

# A FIRST ANALYSIS OF MIMO COMMUNICATION AS A BASIS FOR LOW POWER WIRELESS

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## ABSTRACT

This paper presents a comparison between multiple-input multiple-output (MIMO) systems and a single-input single-output (SISO) system. For a fair comparison the total power dissipation of the radio frequency (RF) front end and the analog-to-digital conversion is kept constant. As a benchmark the outage capacity is used. Monte Carlo simulations show that a MIMO system consisting of low-power low-resolution receivers achieves a higher data-rate and better reliability than a SISO system. However, the scaling of the RF front end should remain within the constraints of the considered semiconductor process. To ensure a more realistic scenario, correlation between the transmit antennas and correlation between the receive antennas is assumed.

## 1. INTRODUCTION

Wireless system designers are facing an increasing demand for low-power high data-rate transceivers. There are several options to improve the data-rate. The option explored here is multiple-input multiple-output (MIMO). The use of MIMO to improve data-rate has been pioneered by Foschini and Gans [1] and Telatar [2], and has been brought to more maturity by many other researchers since, such as; Shuguang Goldsmith and Bahai [3], Alamouti [4], and van Zelst [5].

The desire to achieve higher bit-rates is counter balanced by the desire for lower power consumption. For every new antenna element an entire RF front end and ADC are required. In a MIMO capacity analysis the extra power dissipation, caused by an increase in the number of receive antennas, should be accounted for [3].

In this paper we consider a scenario consisting of a base station and a receiving node with a limited power supply, for example a sensor operating on a battery. We want to determine whether a MIMO system or a single-input single-output (SISO) system has better characteristics in such a scenario. For a fair comparison the total power dissipation of the radio frequency (RF) front end and the analog-to-digital conversion is kept constant.

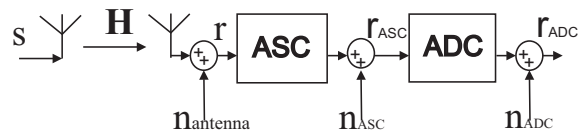


Figure 1: SISO system model.

Next to being low-power and high data-rate the node should also be reliable. Therefore, the outage capacity is used as a benchmark for system performance.

The theoretical model of the system is explained in Section 2. In the analysis of the system we will first introduce the Shannon equation for channel capacity of a MIMO system. Secondly we will take correlation between the transmit and receive antennas into account, since this deteriorates system performance. Thirdly we will extend the model to include the noise figure (NF) of the ASC. Finally we will introduce the quantization noise caused by the ADC. Simulation results are presented in Section 3. The conclusions are given in Section 4.

The research is conducted within the scope of the IOP GenCom project 'MIMO in a Mass-Market' (IGC 0502C), and serves as a first general system analysis.

## 2. MODELING SYSTEM CAPACITY

### 2.1. Received signal at Baseband

Figure 1 shows the system model for one transmit (TX) antenna and one receive (RX) antenna, a single-input single-output (SISO) system. In Figure 1 the received signal  $r$  is first processed by the analog signal conditioning (ASC) block to  $r_{ASC}$  and then digitized by the ADC block to  $r_{ADC}$ . Both the ASC and the ADC contribute noise to the received signal. We denote these noise variables by  $n_{ASC}$  and  $n_{ADC}$  respectively. The addition of noise in the receiver path causes a degradation of the overall system capacity ( $C_{sys}$ ).

## 2.2. MIMO channel capacity

Consider a transmission system that consists of  $N_t$  transmit antennas and  $N_r$  receive antennas. If a narrow band complex transmitted signal  $\mathbf{s}$  is transmitted, the received signal  $\mathbf{r}$  can be expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where  $\mathbf{H}$  is a  $N_r \times N_t$  complex channel-gain matrix and  $\mathbf{n}$  is a complex  $N_r$ -dimensional additive white Gaussian noise (AWGN) vector. For uncorrelated Rayleigh fading, the entries in  $\mathbf{H}$  are independent and identically distributed (i.i.d.), complex, zero-mean Gaussian with unit magnitude variance. The conventional way to calculate MIMO channel capacity is expressed by Foschini and Gans [1] and Telatar [2] as:

$$C = \log_2 \left( \det \left[ \mathbf{I} + \left( \frac{\rho}{N_t} \right) \mathbf{H}\mathbf{H}^\dagger \right] \right) \text{ b/s/Hz}, \quad (2)$$

where  $\rho$  is the average SNR per receive antenna caused by thermal noise at the antenna,  $\dagger$  denotes transpose conjugate and  $\mathbf{I}$  denotes the identity matrix. To ensure a fair comparison of capacity, the total power of the complex transmitted signal  $\mathbf{s}$  is constrained to  $P$ , regardless of the number of transmit antennas,  $\mathbb{E}[\mathbf{s}^\dagger \mathbf{s}] = P$ .

## 2.3. Correlation

To take the correlation between the transmit antennas and the receive antennas into account, the channel matrix  $\mathbf{H}$  can be modeled according to van Zelst [5] method as:

$$\mathbf{H} = \mathbf{R}_{\text{RX}}^{\frac{1}{2}} \mathbf{G} \mathbf{R}_{\text{TX}}^{\frac{1}{2}}, \quad (3)$$

where  $\mathbf{G}$  is a stochastic  $N_r \times N_t$  matrix with i.i.d. complex Gaussian zero-mean unit variance elements.

$\mathbf{R}_{\text{TX}}$  ( $N_t \times N_t$  dimensional) and  $\mathbf{R}_{\text{RX}}$  ( $N_r \times N_r$  dimensional) denote the correlation experienced at the transmitter side and the receiver side, respectively. With  $\mathbf{h}^n$  denoting the  $n^{\text{th}}$  row of  $\mathbf{H}$  and  $\mathbf{h}_m$  denoting the  $m^{\text{th}}$  column of  $\mathbf{H}$ , these correlation matrices can be found by  $\mathbf{R}_{\text{TX}} = E[(\mathbf{h}^n)^\dagger \mathbf{h}^n]$ , for  $n = 1, \dots, N_r$  and  $\mathbf{R}_{\text{RX}} = E[\mathbf{h}_m (\mathbf{h}_m)^\dagger]$ , for  $m = 1, \dots, N_t$ . We assume the correlation between the transmit antennas is independent of the correlation between the receive antennas. The assumed independence between the correlations is justified if the receive antennas and transmit antennas are spaced sufficiently far apart.

Consider a linear antenna array, where the antenna elements at the transmitter and receiver are spaced at an equidistant distance,  $d_{\text{TX}}$  and  $d_{\text{RX}}$ . The correlation matrices  $\mathbf{R}_{\text{TX}}$  and  $\mathbf{R}_{\text{RX}}$  can now be modeled in van Zelst [5] as:

$$\mathbf{R}_{\text{TX}} = \begin{bmatrix} 1 & r_{\text{TX}} & r_{\text{TX}}^2 & \dots & r_{\text{TX}}^{(N_t-1)} \\ r_{\text{TX}} & 1 & r_{\text{TX}} & \ddots & \vdots \\ r_{\text{TX}}^2 & r_{\text{TX}} & 1 & \ddots & r_{\text{TX}}^2 \\ \vdots & \ddots & \ddots & \ddots & r_{\text{TX}} \\ r_{\text{TX}}^{(N_t-1)} & \dots & r_{\text{TX}}^2 & r_{\text{TX}} & 1 \end{bmatrix}, \quad (4)$$

$$\mathbf{R}_{\text{RX}} = \begin{bmatrix} 1 & r_{\text{RX}} & r_{\text{RX}}^2 & \dots & r_{\text{RX}}^{(N_r-1)} \\ r_{\text{RX}} & 1 & r_{\text{RX}} & \ddots & \vdots \\ r_{\text{RX}}^2 & r_{\text{RX}} & 1 & \ddots & r_{\text{RX}}^2 \\ \vdots & \ddots & \ddots & \ddots & r_{\text{RX}} \\ r_{\text{RX}}^{(N_r-1)} & \dots & r_{\text{RX}}^2 & r_{\text{RX}} & 1 \end{bmatrix}, \quad (5)$$

where  $r_{\text{TX}}$  and  $r_{\text{RX}}$  are real-valued correlation coefficients, with  $0 \leq r_{\text{TX}} \leq 1$  and  $0 \leq r_{\text{RX}} \leq 1$ .

## 2.4. Analog signal conditioning

Now we will extend the model to include the ASC. If the number of antennas at the receiver is increased to  $N_r$ , when compared to a SISO system, so is the number of ASCs. If the total power dissipation of ASCs is kept constant, the available power per ASC is now decreased by a factor  $\frac{1}{N_r}$ . Because the available power for the ASC is decreased with a factor  $\frac{1}{N_r}$ , the noise figure NF and therefore the average SNR  $\rho$  of the receiver is changed. This can be accounted for in Equation (2) by adding the factor  $F_{\text{tot}}$ :

$$C_{\text{sys}} = \log_2 \left( \det \left[ \mathbf{I} + \left( \frac{\rho}{F_{\text{tot}} N_t} \right) \mathbf{H}\mathbf{H}^\dagger \right] \right) \text{ b/s/Hz}, \quad (6)$$

A typical ASC receiver is constructed out of several elementary blocks in cascade. The three blocks we consider are; a low-noise amplifier (LNA), a mixer and a filter. A model that gives minimal ASC power dissipation as a function of the overall NF is given by Baltus [6]:

$$P_{\text{min}} = IP3_{\text{tot}} \left( \sqrt{\kappa_n G_{\text{tot}}} + \frac{\left( \sum_{i=1}^{n-1} \sqrt[3]{\kappa_i (F_{i+1} - 1)} \right)^{\frac{3}{2}}}{\sqrt{(F_{\text{tot}} - F_1)}} \right)^2, \quad (7)$$

where  $IP3_{\text{tot}}$  is the third-order intercept point of the ASC,  $\kappa_i$  is the power linearity factor of the  $i^{\text{th}}$  cascade,  $F_i$  the NF of the  $i^{\text{th}}$  cascade,  $F_{\text{tot}}$  is the total NF and  $G_{\text{tot}}$  is the total gain of the ASC. We can use this model to derive the  $F_{\text{tot}}$ .

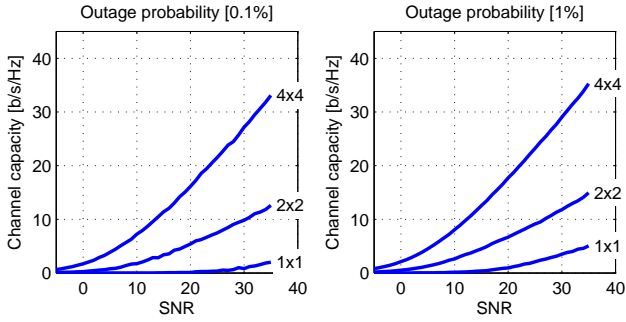


Figure 2: MIMO channel capacity.

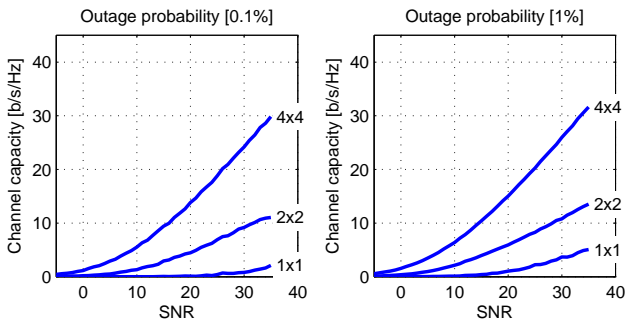


Figure 3: MIMO channel capacity with correlation.

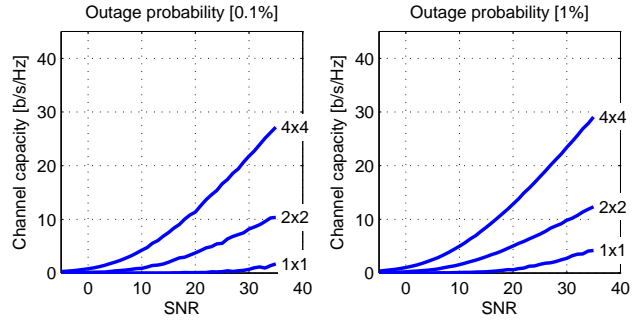


Figure 4: MIMO system capacity with correlation and noise figure.

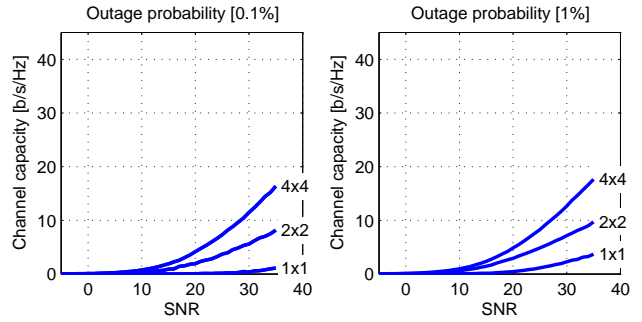


Figure 5: MIMO system capacity with correlation, noise figure and quantization noise.

## 2.5. Analog-to-digital converter

Finally we will extend the model to include the ADC. Again the power dissipation of the front end is kept at a constant level. The noise variance of the quantization noise  $n_{\text{ADC}}$  of a SISO system is  $\sigma_1^2$ . An increase in the number of receive antennas will result in a decrease of the resolution of the ADC to keep power dissipation constant. The variance of the quantization noise of the ADC will therefore increase with the square of the number of receivers  $\sigma^2 = N_r^2 \sigma_1^2$ . The overall system capacity can now be modeled as:

$$C_{\text{sys}} = \log_2 \left( \det \left[ \mathbf{I} + \left( \frac{\rho}{N_t(F_{\text{tot}} + N_r^2 \frac{\rho}{\rho_{\text{ADC}}})} \right) \mathbf{H}\mathbf{H}^\dagger \right] \right), \quad (8)$$

where  $\rho_{\text{ADC}}$  is the SNR of the ADC caused by quantization noise. Since both  $\rho$  and  $\rho_{\text{ADC}}$  depend on the input power the fraction  $\rho/\rho_{\text{ADC}}$  is a constant.

## 3. RESULTS

Since we are most interested in system reliability the outage capacity is used as a benchmark. The outage capacity depends on the allowed outage probability. The event that  $C_{\text{sys}} < C_x$  is called an outage. The outage probability is given by

$$P_{\text{out}} = P_r(C_{\text{sys}} < C_x), \quad (9)$$

which depends on the data rate  $C_x$  and the properties of random variable  $C_{\text{sys}}$ . The outage capacity is expressed as:

$$C_{\text{out}, P_0} = \sup\{C_x : P_{\text{out}} < P_0\}, \quad (10)$$

where  $C_{\text{out}, P_0}$  is the data rate corresponding to an outage probability  $P_0$ . In simulations Equation (8) is used, at each integer number of the SNR 20,000 Monte Carlo simulations are performed. The results of the Monte Carlo simulations are used to derive the outage capacity for given outage probabilities, which have a value of 0.1% and 1%. Simulations are performed for  $1 \times 1$ ,  $2 \times 2$  and  $4 \times 4$  systems.

### 3.1. MIMO channel capacity

First we calculated the outage capacity when we assumed the ASC and AD are ideal and there is no correlation between the antennas,  $F_{\text{tot}} = 1$ ,  $\rho/\rho_{\text{ADC}} = 0$ ,  $r_{\text{TX}} = 0$  and  $r_{\text{RX}} = 0$ . The results are shown in Figure 2. For this idealized scenario the reliability of the MIMO systems is considerably higher than the reliability of the SISO system, even for small SNR.

Table 1: ASC specifications.

	LNA	Mixer	Filter
<b>Power gain</b>	15dB	10dB	40dB
<b>Noise figure</b>	2dB	8dB	15dB
<b>IP3</b>	-10dBm	0dBm	-10dBm
<b>Linearity factor</b>	1.8974	0.7200	0.0048

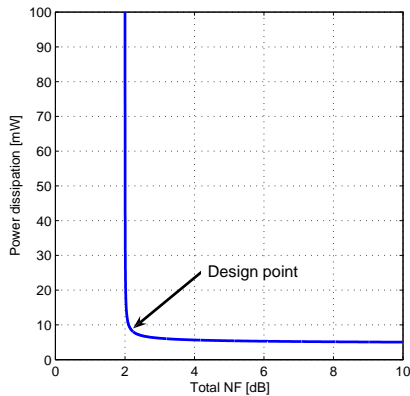


Figure 6: Minimal ASC power dissipation.

### 3.2. Correlation

Now we take correlation into account, since this deteriorates the system performance of the MIMO systems. In simulations the correlation coefficients are;  $r_{TX} = 0.6172$  and  $r_{RX} = 0.5883$ , these values correspond to measured data [5]. The ASC and ADC are assumed to be ideal,  $F_{tot} = 1$  and  $\rho/\rho_{ADC} = 0$ . The results are shown in Figure 3. The performance of the MIMO systems deteriorated, when compared to the scenario without correlation. For example the outage capacity of a  $4 \times 4$  system at an SNR of 30dB and an outage probability of 1% has now decreased from 29dB to 26dB. Although the SISO system has not degraded in performance, the MIMO system still achieves a considerably higher outage capacity.

### 3.3. Analog signal conditioning

Next we will include the NF of the ASC. For the simulations we assume the technology of the RF front end, i.e. of the ASC, is 90 nm CMOS. The specification of the three RF blocks are given in Table 1. We assume a voltage source of 1.2V DC. Figure 6 shows the minimal power dissipation as a function of the NF, as derived from Equation (7). An ASC is designed such that it comes close to the lowest NF with a minimal power dissipation. If the power of the ASC is now halved, the noise figure will rise enormously. Theoretically the NF will go to infinite around  $P_{diss} = 5.17\text{mW}$ , which consequently degrades  $C_{sys}$  to 0 b/s/Hz. The only option to improve power

dissipation is to use other semiconductor technologies or change the specifications. For the considered specifications and technology the total NF is  $F_{tot} = 2.3\text{dB}$  in the optimized point. If we assume the NF of the ASC is dominant and we can afford to loose the most significant bit (MSB) of the ADC, the gain of the ASC can be halved. Halving the gain will halve the power dissipation of the ASC in the optimized point. Loosing the MSB will halve the power dissipation of the ADCs. Thereby, the total RF and AD power dissipation remains constant. It should be noted that this halving of the gain of the ASC can only be performed within the boundary conditions of the considered semiconductor process. Figure 4 shows the outage capacity of different MIMO systems as a function of SNR, taking into account correlation and the noise figure of the ASC. In simulations, we have used the parameter values:  $r_{TX} = 0.6172$ ,  $r_{RX} = 0.5883$ ,  $F_{tot} = 2.3\text{dB}$  and  $\rho/\rho_{ADC} = 0$ . Both the MIMO system and the SISO system have degraded in performance, when compared to the idealized scenario. For example the outage capacity of a  $4 \times 4$  system at an SNR of 30dB and an outage probability of 1% has now decreased from 29dB to 23dB.

### 3.4. Analog-to-digital converter

Finally we assume quantization noise at the ADC. The quantization noise is only relevant when the quantization noise is larger or in the same order of magnitude as the thermal noise. It is assumed the MSB of the ADC is lost and the quantization noise is equal to the thermal noise. The gain of the ASC is halved, reducing the power consumption of the ASC and keeping the NF constant at  $F_{tot} = 2.3\text{dB}$ . The correlation coefficients are set to:  $r_{TX} = 0.6172$  and  $r_{RX} = 0.5883$ . The results are shown in Figure 5. Although the performance of the MIMO systems has degraded even further, when compared to the idealized scenario, they still outperform the SISO system in terms of outage capacity. It should be noted that the average capacity of the SISO system now outperforms the average capacity of the MIMO systems for small SNR. However, due to the shape of the probability density functions of the capacity the reliability of the MIMO systems is still superior. Therefore, since we are interested in reliability rather than average throughput, MIMO systems are still favorable. The superior outage capacity of MIMO systems is due to the resilience of MIMO systems to deep fades in the fading channel.

## 4. CONCLUSIONS

We compared MIMO and SISO systems consisting of a base station and a node with a limited power supply. The total power dissipation of the front end and ADC of the

node is kept constant, and correlation between the antennas is assumed. Simulations show that MIMO systems consisting of several low-power low-resolution receivers, achieve a better reliability. However, the scaling of the RF front end should remain within the constraints of the considered semiconductor process. In future research the power dissipation of the digital signal processing should be accounted for. Furthermore, the model should be extended to include the power dissipation of the transmitter. It should also be explored whether it is economically sensible to use multiple simple receivers instead of a more complex single receiver.

## 5. ACKNOWLEDGEMENTS

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