Statistical path loss law

Most generic system studies address networks in which all terminals have the same antenna gain and height and the same transmit power. For ease of notation, received signal powers and propagation distances can be normalised. In macro-cellular networks (1 km < d < 50 km), the area-mean received power can be written as

$$\bar{\bar{p}} = r^{-b}$$

with r the normalised distance and β the path loss exponent.

Theoretical values are β = 2 and β = 4 for free space and plane, smooth, perfectly conducting terrain, respectively. Typical values for irregular terrain are β = 3 to 3.4. For forestal terrain, propagation can be appropriately described as in free space plus some diffraction losses, but without significant groundwave losses (β ≈ 2.6). If the propagation model has to cover a wide range of distances, β may vary as different propagation mechanisms dominate at different ranges. In microcellular nets, β typically changes from β ≈ 2 to β ≈ 4 at some turnover distance d_g. Experimental values are d_g = 90 to 300 m for h_T between 5 and 20 m and $h_R \approx$ 2 m. A reasonable agreement with the theoretical $d_g \approx 4h_T h_R/\lambda$ is experienced. Harley suggested to model it as a smooth transition

$$\bar{\bar{p}} = r^{-b_1}\left(1 + \frac{r}{r_g}\right)^{-b_2}$$

with r_g the normalised turnover distance. This model neglects the wave-interference pattern that may be experienced at ranges shorter than r_g.

The short-range attenuation exponent is on the order of β_1 = 1.3 to 2.5, with values less than 2 (free space) suggesting guided propagation, e.g. in a canyon-like innercity propagation environment. The long-range exponent $\beta = \beta_1 + \beta_2$ varies from 2 to 7.

For indoor communications, β varies from approximately 2 along corridors and hallways to nearly 6 over highly cluttered and obstructed paths. A typical path loss law is

$$\bar{\bar{p}} = r^{-b} f^{k} w^{m}$$

where k and m are the numbers of floors and wall traversed, respectively. The attenuation factor f per floor is $f \approx$ 0.1 (10 dB) at f_c = 900 MHz and $f \approx$ 0.025 (16 dB) at f_c = 1700 MHz with β ≈ 3.5 to 4. The losses experienced when traversing walls are on the order of 3 to 8 dB for $f_c \approx$ 1.2 GHz.

At higher frequencies ($f_c > 2$ GHz), penetration losses increase, but radio signals become more likely to be guided via stair wells, lift shafts, etc.

Shadowing

The statistical path loss law has limited accuracy and can only predict an area-mean received power $\bar{\bar{p}}$. The more accurate local-mean $\bar{p}$ fluctuates about $\bar{\bar{p}}$ according to a log-normal distribution, that is, the received power $\bar{q}$ in exponential units ($\bar{q} = \ln \bar{p}$) has a normal distribution. Since $\frac{dq}{d\bar{p}} = 1/\bar{p}$, the log-normal distribution has the form

$$f_{\bar{p}}(\bar{p}) = \frac{1}{\sqrt{2\pi}\sigma_s \bar{p}} \exp\left\langle -\frac{1}{2\sigma_s^2} ln^2\left(\frac{\bar{p}}{\bar{\bar{p}}}\right)\right\rangle$$

where σ_s is the standard deviation of ln $\bar{p}$ and $\bar{\bar{p}}$ is the expectation of ln $\bar{p}$.

Exercise: Show that the standard deviation s_S of 10 log $\bar{p}/\bar{\bar{p}}$ (expressed in dB) is found from $\sigma_s \approx 0.23026\, s_S$.

Exercise: Show that the linear mean is $E[\bar{p}] = \bar{\bar{p}} \exp(\sigma_s^2/2)$, the linear variance is $\bar{\bar{p}}^2 \exp(\sigma_s^2)[\exp(\sigma_s^2) - 1]$ and that the median of $\bar{p}$ is $\bar{\bar{p}}$.

The standard deviation σ_s depends on the terrain resolution used to estimate the area mean $\bar{\bar{p}}$. For the statistical path loss law, Egli found the (large-area) accuracy of $s_S = 12$ dB ($\sigma_s = 2.72$).
The CCIR gives $s_S = 8.3$ dB for VHF FM radio broadcasting at 100 MHz. UHF measurements on semicircular routes in Chicago gave large-area standard deviation between 6.5 dB and 10.5 dB, with a median at 9.3 dB. The small-area shadowing, studied over intervals of a few hundred meters, mostly in the order of $s_S \approx 4$ dB.

Mawira (at Netherlands PTT Research) modelled small-area and medium area shadowing as two independent Markovian processes with standard deviation of 4 and 3 dB and coherence distances of 100 and 1200 m, respectively.

Exercise: Design a computer program that simulates this shadowing in discrete time (sampling frequency 1 Hz) for a vehicle travelling at 10 m/s (36 km/h). Use two i.i.d. Gaussian random number generators. Discrete Time (DT), linear time invariant (LTI) filter techniques and an exponential law operation. How would you generate two i.i.d. Gaussian r.v. from two i.i.d. uniform r.v.s? (Hint: think of a complex r.v. with Rayleigh amplitude and uniform argument).

To study a radio network with multiple interfering signals, we are interested in the incoherent (power) sum of *n* i.i.d. shadowed signals. The pdf of the joint power of *n* log normal signals can be found from the (*n*-1) fold convolution of the log-normal pdf. Regretably no simple solution exists for the convolution integrals. However in good approximation, the new pdf is again log-normal. Techniques to estimate the new mean and variance have been proposed by Fenton for s_S < 4 dB and Schwartz and Yeh for s_S between 4 and 12 dB. The method by Schwartz and Yeh is a recursive technique: the mean and variance of *n* signals is found from the mean and variance of the joint pdf of *n*-1 signals and adding a single log-normal signal. Results for n = 1, 2, ..., 6 and s_S is 6, 8.3 and 12 dB are in the book “Narrowband landmobile radio networks”.

Exercise: Compare the mean values in Table 2.1 with $\hat{m}_t = 10 \log n$. Explain intuitively why the mean grows faster than proportional to *n*.

Exercise: Read five scientific paper on the link performance of cellular radio networks and try to find how the accumulation of powers of multiple log normal signals is taken into account.