Compensated estimators for characterizing interference in a Rayleigh fading environment

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***Abstract*—Wireless LANs increasingly experience interference from other users operating in the same frequency band. In this paper, the aim is to develop techniques to learn from the interference environment in order to find the best strategy for radio transmissions. An analytical model is proposed to characterize the radio environment around a particular node. A compensated estimator is derived for the activity of nodes. Simulations show that the proposed estimators are reasonably accurate, particularly for high capture thresholds.**

I. Introduction

Wireless networks are in wide use today, and it is likely that in the future more and more radio devices will have to coexist in the same frequency band. This makes the efficient use of the available spectrum by many users a technical challenge. For devices that operate in the same network, various medium access control (MAC) schemes have been developed to coordinate their transmissions. However, the prevention of interference from conflicting simultaneous transmissions in different networks appears to be more difficult. The term Cognitive Radio [1] has been adopted for communication systems in which devices individually learn about their interference environment by sensing the channel and opportunistically use the radio resources, but treat the spectrum in an ecologically responsible manner according to prescribed rules.

To learn about the wireless environment, cognitive radios observe radio transmissions of other nodes, even when these nodes are not cooperative. The activity of a node and its local-mean received power are of interest, since these quantities are a measure for their interference on other transmissions. An estimator for the local-mean received power from a node can be biased for several reasons. First of all, packets can collide due to simultaneous transmissions of multiple nodes. In these cases, particulary the weaker packets are lost. Secondly, channel conditions fluctuate over time. Both effects can lead to an underestimated activity per node and an overestimated local-mean received power per node.

This paper aims at proposing a better, i.e. compensated, estimation method for the transmission probability. We also investigate whether one can rely on the measured incoming power during a capture event as a reasonable estimate of the local-mean power of a specific node.

II. System model

Throughout this paper we will consider a randomly arriving packet traffic, similar to assumptions used to study slotted ALOHA, although we do not necessarily assume that all packets have a common destination. A packet is send at the beginning of a time slot and consecutive packets do not overlap. Furthermore, we assume that all packets are equal to the slot length. We consider an observing node i separated from a source node *s* by a distance of D_s. This pair is being surrounded by N other possible interfering nodes, each node is indexed as $n = \{1 \dots N\}$. Radio signals propagate by means of reflection, diffraction, and scattering, which result in three effects a radio signal experiences: attenuation, large-scale shadowing, and small-scale fading. For our simulations, we do not consider shadowing. Signal attenuation is mainly based on the location of both source and destination node. The local-mean received power at node i from the source, is linked to the signal attenuation by

$$\bar{P}_s = P_{tx} D_s^{-\alpha}, \tag{1}$$

where α is the pathloss exponent. We do no need to introduce this pathloss in a real-time environment, but for our simulations it is essential.

Small scale fading of a signal is caused by multiple received versions of a transmitted signal with different delay times such that the signal has both time and location varying properties. In our model, the instantaneous received power from a node s, P_s, is exponentially distributed around its local-mean $\bar{P}_s$. We acknowledge that Rayleigh fading is not always an accurate model, particulary not for line of sight communications. However, due to the nice mathematical properties of its exponential distribution we use this model in this paper.

If the instantaneous received power for a specific message exceeds the total power of all other received messages by at least a certain threshold, we model the message to be decoded correctly. We call this the threshold *z*. Thus, a capture event C_s from source node *s* by the observing node *i* occurs when [2]:

$$SINR = \frac{P_s}{P_{N_0} + \sum_{n=1}^{N} P_n} > z. \tag{2}$$

Here, P_{N_0} is the noise power and P_n is the instantaneous received interference power from node n.

III. Capture probability

Given the local-mean received power from node s, the capture probability for a transmitted packet can be determined from:

$$\Pr(C_s|\bar{P}_s) = \Pr\left(\frac{P_s}{P_t} > z|\bar{P}_s\right). \tag{3}$$

Here, P_t is the total received interference power, which equals the noise power plus the received power from all interfering nodes. Assuming Rayleigh fading channels, it was shown in [3] that this conditional capture probability can be expressed as:

$$\mathsf{Pr}(C_s|\bar{P}_s) = \int_0^\infty \exp\left\{-\frac{wz}{\bar{P}_s}\right\} f_{p_t}(w)\,\mathrm{d}w. \tag{4}$$

Since the Laplace transform of a function x is defined by

$$\mathcal{L}_x(s) \triangleq \int_0^\infty x(t)\exp\{(-ts)\}\,\mathrm{d}t, \tag{5}$$

and by using 4 and 5 the conditional capture probability can now be expressed as a multiplication of the Laplace transforms of all Probability Density Functions (PDFs) of all the interfering links individually, namely:

$$\mathsf{Pr}(C_s|N, \bar{P}_s) = \prod_{n=1}^{N} \mathcal{L}\left\{f_{p_n}, \frac{z}{\bar{P}_s}\right\}. \tag{6}$$

The probability that a node n transmits in an arbitrary time slot is $\mathsf{Pr}(T_n)$. We assume that this probability is stationary over all slots, at least during the observation period, and independent for different nodes. The received power is zero when the node does not transmit. So, the Laplace transform of the received power PDF of node n at the observing node i in a point $\frac{z}{\bar{P}_s}$ is:

$$\mathcal{L}\{f_{p_n}, \frac{z}{\bar{P}_s}\} = 1 - \mathsf{Pr}(T_n) + \frac{1}{1+\frac{z}{\bar{P}_s}\bar{P}_n}\mathsf{Pr}(T_n). \tag{7}$$

IV. Estimators for interference

To estimate the transmission probability of node s, we initially count the number of slots in which a packet from node s captures the observation node i and divide it by the total number of observation slots. This approach of estimating the activity of node s is biased since we can only count the number of packets that are recovered. Packets lost, due to collisions or channel conditions, are not taken into account.

Note that the transmission probability, $\mathsf{Pr}(T_s)$, is connected to the capture probability with:

$$\begin{aligned}\mathsf{Pr}(C_s) &= \mathsf{Pr}(C_s, T_s)\\ &= \mathsf{Pr}(C_s|T_s)\mathsf{Pr}(T_s).\end{aligned} \tag{8}$$

A reliable estimate for $\mathsf{Pr}(C_s|T_s)$ is needed if we desire an accurate estimate for $\mathsf{Pr}(T_s)$. Using Eqn. 6 and 7 we get:

$$\begin{aligned}\mathsf{Pr}(C_s|T_s) &= \frac{\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})}{\mathcal{L}_{p_s}(\frac{z}{\bar{p}_s})}\\ &= \frac{\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})}{1-\frac{z}{1+z}\mathsf{Pr}(T_s)}.\end{aligned} \tag{9}$$

Finally, substituting Eqn 9 into Eqn 8 and rearranging the expression to solve for the transmission probability, we get:

$$\mathsf{Pr}(T_s) = \frac{\mathsf{Pr}(C_s)}{\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s}) + \mathsf{Pr}(C_s)\frac{z}{1+z}}. \tag{10}$$

The transmission probability of a certain node can be determined using 10. We assume that z is known, since it is merely a function of the chosen data-rate. Therefore, we need real-time estimations for: $\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})$ and the scalars $\mathsf{Pr}(C_i)$, $\bar{p}_s$.

A. Estimate of $\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})$

Several options exist for estimating the Laplace transform $\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})$. As proposed for the IEEE 802.11k [4] standard, we could use a histogram for the received power to derive an estimate for the PDF of p_t and the corresponding Laplace transform.

Another option is to use an approach which includes the Law of Large Numbers (LLN) [5]. The idea is that we do not need to know the complete function but only the function evaluated at a certain point. Denoting the total received power in a slot as $p_t[m]$, $m = 1, 2, \cdots, M$. then we get:

$$\begin{aligned}\mathcal{L}_{p_t}\left(\frac{z}{p_j}\right) &= \mathbb{E}\left[\exp\left\{-\frac{z}{\bar{p}_s}p_t[m]\right\}\right]\\ &\approx \frac{1}{M}\sum_{m=1}^{M}\exp\left\{-\frac{z}{\bar{p}_s}p_t[m]\right\}.\end{aligned} \tag{11}$$

B. Estimate of the Capture Probability $\mathsf{Pr}(C_s)$

Let us first prove that the Maximum Likelihood (ML) estimator for the capture probability is equal to the number of captured messages (u) divided by the total number of observed slots (v).

$$\begin{aligned}\widehat{\mathsf{Pr}}(C_s) &\triangleq \arg\max_{\mathsf{Pr}(C_s)} \mathsf{Pr}(u|\mathsf{Pr}(C_s))\\ &= \arg\max_{\mathsf{Pr}(C_s)} \log\mathsf{Pr}(u|\mathsf{Pr}(C_s))\\ &= \arg\max_{\mathsf{Pr}(C_s)} \Big\{\log\binom{v}{u} + u\log(\mathsf{Pr}(C_s)) +\\ &\quad (v-u)\log(1-\mathsf{Pr}(C_s)\Big\}.\end{aligned} \tag{12}$$

The first step in these equations is the definition of a ML estimator. The logarithm is introduced so that the binomial distribution in the third step can be expressed as a sum. The maximum of the final expression is found at $\widehat{\mathsf{Pr}}(C_s) = \frac{u}{v}$.

C. Estimate of $\bar{p}_s$

We propose to use the total received power as input for the local-mean power of node s, given that a packet from node s is captured correctly:

$$\begin{aligned}\bar{p}_{t|c_s} &= \mathbb{E}[p_t|C_s]\\ &= \int_0^\infty p_t f_{p_t}(p_t|C_s)\mathrm{d}p_t\\ &\approx \hat{\bar{P}}_s.\end{aligned} \tag{13}$$

If we use this approach to estimate the local-mean power a bias is present. This is because capture typically occurs when the received signal power is high and because interference power is included in the received signal. However, the impact due to the second contribution appears to be low if the threshold is sufficiently large:

$$\begin{aligned}\hat{\bar{P}}_s &\approx \frac{\int_{P_{min}}^\infty P_s f_{P_s}(P_s)\mathrm{d}P_s}{\int_{P_{min}}^\infty f_{P_s}(P_s)\mathrm{d}P_s}\\ &\approx P_{min} + \bar{P}_s.\end{aligned} \tag{14}$$

For our simulations we added both estimation methods for $\Pr(T_s)$:

$$q_1 \triangleq \frac{1}{M}\sum_{m=1}^{M} I\{C_s(m)\} \quad (15)$$

$$q_2 \triangleq \frac{q_1}{\left(\frac{1}{M}\sum_{m=1}^{M}\exp\left\{-\frac{z}{\bar{p}_s}p_t[m]\right\} + q_1\frac{z}{1+z}\right)}, \quad (16)$$

where $I\{C_s(m)\}$ is the event of capture by node s in slot m.

V. Simulations

The network scenarios and the results are depicted in Figures 1-6. We used the Law of Large Numbers to estimate the Laplace transform in a single point ($\mathcal{L}_{p_t}(\frac{z}{\bar{p}_s})$). The first figure illustrates a scenario with relatively low traffic. All interfering nodes transmit with probability 0.1. The observing node estimates the transmission probability of node s and its local-mean received power.
In the second scenario the wireless environment is more hostile. The interfering nodes transmit with probability 0.6 in every time slot. In both scenarios we added the biased as well as the compensated estimator for the transmission probability.

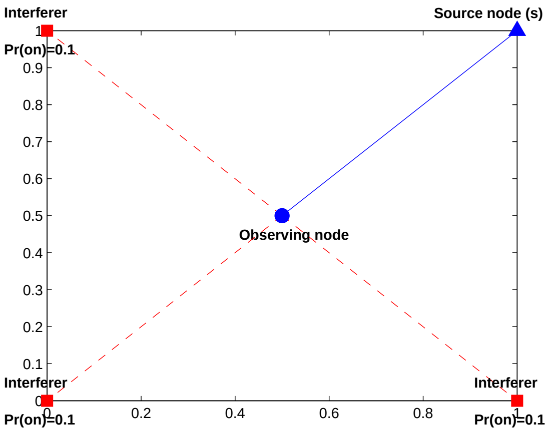

Fig. 1. Low interference probability

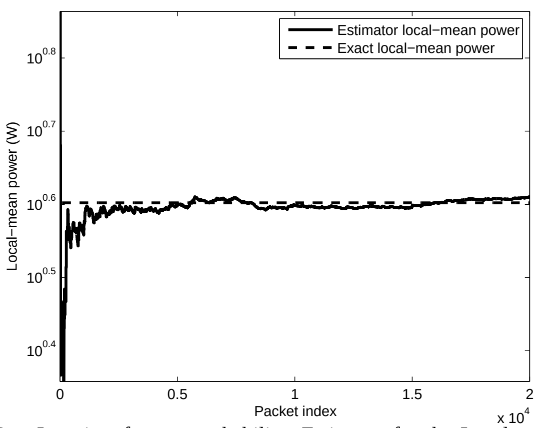

Fig. 2. Low interference probability: Estimator for the Local-mean power received from source node s, $z = 12$, $\Pr(T_s) = 0.2$

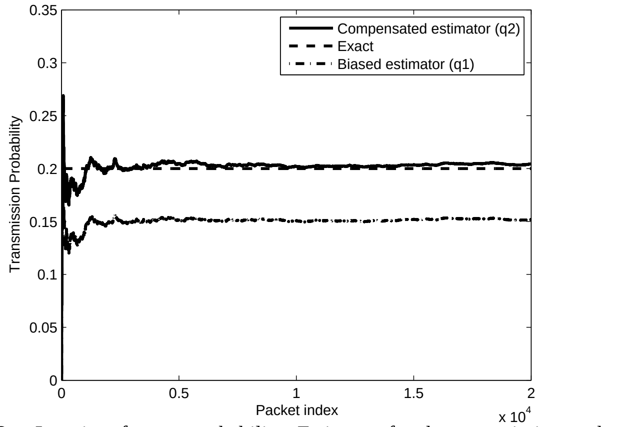

Fig. 3. Low interference probability: Estimator for the transmission probability of source node s, $z = 12$, $\Pr(T_s) = 0.2$

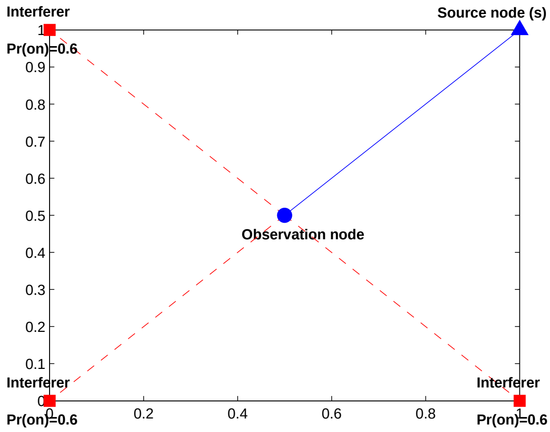

Fig. 4. High interference probability

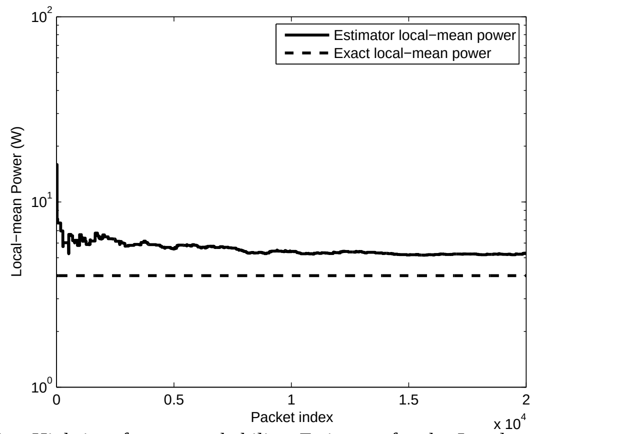

Fig. 5. High interference probability: Estimator for the Local-mean power received from source node s, $z = 12$, $\Pr(T_s) = 0.2$

VI. Conclusions

By measuring channel traffic and performing a compensated algorithm we can estimate the average activity of a node. Although a bias is present, simulations show that the proposed estimator for the local-mean power is reasonable accurate, at least for high capture thresholds. The results are less accurate for low capture thresholds and/or very high interference. Results show that even for situations when only 10% of the traffic for a certain node is observed, we can still estimate its average activity fairly accurately.

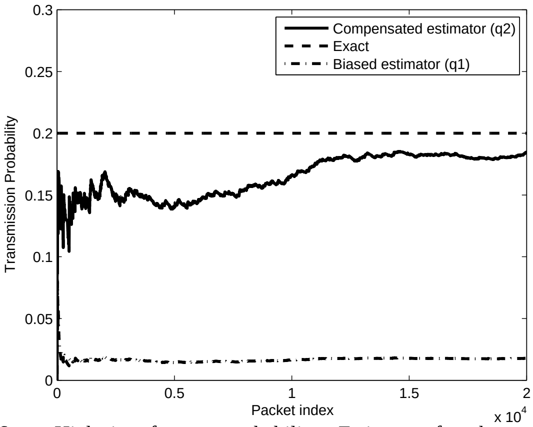

Fig. 6. High interference probability: Estimator for the transmission probability of source node s, $z = 12$, $\mathtt{Pr}(T_s) = 0.2$

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