The Basics of Mobile Propagation

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Mobile Propagation

Path Loss
- Free Space Loss
- Ground Reflections
- Reflections and Diffraction
- Microcellular Propagation
- Indoor propagation

Shadowing

Multipath Reception and Scattering
- Frequency - selectivity (dispersion)
- Time - selectivity (fading)

A few typical questions about propagation

- How does path loss depend on propagation distance?
- Why does radio reception vanish sometimes when you stop for a traffic light?
- Why has the received signal a 'Ricean' amplitude?
- What are the consequences for cell planning?
- Why has DECT reception problems beyond 250 meters?
- Why can antenna diversity improve reception?
- How can error correction, interleaving and retransmission used most effectively?
- How to improve a receiver?

Key Terms of This Section

- Antenna Gain; Free-Space Loss; Ground Reflections; Two-Ray Model; Path Loss; "40 Log d";
- Shadowing; Log-normal fading
- Multipath; Rayleigh Fading; Ricean Fading; Ricean *K*-factor; Bessel Function $I_0(.)$; Outage probability; Diversity

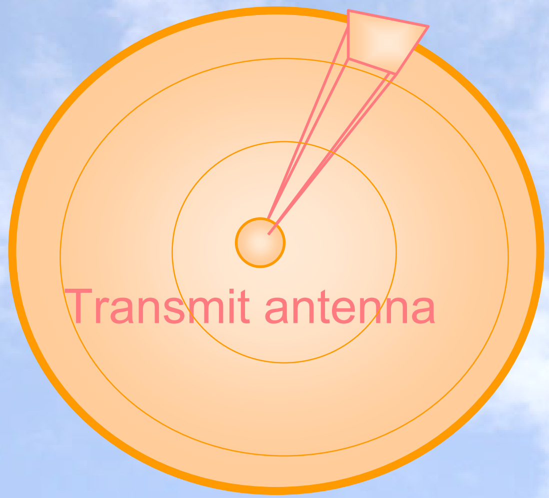
Next section:

•Delay spread; Coherence Bandwidth

•Doppler spread; Scatter Function; Fade durations

Free Space Loss

Isotropic antenna: power is distributed homogeneously over surface area of a sphere.



Received power is power through effective antenna surface over total surface area of a sphere of radius *d*

Free Space Loss

The power density *w* at distance *d* is

$$w = \frac{P_T}{4\pi d^2}$$

where P_T is the transmit power.

The received power is

$$P_R = \frac{A}{4\pi d^2} P_T$$

with *A* the `antenna aperture' or
the effective receiving surface area.

FREE SPACE LOSS, continued

The antenna gain G_R is related to the aperture A according to

$$G_R = \frac{4\pi A}{\lambda^2}$$

Thus the received signal power is

$$P_R = P_T G_R \bullet \frac{\lambda^2}{4\pi} \bullet \frac{1}{4\pi d^2}$$

Received power decreases with distance, $P_R :: d^{-2}$
Received power decreases with frequency, $P_R :: f^{-2}$

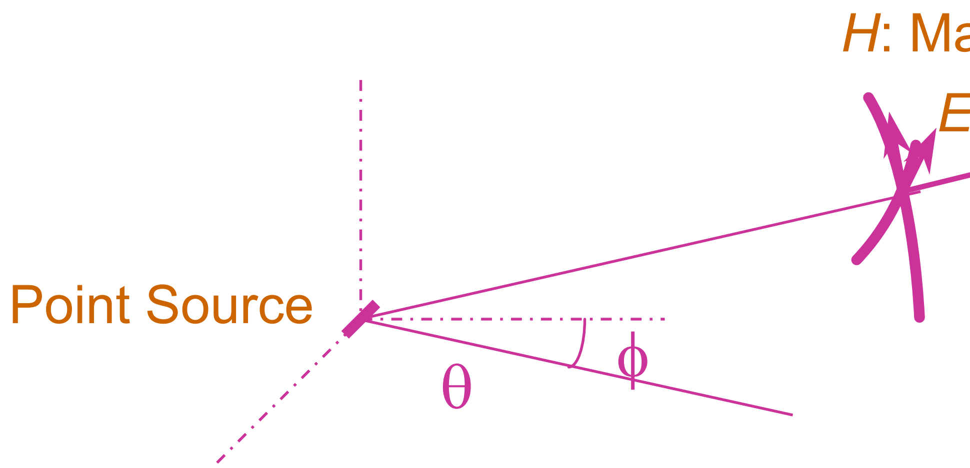
Cellular radio planning: Path Loss in dB:

$$L_{fs} = 32.44 + 20 \log (f / 1 \text{ MHz}) + 20 \log (d / 1 \text{ km})$$

Antenna Gain

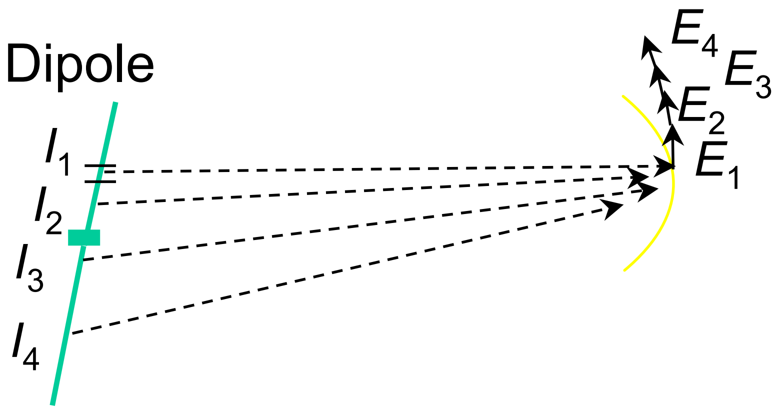
Antenna Gain

$G_T(\phi,\theta)$ is the amount of power radiated in direction (ϕ, θ), relative to an isotropic antenna.



Antenna Gain: derivation

- Starting point: *E* field from basic infinitesimal dipole
- Antenna is sum of many basic dipoles (integral)
- Total field is integral over fields from basic dipoles



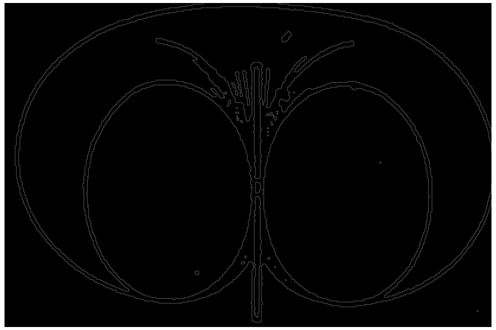
Antenna Gain: Half-Wave Dipole

A theorem about cats:

"An isotropic antenna can not exist."

Half-Wave Dipole: A half-wave dipole has antenna gain

$$G(\theta,\phi) = 1.64\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$



Definition: Effective Radiated Power (ERP) is $P_T\ G_T$

Law of Conservation of Energy

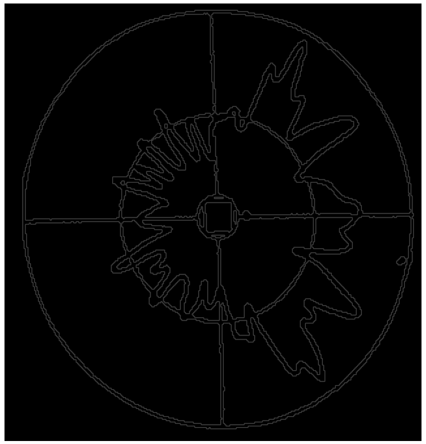
Total power through any sphere centred at the antenna is equal to P_T. Hence,

$$\int_{4\pi} G(\phi,\theta)\, dA = 1$$

A directional antenna can amplify signals from one direction $\{G_R(\phi,\theta) >> 1\}$, but must attenuate signals from other directions $\{G_R(\phi,\theta) < 1\}$.

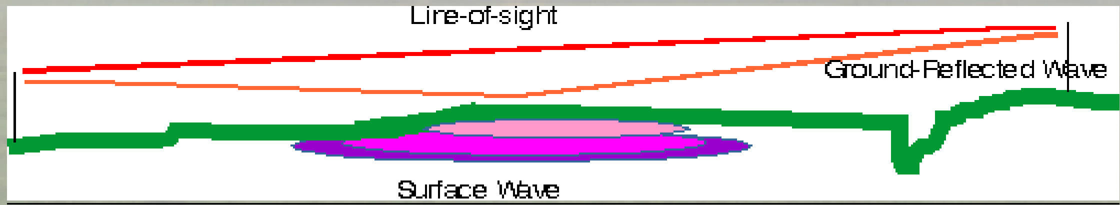
Example: radiation pattern of a base station

- Multipath effects from antenna mast
- Angle-selective fades



Groundwave loss:

Waves travelling over land interact with the earth's surface.



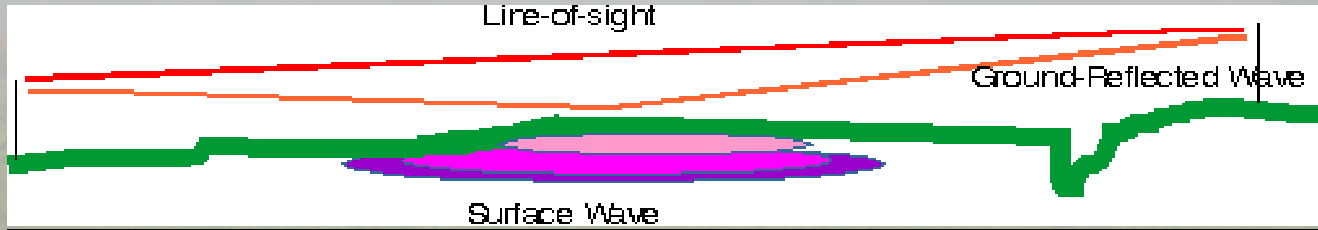
Norton: For propagation over a plane earth,

$$E_i = E_{0_i}\left(1 + R_c e^{j\Delta} + (1 - R_c)\, F(\bullet)\, e^{j\Delta} + \bullet\bullet\bullet\right),$$

where

R_c	is the reflection coefficient,
E_{0i}	is the theoretical field strength for free space
$F(.)$	is the (complex) surface wave attenuation
D	is the phase difference between direct and ground-reflected wave

Three Components



Bullington: Received Electric Field =

- direct line-of-sight wave +
- wave reflected from the earth's surface +
- a surface wave.

Space wave:

- the (phasor) sum of the direct wave, and
- the ground-reflected wave

Space-wave approximation for UHF land-mobile communication

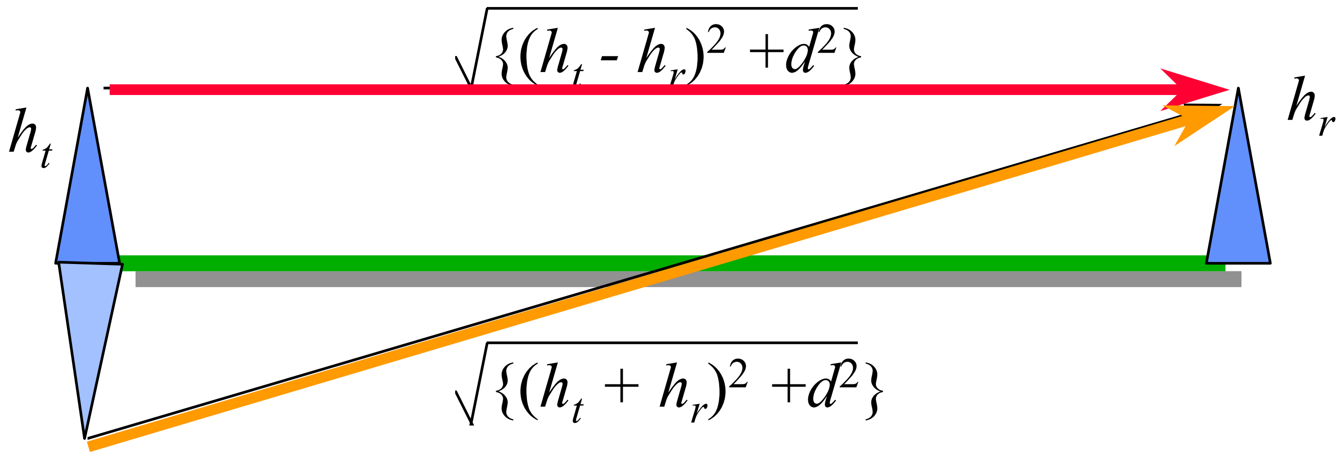
- Received field strength = LOS + Ground-reflected wave.
- Surface wave is negligible, i.e., *F*() << 1, for the usual antenna heights

For LW and MW: surface wave is relevant

The received signal power is

$$P_R = \left(\frac{\lambda}{4\pi d} \left| 1 + \mathrm{R} e^{j\Delta} \right| \right)^2 P_T G_T G_R$$

Space-wave approximation



The phase difference Δ is found from Pythagoras.

- Distance TX to RX antenna = $\sqrt{(h_t - h_r)^2 + d^2}$
- Distance mirrored TX to RX antenna =

$$\sqrt{(h_t + h_r)^2 + d^2}$$

Space-wave approximation

The phase difference Δ is

$$\Delta = \frac{2\pi}{\lambda}\left(\sqrt{d^2 + (h_t + h_r)^2} - \sqrt{d^2 + (h_t - h_r)^2}\right)$$

At large a distance, $d >> 5\ h_t\ h_r$,

$$\Delta \approx \frac{4\pi h_r h_t}{\lambda d}$$

So, the received signal power is

$$P_R = \left(\frac{\lambda}{4\pi d}\left|1 + R\exp\frac{4\pi j h_r h_t}{\lambda d}\right|\right)^2 P_T G_T G_R$$

Space-wave approximation

$$P_R = \left(\frac{\lambda}{4\pi d} \left| 1 + R_c \exp \frac{4\pi j\, h_r h_t}{\lambda d} \right| \right)^2 P_T G_T G_R$$

The reflection coefficient approaches $R_c \to -1$ for

- large propagation distances ($d \to \infty$)
- low antenna heights

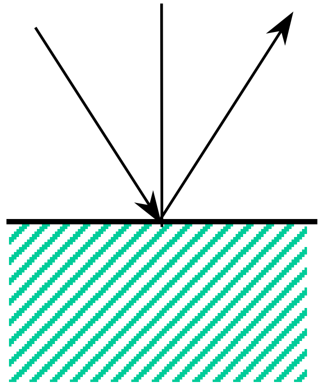
So $\Delta \to 0$, and

LOS and ground-reflected wave cancel!!

Reflection

calculate

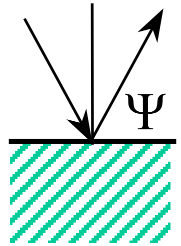
Reflection coefficient



Amplitude and phase depend on:

- Frequency
- Properties of surface (σ, μ, ε)
- Horizontal, vertical polarization
- Angle of incidence (thus, antenna height)

Reflection Coefficient



For a wave incident on the surface of a perfectly smooth earth,

Horizontally polarized

$$R_c = \frac{\sin\Psi - \sqrt{(\varepsilon_r - jx) - \cos^2\Psi}}{\sin\Psi + \sqrt{(\varepsilon_r - jx) - \cos^2\Psi}}$$

Vertically polarized

$$R_c = \frac{(\varepsilon_r - jx)\sin\Psi - \sqrt{(\varepsilon_r - jx) - \cos^2\Psi}}{(\varepsilon_r - jx)\sin\Psi + \sqrt{(\varepsilon_r - jx) - \cos^2\Psi}}$$

ε_r relative dielectric constant of the earth,

Ψ is the angle of incidence (between the radio ray and the earth surface)

$x = \sigma/(2\,\pi\, f_c\, \varepsilon_0)$, with

σ the conductivity of the ground and

ε_0 the dielectric constant of vacuum.

So, $x = \sigma/(\omega\varepsilon_0)$=18 $10^9\sigma/f$.

Propagation Properties of Ground

Surface	Conductivity σ	Rel Dielectric ε_r
Dry Poor Ground	10^{-3}	4-7
Average Ground	$5\ 10^{-3}$	15
Wet Good Ground	$2\ 10^{-2}$	25-30
Fresh Water	10^{-2}	81
Sea Water	5	81

Exercise

calculate

Show that the reflection coefficient tends to -1 for angles close to 0.

$$R_c = \frac{\sin\Psi - \sqrt{(\epsilon_r - jx) - \cos^2\Psi}}{\sin\Psi + \sqrt{(\epsilon_r - jx) - \cos^2\Psi}}$$

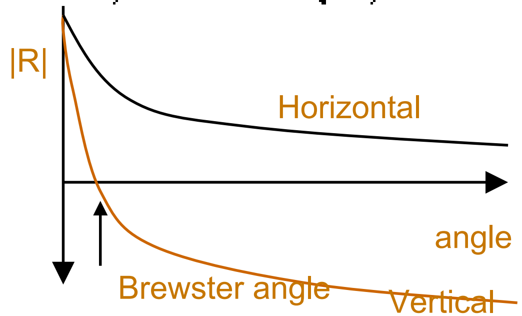
Verify that for horizontal polarization, abs(R_c) > 0.9 for Ψ < 10 degrees.

$$R_c = \frac{(\epsilon_r - jx)\sin\Psi - \sqrt{(\epsilon_r - jx) - \cos^2\Psi}}{(\epsilon_r - jx)\sin\Psi + \sqrt{(\epsilon_r - jx) - \cos^2\Psi}}$$

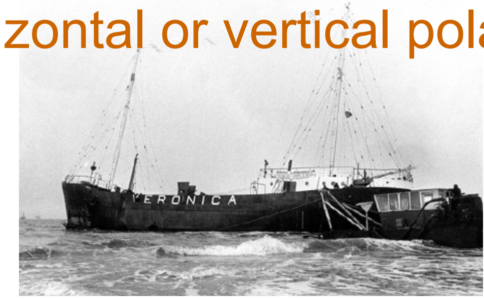
For vertical polarization,

abs(R_c) > 0.5 for Ψ < 5 degrees and

abs(R_c) > 0.9 for Ψ < 1 degree.



Question: you want to operate an AM medium wave station. Would you prefer to use horizontal or vertical polarization?



Jean-Paul Linnartz

Two-ray model

For R_c = -1, the received power is

$$P_R = \left(\frac{\lambda}{4\pi d}\right)^2 \frac{1}{4}\sin^2\frac{2\pi h_r h_t}{\lambda d} G_T G_R P_T$$

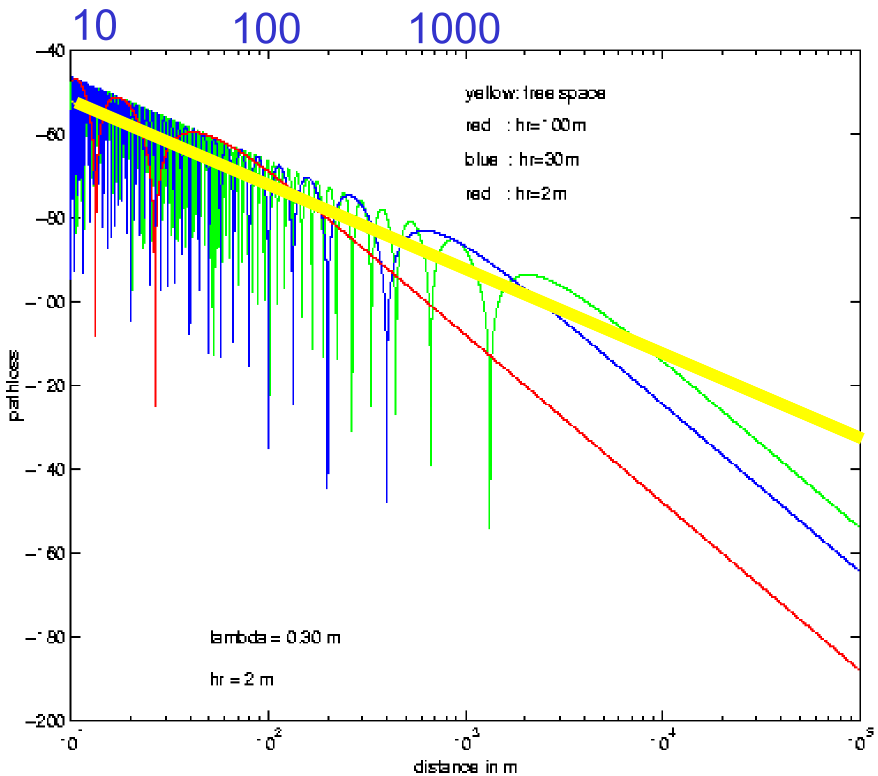
Macro-cellular groundwave propagation:
For small D ($d\lambda >> 4\, h_r\, h_t$), we approximate sin($x$) » x:

$$P_R = \frac{h_r^2 h_t^2}{d^4} P_T G_R G_T$$

Thus, an important turnover point occurs for

$$\frac{2\pi h_r h_t}{\lambda d} = \frac{\pi}{2}$$

Two-Ray Model

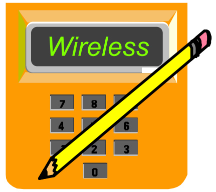


Free space
h_t = 100 meter
h_t = 30 meter
h_t = 2 meter

Observations:

- 40 log *d* beyond a turnover point
- Attenuation depends on antenna height
- Turnover point depends on antenna height
- Wave interference pattern at short range

Egli's semi-empirical model

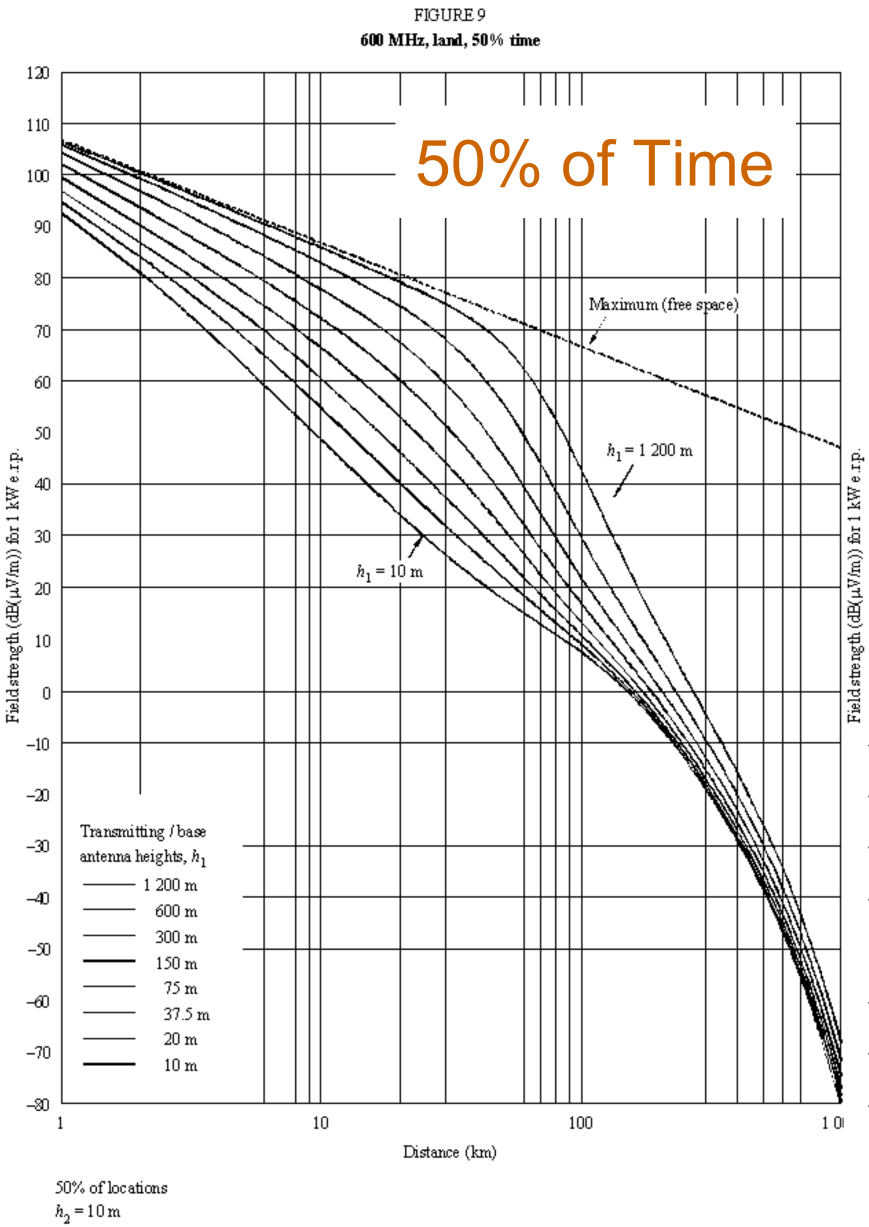


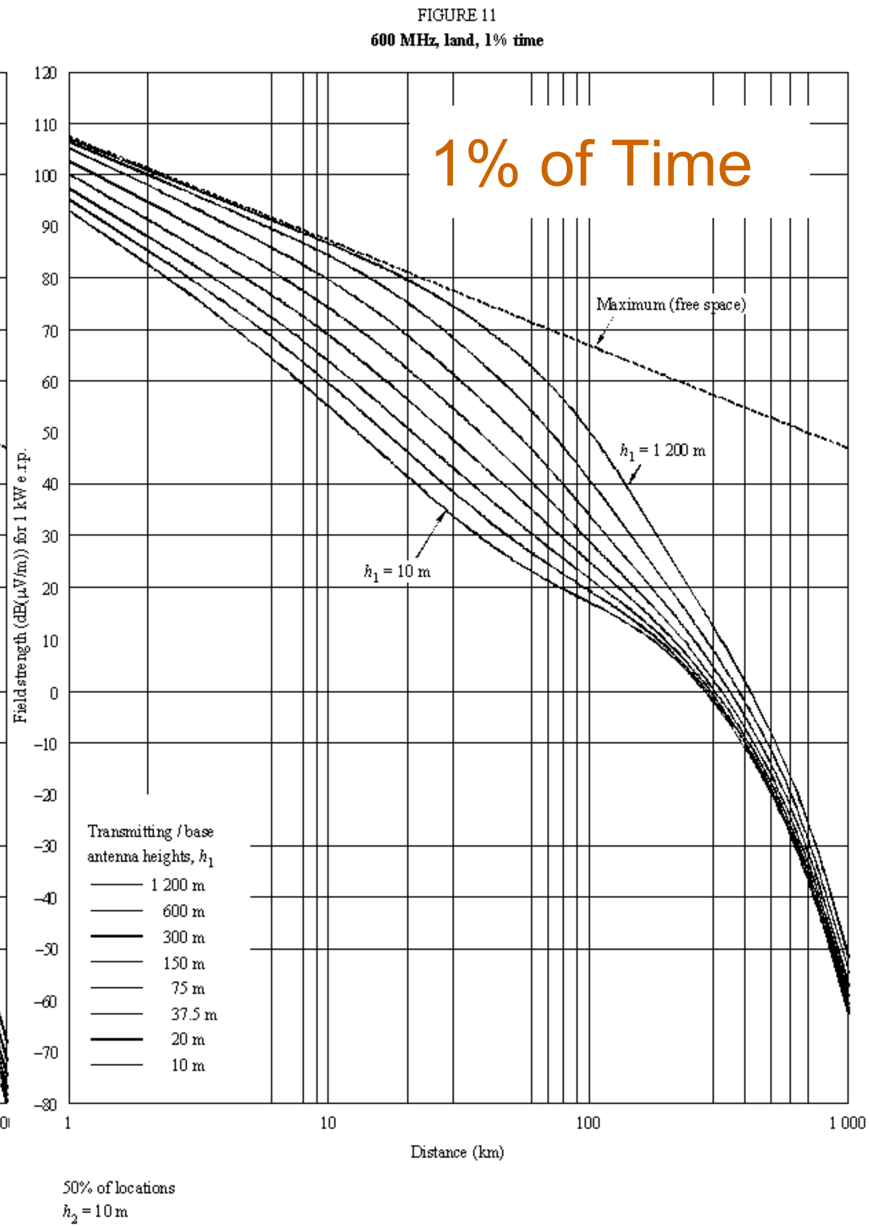
calculate

$$L = 40\log d + 20\log\left(\frac{f_c}{40\text{MHz}}\right) - 20\log h_r h_t \; .$$

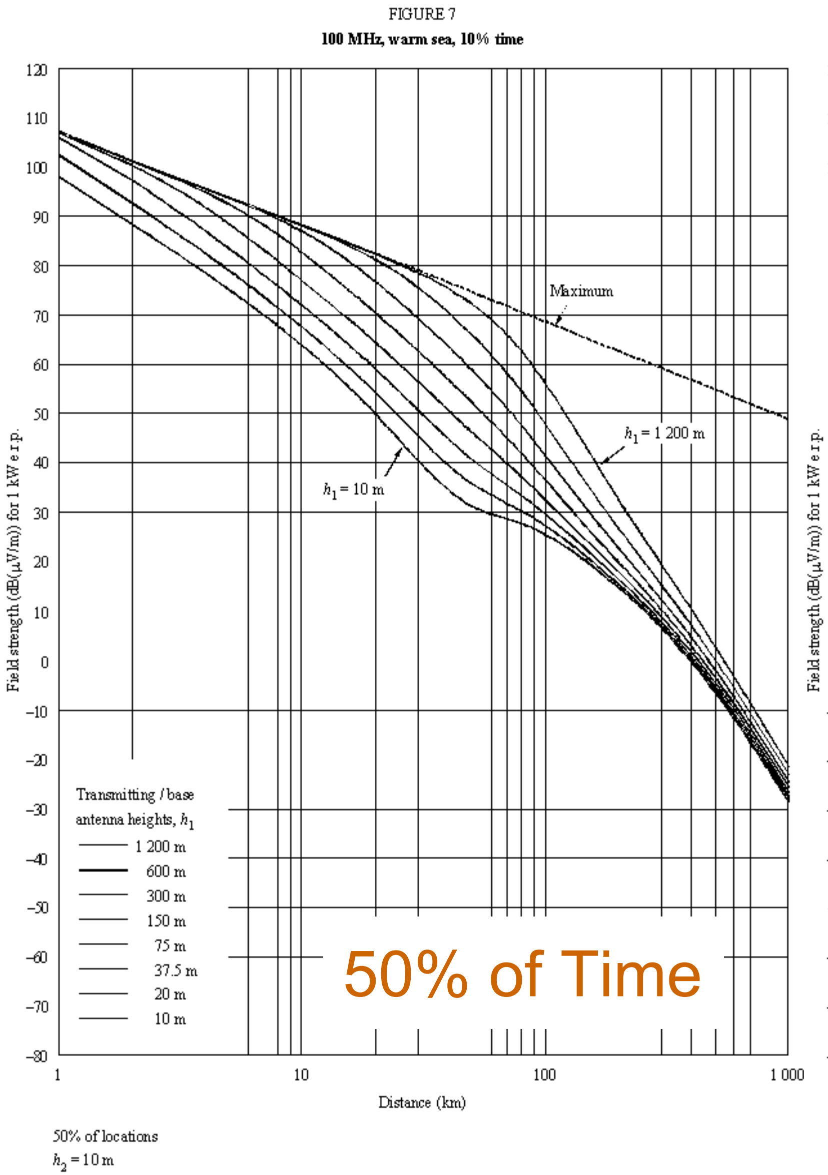
- Loss per distance:................	40 log *d*
- Antenna height gain:.............	6 dB per octave
- Empirical factor:...................	20 log *f*
- Error: standard deviation......	12 dB

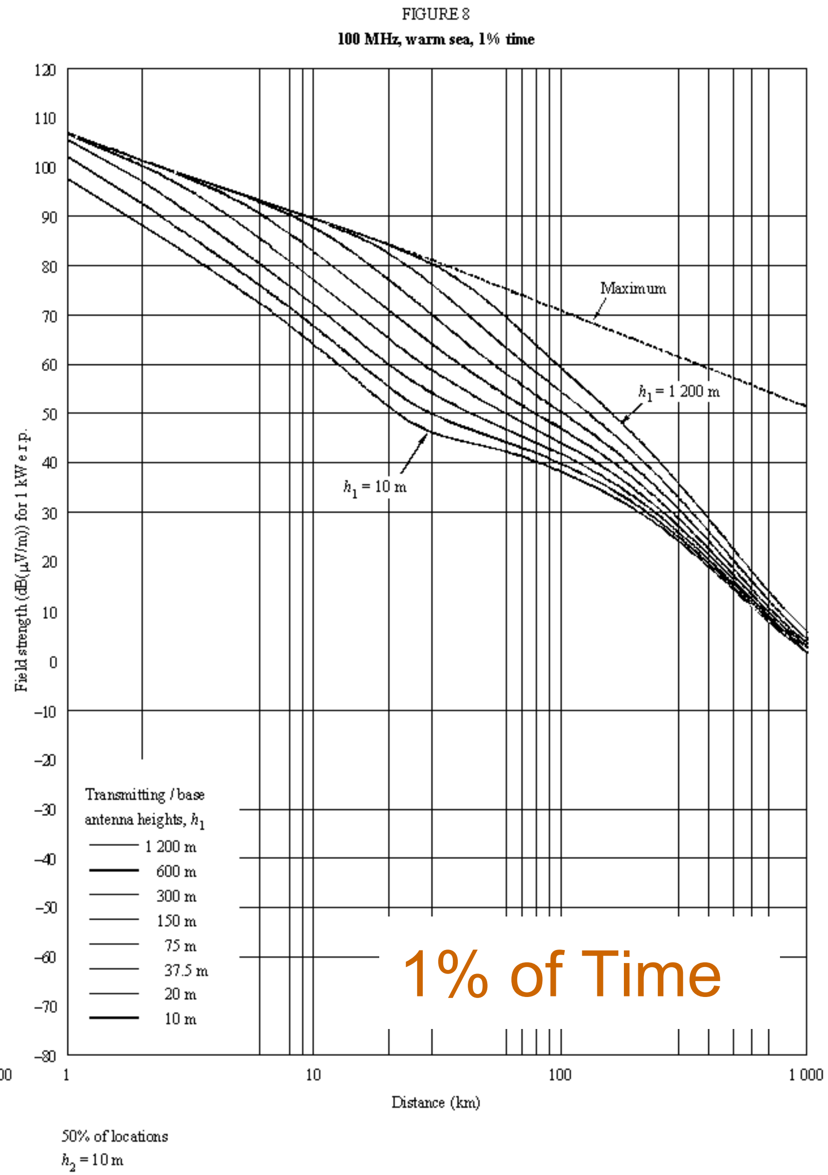
ITU-R Propagation: Land, 600 MHz



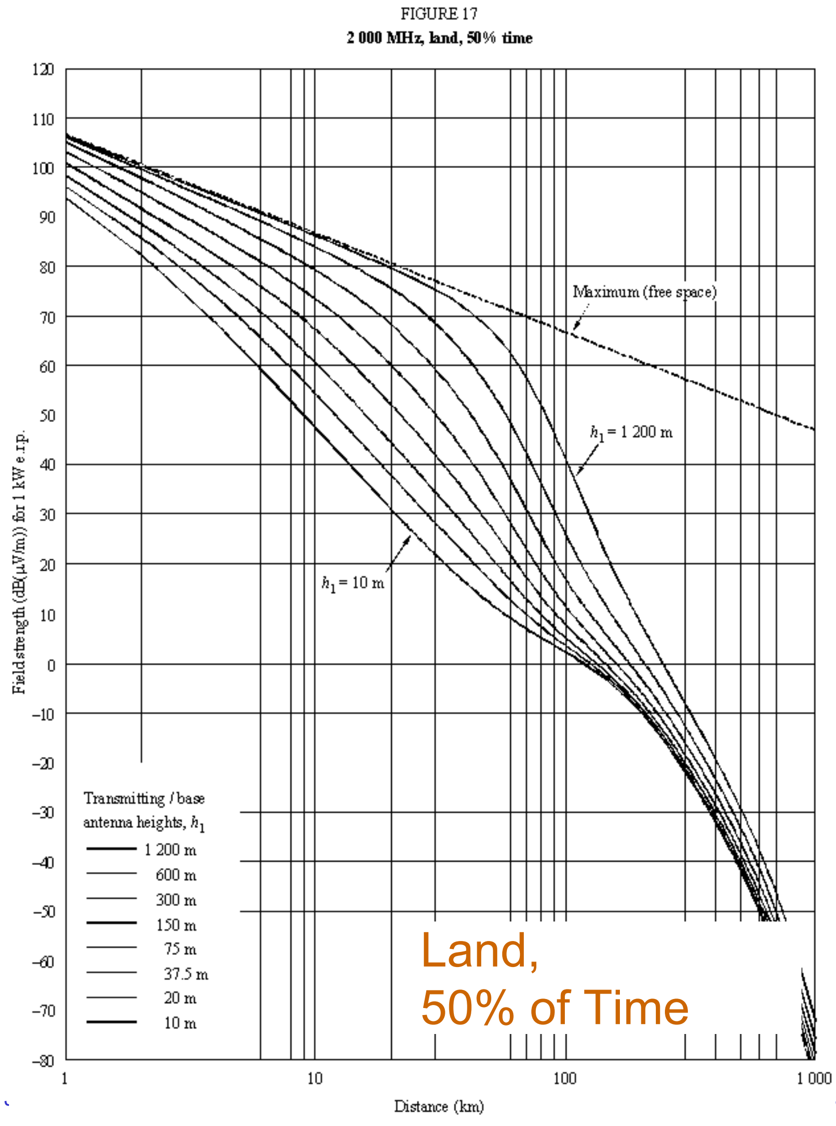


ITU-R Propagation: Warm sea at 100 MHz

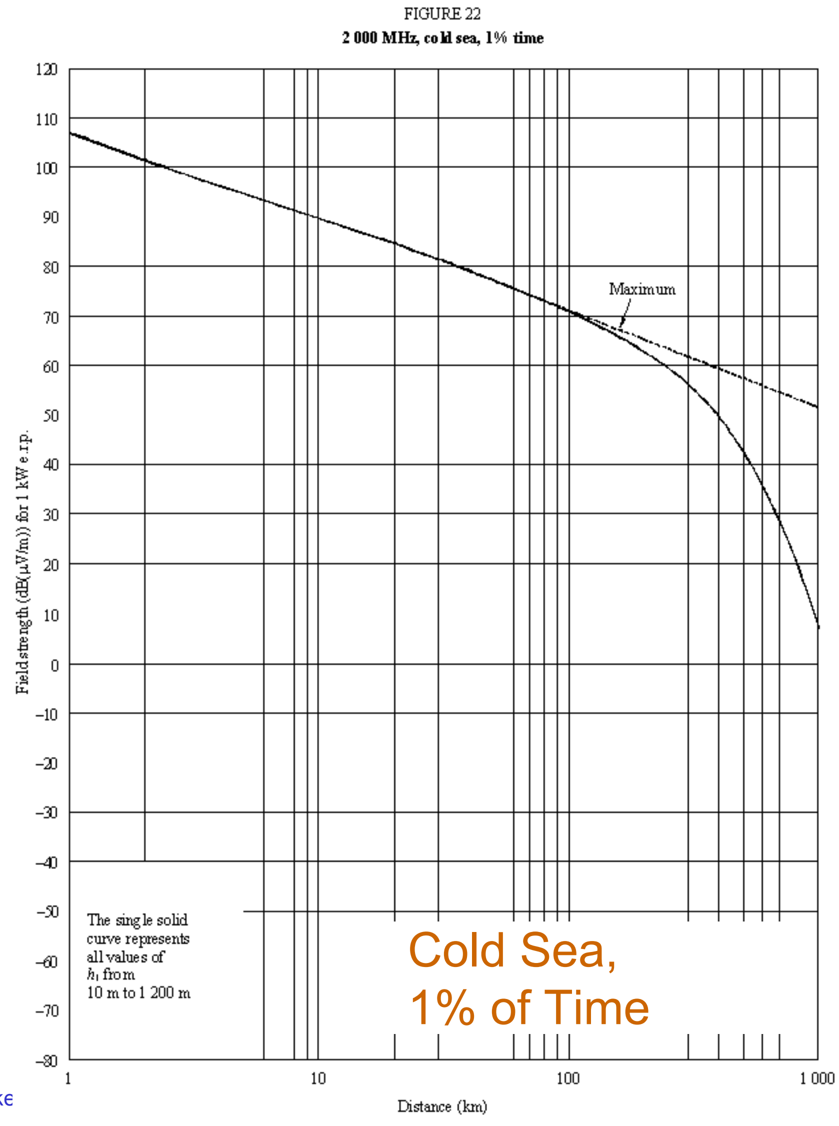




ITU-R Propagation: 2 GHz



50% of locations
h_2 = 10 m



50% of locations
h_2 = 10 m

Overview of Models

	Effect of ant height	Effect of frequency	Effect of distance
Free space	none	20 log *f*	20 log *d*
Theoretical plane earth	6 dB/oct	none	40 log *d*
Egli plane earth	6 db/oct	20 log *f*	40 log *d*
Measured urban	6 dB/oct	20 log *f*	32 log *d*

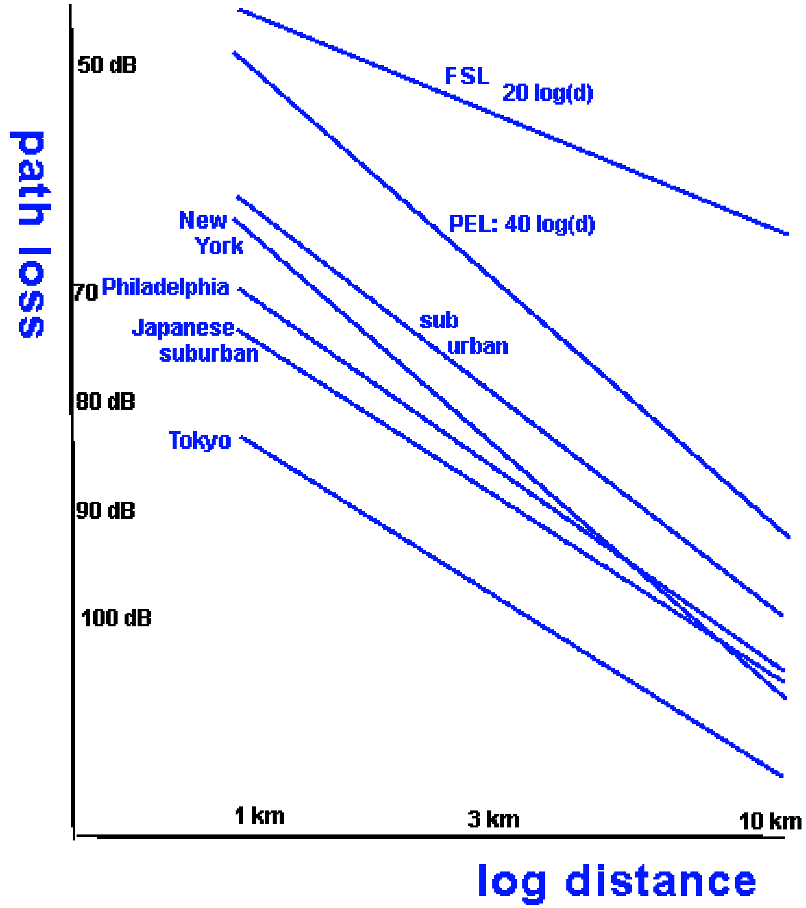
Empirically

$p = r^{-\beta}$, $\beta \approx 2 \ldots 5$ typically $\beta \approx 3.2$

Terrain features hinder ground reflection

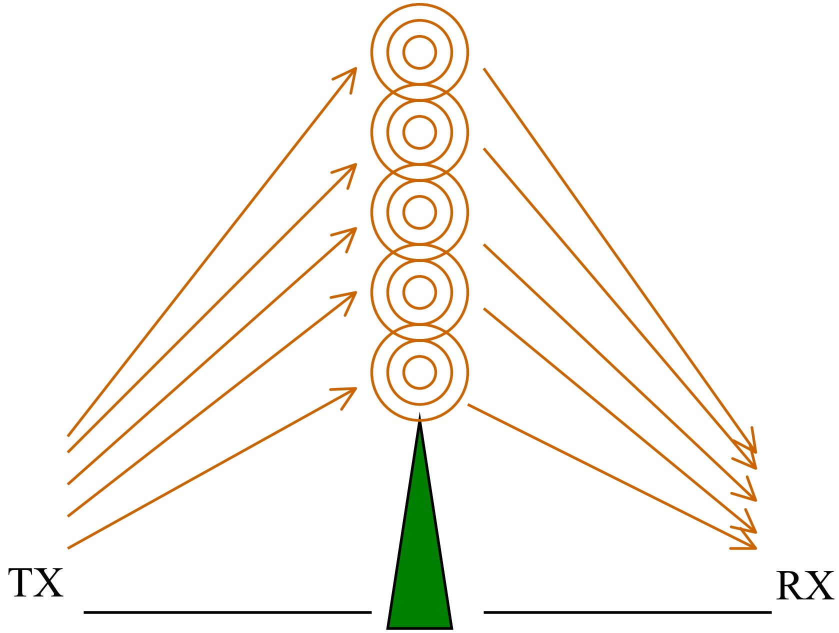
Cancellation effect is less predominant: $\beta < 4$

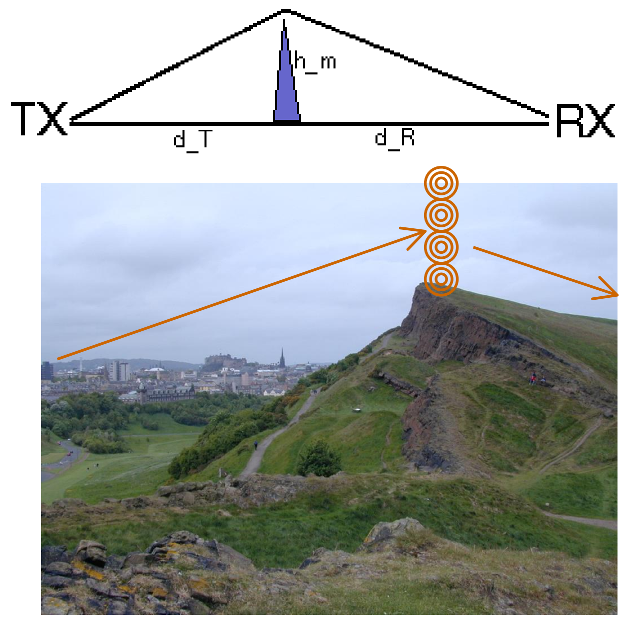
Path Loss versus Distance



Calculate

Diffraction loss: Huygens principle



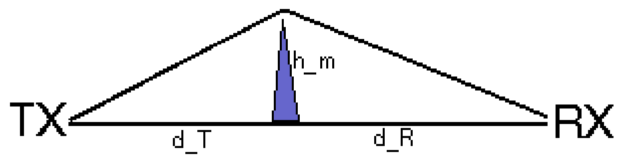


h_m is the height of the obstacle, and

d_t is distance transmitter - obstacle

d_r is distance receiver - obstacle

Diffraction loss



The diffraction parameter *v* is defined as

$$v = h_m \sqrt{\frac{2}{\lambda}\left(\frac{1}{d_t} + \frac{1}{d_r}\right)},$$

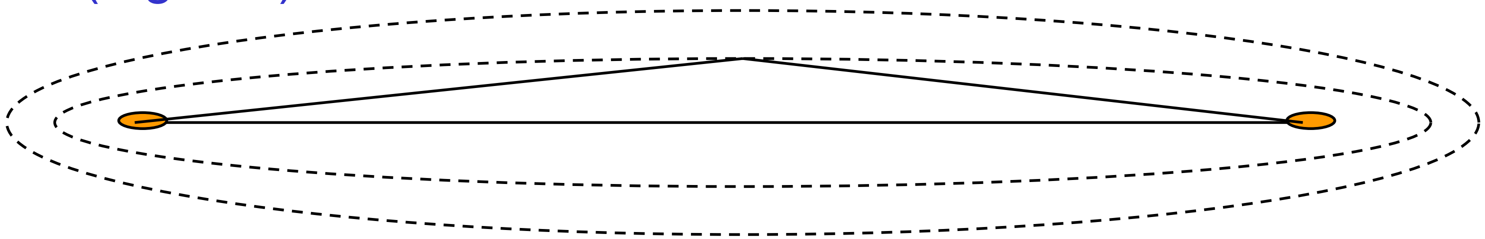
where

h_m is the height of the obstacle, and

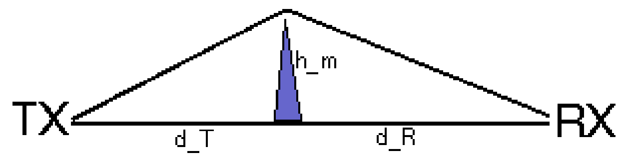
d_t is distance transmitter - obstacle

d_r is distance receiver - obstacle

Fresnel zone: ellipsoid at which the excess path length is constant (e.g. $\lambda/2$)



Diffraction loss



The diffraction parameter *v*

$$v = h_m \sqrt{\frac{2}{\lambda}\left(\frac{1}{d_t} + \frac{1}{d_r}\right)},$$

The diffraction loss L_d, expressed in dB, is approximated by

$$L_d = \begin{cases} 6 + 9v - 1.27v^2 & 0 < v < 2.4 \\ 13 + 20\log v & v > 2.4 \end{cases}$$

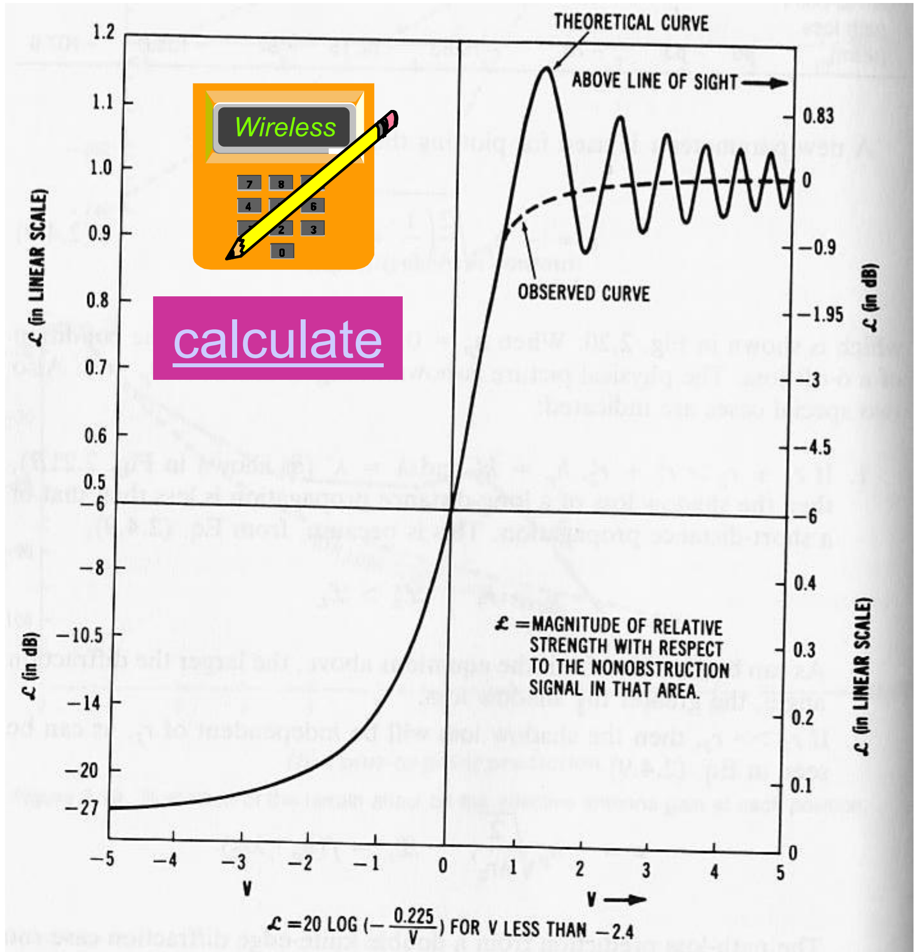
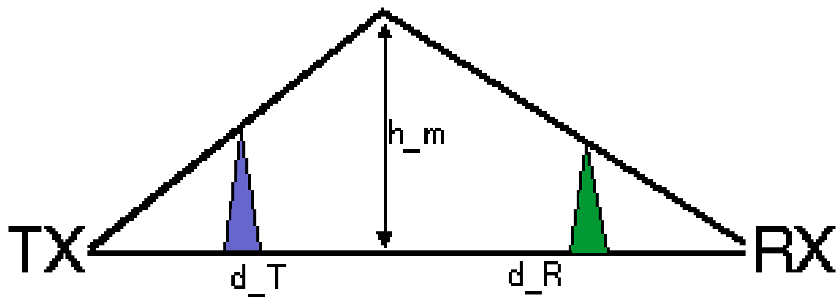


Figure 2.20. Shadow-loss prediction.

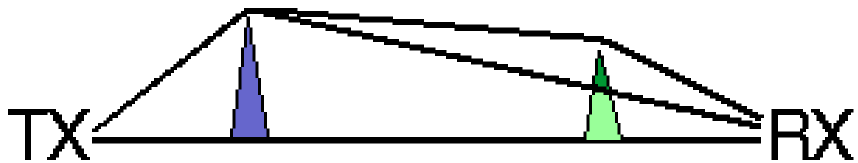
Multiple knife edges

How to model
multiple hills?

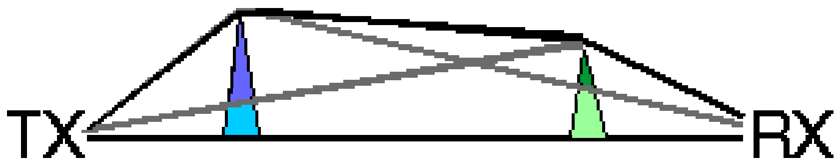
Bullington



Deygout



Epstein



Typical terrain

Propagation models consider a full terrain profile

- multiple knife edges or rounded edges
- groundreflections

Micro-cellular models

Statistical Model

- At short range, R_c may not be close to -1. Therefor, nulls are less prominent than predicted by the simplified two-ray formula.
- UHF propagation for low antenna's (h_t = 5 .. 10 m)

$$p = r^{-\beta_1} \left(1 + \frac{r}{r_g} \right)^{-\beta_2}$$

Deterministic Models:

- Ray-tracing (ground and building reflection, diffraction, scattering)

Indoor Models

calculate

- Difficult to predict exactly
- Ray-tracing model prevail
- Some statistical Models, e.g.

  COST 231: 800 MHz and 1.9 GHz

Environment	Exponent *n*	Propagation Mechanism
Corridors	1.4 - 1.9	Wave guidance
Large open rooms	2	Free space loss
Furnished rooms	3	FSL + multipath
Densely furnished rooms	4	Non-LOS, diffraction, scattering
Between different floors	5	Losses during floor / wall traverses

Attenuation by Constructions

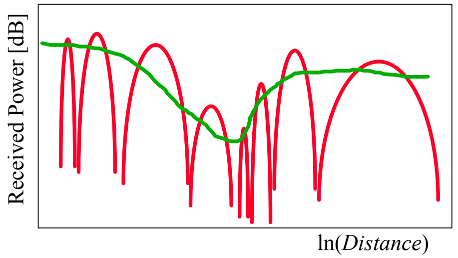
900 MHz

– 20 cm concrete	7 dB (σ = 1 dB)
– wood and brick siding	3 dB (σ = 0.5 dB)
– Aluminum siding	2 dB (σ = 0.5 dB)
– metal walls	12 dB (σ = 4 dB)
– office furnishing	1 dB (σ = 0.3 dB)

2.4 GHz

– Plasterboard wall	3 dB
– Glass wall with metal frame	6 dB
– Cinder block wall	4 dB
– Office window	3 dB
– Metal door	6 dB
– Metal door in brick wall	12 dB

Statistical Fluctuations

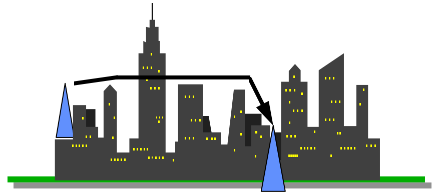


- Area-mean power
  - is determined by path loss
  - is an average over 100 m - 5 km
- Local-mean power
  - is caused by local 'shadowing' effects
  - has slow variations
  - is an average over 40 λ (few meters)
- Instantaneous power
  - fluctuations are caused by multipath reception
  - depends on location and frequency
  - depends on time if antenna is in motion
  - has fast variations (fades occur about every half a wave length)

Relevant to operator

Relevant to manufacturer

Shadowing



Local obstacles cause random shadow attenuation

Model: Normal distribution of the received power P_{Log} in logarithmic units (such as dB or neper),

Probability Density:

$$f_{\bar{p}}(\bar{p}) = \frac{1}{\sqrt{2\pi}\,\sigma\,\bar{p}} \exp\left\{-\frac{1}{2\sigma^2}\ln^2\left(\frac{\bar{p}}{\bar{\bar{p}}}\right)\right\},$$

where

σ is 'log. standard deviation' in neper (σ_{dB}= 4.34 σ).

P_{Log} = ln [local-mean power / area-mean power

Shadowing: σ = 3 .. 12 dB

"Large-area Shadowing":

- Egli: Average terrain: 8.3 dB for VHF and 12 dB (UHF)
- Semi-circular routes in Chicago: 6.5 dB to 10.5 dB

"Small-area shadowing": 4 .. 7 dB

Combined model by Mawira (KPN Research, NL):

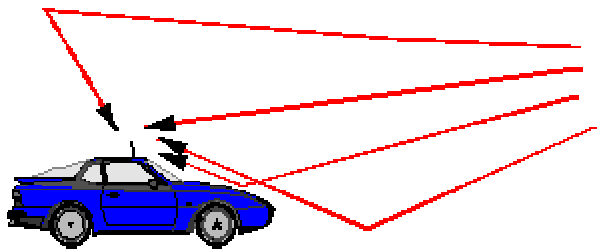
- Two superimposed Markovian processes:
  - 3 dB with coherence distance over 100 m, plus
  - 4 dB with coherence distance 1200 m

How do systems handle shadowing?

- GSM
  - Frequency planning and base station locations
  - Power control
- DECT
  - Select good base station locations
- IS95
  - Power control
  - Select good base station locations
- Digital Audio Broadcasting
  - Single frequency networks

Multipath fading

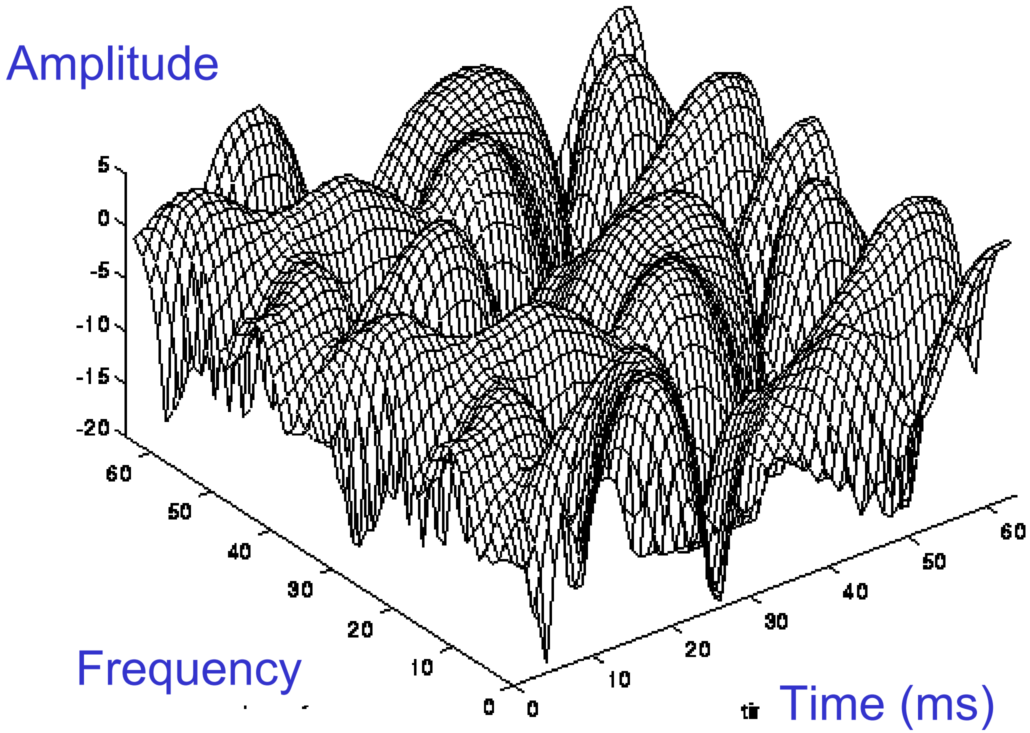
Multiple reflected waves arrive at the receiver



Narrowband model

- Different waves have different phases.
- These waves may cancel or amplify each other.
- This results in a fluctuating ("fading") amplitude of the total received signal.

Rayleigh Multipath Reception



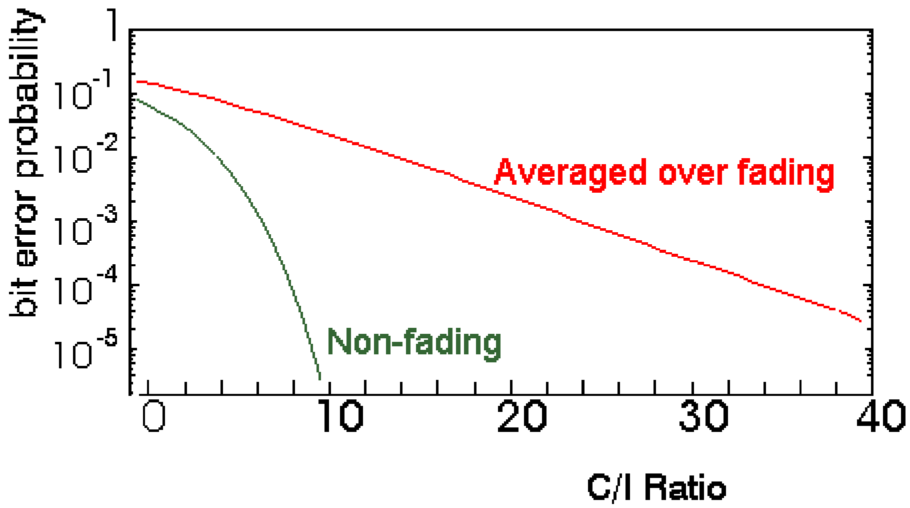
The received signal amplitude depends on location and frequency

If the antenna is moving, the location x changes linearly with time t ($x = v\,t$)

Parameters:

- probability of fades
- duration of fades
- bandwidth of fades

Effect of Flat Fading



- In a fading channel, the BER only improves very slowly with increasing C/I
- Fading causes burst errors
- Average BER does not tell the full story
- Countermeasures to improve the slope of the curve

Preliminary math: I-Q phasor diagram

Any bandpass signal $s(t)$ can be composed into an <u>inphase</u> I and a <u>quadrature</u> Q component, $s_I(t)$ and $s_Q(t)$, respectively.

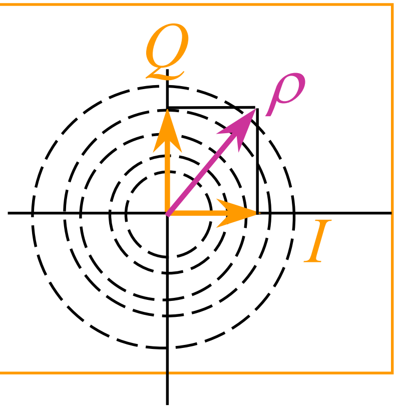
$$s(t) = s_I(t)\cos(\omega_c t) - s_Q(t)\sin(\omega_c t)$$

$s_I(t)$ and $s_Q(t)$ are lowpass baseband signals

Example:

$$s(t) = \rho\cos(\omega_c t + \phi)$$
$$= \rho\cos(\phi)\cos(\omega_c t) - \rho\sin(\phi)\sin(\omega_c t)$$

Then $s_I(t) = \rho\cos(\phi)$ and $s_Q(t) = \rho\sin(\phi)$



Preliminary math:

Examples for analog tone modulation (AM)

$$AM: s(t) = A_c\,(1 + c\,m(t))\cos(\omega_c t)$$

where c is the modulation index ($0 < c < 1$)

For full (c=1) tone modulation $m(t) = \cos(\omega_m t)$,

we get

$$s(t) = A_c\,(1 + \cos(\omega_m t))\cos(\omega_c t)$$

So

$$s_I(t) = A_c + A_c \cos(\omega_m t) \text{ and } s_q(t) = 0.$$

Preliminary math:

Examples for analog tone modulation of AM, PM, FM

AM: $s(t) = A_c\,(1 + c\,m(t))\cos(\omega_c t)$

Let's now see whether we can also study each individual spectral component in the I and Q diagram. The spectrum is:

$$s(t) = A_c \cos(\omega_c t) + A_c/2 \cos((\omega_c - \omega_m)t) + A_c/2 \cos((\omega_c + \omega_m)t)$$

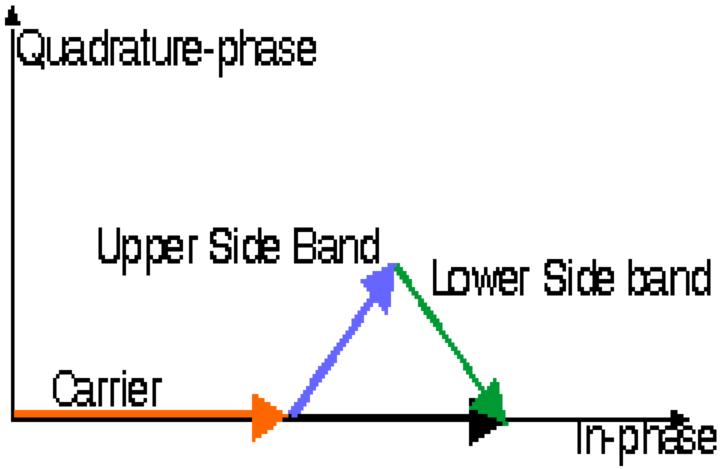
Each can be decomposed into *I* and *Q* component, using

$$\cos((\omega_c + \omega_m)t) = \cos(\omega_m t)\cos(\omega_c t) - \sin(\omega_m t)\sin(\omega_c t)$$

So

$$s_I(t) = A_c + A_c/2 \cos(\omega_m t) + A_c/2 \cos(\omega_m t)$$

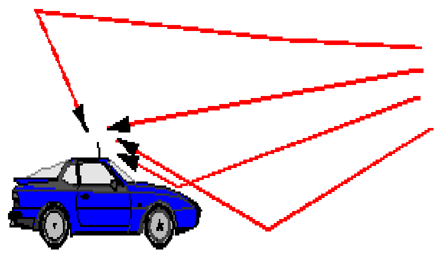
$$s_q(t) = \qquad A_c/2 \sin(\omega_m t) - A_c/2 \sin(\omega_m t)$$



Models for Multipath Fading

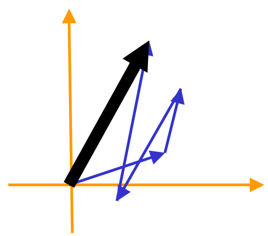
Rayleigh fading

- (infinitely) large collection of reflected waves
- Appropriate for macrocells in urban environment
- Simple model leads to powerful mathematical framework



Transmit $s(t) = \cos(\omega_c t),$

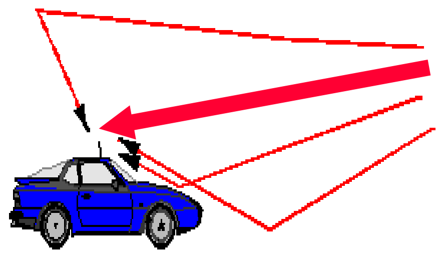
Receive $v(t) = \sum_{n=1}^{N} \rho_n s(t - T_n),$



N=4 waves with amplitude ρ_n and phase shift due to delay T_n

Ricean fading

- (infinitely) large collection of reflected waves plus line-of sight
- Appropriate for micro-cells
- Mathematically more complicated



Rayleigh Model

Use Central Limit Theorem

inphase $s_I(t) = \zeta$ and quadrature $s_Q(t) = \xi$ components are zero-mean independently identically distributed (i.i.d.) jointly Gaussian random variables

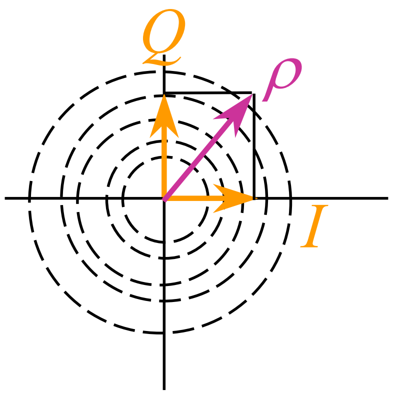
PDF:

$$f(\xi, \zeta) = \frac{1}{2\pi\sigma^2} \exp\left\{ -\frac{\xi^2 + \zeta^2}{2\sigma^2} \right\}$$

Conversion to polar co-ordinates:

Received amplitude ρ: $\rho^2 = \zeta^2 + \xi^2$.

$\zeta = \rho \cos \phi$; $\xi = \rho \sin \phi$,



PDF of Rayleigh Amplitude

After conversion to polar co-ordinates: !!

$$f_{\mathrm{P},\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2}\exp\left\{-\frac{\rho^2}{2\sigma^2}\right\}$$

Integrate this PDF over ϕ from 0 to 2π:

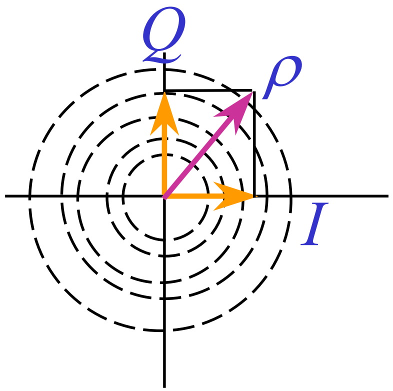
Rayleigh PDF of ρ

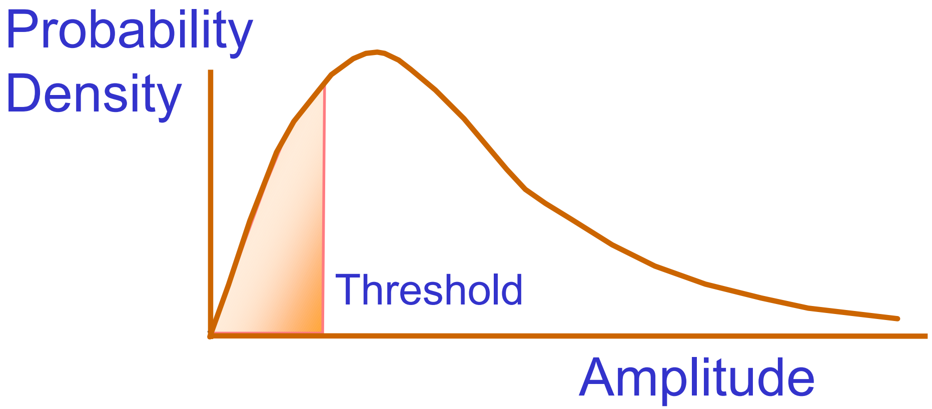
$$f_{\rho}(\rho) = \frac{\rho}{\overline{p}}\exp\left(-\frac{\rho^2}{2\overline{p}}\right)$$

where

– p is the local mean power total scattered power ($p = \sigma^2$).

Received Amplitudes





$$f_{\mathrm{P}}(\rho) = \frac{\rho}{\bar{p}} \exp\left\{ -\frac{\rho^2}{2\bar{p}} \right\}.$$

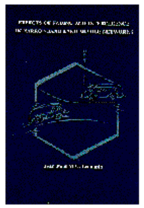
Received Power

Conversion from amplitude to power ($p = \rho^2/2$) gives the exponential distribution:

$$f_p(p) = f_\rho(\rho)\left|\frac{d\rho}{d\,\bar{p}}\right| = \frac{1}{\bar{p}}\exp\left\{-\frac{p}{\bar{p}}\right\} .$$

Exponential distributions are very convenient to handle mathematically.

Example: If one computes the average channel behaviour, one integrates of the exponential distribution, thus basically does a Laplace transform.



Who was Rayleigh?

The basic model of Rayleigh fading assumes a received multipath signal to consist of a (theoretically infinitely) large number of reflected waves with independent and identically distributed inphase and quadrature amplitudes.

This model has played a major role in our understanding of mobile propagation.

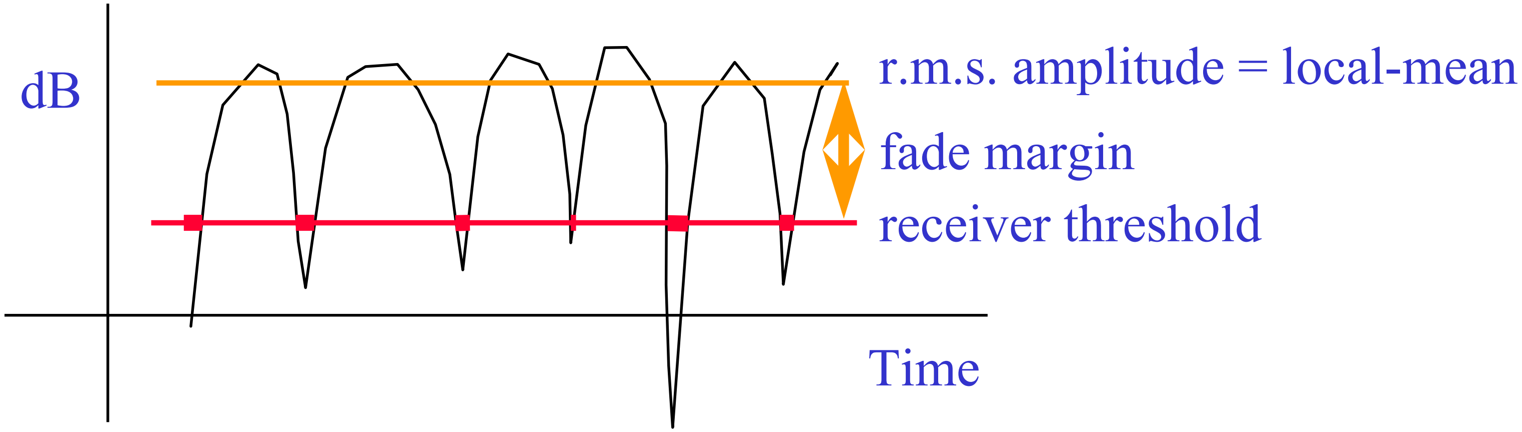
The model was first proposed in a comment paper written by Lord Rayleigh in 1889, describing the resulting signal if many violinists in an orchestra play in unison, long before its application to mobile radio reception was recognized.

[1] Lord Rayleigh, "On the resultant of a large number of vibrations of the same pitch and of arbitrary phase", Phil. Mag., Vol. 10, August 1880, pp. 73-78 and Vol. 27, June 1889, pp. 460-469.

Lord Ravleigh (John William Strutt) was an English physicist (1877 - 1919) and a Nobel Laureate (1904) who made a number of contributions to wave physics of sound and optics.

Fade Margin

Fade margin is the ratio of the average received power over some threshold power, needed for reliable communication.



PDF of signal amplitude

Fade margin

Outage probability

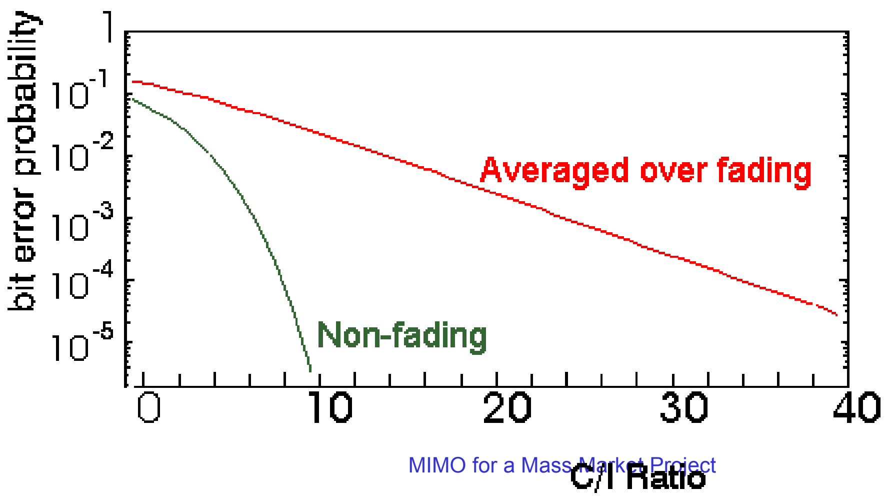
Average BER

The BER for BPSK with known instantaneous power *p*

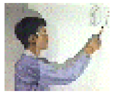
$$P = \frac{1}{2} erfc \sqrt{\frac{E_b}{N_0}}$$

The BER averaged over an exponential distribution

$$\overline{P} = \int_0^\infty \frac{1}{\overline{p}} \exp\left\{-\frac{p}{\overline{p}}\right\} \frac{1}{2} erfc \sqrt{\frac{pT_b}{N_0}} dp = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p}T_b}{N_0 + \overline{p}T_b}}$$



calculate



Exercise: Outage Probability

- Find the probability that the instantaneous power of a Rayleigh-fading signal is x dB or more below its local-mean value.

- If the receiver can choose the strongest signal from *L* antennas, each receiving an independent signal power, what is the probability that the signal is x dB or more below the threshold

Solution

Define fade margin η as $\eta = p_{\text{local-mean}}/p_{\text{threshold}}$

Define the fade margin x in dB, where $\eta = 10^{x/10}$

$$\eta = \frac{\bar{p}}{p_T}.$$

The signal outage probability is

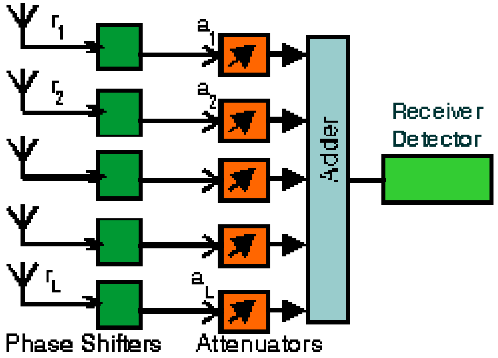
$$\Pr(p < p_T) = \int_0^{p_T} \frac{1}{\bar{p}} \exp\left\{-\frac{p}{\bar{p}}\right\} dp = 1 - \exp\left\{-\frac{p_T}{\bar{p}}\right\}.$$

$$\Pr(p < p_T) = 1 - \exp\left\{-\frac{1}{\eta}\right\}. \xrightarrow{\text{large } \eta} \frac{1}{\eta}$$

Solution, Part II: Diversity

Diversity rule:

Select strongest signal.



Outage probability for selection diversity:

$$\Pr(\max(p) < p_{\mathrm{thr}}) = \Pr(\mathrm{all}(p) < p_{\mathrm{thr}}) = \Pi_i \Pr(p_i < p_{\mathrm{thr}})$$

For L-branch selection diversity in Rayleigh fading:

$$\Pr\left(\max(p) < \overline{p}/\eta\right) = \left[1 - \exp\left\{-1/\eta\right\}\right]^L \rightarrow \frac{1}{\eta^L}$$

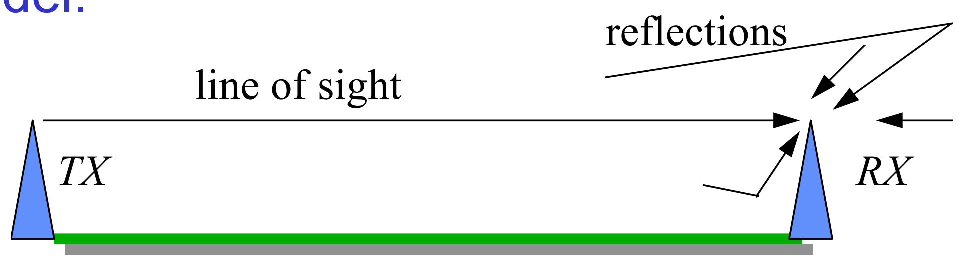
Outage Probability Versus Fade Margin

- Performance improves very slowly with increased transmit power
- Diversity Improves performance by orders of magnitude
- Slope of the curve is proportional to order of diversity
- Only if fading is independent for all antennas

Better signal combining methods exist:
Equal gain, Maximum ratio, Interference Rejection Combining

Ricean Multipath Reception

Narrowband propagation model:



Transmitted carrier $s(t) = \cos(\omega_t\, t)$

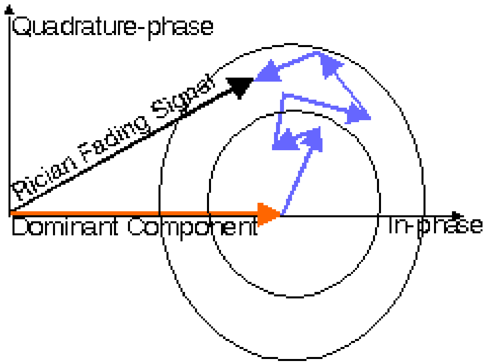
Received carrier

$$v(t) = C \cos \omega_c t + \sum_{n=1}^{N} \rho_n \cos(\omega_c t + \phi_n),$$

where

- C is the amplitude of the line-of-sight component
- ρ_n is the amplitude of the n-th reflected wave
- ϕ_n is the phase of the n-th reflected wave

Ricean Multipath Reception



Received carrier:

$$v(t) = C \cos \omega_c t + \sum_{n=1}^{N} \rho_n \cos(\omega_c t + \phi_n) ,$$

where

- ζ is the in-phase component of the reflections
- ξ is the quadrature component of the reflections.
- I is the total in-phase component ($I = C + \zeta$)
- Q is the total quadrature component ($I = C + \zeta$)

calculate

Ricean Amplitude

After conversion to polar co-ordinates:

$$f_{\mathrm{P},\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2}\exp\left\{-\frac{\rho^2 + C^2 - 2\rho C\cos\phi}{2\sigma^2}\right\}$$

Integrate this PDF over ϕ from 0 to 2π: Ricean PDF of ρ

$$f_{\rho}(\rho) = \frac{\rho}{\bar{q}}\exp\left(-\frac{\rho^2 + C^2}{2\bar{q}}\right) I_0\left(\frac{\rho C}{\bar{q}}\right),$$

where

- $I_0(.)$ is the modified Bessel function of the first kind and zero order
- q is the total scattered power ($q = \sigma^2$).

Ricean Phase

After conversion to polar co-ordinates:

$$f_{P,\Phi}(\rho,\phi) = \frac{\rho}{2\pi\sigma^2} \exp\left\{-\frac{\rho^2 + C^2 - 2\rho C \cos\phi}{2\sigma^2}\right\}$$

Integrate this PDF over ρ

Special case: $C = 0$ .... $f_{\Phi}(\phi) = \frac{1}{2\pi}$

Special case: large C .... $f_{\phi}(\phi) = \frac{C}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{C^2\phi^2}{2\sigma^2}\right\}$

$\phi \approx \arctan(\zeta/C) \approx \zeta/C$

Ricean K-factor

calculate

Definition: K = direct power $C^2/2$ over scattered power q

Measured values

K = 4 ... 1000 (6 to 30 dB) for micro-cellular systems

Light fading (K -> infinity)

- Very strong dominant component
- Ricean PDF approaches Gaussian PDF with small σ

Severe Fading (K = 0):

- Rayleigh Fading

How do systems handle outages?

- Analog
  - Fast moving User experiences a click
  - Slow moving user experiences a burst of noise
- GSM
  - Speech extrapolation
- DECT
  - Handover to other base station if possible
  - Handover to different frequency
- WLAN / cellular CDMA
  - Large transmit bandwidth to prevent that the full signal vanishes in a fade

Other fading models

- Rayleigh
- Ricean
- Nakagami
- Weibull

Nakagami Math

The distribution of the amplitude and signal power can be used to find probabilities on signal outages.

- If the envelope is Nakagami distributed, the corresponding instantaneous power is gamma distributed.
- The parameter m is called the 'shape factor' of the Nakagami or the gamma distribution.
- In the special case $m = 1$, Rayleigh fading is recovered, with an exponentially distributed instantaneous power
- For $m > 1$, the fluctuations of the signal strength reduce compared to Rayleigh fading.

Nakagami

The Nakagami fading model was initially proposed because it matched empirical results for short wave ionospheric propagation.

$$f_{p_i}(p_i) = \frac{1}{\Gamma(m)} \left(\frac{m}{\overline{p}_i} \right)^m p_i^{m-1} \exp\left\{ - \frac{m p_i}{\overline{p}_i} \right\}.$$

where $\Gamma(m)$ is the gamma function, with $\Gamma(m + 1) = m!$ for integer shape factors m.

In the special case that $m = 1$, Rayleigh fading is recovered, while for larger m the spread of the signal strength is less, and the pdf converges to a delta function for increasing m.

When does Nakagami Fading occur?

- **Amplitude after maximum ratio diversity combining.**
  After *k*-branch MRC of Rayleigh-fading signals, the resulting signal is Nakagami with $m = k$.
  MRC combining of *m*-Nakagami fading signals in *k* branches gives a Nakagami signal with shape factor *mk*.
- **The power sum** of multiple independent and identically distributed (i.i.d.) **Rayleigh-fading signals** have a Nakagami distributed signal amplitude. This is particularly relevant to model interference from multiple sources in a cellular system.
- The Nakagami distribution matches some empirical data better than other models
- Nakagami fading occurs for multipath scattering with relatively large delay-spreads with different clusters of reflected waves. Within any one cluster, the phases of individual reflected waves are random, but the delay times are approximately equal for all waves. As a result the envelope of each cumulated cluster signal is Rayleigh distributed. The average time delay is assumed to differ significantly between clusters. **If the delay times also significantly exceed the bit time** of a digital link, the different clusters produce serious intersymbol interference. The multipath self-interference then approximates the case of co-channel interference by multiple incoherent Rayleigh-fading signals.

Approximations

The models by Rice and Nakagami behave approximately equivalently near their mean value.

This observation has been used in many recent papers to advocate the Nakagami model as an approximation for situations where a Rician model would be more appropriate.

While this may be accurate for the main body of the probability density, it becomes highly inaccurate for the tails.

Bit errors or outages mainly occur during deep fades

Performance is mainly determined by the tail of the probability density function (for probability to receive a low power).

Approximations

The Nakagami model is sometimes used to approximate the pdf of the power of a Rician fading signal.

Matching the first and second moments of the Rician and Nakagami pdfs gives

$$m = \frac{K^2 + 2K + 1}{2K + 1}$$

which tends to $m = K/2$ for large K.

For Ricean fading, the probability distribution at small powers is

$$F_{p_i}(p_{th}) \rightarrow (1 + K)e^{-K}\frac{p_{th}}{\bar{p}_i} + O(p_{th}^2) = O(p_{th}).$$

For Nakagami fading,

$$F_{p_i}(p_{th}) \rightarrow O(p_{th}{}^{m}) .$$

Summary

- Three mechanisms: Path loss, shadowing, multipath
- Rapid increase of attenuation with distance helps cellular system operators
- Multipath fading: Rayleigh and Ricean models
- Fading has to be handled within user terminal

Exercises: See “Wireless Communication CD-ROM”

- Plane Earth Loss
- Quiz questions