Fundamentals of Diversity Reception

What is diversity?

Diversity is a technique to combine several copies of the same message received over different channels.

Why diversity?

To improve link performance

Methods for obtaining multiple replicas

- Antenna Diversity
- Site Diversity
- Frequency Diversity
- Time Diversity
- Polarization Diversity
- Angle Diversity

Antenna (or micro) diversity.

- at the mobile (antenna spacing > $\lambda/2$)

  Covariance of received signal amplitude

$$J_0^2(2\pi f_D \tau) = J_0^2(2\pi d/\lambda).$$

- at the base station (spacing > few wavelengths)

  Covariance of received signal amplitude

$$J_0^2\left(\frac{2\pi k d}{\lambda}\sin\xi\right) \quad J_0^2\left(\pi k^2 \frac{d}{\lambda}\sqrt{1-\frac{3}{4}\cos^2\xi}\right)$$

where

ξ	angle of arrival of LOS
d	is the antenna spacing
k	(k << 1) is the ratio of radius a of scattering objects and distance between mobile and base station. Typically, a is 10 .. 100 meters.

Site (or macro) diversity

- Receiving antennas are located at different sites.
  For instance, at the different corners of hexagonal cell.
- Advantage: multipath fading, shadowing, path loss and interference are "independent"

Polarization diversity

- obstacles scatter waves differently depending on polarization.

Angle diversity

- waves from different angles of arrival are combined optimally, rather than with random phase
- Directional antennas receive only a fraction of all scattered energy.

Frequency diversity

- Each message is transmitted at different carrier frequencies simultaneously
- Frequency separation >> coherence bandwidth

Time diversity

- Each meesage is transmitted more than once.
- Useful for moving terminals
- Similar concept: Slow frequency hopping (SFH): blocks of bits are transmitted at different carrier frequencies.

Selection Methods

- Selection Diversity
- Equal Gain Combining
- Maximum Ratio Combining
- Wiener filtering
  - if interference is present
- Post-detection combining:
  - Signals in all branches are detected separately
  - Baseband signals are combined.
  - For site diversity: do error detection in each branch

Pure selection diversity

- Select only the strongest signal
- In practice: select the highest signal + interference + noise power.
- Use delay and hysteresis to avoid excessive switching
- Simple implementations: Threshold Diversity

  - Switch when current power drops below a threshold

  - This avoids the necessity of separate receivers for each diversity branch.

PDF of C/N for selection diversity

One branch with Rayleigh fading

The signal-to-noise ratio γ has distribution

$$F_{\gamma_i}(\gamma_0) = P(\gamma_i \le \gamma_0) = 1 - \exp\left\{-\frac{\gamma_0}{\overline{\gamma}_i}\right\}$$

where

$\overline{\gamma}_i$ is local-mean signal-to-noise ratio

$(\overline{\gamma}_i = \overline{\gamma} = \overline{p} / N_0 B_T)$

L brances with i.i.d. Rayleigh fading

The probability that the signal-to-noise ratio γ_R is below γ_0 is

$$F_{\gamma_R}(\gamma_0) \triangleq P(\gamma_R \le \gamma_0) = \left[1 - \exp\left\{-\frac{\gamma_0}{\overline{\gamma}_i}\right\}\right]^L$$

Selection Diversity

Expectation of received signal-to-noise ratio

$$E\gamma_R = \bar{\gamma}\,[1 + 1/2 + 1/3 + \ldots\ 1/L].$$

Outage probability

- Insert $\gamma_0 = z$ in distribution.
- For large fade margins ($\bar{\gamma} >> z$), outage probability tends to $(z/\bar{\gamma})^L$.

PDF of C/N ratio γ_R

Derivative of the cumulative distribution

$$f_{\gamma_R}(\gamma) = \frac{L}{\bar{\gamma}}\left[1 - \exp\left\{-\frac{\gamma}{\bar{\gamma}}\right\}\right]^{L-1} \exp\left\{-\frac{\gamma}{\bar{\gamma}}\right\}$$

Diversity Combining Methods

Each branch is co-pahased with the other branches and weighted by factor a_i

- Selection diversity
  $a_i = 1$ if $\rho_i, > \rho_j$, for all $j \neq i$ and 0 otherwise.

- Equal Gain Combining: $a_i = 1$ for all i.

- Maximum Ratio Combining: $a_i = \rho_i$.

PDF of C/N for diversity reception

- Signal in branch i with amplitude ρ_i is multiplied by a diversity combining gain a_i.
- Signals are then co-phased and added.

Combined received signal amplitude is

$$\rho_R = \sum_{i=1}^{L} a_i \rho_i$$

The noise power N_R in the combined signal is

$$N_R = N \sum_{i=1}^{L} a_i^2$$

where N is the (i.i.d.) Gaussian noise power in each branch.

The signal-to-noise ratio in the combined signal is

$$\gamma_R = \frac{\rho_t^2}{2N_t} = \frac{\left(\sum_{i=1}^{L} a_i \rho_i\right)^2}{2N \sum_{i=1}^{L} a_i^2}$$

Optimum branch weight coefficients a_i

Cauchy's inequality

$$(\Sigma a_i\, r_i)^2 \quad \le \quad \Sigma a_i^2\, \Sigma\, r_i^{\,2}$$

is an equality for a_i is a constant times r_i . Hence,

$$\gamma_R = \frac{\left(\sum_{i=1}^{L} a_i \sqrt{N} \frac{\rho_i}{\sqrt{N}}\right)^2}{2N \sum_{i=1}^{L} a_i^2} \le \frac{\sum_{i=1}^{L} a_i^2 N \sum_{i=1}^{L} \frac{\rho_i^2}{N}}{2N \sum_{i=1}^{L} a_i^2} = \sum_{i=1}^{L} \gamma_i$$

where

γ_i is instantaneous signal-to-noise ratio in i-th branch

($\gamma_i \triangleq p_i / N_0 B_T$).

Optimum: Maximum Ratio Combining.

We conclude that γ_R is maximized for $a_i = \rho_i$.

Maximum Ratio Combining

SNR of combined signal is sum of SNR's

Inserting $a_i = \rho_i$ gives

$$\gamma_R = \sum_{i=1}^{L} \frac{\rho_i^2}{2N} = \sum_{i=1}^{L} \gamma_i$$

I.I.D. Rayleigh-fading channel

PDF of the combined SNR is Gamma distributed, with

$$f_{\gamma_R}(\gamma) = \frac{\gamma^{L-1}}{\overline{\gamma}^L (L-1)!} \exp\left\{-\frac{\gamma}{\overline{\gamma}}\right\}$$

MRC

Distrubution

$$F_{\gamma_R}(\gamma_0) = 1 - \exp\left\{-\frac{\gamma_0}{\overline{\gamma}}\right\} \sum_{l=1}^{L} \frac{1}{(L-1)!}\left(\frac{\gamma_0}{\overline{\gamma}}\right)^{L-1}$$

For large fade margins ($\gamma_0 = z << \overline{\gamma}$), this closely approaches

$$F_{\gamma_R}(\gamma_0) \rightarrow \frac{1}{L!}\left(\frac{\gamma_0}{\overline{\gamma}}\right)^{L}$$

Equal Gain Combining

For EGC, weight $a_i = 1$ irrespective of ρ_i,.

The combined-signal-to-noise ratio is

$$\gamma_R = \frac{\left(\sum_{i=1}^{L} \rho_i\right)^2}{2NL}$$

Combined output is the sum of L Rayleigh variables.

- No closed form solution, except for $L = 1$ or 2.

EGC

- Approximate pdf (Schwartz): for L = 2, 3,... and large fade margins ($\gamma_0 = z << \bar{\gamma}$)

$$F_{\gamma_R}(\gamma_0) = P(\gamma \le \gamma_0) = \frac{\sqrt{\pi}\left(\frac{L}{2}\right)^L}{(L-\frac{1}{2})!L!} \left(\frac{\gamma_0}{\bar{\gamma}}\right)^L$$

where

$(L - 1/2)! \triangleq \Gamma(L + 1/2) = (1.3...(2L - 1))\sqrt{\pi}/2^L$.

EGC performs slightly worse than MRC.

For large fade margins,

outage probabilities differ by a factor $\sqrt{\pi}(L/2)^L / \Gamma(L + 1/2)$.

Average SNR for EGC

The local-mean combined-signal-to-noise ratio $\bar{\gamma}_R$ is

$$\bar{\gamma}_R = \frac{\mathrm{E}\sum_{i=1}^{L}\sum_{j=1}^{L}\rho_j\rho_i}{2NL}$$

Since

$\mathrm{E}\rho_i\rho_i = 2\bar{p}$ and

$\mathrm{E}\rho_i\rho_j = \pi\bar{p}/2$ for $i \neq j$,

this becomes

$$\bar{\gamma}_R = \bar{p}\frac{\left(2L+L(L-1)\frac{\pi}{2}\right)}{2NL} = \bar{\gamma}\left(1+(L-1)\frac{\pi}{4}\right)$$

For $L \to \infty$, this is 1.05 dB below the mean C/N for MRC.

Comparison

i.i.d. Rayleigh fading in L branches.

Technique:	Circuit Complexity:	C/N improvement factor:
Threshold	simple, cheap single receiver	$1 + \gamma_T/\Gamma \exp(-\gamma_T/\Gamma)$ for $L = 2$ optimum for γ_T/Γ: $1 + e \approx 1.38$
Selection	L receivers	$1 + 1/2 + .. + 1/L$
EGC	L receivers co-phasing	$1 + (L - 1)\,\pi/4$
MRC	L receivers co-phasing channel estimator	L

Compared to simple, inxpensive selection diversity, the average SNR is much better if MRC is used .

However if one compares the probability of a deep fade of the output signal, selection diversity appears to perform reasonably well, despit its relative simplicity.