RADIO PROPAGATION MODELS

Radio Propagation Models

1 Path Loss

- Free Space Loss
- Ground Reflections
- Surface Waves
- Diffraction
- Channelization

2 Shadowing

3 Multipath Reception and Scattering

- Dispersion
- Time Variations

Key Questions about Propagation

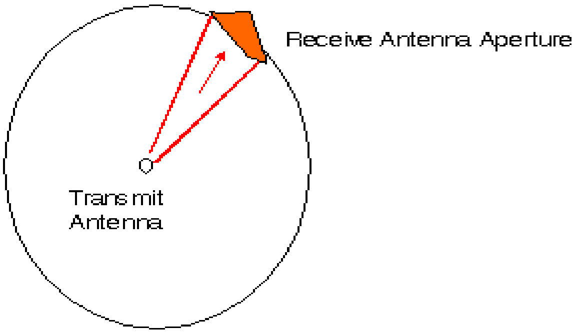
- Why may radio reception vanish while waiting for a traffic light?
- How does path loss depend on propagation distance?
- What are the consequences for cell planning?
- Why has the received amplitude a ‘Rician’ amplitude?
- What can we do to improve the receiver?

Key Terms

- Antenna Gain; Free-Space Loss; Ground Reflections; Two-Ray Model; "40 Log d"; Shadowing; Rician Fading; Bessel Function $I_0(.)$; Rician K-Ratio; Rayleigh Fading

Free Space Loss

Isotropic antenna: power is distributed homogeneously over surface area of a sphere.



The power density w at distance d is

$$w = \frac{P_T}{4\pi d^2}$$

where P_T is the transmit power.

The received power is

$$w = \frac{A}{4\pi d^2} P_T$$

with A the `antenna aperture' or the effective receiving surface area.

FREE SPACE LOSS, continued

The antenna gain G_R is related to the aperture A according to

$$G_R = \frac{4\pi A}{\lambda^2}$$

Thus the received signal power is

$$P_R \;=\; P_T\, G_R \bullet \frac{\lambda^2}{4\pi} \bullet \frac{1}{4\pi\, d^2}$$

- The received power decreases with distance, $P_R :: d^{-2}$
- The received power decreases with frequency, $P_R :: f^{-2}$

Cellular radio planning

Path Loss in dB:

$$L_{fs} = 32.44 + 20 \log (f / 1 \text{ MHz}) + 20 \log (d / 1 \text{ km})$$

Broadcast planning (CCIR)

Field strength and received power: $E_0 = \sqrt{(120\, \pi\, P_R)}$

In free space: $E_0 \;=\; \dfrac{\sqrt{30\;\; P_T\, G_T}}{\sqrt{4\pi} d}$

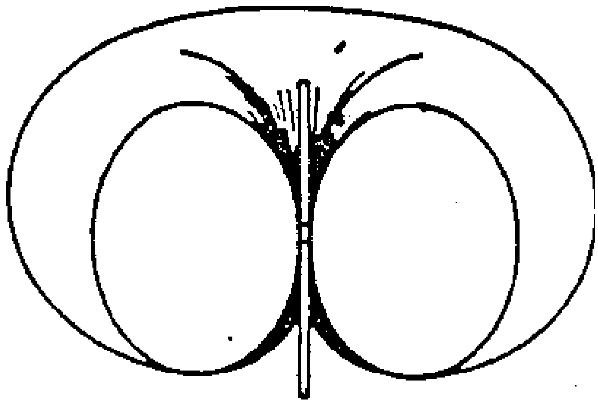
Antenna Gain

A theorem about cats: An isotropic antenna can not exist.

Antenna Gain

G_T (ϕ,θ) is the amount of power radiated in direction (ϕ,θ), relative to an isotropic antenna.

Definition: Effective Radiated Power (ERP) is $P_T\, G_T$



Half-Wave Dipole: A half-wave dipole has antenna gain

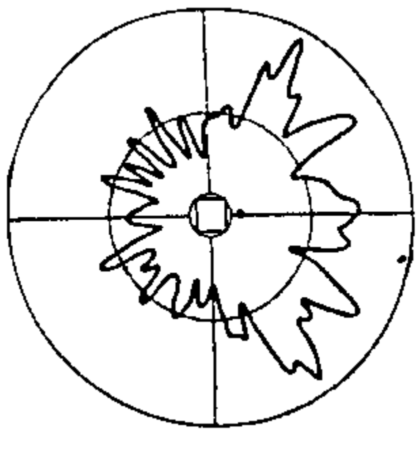
$$G(\theta,\phi) = 1.64\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$

Law of Conservation of Energy

Total power at distance d is equal to P_T

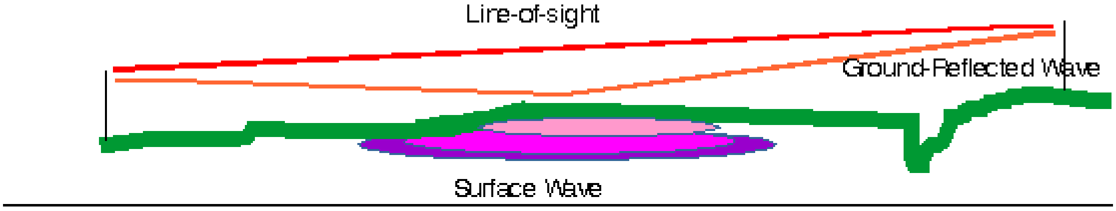
$$\int_{4\pi} G(\phi,\theta)\, dA \quad = \quad 1$$

$\Rightarrow$ A directional antenna can amplify signals from one direction $\{G_R(\phi,\theta) >> 1\}$, but must attenuate signals from other directions $\{G_R(\phi,\theta) < 1\}$.



Groundwave loss:

Waves travelling over land interact with the earth's surface.



Norton: For propagation over a plane earth,

$$E_i = E_{0_i}\left(1 + R_c e^{j\Delta} + (1 - R_c)\, F(\bullet) e^{j\Delta} + \bullet\bullet\bullet\right),$$

where

R_c is the reflection coefficient,

E_{0i} is the theoretical field strength for free space

$F(\cdot)$ is the (complex) surface wave attenuation

Δ is the phase difference between direct and ground-reflected wave

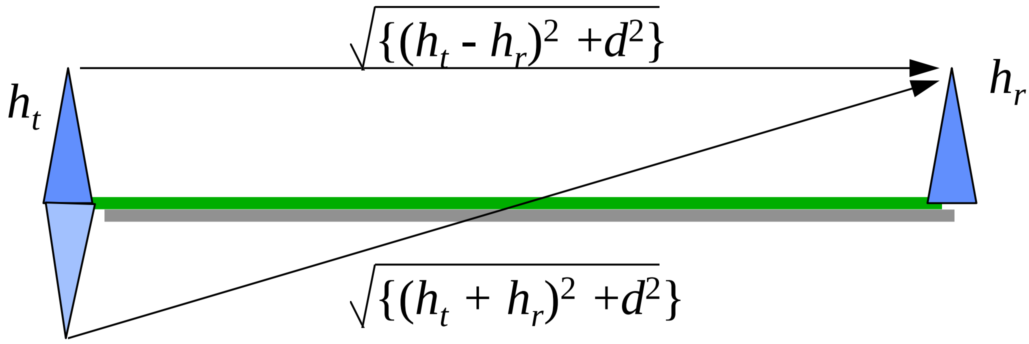
Bullington: Received Electric Field = direct line-of-sight wave + wave reflected from the earth's surface + a surface wave.

Space wave The (phasor) sum of the direct wave and the ground-reflected wave is called 'space wave'

Space-wave approximation for UHF land-mobile communication:

- Received field strength ≈ LOS + Ground-reflected wave.

Surface wave is negligible, i.e., $|F(\cdot)| << 1$, for the usual values of h_t and h_r.



The received signal power is

$$P_R = \left(\frac{\lambda}{4\pi d} \left| 1 + Re^{j\Delta} \right| \right)^2 P_T G_T G_R$$

The phase difference Δ is found from Pythagoras. Distance between TX and RX antenna = $\sqrt{\{(h_t - h_t)^2 + d^2\}}$

Distance between TX and mirrored RX antenna =

$\sqrt{\{(h_t + h_t)^2 + d^2\}\}}$

Space-wave approximation

The phase difference Δ is

$$\Delta = \frac{2\pi}{\lambda}\left(\sqrt{d^2 + (h_t + h_r)^2} - \sqrt{d^2 + (h_t - h_r)^2}\right)$$

At large a distance, $d >> 5h_t\, h_r$,

$$\Delta \approx \frac{4\pi\, h_r\, h_t}{\lambda d}$$

so, the received signal power is

$$P_R = \left(\frac{\lambda}{4\pi d}\left|1 + R \exp\ \frac{4\pi j\, h_r\, h_t}{\lambda d}\right|\right)^2 P_T\, G_T\, G_R$$

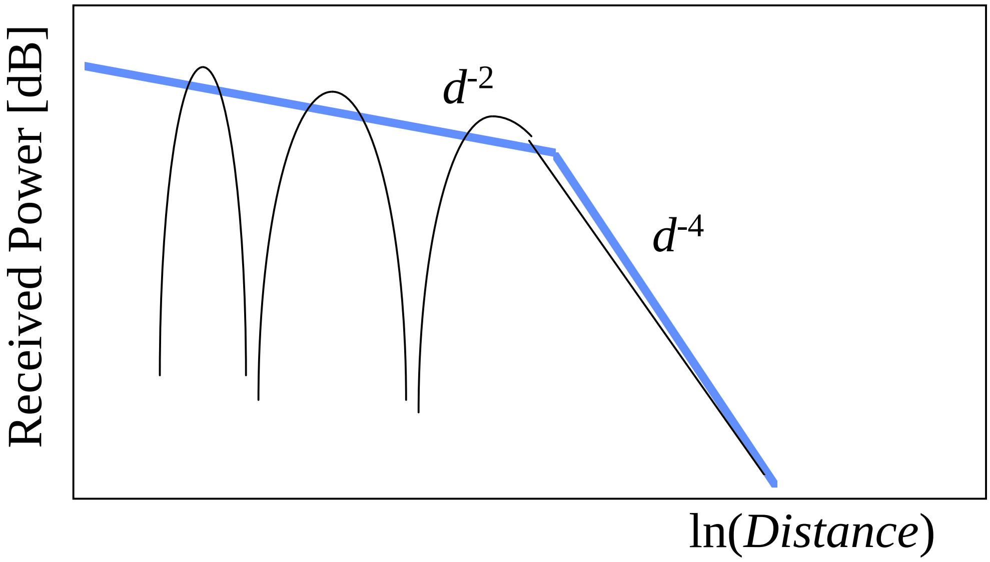
The reflection coefficient approaches $R_c \rightarrow -1$ for

- large propagation distances
- low antenna heights

For large distances $d \rightarrow \infty$: $\Delta \rightarrow 0$ and $R_c \rightarrow -1$.

In this case, LOS and ground-reflected wave cancel!!

Two-ray model (space-wave approximation)



For R_c = -1 and approximate Δ, the received power is

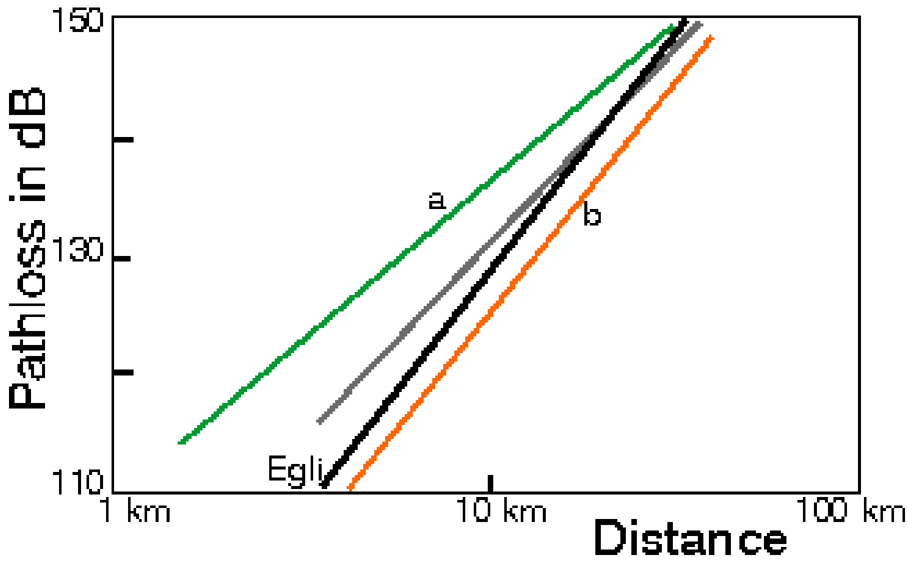
$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 4\sin^2\left(\frac{2\pi h_r h_t}{\lambda d}\right) G_R \; G_T P_T$$

N.B. At short range, R_c may **not** be close to -1. Therefor, nulls are less prominent as predicted by the above formula.

Macro-cellular groundwave propagation

For $d\lambda >> 4\ h_r\ h_t$, we approximate $\sin(x) \approx x$:

$$P_R - \frac{h_r^2 h_t^2}{d^4} P_T G_R G_T$$



Egli [1957]: semi-empirical model for path loss

$$L = 40 \log d + 20 \log\left(\frac{f_c}{40\,\mathrm{MHz}}\right) - 20 \log h_r h_t\ .$$

- Loss per distance:................ 40 log *d*
- Antenna height gain:............. 6 dB per octave
- Empirical factor:................... 20 log *f*
- Error: standard deviation...... 12 dB

Generic path-loss models

- p is normalized power
- r is normalized distance

Free Space Loss: "20 log d" models

$$p = r^{-2}$$

Groundwave propagation: "40 log d" models

$$p = r^{-4}$$

Empirical model:

$$p = r^{-\beta}, \qquad \beta \approx 2 \ldots 5$$

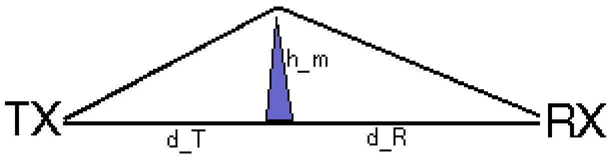
$$\beta \approx 3.2$$

Micro-cellular models

VHF/UHF propagation for low antenna height ($h_t = 5 \cdot\cdot 10$ m)

$$p = r^{-\beta_1} \left(1 + \frac{r}{r_g} \right)^{-\beta_2}$$

Diffraction loss



The diffraction parameter v is defined as

$$v = h_m \sqrt{\frac{2}{\lambda}\left(\frac{1}{d_t} + \frac{1}{d_r}\right)},$$

where

h_m is the height of the obstacle, and

d_t is distance transmitter - obstacle

d_r is distance receiver - obstacle

The diffraction loss L_d, expressed in dB, is approximated by

$$L_d = \begin{cases} 6 + 9v - 1.27v^2 & 0 < v < 2.4 \\ 13 + 20\log v & v > 2.4 \end{cases}$$

How to combine ground-reflection and diffraction loss?

Obstacle gain:

- The attenuation over a path with a knife edge can be smaller than the loss over a path without the obstacle!
- "Obstacles mitigate ground-reflection loss"

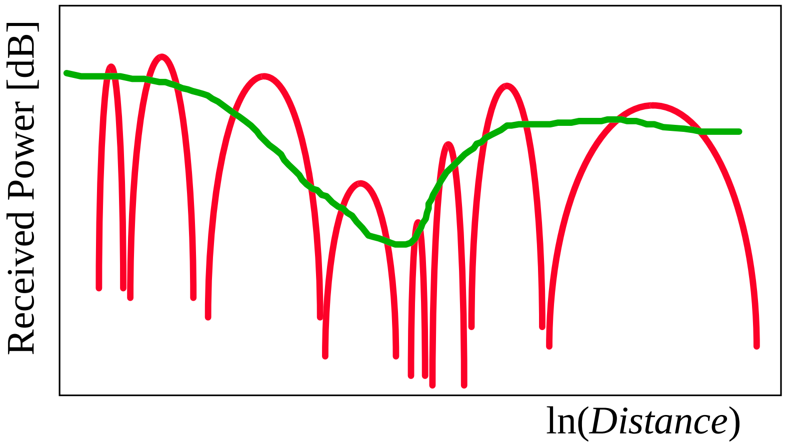
Bullington: "add all theoretical losses"

$$L_K = L_{fs} + L_d + L_R \,,$$

Blomquist:

$$L_K = L_{fs} + \sqrt{L_d^2 + L_R^2} \,,$$

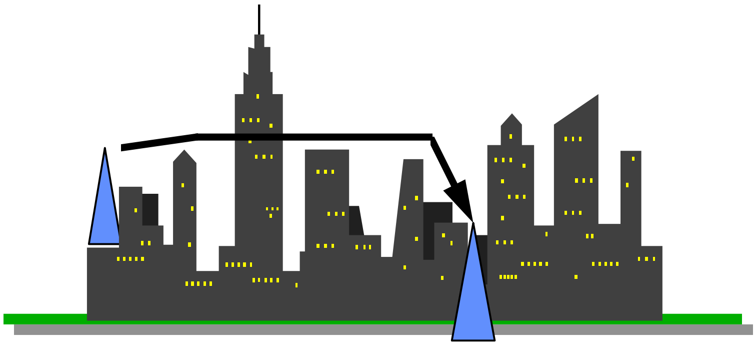
Statistical Fluctuation: Location Averages



- Area-mean power
  - is determined by path loss
  - is an average over 100 m - 5 km
- Local-mean power
  - is caused by local 'shadowing' effects
  - has slow variations
  - is an average over 40 λ (few meters)
- Instantaneous power
  - fluctuations are caused by multipath reception
  - depends on location and frequency
  - depends on time if antenna is in motion
  - has fast variations (fades occur about every half a wave length)

Shadowing

Local obstacles cause random shadow attenuation



Model: Normal distribution of the received power P_{Log} in logarithmic units (such as dB or neper),

Probability Density:

$$f_{\bar{p}_{\mathrm{Log}}}\left(\bar{p}_{\mathrm{Log}}\right) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2\sigma^2} \bar{p}_{\mathrm{Log}}^2\right\}$$

where

σ is the 'logarithmic standard deviation' in natural units.

P_{Log} = ln [local-mean power / area-mean power]

The standard deviation in dB is found from $s = 4.34\,\sigma$

The log-normal distribution

Convert 'nepers' to 'watts'. Use

$$\overline{p}_{\mathrm{Log}} = \ln \frac{\overline{p}}{\overline{\overline{p}}}$$

and

$$\left| f_{\overline{p}}(\overline{p})\, d\overline{p} \right| = \left| f_{\overline{p}}\left(\overline{p_{Log}}\right) d\,\overline{p_{Log}} \right|_{\overline{p_{Log}} = \ln\left(\frac{\overline{p}}{\overline{\overline{p}}}\right)}$$

The *log-normal* distribution of received (local-mean) power is

$$f_{\overline{p}}(\overline{p}) = \frac{1}{\sqrt{2\pi}\,\sigma_s \overline{p}} \;\exp\left\{ -\frac{1}{2\sigma^2} \ln^2\left(\frac{\overline{p}}{\overline{\overline{p}}} \right) \right\},$$

Area-mean and local-mean power

- The area-mean power   is the *logarithmic* average of the local-mean power

- The *linear* average and higher-order moments of local-mean power are

$$\mathrm{E}\left[\bar{p}^{m}\right] = \int_{0}^{\infty} \bar{p}^{m} \; f_{\bar{p}}\left(\bar{p}\right) d\bar{p} \; = \; \bar{\bar{p}}^{m} \exp\left\{m^{2}\frac{\sigma^{2}}{2}\right\}.$$

N.B. With shadowing, the interference power accumulates rapidly!! Average of sum of 6 interferers is larger than sum of area means.

Depth of shadowing: sigma = 3 .. 12 dB

"Large-area Shadowing":

· Egli: Average terrain: 8.3 dB for VHF and 12 dB for UHF

· Marsan, Hess and Gilbert:

Semi-circular routes in Chicago: 6.5 dB to 10.5 dB, with a median of 9.3 dB.

"Small-area shadowing"

· Marsan et al.: 3.7 dB

· Preller & Koch: 4 .. 7 dB

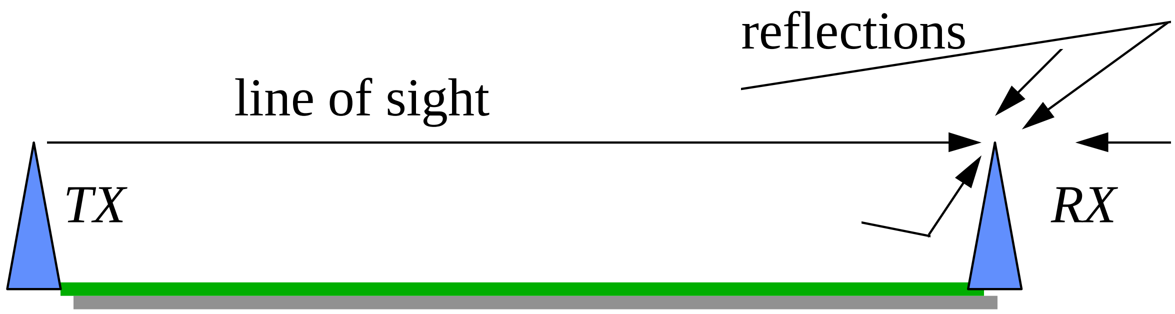
Combined model by Mawira (PTT Research):

Two superimposed Markovian processes:

3 dB with coherence distance over 100 m, plus

4 dB with coherence distance 1200 m

Rician multipath reception



Narrowband propagation model:

- Transmitted carrier

$$s(t) \;=\; \cos\omega_c t$$

- Received carrier

$$v(t) = \; C\cos\omega_c t + \sum_{n=1}^{N} \rho_n \cos(\omega_c t + \phi_n) \;,$$

where

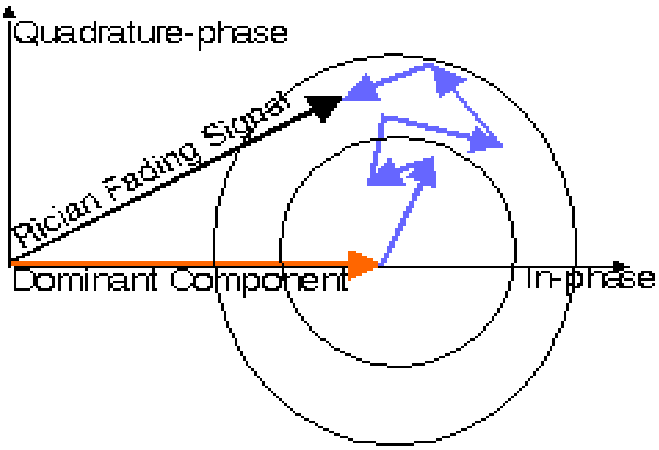
C is the amplitude of the line-of-sight component

ρ_n is the amplitude of the n-th reflected wave

ϕ_n is the phase of the n-th reflected wave

Rayleigh fading: $C = 0$

Rician fading: *I*-*Q* Phasor diagram



Received carrier:

$$v(t) = C\cos\omega_c t + \sum_{n=1}^{N} \rho_n \cos(\omega_c t + \phi_n)\ ,$$

where

ζ is the in-phase component of the reflections

ξ is the quadrature component of the reflections.

I is the total in-phase component ($I = C + \zeta$)

Q is the total quadrature component ($Q = \xi$)

Central Limit Theorem

ζ and ξ are zero-mean independently identically distributed (i.i.d.) **jointly Gaussian** random variables

PDF:

$$f_{I,Q}(i,q) = \frac{1}{2\pi\sigma^2} \exp\left\{-\frac{i^2 + (q - C)^2}{2\sigma^2}\right\}$$

Conversion to polar coordinates:

- Received amplitude ρ: $\rho^2 = i^2 + q^2$.
- $i = \rho \cos \phi$; $q = \rho \sin \phi$,

$$f_{\mathrm{P},\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2} \exp\left\{-\frac{\rho^2 + C^2 - 2\rho C \cos\phi}{2\sigma^2}\right\}$$

Rician Amplitude

Integrate joint PDF over ϕ from 0 to 2π: Rician PDF of ρ

$$f_{\rho}(\rho) \;=\; \frac{\rho}{\overline{q}}\exp\left(-\frac{\rho^{2}+C^{2}}{2\overline{q}}\right)\; I_{0}\left(\frac{\rho C}{\overline{q}}\right),$$

where

$I_0(\cdot)$ is the modified Bessel function of the first kind and zero order

is the total scattered power (= σ^2).

Rician K-ratio

K = direct power $C^2/2$ over scattered power

Measured values

K = 4 ... 1000 (6 to 30 dB) for micro-cellular systems

Light fading ($K \rightarrow \infty$)

- Very strong dominant component
- Rician PDF $\rightarrow$ Gaussian PDF

Severe Fading: Rayleigh Fading

- Direct line-of-sight component is small ($C \rightarrow 0$, $K \rightarrow 0$).
- The variances of ζ and ξ are equal to local-mean power
- PDF of amplitude ρ is Rayleigh

$$f_{\mathrm{P}}(\rho) = \frac{\rho}{\bar{p}} \exp\left\{-\frac{\rho^2}{2\bar{p}}\right\}.$$

- The instantaneous power p ($p = {}^1/_2\rho^2 = {}^1/_2\zeta^2 + {}^1/_2\xi^2$) is exponential

$$f_p(p) = f_\rho(\rho) \left|\frac{d\rho}{d\bar{p}}\right| = \frac{1}{\bar{p}} \exp\left\{-\frac{p}{\bar{p}}\right\}.$$

Nakagami fading

- The sum of m exponentially distributed *powers* is Gamma distributed.

$$f_{p_t}(p_t) = \frac{1}{\bar{p}\Gamma(m)}\left(\frac{p_t}{\bar{p}}\right)^{m-1} \exp\left\{-\frac{p_t}{\bar{p}}\right\}.$$

where

$\Gamma(m)$	is the gamma function; $\Gamma(m+1) = m!$
m	is the 'shape' factor

- The local-mean power E $[p_t] = m$.
- The amplitude is Nakagami m-distributed

$$f_\rho(\rho) = \frac{\rho^{2m-1}}{\Gamma(m) 2^{m-1} \bar{p}^m} \exp\left(-\frac{\rho^2}{2\bar{p}}\right)$$

- Application of this model:
  - Joint interference signal (not constant envelope!!)
  - Dispersive fading; self interference

N.B. The sum of m Rayleigh phasors is again a Rayleigh phasor.