Calculation of the Spatial Reservation Area for the RTS/CTS Multiple Access Scheme

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Abstract

The Request-to-Send/Clear-to-Send (RTS/CTS) protocol is a multiple access scheme that reserves the channel over space and time in order to transmit a payload (PAY). In this paper, exact expressions for the probability of correctly receiving the RTS, CTS and PAY are derived. Unlike other approaches, the power capture effect is considered for each packet type separately. Numerical results reveal an inter-play of the different capture probability over time and space, leading to an improved understanding of the scheme in the wireless network.

1 Introduction

Carrier sensing is a multiple access scheme that inhibits transmission when an ongoing transmission is detected at the transmitter [1]. However, in some situations users are inhibited unnecessarily [2]. This could be solved by virtual carrier-sensing in which the source first asks the destination whether the channel is idle by a Request-to-Send (RTS) packet, and the destination confirms this with a Clear-to-Send (CTS) packet [2]. Remote stations that recover the RTS or CTS packet are inhibited to transmit during some specified time, hence increasing the probability of a successful transmission.

Two types of packet detection have been assumed when considering the RTS/CTS scheme. In the collision model, a packet is correctly received if and only if there is no other concurrent transmission (e.g. [3]). A improved model is based on the power capture effect: a packet is correctly received (even if there are interfering signals) if and only if the signal-to-noise plus interference ratio (SINR) is greater than a capture ratio (e.g. [4, 5]). In the literature, the RTS/CTS scheme has been modeled by assuming that if the RTS is recovered by the destination, then the CTS, payload (PAY) and acknowledgement (ACK) will be received correctly too. This is in fact an approximation that violates the principle of the capture effect. As a result, the causal effect of the RTS, CTS, PAY and ACK in time is ignored, and the spatial reservation of the RTS and CTS cannot be highlighted. It is also not known how the scheme can be optimized to achieve a high system throughput with capture effect.

In this paper, the capture probabilities of the various packet types are derived and used to investigate the spatial and temporal impacts of the RTS/CTS scheme on the network. Although the capture probability depends on many system parameters, in particular the protocol designer can select the rate that is used to encode the packet. In principle at least, with proper rate adaptations of the different types of packets, their capture probabilities could be jointly optimized to achieve a high system throughput.

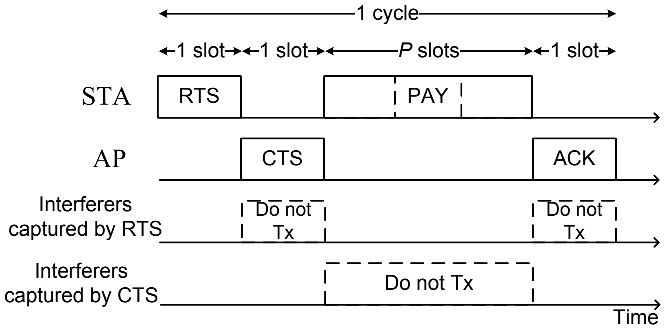


Figure 1: RTS/CTS multiple access scheme.

2 Model

2.1 RTS/CTS Scheme

The test station (STA) will be referred to simply as STA and the other STAs as interferers. We assume that the STA uses RTS/CTS for channel access to the intended destination which we assume to be the access point (AP). On the other hand, the interferers use slotted ALOHA for multiple access of unit slot length.

The detailed RTS/CTS scheme considered here is shown in Fig. 1. A transmission cycle consists of an RTS packet, a CTS packet, a PAY (data) packet and an ACK packet; for conciseness the word "packet" is subsequently omitted when referring to the different packet types. For ease of analysis, we assume the system to be time slotted. Each RTS, CTS and ACK uses one slot, and the PAY uses P slots. As illustrated, If the interferers received the RTS (CTS) correctly, they are inhibited to transmit during the CTS and ACK periods (PAY period, respectivly). In effect, the inhibition period is kept to a minimum; this is in contrast to the practice of IEEE 802.11, where any inhibition is carried out continuously until the cycle completes. Furthermore, we assume:

1. *No carrier sensing:* In contrast to common practice in IEEE 802.11, we explicitly do not consider that carrier sensing is carried out, but only virtual carrier sensing via the RTS/CTS mechanism. This eliminates the *exposed node problem* associated with carrier sensing.

2. *Only the STA is fully RTS/CTS capable:* For analytical tractability, we assume that only the STA uses RTS/CTS for multiple access. The interferers uses slotted ALOHA. Yet, they obey the RTS/CTS rules, that is, they temporarily refrain from transmission after they receive any RTS or CTS correctly.

3. *Slotted Data Detection:* Data is (transmitted and) detected during PAY on a slot by slot basis, independently of other slots. As a result, the ACK comprises of multiple bits of acknowledgement, one for each slot of data. This assumption allows the capture model to be applied for each slot independently.

2.2 Capture, Channel and Network Models

The instantaneous SINR can be represented as

$$\mathsf{SINR} = \frac{p_0}{p_t}, \quad p_t = N_o + \sum_{i=1}^{N} p_i \tag{1}$$

where p_0, p_i is the instantaneous signal power of the STA and the i^{th} interferer, respectively, while N_o is the noise power and p_t the sum interference and noise power. From information theory, when the interference and the data is independent Gaussian distributed, an achievable instantaneous data rate, in bits/symbol, is given by the mutual information $\mathcal{I}(\mathsf{SINR}) = \log(1 + \mathsf{SINR})$. For a given rate R, an *information outage* is said to occur if the instantaneous mutual information is smaller than R, i.e. when $\mathsf{SINR} < 2^R - 1$.

The capture, channel and network are modeled as follows.

- *Capture Effect:* The capture effect is modeled by assuming that the packet is correctly decoded if and only if there is no information outage, hence relating the capture ratio to the rate as $z(R) \triangleq 2^R - 1$. This assumption is accurate to within a few dBs of SNR (or SINR in this case) for large codeword size, see e.g. [6].
- *Path Loss:* All transmitters use the same power. However, each signal experiences path loss depending on the propagation distance, a. We model the local mean power according to the path loss law $\bar{p} = a^{-\beta}$ where $\beta = 4$ is the path loss exponent used in this paper.
- *Channel:* Each channel exhibits quasi-static flat Rayleigh fading during one RTS, CTS, PAY or ACK transmission, but independent for different transmission links and slots. Hence, the probability density function (pdf) of the power p is $f_p(p) = \frac{1}{\bar{p}} \exp\left(-\frac{p}{\bar{p}}\right)$.
- *Interferers' Positions:* The x- and y-coordinates of the STA and AP are denoted by position vectors $\mathbf{a}_{\mathsf{STA}}, \mathbf{a}_{\mathsf{AP}}$, respectively. When the RTS/CTS scheme is not implemented, interfering packets are transmitted according to a spatial homogenous Poisson process with intensity $\mathcal{G}(\mathbf{a}) = G_o$ packets per time slot per unit area, where $0 < G_o < \infty$ and $\mathbf{a}$ is in an operating region $\mathcal{A}$. The total traffic rate is $G_t = \int_{\mathbf{a}\in\mathcal{A}} \mathcal{G}(\mathbf{a}) d\mathbf{a}$. In this paper, we let the region $\mathcal{A}$ grows infinitely large.

3 Capture Probability

In this section, we give the derivation of a generic capture probability, the building block to compute the capture probability of the data. A packet captures the destination if and only if the instantaneous SINR is at least $z(R)$. Consider a source at $\mathbf{a}_{\text{source}}$ transmitting a packet of length L and rate R to a destination at $\mathbf{a}_{\text{dest}}$. The traffic intensity is $\mathcal{G}(\mathbf{a})$. The event that the packet is received correctly, i.e., that the packet *captures* the receiver, is denoted as

$$\mathcal{E}_{\text{cap}}(\mathbf{a}_{\text{source}}, \mathbf{a}_{\text{dest}}, R, L, \mathcal{G}).$$

For notational convenience, the arguments of z and $\mathcal{E}_{\text{cap}}$ will be dropped when there is no ambiguity.

Our analysis exploits the concept of the vulnerability weight function, defined as

$$W(2^R - 1, |\mathbf{a}_{\text{source}} - \mathbf{a}_{\text{dest}}|, |\mathbf{a}_i - \mathbf{a}_{\text{dest}}|) = 1 - \mathcal{L}_{f_{p_i}}((2^R - 1)/\bar{p})$$

where f_{p_i} denotes the (exponential) pdf of the power of the i^{th} interferer and $\mathcal{L}_f(s)$ the Laplace transform of function f evaluated at s. It has been shown in [7] that the *capture probability* can be simplified into

$$\Pr\{\mathcal{E}_{\text{cap}}(\mathbf{a}_{\text{dest}})\} = \exp\left\{-\frac{zN_o}{\bar{p}} - L\int_{\mathbf{a}_i\in\mathcal{A}} \mathcal{J}(\mathbf{a}_i)\, d\mathbf{a}_i\right\}, \quad (2)$$

$$\mathcal{J}(\mathbf{a}_i) \triangleq W(z, |\mathbf{a}_{\text{source}} - \mathbf{a}_{\text{dest}}|, |\mathbf{a}_i - \mathbf{a}_{\text{dest}}|)\mathcal{G}(\mathbf{a}_i). \quad (3)$$

4 Data Capture Probability

4.1 Preliminaries

Let $\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P}$ and $\mathcal{E}_{\rm A}$ denote the respective capture events that a slot of the RTS, CTS, PAY and ACK are received correctly. Note that for $P > 1$, $\mathcal{E}_{\rm P}$ is the capture event of a given slot during PAY, but is statistically the same for any slot due to the assumption that the channel is independent for every slot. The event that the data in a slot carried by the PAY is received successfully, $\mathcal{E}_{\rm data}$, occurs if all the above four capture events occurs[1]. Hence, the probability that the data in a PAY slot is transported from STA to AP successfully, i.e., the *data capture probability*, can be expressed using the chain rule as

$$\begin{aligned} \Pr(\mathcal{E}_{\rm data}) &= \Pr(\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P}, \mathcal{E}_{\rm A}) \\ &= \Pr(\mathcal{E}_{\rm R}) \Pr(\mathcal{E}_{\rm C}|\mathcal{E}_{\rm R}) \Pr(\mathcal{E}_{\rm P}|\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}) \Pr(\mathcal{E}_{\rm A}|\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P}). \end{aligned} \tag{4}$$

Unlike the RTS and CTS which inhibits other users, reducing the ACK rate does not incur any penalty on the network. Often, the ACK is transmitted at the lowest possible rate to ensure reliable communication. To make the analysis concise, we approximate $R_{\rm A}$ as (effectively) zero and so[2]

$$\Pr(\mathcal{E}_{\rm A}|\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P}) = 1. \tag{5}$$

Hence, to compute $\Pr(\mathcal{E}_{\rm data})$ then only requires computing the first three factors of (4).

4.1.1 Notations

We use $\Phi \in \{\rm R, C, P, A\}$ to denote a generic packet type. In our subsequent discussion, the following notations are used frequently:

- R_Φ is the rate used for transmitting Φ,
- $z_\Phi \triangleq 2^{R_\Phi} - 1$ is the capture ratio,
- $\mathcal{G}_{\Phi|\mathcal{E}}(\mathbf{a})$ is the spatial traffic intensity at $\mathbf{a}$ when Φ is transmitted conditioned on event $\mathcal{E}$, and
- $\mathcal{P}_{\Phi|\mathcal{E}}(\mathbf{a})$ is the probability that Φ captures a receiver at $\mathbf{a}$ conditioned on event $\mathcal{E}$.

Here, $\mathbf{a}$ is an arbitrary position referring to the position of a source for the traffic intensity or a destination for the capture probability. Furthermore, $\mathcal{E}$ refers to any or some of $\{\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P}, \mathcal{E}_{\rm A}, \emptyset\}$, where $\emptyset$ denotes a null set, i.e., a non-event.

4.1.2 Relationship of Traffic Intensity and Capture Probability

Computing any capture probability can be carried out using (2) if the traffic intensity is known. Hence, the problem of finding the capture probability is reduced to finding the required traffic intensity. Based on the RTS/CTS scheme described in Sect. 2.1, the capture probability for different packet types at an arbitrary position is

$$\mathcal{P}_{\rm R|\mathcal{E}}(\mathbf{a}) = \Pr\left\{\mathcal{E}_{\rm cap}(\mathbf{a}_{\sf STA}, \mathbf{a}, R_{\rm R}, \mathcal{G}_{\rm R|\mathcal{E}})\right\}, \tag{6}$$

$$\mathcal{P}_{\rm C|\mathcal{E}}(\mathbf{a}) = \Pr\left\{\mathcal{E}_{\rm cap}(\mathbf{a}_{\sf AP}, \mathbf{a}, R_{\rm C}, \mathcal{G}_{\rm C|\mathcal{E}})\right\}, \tag{7}$$

$$\mathcal{P}_{\rm P|\mathcal{E}}(\mathbf{a}) = \Pr\left\{\mathcal{E}_{\rm cap}(\mathbf{a}_{\sf STA}, \mathbf{a}, R_{\rm P}, \mathcal{G}_{\rm P|\mathcal{E}})\right\}. \tag{8}$$

[1] Recall that the ACK contains acknowledgement bits for all PAY slots

[2] Although not done here, it is also possible to calculate $\Pr(\mathcal{E}_{\rm A}|\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}, \mathcal{E}_{\rm P})$ exactly by following the same methodology to be outlined.

$$\mathcal{G}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) \xrightarrow{(6)} \mathcal{P}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) \xrightarrow{(15)} \mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) \xrightarrow{(7)} \mathcal{P}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) \xrightarrow{(10)} \Pr(\mathcal{E}_{\mathrm{C}}|\mathcal{E}_{\mathrm{R}})$$

Figure 2: Relationship of capture probabilities and traffic intensities over time and space in calculating $\Pr(\mathcal{E}_{\mathrm{C}}|\mathcal{E}_{\mathrm{R}})$.

$$\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}) \xrightarrow{(7)} \mathcal{P}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}) \xrightarrow{(17)} \mathcal{G}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}) \xrightarrow{(8)} \mathcal{P}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}) \xrightarrow{(11)} \Pr(\mathcal{E}_{\mathrm{P}}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}).$$

Figure 3: Relationship of capture probabilities and traffic intensities over time and space in calculating $\Pr(\mathcal{E}_{\mathrm{P}}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}})$.

For example, the (conditional) capture probability (6) is obtained since the RTS is transmitted at $\mathbf{a}_{\mathsf{STA}}$ and received at an arbitrary position $\mathbf{a}$. Moreover, the rate used is R_{R} and the corresponding traffic intensity is $\mathcal{G}_{\mathrm{R}|\mathcal{E}}$.

The individual capture probability of (4) can then be expressed concisely as special cases of (6), (7), (8), respectively, in the following ways:

$$\begin{aligned}
\Pr(\mathcal{E}_{\mathrm{R}}) &= \mathcal{P}_{\mathrm{R}}(\mathbf{a}_{\mathsf{AP}}), &(9)\\
\Pr(\mathcal{E}_{\mathrm{C}}|\mathcal{E}_{\mathrm{R}}) &= \mathcal{P}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}_{\mathsf{STA}}), &(10)\\
\Pr(\mathcal{E}_{\mathrm{P}}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}) &= \mathcal{P}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}_{\mathsf{AP}}). &(11)
\end{aligned}$$

4.2 Capture Probability of RTS

We first consider the capture probability of the RTS at an arbitrary position. The traffic intensity is $\mathcal{G}_{\mathrm{R}}(\mathbf{a}) = G_o$ for all $\mathbf{a}$ since during the RTS period no inhibition is active yet. Using (6), the exact capture probability can be expressed, after some simplifications, in closed form as

$$\mathcal{P}_{\mathrm{R}}(\mathbf{a}) = \exp\left\{-z_{\mathrm{R}}N_o a^4 - a^2\pi^2 G_o\sqrt{z_{\mathrm{R}}}/2\right\}, \qquad (12)$$

where a is the distance between STA and the receiver. This corresponds to the results derived for slotted ALOHA [7]. When the destination is AP, using (9) allows us to obtain the capture probability of the RTS by AP

$$\Pr(\mathcal{E}_{\mathrm{R}}) = \exp\left\{-z_{\mathrm{R}}N_o a_s^4 - a_s^2\pi^2 G_o\sqrt{z_{\mathrm{R}}}/2\right\}. \qquad (13)$$

where a_s is defined as the distance between STA and AP.

4.3 Capture Probability of CTS

We first present an important property to clarify how an inhibition affects a future traffic intensity. Mathematically, the memory of the RTS/CTS scheme within a cycle is attributed to this inhibition property.

Property 1 (Inhibition) *Consider a packet of type Θ that inhibits receivers that it captures from transmitting in the time period when Φ is active, where Φ occurs latter in time than Θ. From Fig. 1, if $\Theta = \mathrm{R}$, then $\Phi \in \{\mathrm{C},\mathrm{A}\}$, and if $\Theta = \mathrm{C}$, then $\Phi = \mathrm{P}$. Without inhibition, the traffic is a homogenous Poisson process with intensity*

G_o. Due to the inhibition, the traffic conditioned on $\mathcal{E}$ when Φ is transmitted is then an inhomogeneous Poisson process with intensity

$$\mathcal{G}_{\Phi|\mathcal{E}}(\mathbf{a}) \;=\; G_o\left(1-\mathcal{P}_{\Theta|\mathcal{E}}(\mathbf{a})\right) \tag{14}$$

where $\mathcal{P}_{\Theta|\mathcal{E}}(\mathbf{a})$ is the capture probability when Θ is sent conditioned on $\mathcal{E}$.

Proof 1 *Note that the transmission of the interferers are mutually independent and also independent of STA, AP without inhibition. Then, the property holds immediately by* thinning *the original homogenous Poisson process with probability $\left(1-\mathcal{P}_{\Theta|\mathcal{E}}(\mathbf{a})\right)$ [8].*

We are now ready to compute $\Pr(\mathcal{E}_{\mathrm{C}}|\mathcal{E}_{\mathrm{R}})$. We proceed in a reverse time order. As seen from (10) and (7), we need to first find $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$. If an interferer receives the RTS correctly, it is inhibited to transmit during the CTS, and the intensity is reduced. From Property 1, we get

$$\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) = G_o\left(1-\mathcal{P}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})\right). \tag{15}$$

The capture probability $\mathcal{P}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$, in turn, depends on the traffic intensity $\mathcal{G}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$, via (6). This traffic intensity is conditioned on $\mathcal{E}_{\mathrm{R}}$ and hence can be interpreted as an *a posteriori traffic intensity.* In contrast, when no additional information is provided, the a priori traffic intensity is $\mathcal{G}_{\mathrm{R}}(\mathbf{a}) = G_o$. To complete the analysis, $\mathcal{G}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$ is provided by (16) in Proposition 1. Proofs are omitted due to lack of space; see [9].

Proposition 1 *Consider a two-dimensional network with traffic modeled as a homogenous Poisson process with spatial traffic intensity G_o. Given that the RTS captures AP, the traffic intensity when the RTS is sent is given by*

$$\mathcal{G}_{\mathrm{R}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}) = G_o(1-W(z_R,|\mathbf{a}_{\mathsf{STA}}-\mathbf{a}_{\mathsf{AP}}|,|\mathbf{a}-\mathbf{a}_{\mathsf{AP}}|)). \tag{16}$$

In summary, the sequence of computations (and their relationship) in the forward time order is illustrated from left to right in Fig. 2.

4.4 Capture Probability of PAY

The calculation of $\Pr(\mathcal{E}_{\mathrm{P}}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}})$ is conceptually similar to $\Pr(\mathcal{E}_{\mathrm{C}}|\mathcal{E}_{\mathrm{R}})$. First, note that if an interferer receives the CTS correctly, it is inhibited to transmit during the PAY period. Using Property 1, the traffic intensity during this period is

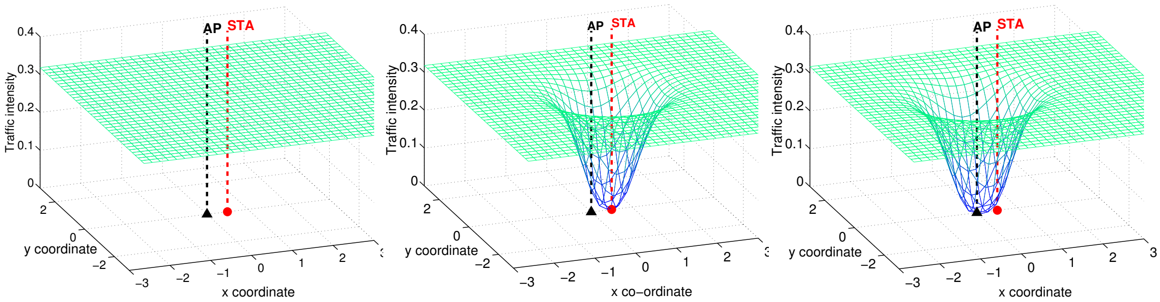
$$\mathcal{G}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}) = G_o\left(1-\mathcal{P}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a}))\right). \tag{17}$$

Following the same arguments as before, we arrive at a sequence of computations shown in Fig. 3. Finally, what is left is to determine $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a})$, provided by Proposition 2.

Proposition 2 *Assume the same two-dimensional network given in Proposition 1. Given the events $\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}$, the traffic intensity when the CTS is sent is given by*

$$\begin{aligned}&\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a})\\ =\;&(1-W(z_{\mathrm{C}},|\mathbf{a}_{\mathsf{AP}}-\mathbf{a}_{\mathsf{STA}}|,|\mathbf{a}-\mathbf{a}_{\mathsf{STA}}|))\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a}).\end{aligned} \tag{18}$$

The proof is similar to that given for Proposition 1; see also [9]. Note that $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$ is required for the computation in (18), which is given by (15). Hence, to compute $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a})$ further requires computing the first three spatial functions in Fig. 2.



(a) $\mathcal{G}_{\mathrm{R}}(\mathbf{a})$: no spatial reservation is carried out since the RTS/CTS scheme has not yet been employed.

(b) $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$: a spatial reservation has been carried out around STA using the RTS in anticipation of the CTS.

(c) $\mathcal{G}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a})$: a spatial reservation has been carried out around AP using the CTS in anticipation of the PAY.

Figure 4: Plot of traffic intensity where AP is at ▲ and STA is at ●.

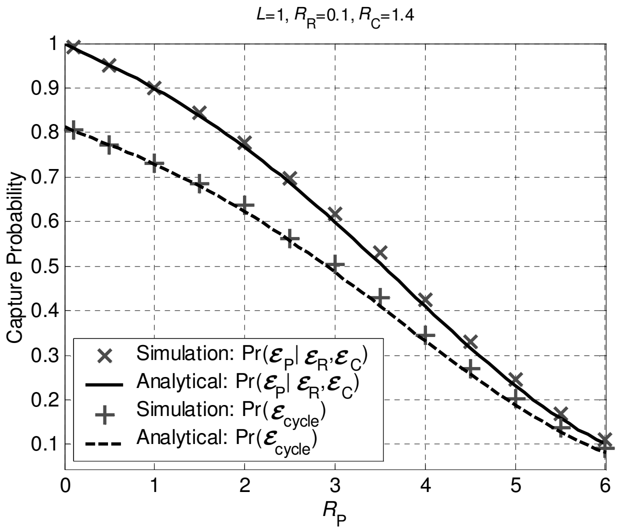


Figure 5: Simulations and analytical results for two exemplary capture probabilities.

5 Numerical Results

The temporal and spatial effects of the RTS/CTS scheme as successive captures occur is not immediately apparent from Fig. 2 and Fig. 3. We highlight the combined spatial and temporal effects by plotting the traffic intensities at different time, obtained by numerical computations. Noise is neglected for all our numerical studies to emphasize the effects of interference.

Consider an exemplary case with parameters $R_{\mathrm{R}} = 1 = R_{\mathrm{C}} = 1, a_S = 0.5, G_o = 1/\pi \approx 0.32$. If no spatial reservation is carried out using the RTS, the traffic intensity is uniform at G_o; see Fig. 4(a). When the RTS is used to reserve the channel in anticipation of the CTS, the traffic $\mathcal{G}_{\mathrm{C}|\mathcal{E}_{\mathrm{R}}}(\mathbf{a})$ is reduced mostly in the vicinity of the STA; see Fig. 4(b). Hence, the CTS capture probability is improved. After the RTS is received correctly, the CTS is used to reserve the channel in anticipation of the PAY. The traffic intensity $\mathcal{G}_{\mathrm{P}|\mathcal{E}_{\mathrm{R}},\mathcal{E}_{\mathrm{C}}}(\mathbf{a})$ is illustrated in Fig. 4(c). Similarly, the PAY capture probability is improved. Furthermore, as suggested by Fig. 4(b) and Fig. 4(c), the amount of inhibition can be considered appropriate since reservation is only activated when needed and mostly over the region close to the receivers.

Simulations have also been carried out to verify the analysis. The number of potentially transmitting interferers are first generated using the Poisson distribution, independently for different slots, before the simulation of a cycle starts. Then, the positions of the interferers within a square of length 20 is generated independently using a uniform spatial pdf. The Rayleigh channel is also generated independently for every slot and transmission link. If the cycle is not prematurely aborted (due to a failed transmission), the pre-determined interferers are inhibited accordingly after captures occurs. Otherwise, the cycle restarts.

From Fig. 5, it is observed that the simulation results (using 10,000 runs) match the analytical results well, with the simulated probabilities $\Pr(\mathcal{E}_{\rm P}|\mathcal{E}_{\rm R}, \mathcal{E}_{\rm C}), \Pr(\mathcal{E}_{\rm data})$ slightly higher than the analytical ones. From simulation trials, the gap decreases as the network area increases, but at an increase in simulation time. Hence, the simulations corroborate the analysis which assumes an infinitely large operation region.

6 Conclusion

An analytical description of the behavior of the RTS/CTS multiple access scheme taking capture effect into consideration has been developed. We have derived new exact expressions for the capture probabilities of the RTS, CTS and payload as functions of their transmission rates. Numerical results show that the scheme makes a spatial and temporal reservation of the channel only when required to. In subsequent research [9], rate adaptation will be carried out to perform throughput optimization and to illustrate the advantage of the RTS/CTS scheme.

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