmitted signal, $\mathbf{n}$ is the AWGN noise and $\mathbf{H}$ is the $N_r \times N_t$ channel matrix with $H_{i,j}$ being the channel response between the jth transmit antenna and the ith receive antenna. Each element of $\mathbf{H}$ is modeled as a complex Gaussian random variable. In practice, MIMO channels are usually correlated in the spatial domain. Such correlation can be modeled as [11]

$$\mathbf{H} = [\mathbf{R_{rx}}]^{1/2}\, \mathbf{H}_{iid} \left([\mathbf{R_{tx}}]^{1/2}\right)^T, \tag{2}$$

where $\mathbf{H}_{iid}$ is the channel matrix generated using zero mean unit variance i.i.d complex Gaussian random variables, and $\mathbf{R_{tx}}$ and $\mathbf{R_{rx}}$ are the transmit and receive correlation matrices, respectively. For a 3×3 channel, the latter matrices take on the following form [11]

$$\mathbf{R_{tx}} = \begin{bmatrix} 1 & \rho_{tx,1,2} & \rho_{tx,1,3} \\ \rho_{tx,2,1} & 1 & \rho_{tx,2,3} \\ \rho_{tx,3,1} & \rho_{tx,3,2} & 1 \end{bmatrix}$$
$$\mathbf{R_{rx}} = \begin{bmatrix} 1 & \rho_{rx,1,2} & \rho_{rx,1,3} \\ \rho_{rx,2,1} & 1 & \rho_{rx,2,3} \\ \rho_{rx,3,1} & \rho_{rx,3,2} & 1 \end{bmatrix}, \tag{3}$$

where $\rho_{tx,i,j}$ is the correlation coefficient between the ith and jth transmit antennas and $\rho_{rx,i,j}$ is the correlation coefficient between the ith and jth receive antennas. In [11], the complex correlation coefficients are calculated based on the PAS, the mean angle of departure (AOD) of the transmit antennas and the mean angle of arrival (AOA) of the receive antennas. All these parameters depend on the propagation environment and are determined from practical measurement. In this paper, we assume a uniform PAS over 360^o, i.e.

$$p(\phi) = \frac{1}{2\pi} \qquad \text{for} \quad \phi \in [-\pi, \pi). \tag{4}$$

In this case, it is shown in [12] that the correlation coefficient between the two antennas is given by

$$\rho_{i,j}(d) = J_0(2\pi d/\lambda), \tag{5}$$

where d is the spacing between the two antennas and J_0 is the zero-th order Bessel function of the first kind.

3 Effect of mutual coupling

In this section, we build on the model in Section 2 by including the effect of the mutual coupling in the correlation model. Here, we use two dipole antennas with the following parameters

- λ: wavelength of the signal, $\lambda = \frac{c}{f_c}$;
- l: length of the dipole antenna;
- D: spacing between the two antennas, normalized with respect to λ, $D = \frac{d}{\lambda}$;
- $Z_s = R_s + jX_s$: self impedance of the antenna;
- $Z_m = R_m + jX_m$: mutual impedance between the antennas;
- $Z_{\text{load}} = R_{\text{load}} + jX_{\text{load}}$: loading impedance of the antennas.

From [7], the effect of mutual coupling can be modeled in the following matrix form

$$\mathbf{H}_{C,Z_{\text{load}}} = \mathbf{C_p}\mathbf{H}_{U,Z_{\text{load}}} = (Z_{\text{load}} + Z_s)\begin{bmatrix} Z_{\text{load}} + Z_s & Z_m \\ Z_m & Z_{\text{load}} + Z_s \end{bmatrix}^{-1} \mathbf{H}_{U,Z_{\text{load}}} \tag{6}$$

where $\mathbf{H}_{C,Z_{\text{load}}}$ and $\mathbf{H}_{U,Z_{\text{load}}}$ are the coupled and uncoupled channel matrices with the loading impedance Z_{load}, and $\mathbf{C}_p$ is the coupling matrix. In this paper, we assume the loading impedance is matched to the source impedance of the antenna, i.e. $Z_{\text{load}} = Z_s^*$. The mutual impedance Z_m is a function of the dipole length l, the antenna spacing D and the antenna placement configuration. The mutual impedance can be calculated using the induced ElectroMagnetic Fields (EMF) method [13]. For the special case of $d = 0$, the two antennas become one. In this case, the mutual impedance is the same as the source impedance of the antenna, i.e. $Z_m = Z_s$.

If we can model the correlation from the environment in (5) and the effect of coupling in (6) independently, we can write [14]

$$\mathbf{R_{tx}} = \mathbf{C_{tx}}\mathbf{A_{tx}}\mathbf{C_{tx}}^H; \mathbf{R_{rx}} = \mathbf{C_{rx}}\mathbf{A_{rx}}\mathbf{C_{rx}}^H, \tag{7}$$

where $\mathbf{C_{tx}}$ and $\mathbf{C_{rx}}$ are the mutual coupling matrices for the transmit and receive antennas and $\mathbf{A_{tx}}$ and $\mathbf{A_{rx}}$ are the correlation matrices of transmit and receive antennas derived from the PAS. For uniformly distributed AOA, we have $a_{i,j}(d) = J_0(2\pi D)$. Using this in (7), we can write the transmit correlation matrix for a 2 transmit antenna system as

$$\begin{aligned} \mathbf{R_{tx}} &= \begin{bmatrix} C_{tx,1,1} & C_{tx,1,2} \\ C_{tx,2,1} & C_{tx,2,2} \end{bmatrix} \begin{bmatrix} 1 & J_0(2\pi D) \\ J_0(2\pi D) & 1 \end{bmatrix} \begin{bmatrix} C^*_{tx,1,1} & C^*_{tx,2,1} \\ C^*_{tx,1,2} & C^*_{tx,2,2} \end{bmatrix} \\ &= P_{tx}\begin{bmatrix} 1 & \tilde{\rho}_{tx,1,2} \\ \tilde{\rho}_{tx,2,1} & 1 \end{bmatrix}, \end{aligned} \tag{8}$$

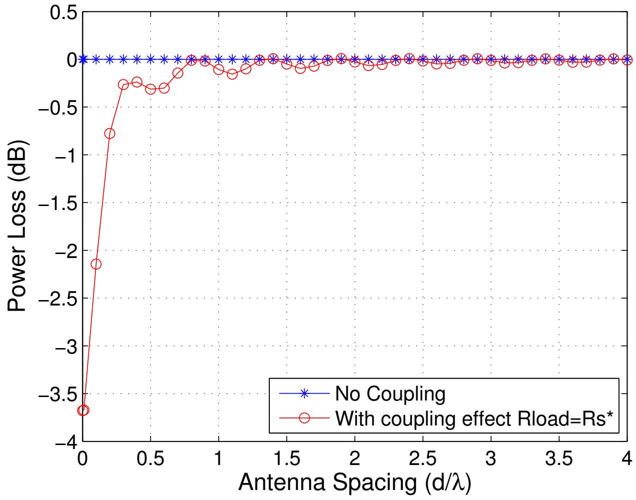

Figure 1: Power loss due to coupling for two side-by-side $\lambda/2$ dipole antennas with $Z_{\text{load}} = Z_s^*$.

where P_{tx} is the power loss due to mutual coupling and $\tilde{\rho}_{i,j}$ is the combined correlation coefficients due to coupling and propagation environment. The same applies to the receive correlation matrix $\mathbf{R_{rx}}$.

The power loss as a function of the antenna spacing for a 2 antenna system is shown in Figure 1. Here, we assume two dipole antennas with length $l = 0.5\lambda$ placed side by side. For such antennas, the self impedance is $Z_s = (73 + j42)\,\Omega$ [13]. We can see that the system suffers significant power loss if the two antennas are placed too close. The power loss becomes insignificant when the antenna spacing is about 1λ. The effective correlation is depicted in Figure 2. With mutual coupling, we can see that the spatial correlation between the antennas is reduced. In summary, the effect of mutual coupling is twofold. Firstly it reduces the correlation between the antennas, which is a desirable effect. On the other hand, it introduces extra power loss, which is undesirable. In the next section, we will study the combination of these two effects on the outage capacity of the MIMO systems.

4 Simulation Results

The capacity of the MIMO channel is given by [1]

$$C\left(\gamma, \mathbf{H}\right) = \log_2 \left[\det\left(\mathbf{I} + \frac{\gamma}{N_t}\mathbf{H}\mathbf{H}^H\right)\right] \text{ b/s/Hz}, \tag{9}$$

where $\mathbf{I}$ is the identity matrix, γ is the average SNR per receive antenna and superscript H indicates matrix Hermitian. The event when the channel capacity $C\left(\gamma, \mathbf{H}\right)$ falls below a given capacity C_b is called an outage. The probability of outage is defined as

$$P(\gamma, C_b) = \Pr\left[C\left(\gamma, \mathbf{H}\right) < C_b\right]. \tag{10}$$

Given an SNR γ and a maximum outage probability p_o, the outage capacity is defined as

$$C_o\left(\gamma, p_o\right) = \sup\left\{C_b : P(\gamma, C_b) < p_o\right\}. \tag{11}$$

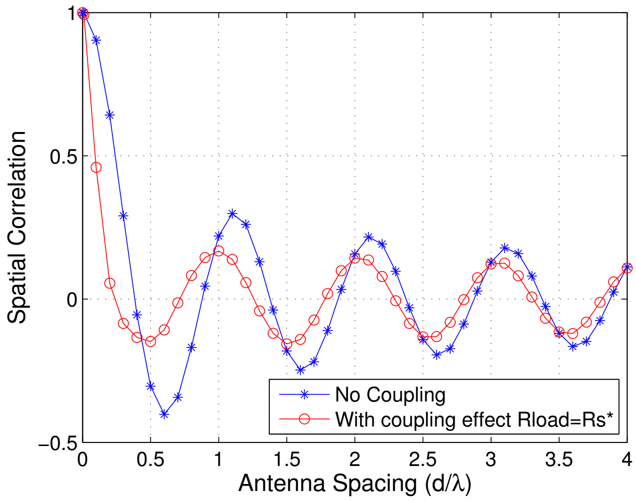

Figure 2: Effective spatial correlation due to coupling for two side-by-side $\lambda/2$ dipole antennas with $Z_{\text{load}} = Z_s^*$.

It can be interpreted as the maximum data rate achievable for the given SNR and the outage probability.

To obtain the outage capacity numerically, we use 200,000 channel realizations for a 2x2 MIMO channel. The correlated channels are generated using (2). Here, we assume that the antenna spacings between the transmit and receive antennas are the same. We present two sets of curves. Firstly we fix the SNR value and plot the capacity as a function of the antenna spacing. This gives us information on how far we should place the antennas apart. Secondly we fix the antenna spacing and plot the capacity as a function of the SNR values. This tell us the degradation in SNR for each simulation setup. For each set of curves, we compared the results of the following three setups

- i.i.d. channels;
- Correlated channel due to propagation environments;
- Correlated channel due to propagation environments and mutual coupling.

Figure 3 and Figure 4 show the outage capacity for these three cases for SNR of 10 dB and 20 dB respectively. We can see that the capacity drops significantly when the two antennas are spaced below 0.5λ. The degradation becomes insignificant when the antenna spacing is above 1λ.

Now we fix the antenna spacing to 0.5 and 1 λ. The corresponding outage capacity vs SNR curves are shown in Figure 5 and 6 respectively. We can see that for an antenna spacing of $\lambda/2$, the uniform AOA introduces 0.3 dB of performance degradation. The coupling effect introduces another 0.38 dB degradation. For an antenna spacing of 1λ, the degradation due to propagation environment is 0.2 dB while mutual coupling introduces another 0.15 dB degradation.

5 Conclusions

In this paper, we studied the effect of the mutual coupling between the antennas on the performance of the MIMO systems. We modeled the effect of the coupling as

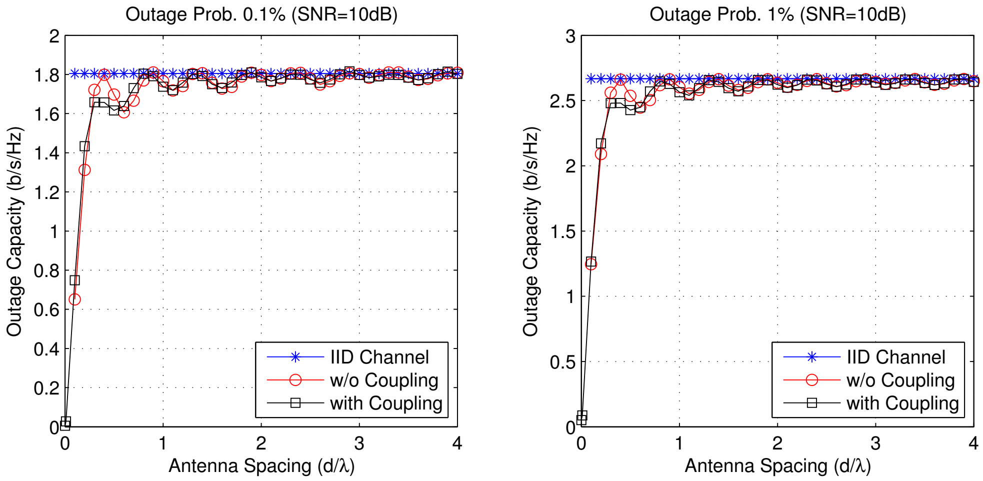

Figure 3: Outage capacity as a function of the antenna spacing (SNR=10dB) .

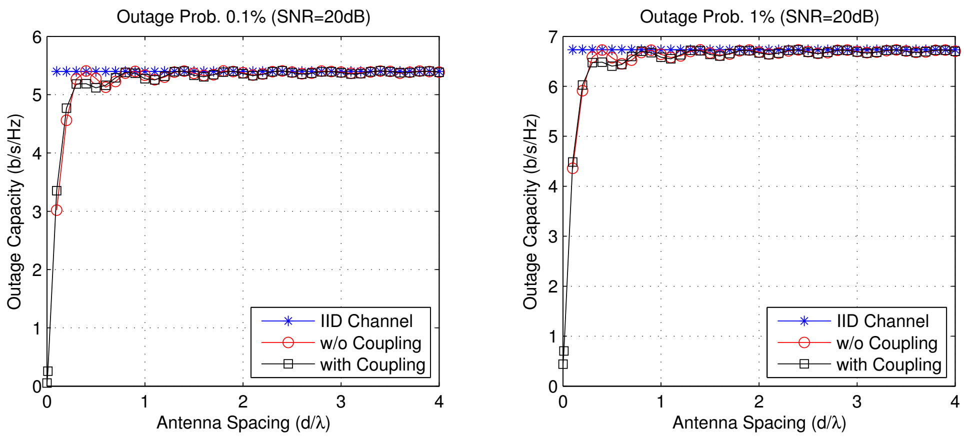

Figure 4: Outage capacity as a function of the antenna spacing (SNR=20dB) .

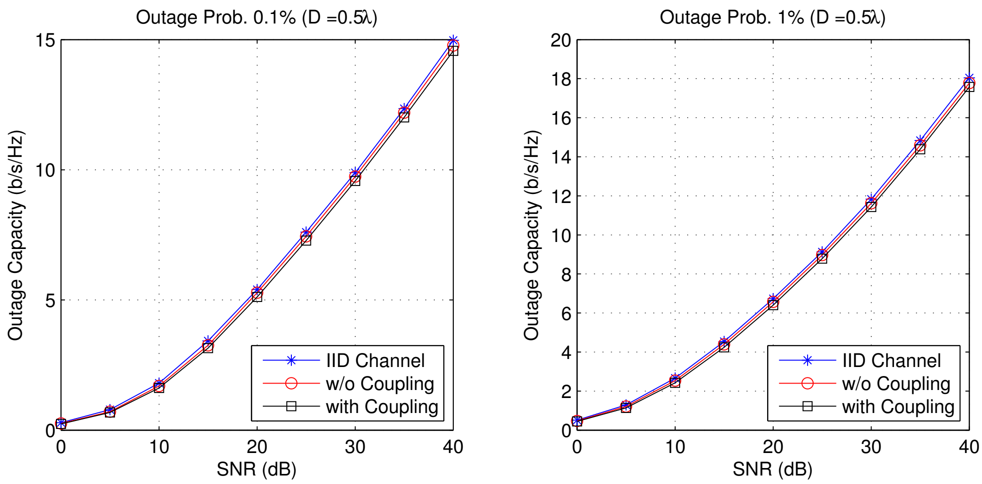

Figure 5: Outage capacity as a function of the SNR (D=0.5) .

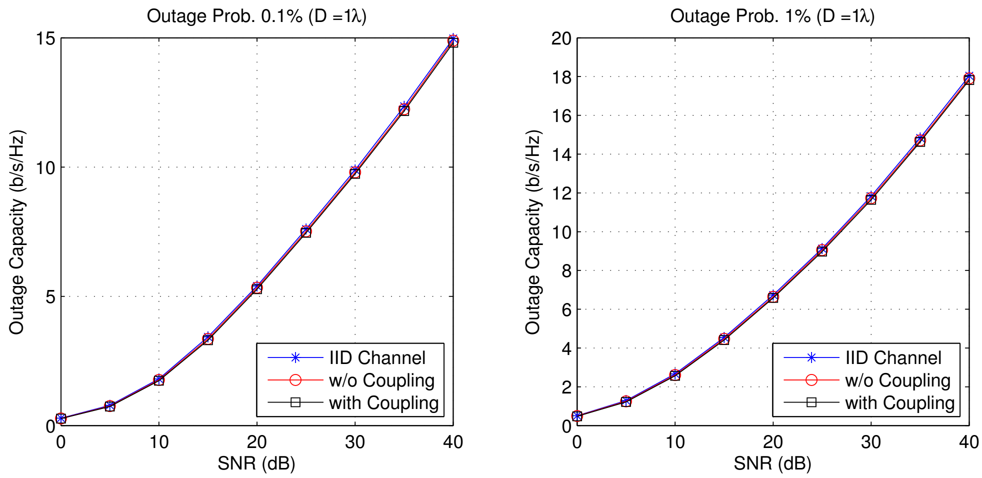

Figure 6: Outage capacity as a function of the SNR (D=1) .

an extra coupling matrix on top of the channel correlation matrix due to propagation environment. From the study, we find that the mutual coupling adds extra degradation to the outage capacity of the MIMO systems. Therefore, to guarantee minimal loss in capacity, the antennas should be spaced about 1 wavelength apart.

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