Random Access

- Many terminals communicate to a single base station
- Fixed multiple access methods (TDMA, FDMA, CDMA) become inefficient when the traffic is bursty.
- Random Access works better for
  - many users, where ..
  - each user only occasionally sends a message

Suitable Protocols

- ALOHA
- Carrier Sense
- Inhibit Sense
- Collision Resolution
  - Stack Algorithm
  - Tree Algorithm
- Reservation methods
  - Reservation ALOHA
  - Packet Reservation Multiple Access

ALOHA Protocol

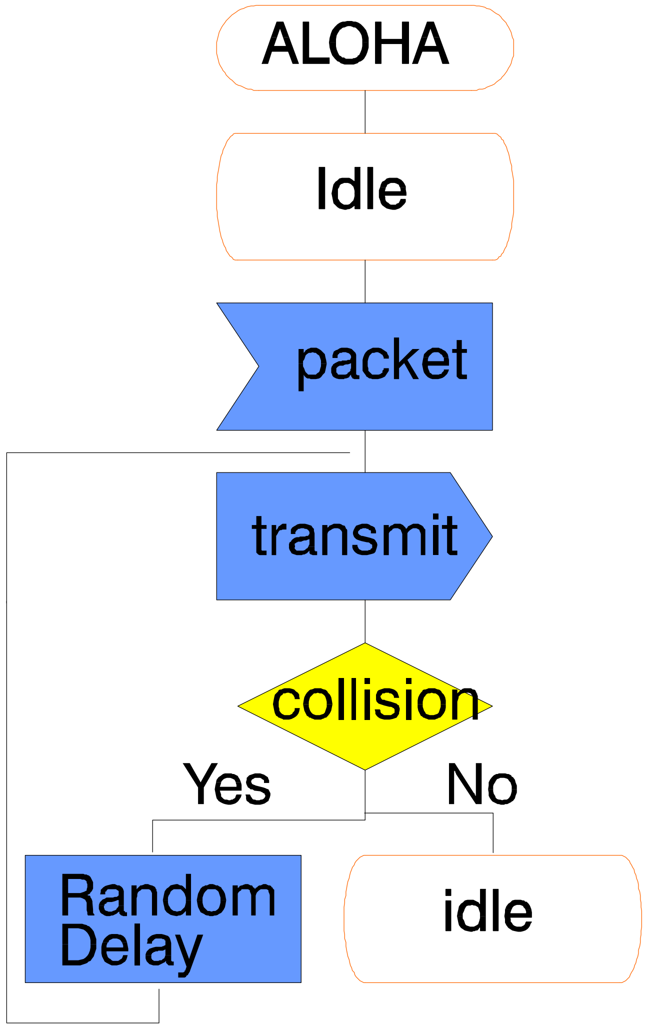
- Developed early 70s at University of Hawaii
- First realization used radio links to connect terminals on islands with main computer
- Basic idea is very simple but many modifications exist (to optimize retransmission policy)
- Any terminal is allowed to transmit without considering whether channel is idle or busy
- If packet is received correctly, the base station transmits an acknowledgement.
- If no acknowledgement is received by the mobile,

  1) it assumes the packet to be lost

  2) it retransmits the packet after waiting a *random* time
- Critical performance issue: "How to choose the retransmission parameter?"
  - Too long: leads to excessive delay
  - Too short: stirs instability
- Unslotted ALOHA: transmission may start anytime

  Slotted ALOHA: packets are transmitted in time slots

ALOHA Algorithm: Terminal Behavior



Carrier Sense Multiple Access : CSMA

- " Listen before talk "
- No new packet transmission is initiated when the channel is busy
- Reduces collisions
- Performance is very sensitive to delays in Carrier Sense mechanism
- CSMA is usefull if channel sensing is much faster than packet transmission time
  - satellite channel with long roundtrip delay: just use ALOHA
- Hidden Terminal Problem:
  mobile terminal may not be aware of a transmission by another (remote) terminal.
  Solution: Inhibit Sense Multiple Access (ISMA)
- Decision Problem: how to distinguish noise and weak transmission?
  Solution: Inhibit Sense Multiple Access (ISMA)

Inhibit Sense Multiple Access : ISMA

Busy Tone Multiple Access : BTMA

- If busy, base station transmits a "busy" signal to inhibit all other mobile terminals from transmitting

- Collisions still occur, because of

  Signalling delay

  - New packet transmissions can start during a delay in the broadcasting of the inhibit signal,

  Persistent terminals

  - after the termination of transmission, packets from persistent terminals, awaiting the channel to become idle, can collide.

Transmission Attempt Persistency in CSMA

Non-persistent

- Random waiting time after sensing the channel busy
- High throughput, but long delays

1-Persistent

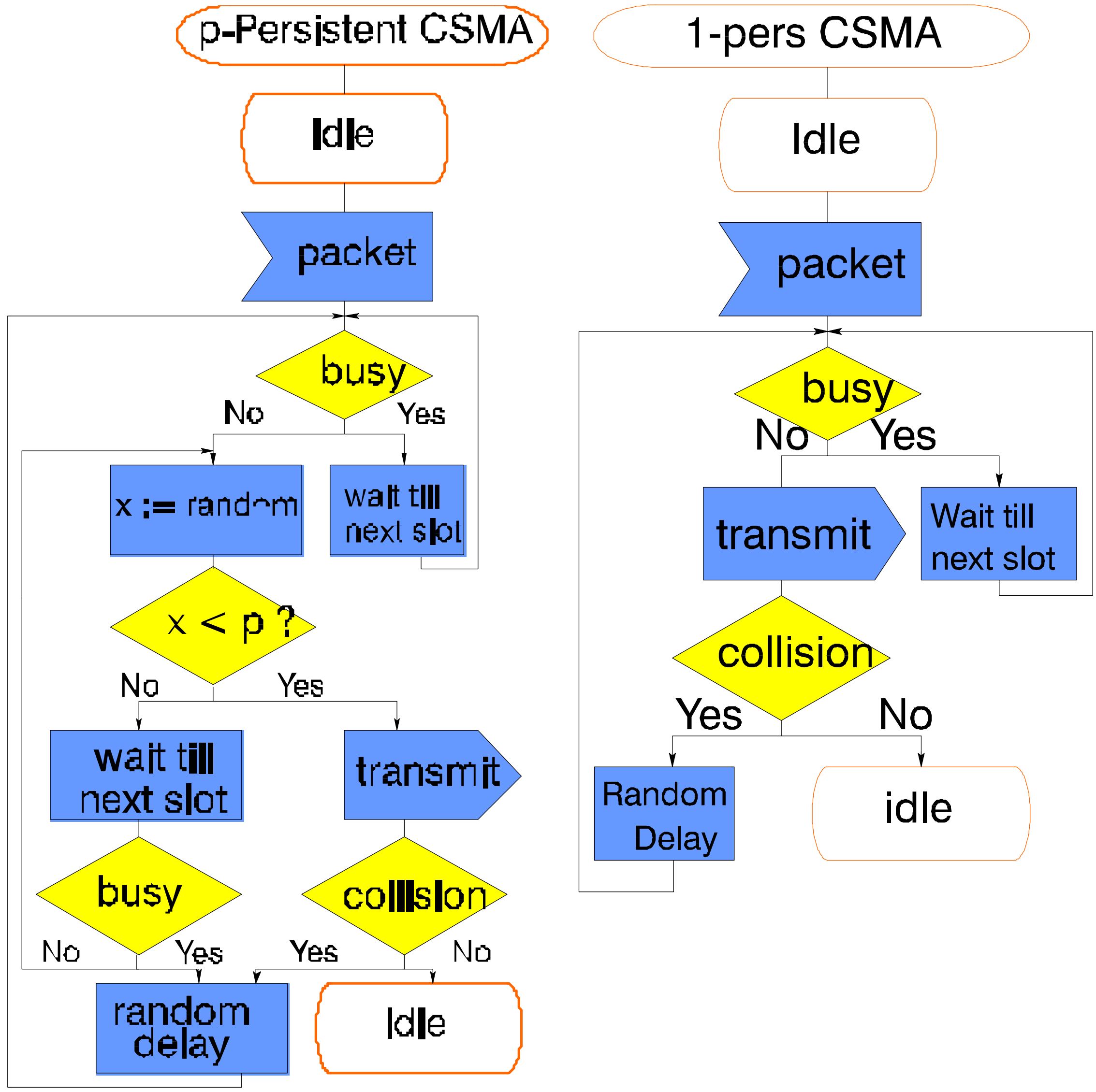
- waiting terminal may start transmitting as soon as previous transmission is terminated
- Short delays, but more severe stability problems

***p*-Persistent**

- The channel has mini-slots, much shorter than packet duration
- Transmission attempt takes place with probability *p*

NB: One may combine a very persistent channel sensing method with a more sophisticated Collision Resolution method

1 and *p*-Persistent CSMA Algorithm: Terminal Behavior



Parameter Definitions and Notation

G_t, G — Total offered traffic
Expected number of transmitted packets per slot

$G(r)$ — Offered traffic per unit area at distance r

S_t, S — Total throughput
Expected number of successfully received packets packet per slot

Approaches for Performance Analysis

- Input traffic λ (packets per second), T is packet duration
- Realistic model

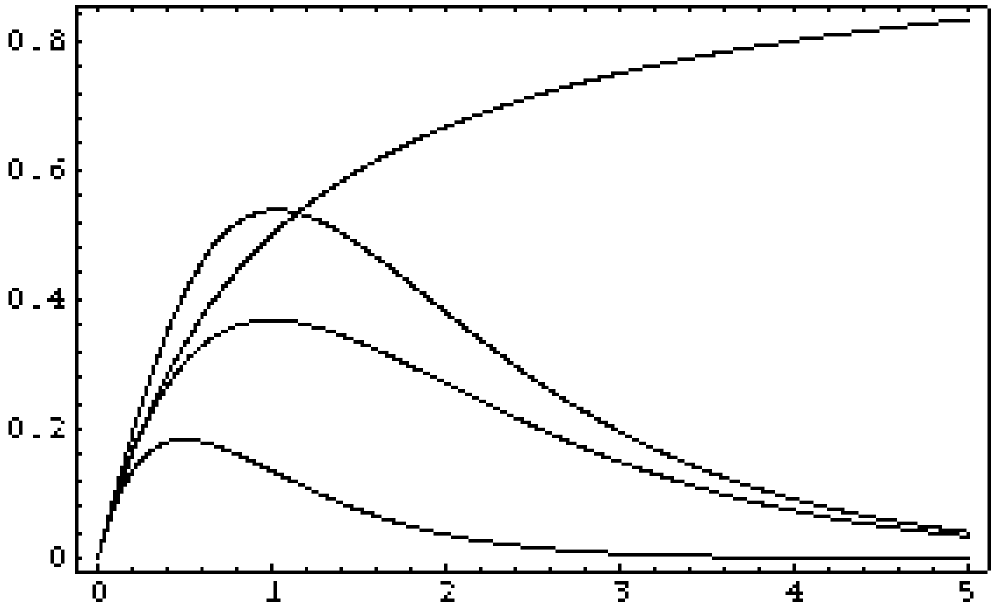
  $S = \lambda T$

  All unsuccessful traffic is retransmitted
- Simple Model

  $G = \lambda T$

  Attempted Traffic is fixed, as e.g. in telemetry.

Throughput - Offered Traffic (*S*-*G*) Relation



Throughput *S* versus Attempted Traffic *G*

The ideal wired (LAN) channel:

- No packet is lost, unless a collision occurs (no fading, no ISI, no noise)
- All packets involved in a collision are lost
- Perfect feedback

Common Performance Analysis Assumptions

- All packets are of uniform duration,
  unit of time = packet duration + guard time
- Acknowledgements are never lost
- Steady-state operation (stability)
- Poisson distributed attempts

Steady-state operation:

- Random waiting times need to be long enough to ensure uncorrelated interference during the initial and successive transmission attempts.
- This is an approximation: dynamic retransmission control is needed in practice
- N.B. ALOHA without capture, with infinite population is always unstable

Throughput Curves

Unslotted ALOHA

$$S_t = G_t \exp(-2G_t)$$

Slotted ALOHA

$$S_t = G_t \exp(-G_t)$$

Carrier Sense Multiple Access

- Non-persistent

$$S_t = \frac{G_t}{1 + G_t}$$

- 1-Persistent

$$S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)}$$

WIRELESS RANDOM-ACCESS

Probability of successful reception

.. depends on

- Receiver capture performance
- Distance from the central receiver,
  path loss
- Channel fading and dispersion
- Shadowing
- Contending packet traffic (from same cell)
- Interference from co-channel cells
- Channel noise
- Modulation method
- Type of coding
- Signal processing at the receiver (diversity, equalization, ...)
- Initial Access protocol:
  slotted ALOHA, Carrier Sense (CSMA) or Inhibit Sense Multiple Access (ISMA)
- Retransmission policy

Useful probabilities of successful reception:

Q(r) **probability of successful reception of a particular *test* packet**

- Packet is generated at a distance *r*
- Taking account of the probability of permission to transmit
- Averaged over the number of interfering packets
- Averaged over the unknown positions of the interfering terminals.
- Sometimes called "near-far effect"
- Determines *fairness* of system

$q_n(r)$ **probability of correct reception of a particular test packet**

- Transmitted from a distance *r*
- Given the number of interfering packets *n*,
- Averaged over the unknown positions of interfering signals.
- Can usually be calculated

C_{n+1} Expected number of successful packets

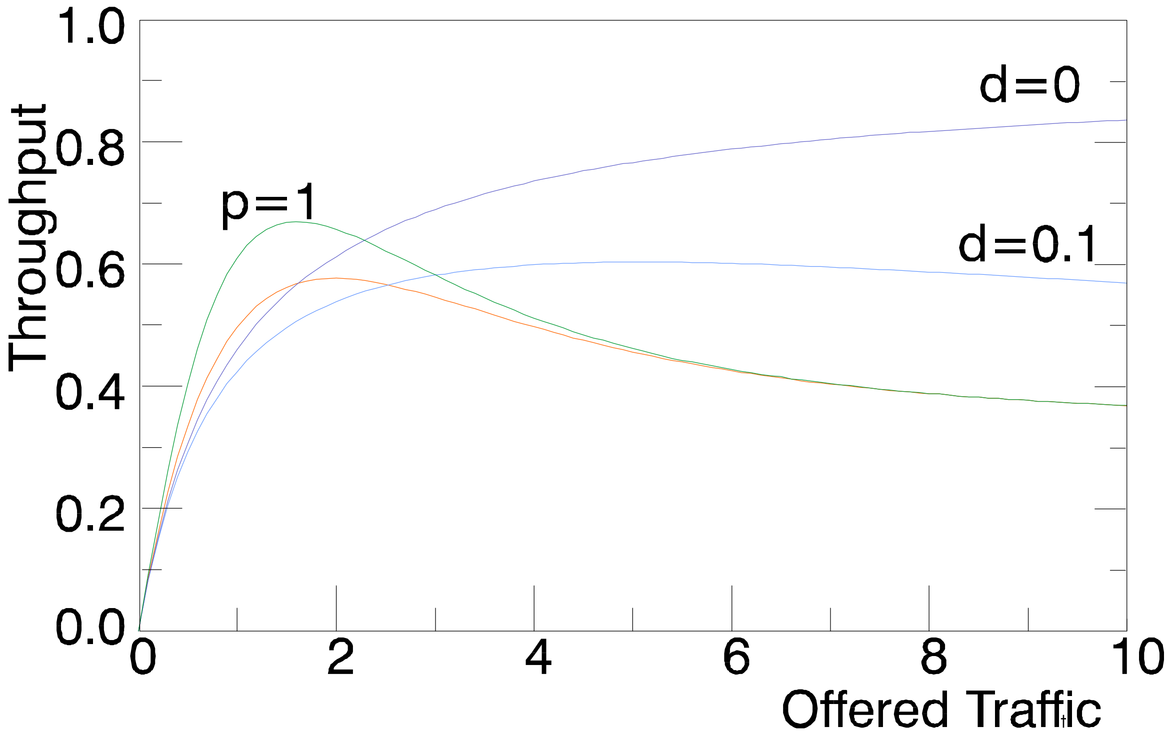
- given that n+1 packets collide.
- If receiver capture is mutually exclusive (no multi-signal detection), $C_{n+1} < 1$.
- Often equals $n + 1$ time probability of success for one particular packet.
- Typically decreases with $n + 1$
- Behavior is critical for stability

S_t Total throughput

- expected number of successful packets per unit of time

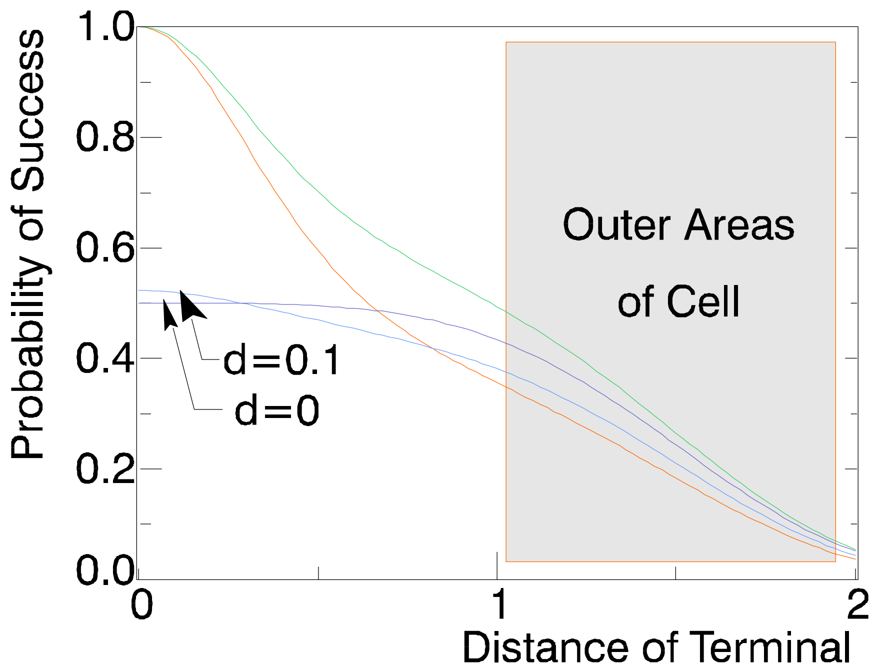
$$S_t = \int_0^\infty 2\pi r\, Q(r)\, G(r)\, dr$$

Total Throughput versus Offered Traffic



- ALOHA (orange), 1-persistent CSMA (green), non-persistent CSMA (blue)
- *d* : Carrier Sensing Delay, relative to packet time
- Unit of throughput: packets per slot time
- Mobile slowly Rayleigh-fading channel
- Plane-earth path loss
- Quasi-uniform distribution of terminals in circular area
- Capture threshold $z = 4$ (6 dB C/I ratio needed)

Probability of Successful Transmission



- ALOHA (orange), 1-persistent CSMA (green), non-persistent CSMA (blue)
- *d* : Carrier Sensing Delay, relative to packet time
- Offered Traffic: average of 1 packet per slot time
- Mobile slowly Rayleigh-fading channel
- Plane-earth path loss
- Uniform distribution of terminals in circular area
- Capture threshold $z = 4$ (6 dB C/I ratio needed)
- For non-persistent CSMA, some attempts do not lead to transmission:

  P(success) is not unity for terminal near base station

Packet Success Probability in Slotted ALOHA

- Fundamental property of independent (Poisson) arrivals:

  Probability of a total of *n* packets =

  Probability of *n* packets interfering with test packet (total *n*+1 packets)

Poisson probability $P_n(n)$ of n contending signals in same slot is

$$P_n(n) = \frac{G_t^n}{n!} \exp(-G_t).$$

The probability $Q(r)$ of a successful transmission is

$$Q(r) = \sum_{n=0}^{\infty} P_n(n)\, q_n(r).$$

The total packet throughput is

$$S_t = G_t \sum_{n=0}^{\infty} P_n(n) q_n = \sum_{i=1}^{\infty} P_n(i)\, C_i$$

where

q_n is the probability that one test packet captures,

while C_i is the probability that one out of i captures

Throughput of Slotted ALOHA

If no capture

$q_n(r) = q_n = 0$ if $n = 1, 2, \ldots\ldots$

$q_0 = 1$

The probability $Q(r)$ of a successful transmission is

$$Q(r) = P_n(0) = \exp(-G_t)$$

The total packet throughput is

$$S_t = G_t P_n(0) = P_n(1)$$

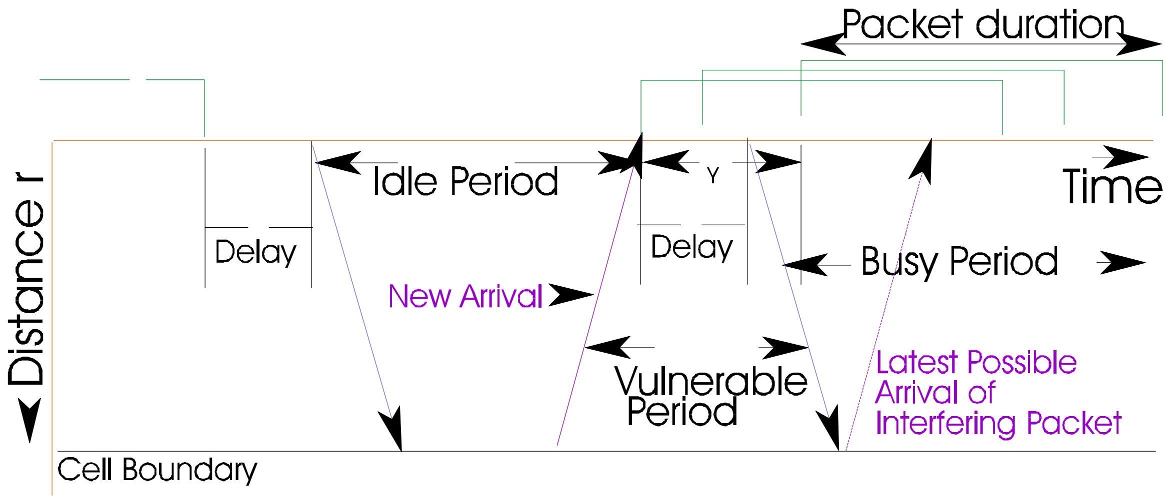
Both methods give the classical expression

$$S_t = G_t \exp(-G_t)$$

INHIBIT SENSE MULTIPLE ACCESS

Outbound signalling channel:

- receiver status: *busy* or *idle*.
- acknowledgements



Time-Space diagram for ISMA

A **Busy period** contains an

- Inhibited period

  = (period in which the base station sends busy signal)

plus a

- Vulnerable period

  = (packet has arrived but no busy tone yet)

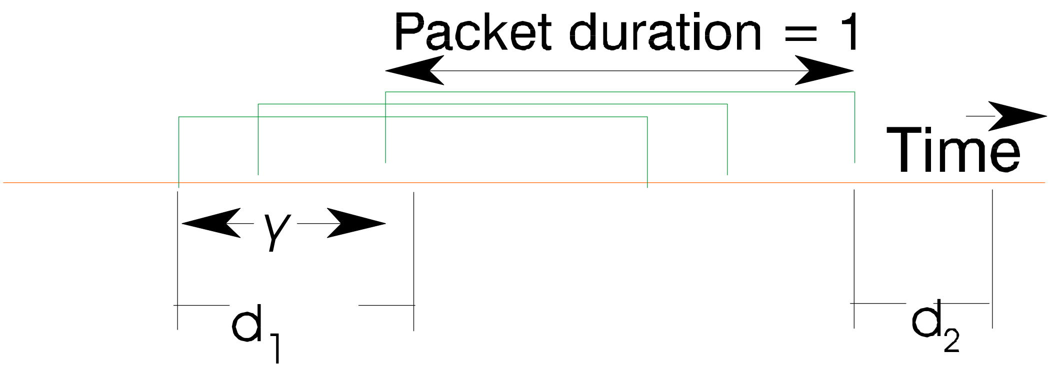
  ■ Duration: signalling delay *d*.

Length of busy period

- Assumption: same delay for all terminals

Add:

+ one packet time (unity duration)
+ busy-tone turn-off delay (d)
+ additional duration because of colliding packets
  (d - length of period with no arrivals)



The busy period has average duration

$$E\ B = 1 + 2d - \frac{1}{G_t}[1 - \exp(-dG_t)].$$

Idle period

- Idle period is the time interval from end of busy tone till arrival of new packet
- For Poisson arrivals and no *propagation* delays:

  · Memoryless property of Poisson arrivals:

  · Expected duration I of idle period

  = the average time until a new packet arrival occurs,

- Thus, E $I = G_t^{-1}$

Cycle

one cycle = idle period + busy period

Renewal Reward Theorem

Throughput per unit of time =

$$\frac{\text{Expected throughput per cycle}}{\text{Expected length per cycle}}$$

Non-p. ISMA without Delay without Capture

If a packet arrives when the base station transmits a "busy" signal

- The attempt fails.
- The packet is rescheduled for later transmission.
- It contributes to G, but not to S
- Retransmissions also contribute to G

If a packet arrives in the idle period

- The transmission is successful
  - No interference can occur ($d = 0$)
  - Channel is assumed perfect
- This occurs with probability $\mathrm{E}I/(\mathrm{E}I + \mathrm{E}B)$

Using the renewal reward theorem, the throughput becomes

$$S_t = \frac{\mathrm{E}B}{\mathrm{E}I + \mathrm{E}B} = \frac{1}{\frac{1}{G_t} + 1}$$

Non-persistent ISMA in Mobile Channel

Probability of successful transmission *Q(r)*

- Take account of the three possible events
    - Arrival in idle period
    - Arrival in vulnerable period
    - Arrival in busy period

If a packet arrives when the base station transmits a "busy" signal

- The attempt fails.

If a packet arrives in the idle period

- This occurs with probability E*I*/(E*I* + E*B*)
- We call this packet an "initiating packet"
- A collision occurs if other terminals start transmitting during delay *d* of the inhibit signal.
- Probability of *n* interfering transmissions is Poissonian, with

$$\frac{(dG_t)^n}{n!}\exp(-dG_t).$$

Non-persistent ISMA: Probability of success

If a packet arrives in the vulnerable period

- Channel is "busy" but seems "idle
- It occurs with probability *d/(*E*B* + E*I)*.
- Packet is NOT inhibited
- It always interferers with the initiating packet
- This packet experiences interference from at least one other packet
- Additional *n* - 1 contending signals are Poisson distributed. Conditional probability of *n* interferers is

$$\frac{(dG_t)^{n-1}}{(n-1)!}\exp(-dG_t)$$

with $n = 1, 2, \ldots$.

Total Throughput of non-persistent ISMA

Use the following results:

- Average cycle length EI + EB
- Initiating packet plus Poisson arrivals during period d

So

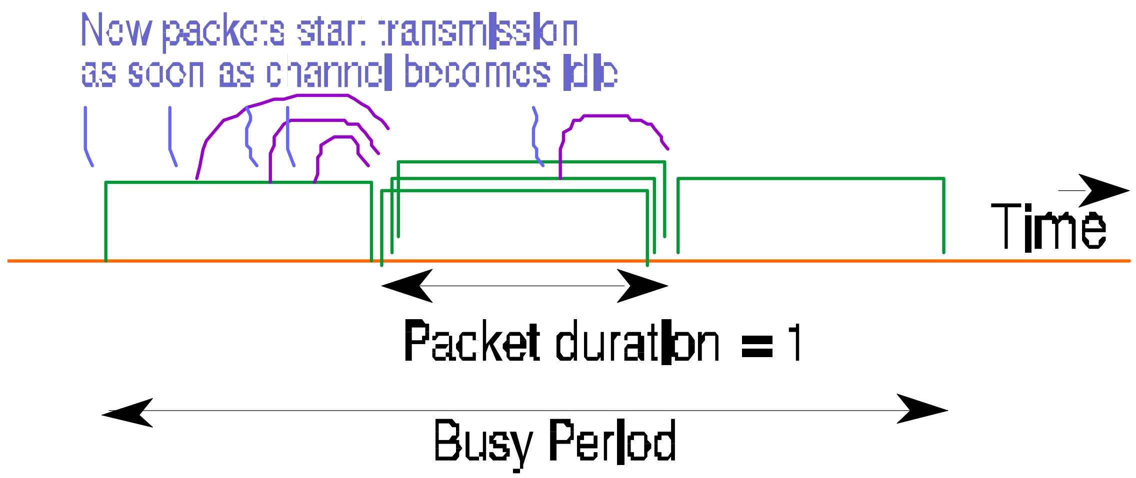
$$S_t = \frac{\exp(-dG_t)}{\mathrm{E}B + \mathrm{E}I} \sum_{n=0}^{\infty} \frac{d^n G_t^n}{n!} C_{n+1}$$

Special case : instantaneous inhibit signalling ($d \to 0$)

- collisions can never occur in non-persistent ISMA.
- $S_t \to G_t (1 + G_t)^{-1}$.
- $S_t \to 1$ for $G_t \to \infty$.

1-Persistent unslotted ISMA

- Waiting terminal may start transmitting as soon as previous transmission is terminated
- Busy period can consist of a number of packet transmissions in succession
- We consider no signalling delay (d = 0).
- For large offered traffic ($G \rightarrow \infty$), throughput rapidly decreases (with exp{-G})



Transmission cycle in 1-p ISMA

Throughput of 1-Persistent unslotted ISMA

Cycle-initiating packet

- If a packet arrives during idle period
- Probability of correct reception is $q_0(r)$.

During transmission of (initiating) packet

- A random number of k terminals sense the channel busy
- k is Poissonian with probability $P_n(k)$.
- When the channel goes idle, k terminals start transmitting

Probability that busy period terminates

- Probability that no terminals starts transmitting, (k = 0) is exp ($-G_t$)
- Probability $P_m(m)$ of transmissions during m units of time, concatenated to initiating packet is

$$P_m(m) = \exp(-G_t)\ [1 - \exp(-G_t)]^m.$$

- Average duration of busy period

$$\mathrm{E}B = E[1 + mP_m(m)] = e^{G_t}$$

Probability of a successful transmission *Q(r)*

Successful packet arrive in idle or vulnerble period

Capture probability:

$$Q(r) = \frac{\mathrm{E}I}{\mathrm{E}B + \mathrm{E}I} q_0(r) + \frac{\mathrm{E}B}{\mathrm{E}B + \mathrm{E}I} \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} q_n(r).$$

Inserting EB and EI=G_t^{-1} and capture probabilities gives

$$Q(r) = \frac{q_0(r) + G_t \sum_{n=0}^{\infty} \frac{G_t^n}{n!} q_n(r)}{1 + G_t \exp(G_t)}$$

Throughput of 1-Persistent unslotted ISMA

Total channel throughput S_t

$$S_t = G_t \frac{C_1 + \sum_{i=1}^{\infty} \frac{G_t^{\,i}}{i!} C_i}{1 + G_t \exp(G_t)}$$

where

C_1 is probability of success if no interference is present

C_i is probability of success when i packets collide

Special case

- 1-persistent CSMA on wired channels
  (q_0=1 and q_n=0 for n = 1, 2, ...)

$$S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)}$$

A Capture Model : C/I Ratio Threshold

- Successful reception if C/I is above threshold z
- The probability of capture $q_n(r)$
  - given location of test packet
  - given n interferers

$$q_n(r) \triangleq \Pr\left(\frac{p_0}{P_t} > z \mid \overline{p_0}, n\right)$$

$$= \int_0^\infty f_{P_t}(x) \int_{zx}^\infty f_{p_0}(y) \; dydx$$

$$= \int_0^\infty \exp\left(-\frac{yz}{\overline{p_0}}\right) f_{P_t}(y)\, dy$$

- Recognize that this is a Laplace Transform
- This can also be written as

$$q_n(r) = \left[\frac{1}{G_t}\int_0^\infty \frac{x^4}{x^4 + zr^4} 2\pi x\, G(x) dx\right]^n.$$

- Capture Probability, directly expressed in terms of traffic intensity $G(r)$

Probability of successful transmission

Slotted ALOHA:

$$Q(r) = \exp\{-G_t(1 - q_1(r))\}$$

$$= \exp\{-\int_{\text{area}} \frac{x^4}{x^4 + zr^4} G(x)dx\}$$

Non-persistent ISMA:

$$Q(r) = \frac{\exp\{-dG_t(1 - q_1(r))\}\,(1 + q_1(r)dG_t)}{G_t(1 + 2d) + e^{-dG_t}}$$

1-persistent ISMA with zero signalling delay

$$Q(r) = \frac{1 + G_t \exp\{G_t q_1(r)\}}{1 + G_t \exp(G_t)}$$

Discussion of results

- Performance of access protocols depends on the channel
- In typical mobile networks, data packet arriving without interference experiences an outage probability of a few percent
- In radio systems, capture occurs
- For best performance, keep packets short. C/I ratios are small, particularly during collisions.
- Models for packet error rates produce largely different estimates of the probability capture.
- Slotted ALOHA results in the most significant near-far unfairness
- Non-persistent ISMA without delay (d=0) gives a uniform probability of access
- Signalling delay degrades average network performance.
- Nonetheless, nearby users benefit from a small signalling delay.
- For low offered traffic loads ($G_t < 1$ *ppt*), slotted ALOHA and non-persistent ISMA (or $p < 0.1$) have almost equal performance.

- For exceptionally high traffic loads, the total channel throughput approaches an identical non-zero limit for 1-persistent ISMA and slotted ALOHA.
- For reasonably high traffic loads ($3 < G_t < 10ppt$), non-persistent ISMA outperforms slotted ALOHA and 1-persistent ISMA.

Capture Probability

- Finite population of N terminals with known positions

- Transmissions are independent from slot to slot
- Conditional on local-mean power of *test* packet

Capture probabability for test packet j is

$$\Pr(capt_j \mid \overline{p}_j) = \prod_{\substack{k=1 \\ k \neq j}}^{N} \mathcal{L}\left\{ f_{p_k}, \frac{z}{\overline{p}_j} \right\}$$

where

- f_{p_k} is (unconditional) PDF of interference power, considering

  · probability P(k_{OFF}) that terminal is idle ($p_k = 0$ if k_{OFF})

  · path loss variations, shadowing and multipath fading.

Multiple interfering signals

- Incoherent cumulation

  Interference power = sum of powers for interferers
- PDF of interference power is *n*-fold convolution of PDF of power of single signal
- Laplace image is *n*-th power

For *n* interferers, the capture probability $q_n(r_j)$ is

$$q_n(r_j) = \left[\frac{1}{G_t} \int_0^{\infty} \frac{r^{\beta}}{r^{\beta} + z r_j^{\beta}} 2\pi r \, G(r) dr \right]^n$$

- Note: $q_n(r) = q_1^n(r)$
- Note: This is an integral transform of the offered traffic *G(r)*

Poisson field of interferers

Poisson distributed number of interfering packets,

Capture probability is

$$Q(r) = \sum_{n=0}^{\infty} \frac{G_t^{\,n}}{n!} \exp\left(-G_t\right) q_n(r)$$

$$= \exp\left\{-G_t\right\} \quad \exp\left\{+G_t q_1(r))\right\} \ .$$

This can also be written as

$$Q(r) = \exp\left\{-\int_{area} W(r,x) G(x)\, dx\right\} \ .$$

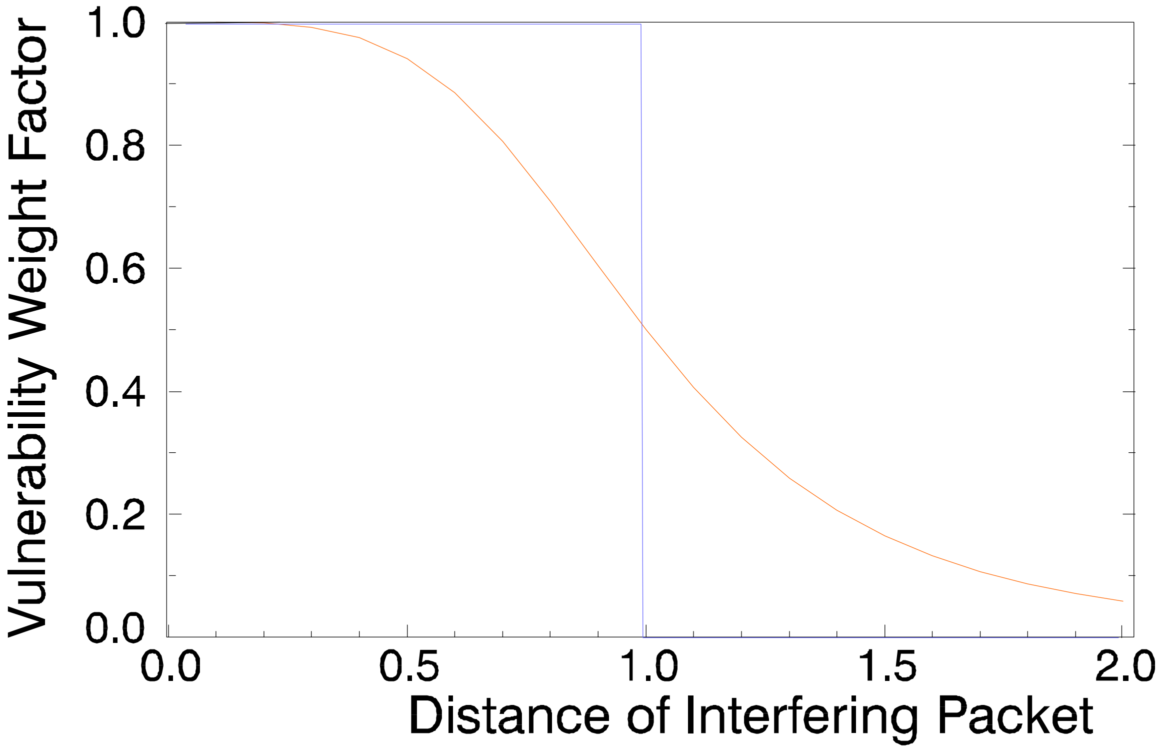
Interpretation:

- Interfering traffic intensity $G(r)$ is multiplied by a weight factor
- This factor is determined by propagation attenuation and receiver capture ratio z
- Interference from remote areas ($r_i >> r_j$) is weak: $W(r_j,r_i) \rightarrow 0$
- Nearby interference causes destructive collisions: weigh by unity ($W(r_j,0) = 1$).

Vulnerability Weight Function

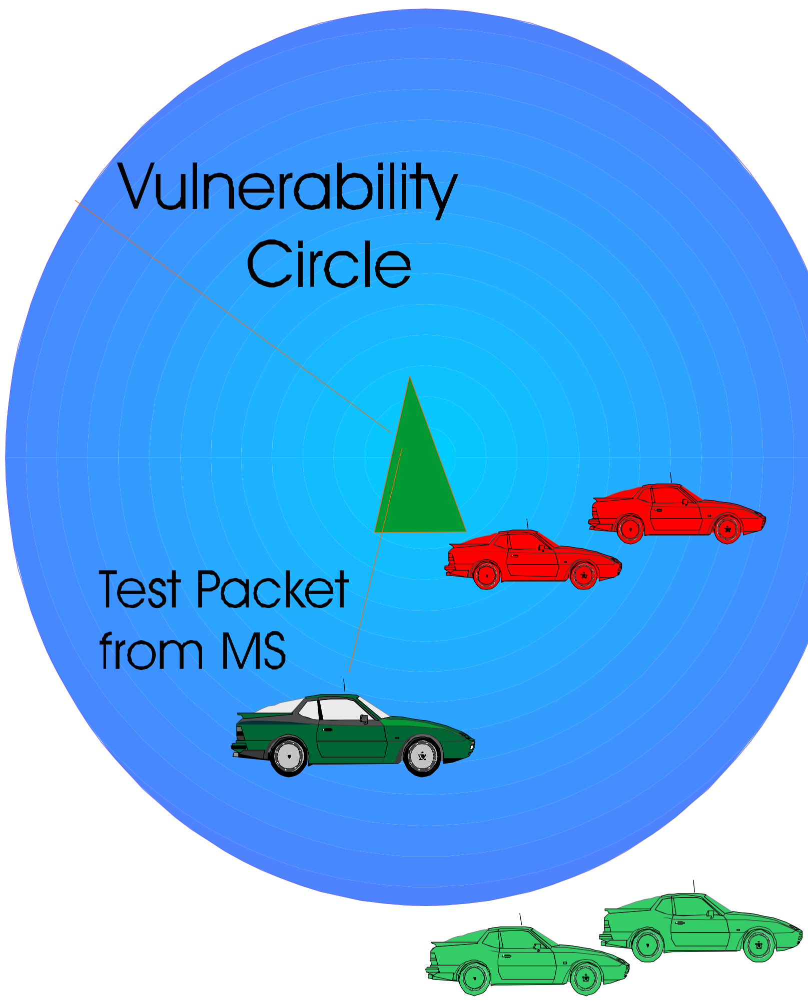
Interpretation:

a test signal from distance r_j is vulnerable to interference k from distance r_k to an extent quantified by $W(r_j, r_k)$



Factor $W(1, r_k)$ to weigh the vulnerability of a test packet from unity distance (r_j = 1) to an interfering signal from r_k. Receiver threshold z = 1 (0 dB).

Vulnerability circle



- Test Packet is lost if and only if interference occurs within vulnerability circle
- Proposed by Abramson (1977)
- Weight function is replaced by a step function: Interference is harmful, only iff transmitted from within vulnerability circle

Ring distribution of offered traffic

- All signals have the same local-mean power
  thus, no near-far effect and shadowing
- Realistic model for (slow) adaptive power control

- Insert spatial distribution

$$G(r) = \frac{G_t}{2\pi r}\,\delta(r-1)$$

This gives

$$q_n = q_n(1) = \frac{1}{(z+1)^n}$$

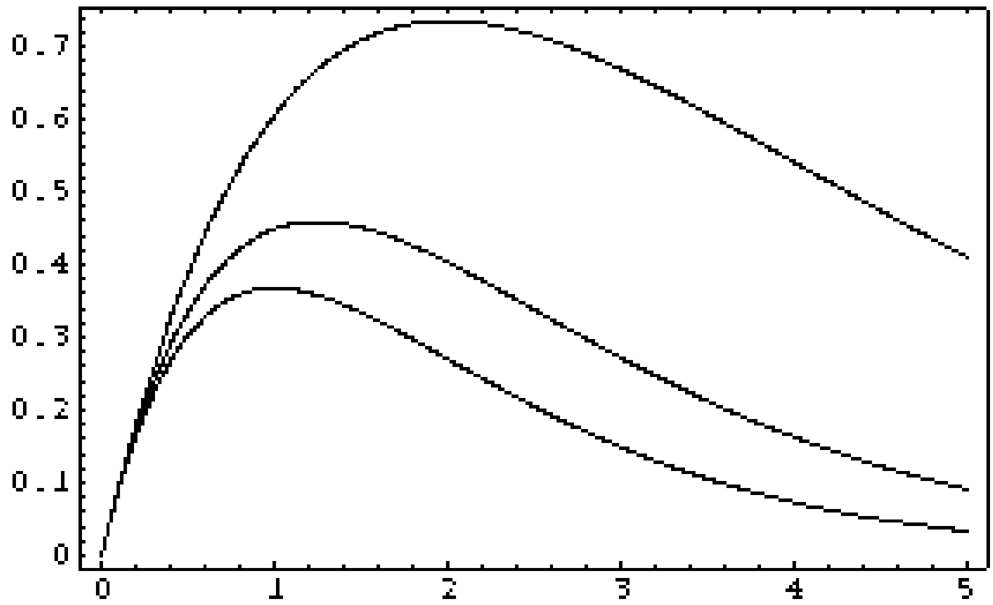
Total throughput is [Verhulst et al.], [Arnbak et al.]

$$S_t = G_t \exp\left\{-\frac{z}{z+1}G_t\right\} .$$

Ring distribution of offered traffic

Total throughput

$$S_t = G_t \exp\left\{-\frac{z}{z+1} G_t\right\} \ .$$



Throughput *S* versus offered traffic *G*,

- Rayleigh fading channel
- Receiver threshold z = 1, 4 and infinity (no capture)

Uniform distribution with infinite extension

- offered traffic $G(r) \equiv G_0$ everywhere $(0 < r < \infty)$
- Note: total offered traffic G_t is unbounded.
- Example: uncontrolled burst transmissions in ISM bands;

Probability of a successful transmission from distance r is

$$Q(r) = \exp\left\{ -\frac{2\pi^2 G_0 z^{\frac{2}{\beta}}}{\beta \sin\frac{2\pi}{\beta}} r^2 \right\} .$$

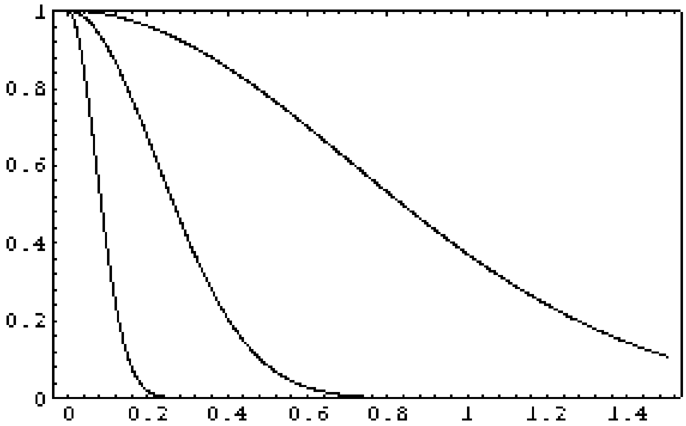
Special case: plane earth loss ($\beta = 4$)

$$Q(r) = \exp\left\{ -\frac{\pi^2}{2} G_0 \sqrt{z}\, r^2 \right\} .$$

Effect of traffic load for uniform G(r)

Probability of successful access

$$Q(r) = \exp\left\{-\frac{\pi^2}{2} G_0 \sqrt{z}\, r^2\right\} .$$



Probability of successful reception versus location of terminal

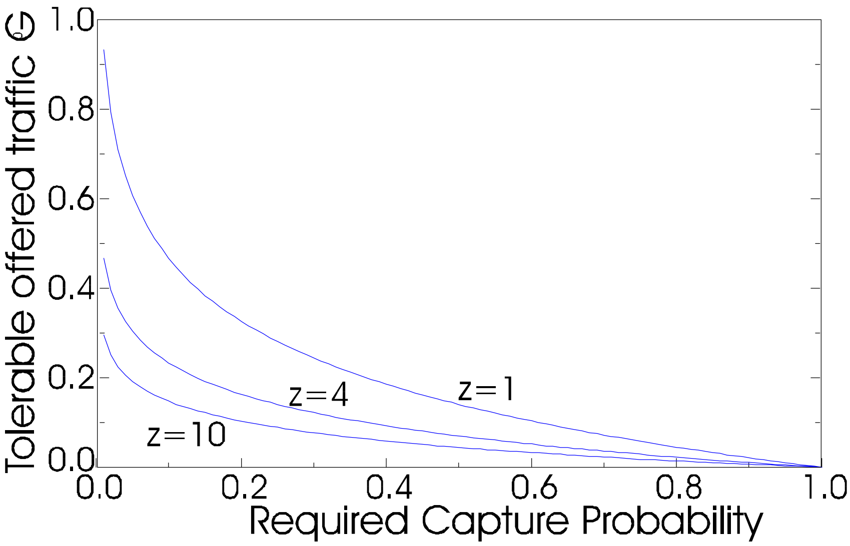
- Rayleigh-fading channel, Plane earth loss, Receiver capture threshold z = 4, Attempted traffic G_0 = 0.1, 1 and 10 packets per slot per unit of area, uniform offered traffic
- Example:
  Intelligent Vehicle Highway System: Collecting travel times from probe vehicles

Maximum traffic capacity

- minimum success rate $Q(r) \geq Q_{MIN}$ for any $0 \leq r \leq 1$

Packet traffic per unit of area offered to the network must be bounded by

$$G_0 < -\frac{2}{\pi^2 \sqrt{z}} \ln Q_{MIN} \ .$$



- Slow Rayleigh fading
- PEL path loss (40 log d)
- Infinitely large Poisson field of interferers

Total Throughput

- Special case: plane earth loss $\beta = 4$

The total throughput S_t is

$$S_t = \frac{2}{\pi\sqrt{z}} \approx \frac{0.64}{\sqrt{z}}$$

Typically, $z = 4$. Then $S_t = 0.32$ packets per slot

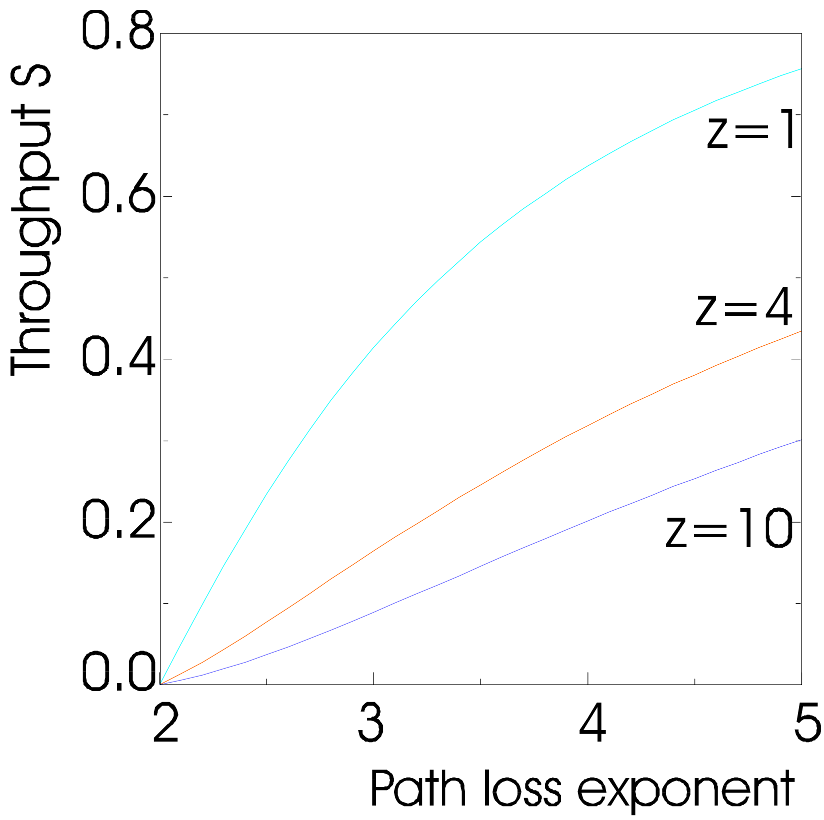
Effect of noise:

- reduces throughput at cell fringe
- reduces total throughput
- leads to more attempts

Total Throughput

If no noise: The total throughput is

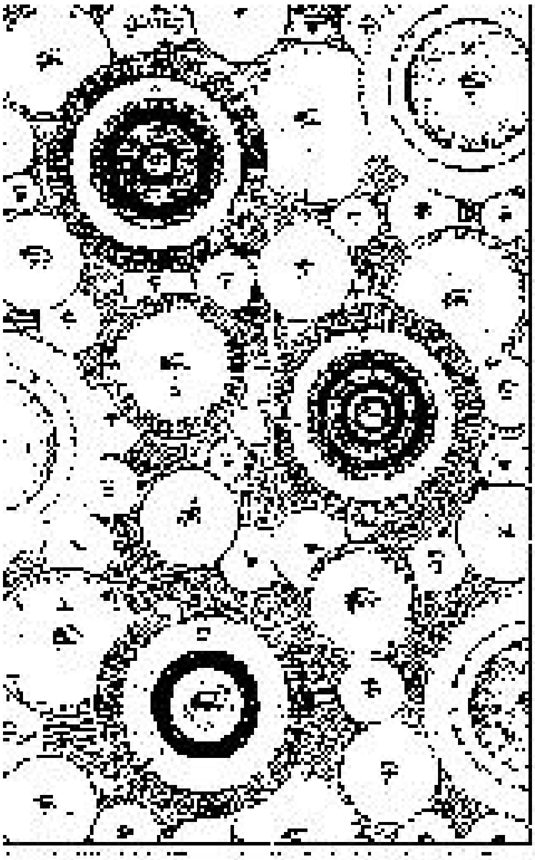
$$S_t = \frac{\beta}{2\pi\, z^{\frac{2}{\beta}}} \sin\frac{2\pi}{\beta} \; .$$



- Throughput as a function of path loss exponent β
- Slowly Rayleigh-fading channel
- Infinitely large Poisson field of interferers
- Various receiver thresholds z

For free-space propagation ($\beta = 2$),

The throughput degrades to zero.



- Why? The amount of interference accumulates to infinity
- Why does it get dark at night?

  Why does the amount of light from all the stars remain finite?

Uniform distribution in circular band

- Offered traffic uniformly distributed with intensity G_0 between radius r_1 and r_2, i.e.,

$$G(r) = \begin{cases} G_0 = \dfrac{G_t}{\pi\left(r_2^2 - r_1^2\right)}, & r_1 < r < r_2 \\ 0, & \text{elsewhere} \end{cases},$$

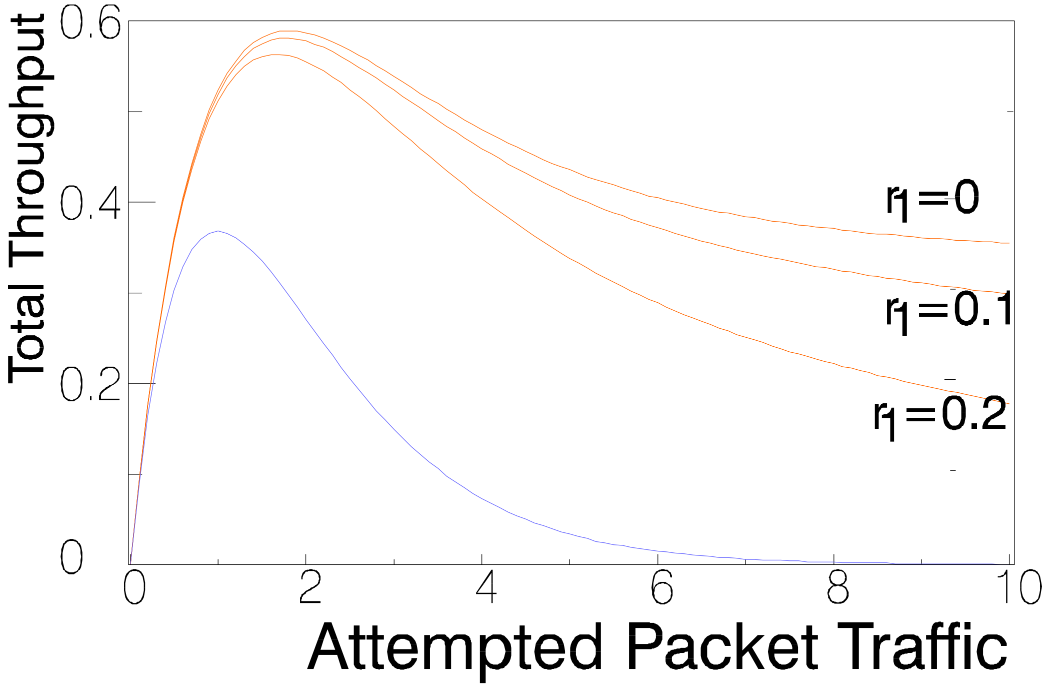
The probability of a successful transmission becomes

$$Q(r) = \exp\left\{ -\pi G_0 \int_{\lambda = r_1}^{r_2} \frac{z r^{\beta}}{\lambda^{\beta} + z r^{\beta}} d\lambda^2 \right\} .$$

For plane-earth loss $\beta = 4$,

$$Q(r) = \exp\left\{ -\sqrt{z}\pi r^2 G_0 \arctan\left(\frac{\sqrt{z} r^2 (r_2^2 - r_1^2)}{z r^4 + r_1^2 r_2^2} \right) \right\} .$$

Throughput for uniform distribution in circular band



- Slow Rayleigh fading, PEL path loss
- Blue: no capture, Orange: receiver threshold $z = 4$
- $r_1 = 0$:

  Terminals can come arbitrarely close to base station

  Throughput $S_t \rightarrow 2 / (\pi\sqrt{z})$
- $r_1 > 0$:

  Throughput decreases to zero for large ffered traffic

Uniform distribution within unit cell

$$G(r) = \begin{cases} G_0 = \frac{G_t}{\pi}, & 0 < r < 1 \\ 0, & \text{elsewhere} \end{cases}$$

The throughput is

$$Q(r) = \exp\left\{ -\sqrt{z}\, r^2 G_t \arctan\left(\frac{1}{\sqrt{z} r^2} \right) \right\} .$$

Minimum success rate Q_{MIN} at cell boundary

- Packet traffic must be bounded by

$$G_0 < \frac{-\ln Q_{MIN}}{\pi \sqrt{z} \arctan\left(\frac{1}{\sqrt{z}} \right)} .$$

- For near-perfect capture ($1 < z < 4$), arctan(1) = ¼√π.
- Thus: Maximum offered traffic is (slightly less than) twice the traffic for uniform spatial distribution with infinite extension ($r_2 \rightarrow \infty$)

What does this mean for frequency reuse?

- Optimum reuse factor is $C = 1$

Limit for high offered traffic ($G_t \to \infty$)

- Uniform offered traffic gives non-zero limits
- only nearby packets contribute to the throughput

Slotted ALOHA and *p*-persistent ISMA *(p* > 0*)*

without signalling delay:

Throughput $S_t \to 2 / (\pi\sqrt{z})$

ISMA with a propagation delay

$$\lim_{G_t \to \infty} S_t = \frac{2}{\sqrt{z}\,\pi\,(1+2d)}$$

- Delay reduces the throughput
- Throughput ISMA (with *d*) is less than for slotted ALOHA
- Theoretical limit is approached very slowly.
- For reasonably high traffic loads ($3 < G_t < 10$) and small delay, non-persistent ISMA outperforms slotted ALOHA.

Uniform throughput

- Remote terminal:
  capture probability is less
  more retransmissions needed
- Attempted traffic increases with distance

Define *S(r)*

Expected number of packets per slot per unit area transmitted from distance *r*.

Take uniform throughput within a cell

$S(r) = S_0$ for $0 < r < 1$

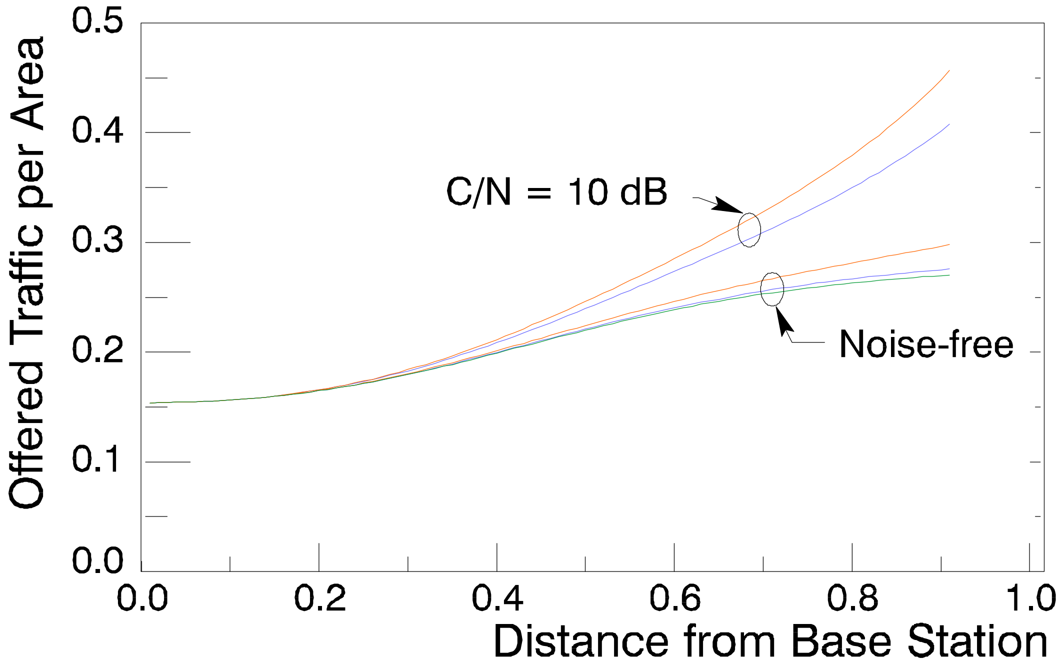
Mathematical Solution: Find *G(r)* such that

$$S_0 = G(r) \exp\left\{ -\int_0^\infty 2\pi \frac{zr^\beta}{\lambda^\beta + zr^\beta} G(\lambda)\, \lambda d\lambda \right\} .$$

Do recursive estimation of *G(r)*

Uniform throughput

- Uniform throughput
- Total throughput $S_t = 0.4$ packet per slot
- Noise mainly affects remote users



cluster size C = 4: orange
cluster size C = 9: blue
cluster size C infinite (no co-ch. interference): green

Cellular Reuse for ALOHA system

A simple case study

- Consider two cells
- Arrival rate per second per cell is λ
- Bandwidth is such that we can transmit one packet in T seconds

Case I: Each cells has its own channel

- Each cell has only half the bandwith
- Packet transmission time is $2T$
- Success probability (no capture) is $\exp\{-\lambda\, 2T\}$
- Delay proportional to $2T \exp\{-2\lambda T\}$

Case II: The cells share the same channel

- Each cell has the full bandwith
- Packet transmission time is T
- Success probability (no capture) is $\exp\{-2\lambda\, T\}$
- Delay proportional to $T \exp\{-2\lambda T\}$

Conclusion:

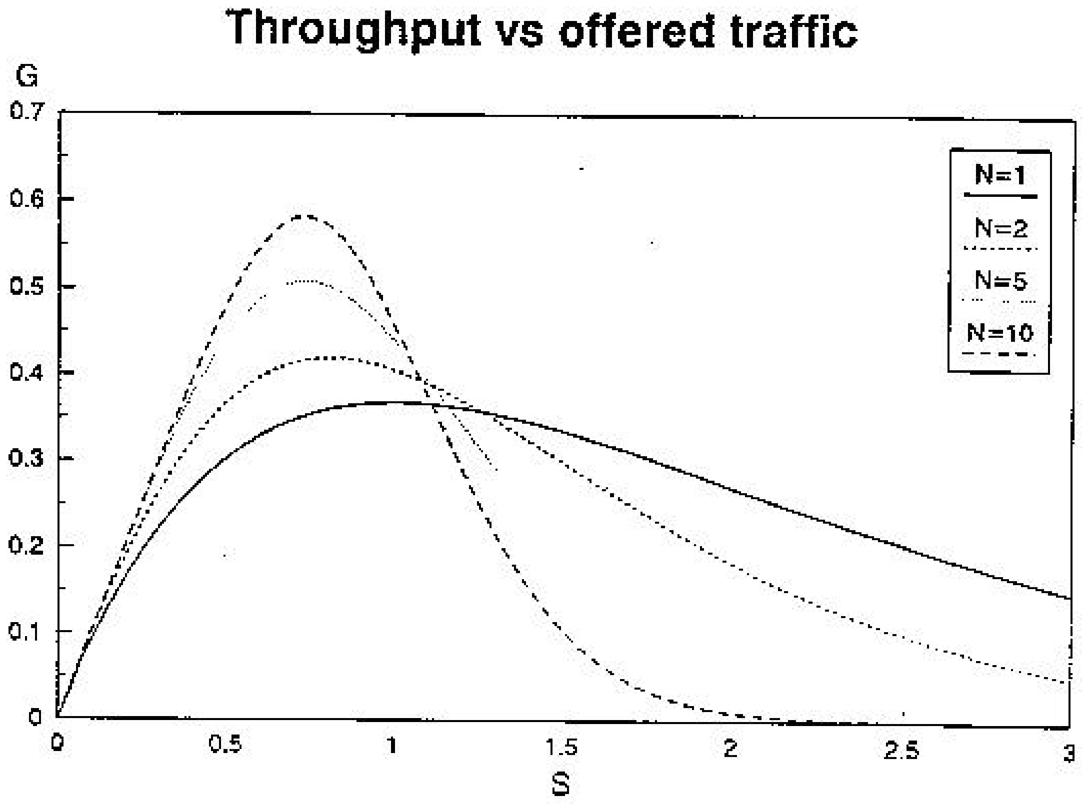
- Contiguous Frequency Reuse gives best performance

DS-CDMA ALOHA Network

- Under ideal signal separation conditions, DS-CDMA can enhance the capacity
- Make a fair comparison!
  Spreading by *N* in the same *transmit* bandwidth implies slot that are N times longer. The arrival rate per slot is *N* times larger
- Assumption for simple analysis:
  All packets in a slot are successful iff the number of packets in that slot does not exceed the speading gain.
- Probability of success = Prob($n \le N$) =

$$\mathrm{P(capt)} = \sum_{n=1}^{N} \frac{(NG)^n}{n!} \exp(-NG)$$

DS-CDMA ALOHA THROUGHPUT



- Spread Factors 1, 2, 5 and 10
- Perfect Capture; perfect signal separation
- Throughput seems to increase with spread factor

Intuition

- Compare the ALOHA system with an embarkment quay
- People arrive with Poisson arrival rate $\lambda < 1$ person per unit of time
- Boats of seat capacity N at regular intervals of duration N
- Thus: total seat capacity is 1 person per unit of time
- The boat sinks and the passengers drown if the number of people exceeds N

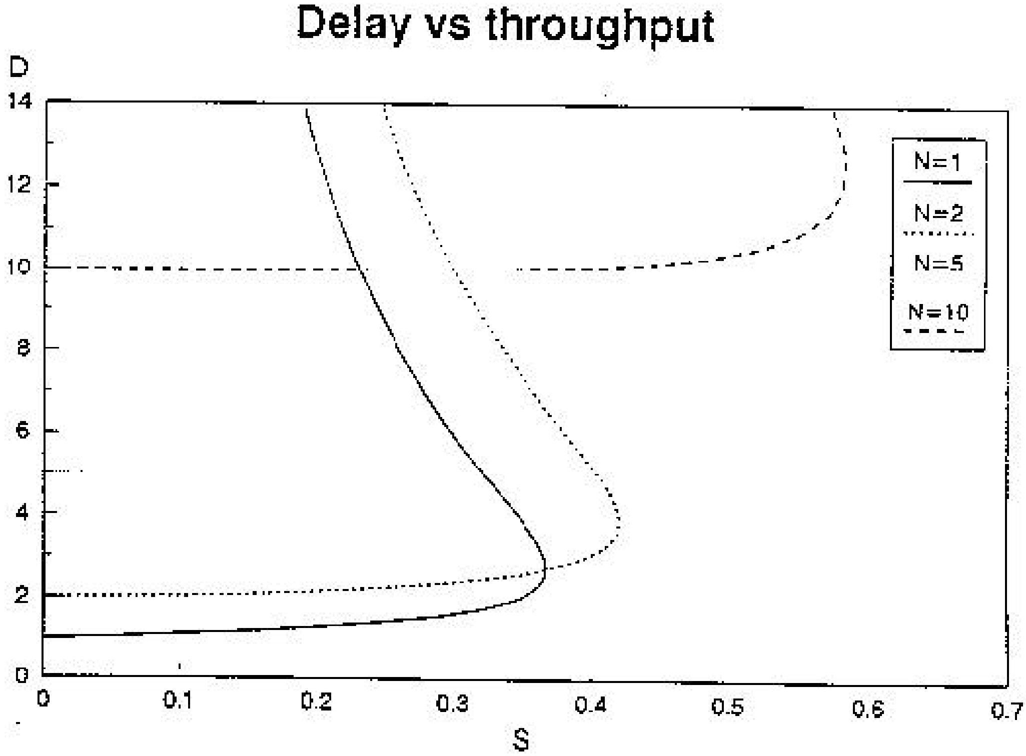
Case I: $N = 1$ (ALOHA without spreading)

- Boats arrive very frequently
- Probability of survival is $\exp\{-\lambda\}$

Case II: Large N

- Fewer but larger boats arrive
- Average waiting time is N times larger
- Probability of survival is larger, because of the law of large numbers

DS-CDMA ALOHA Delay



- Spread Factors N = 1, 2, 5 and 10
- Perfect Capture; perfect signal separation
- Small load: small N preferable
- Large load: high N preferable

Direct sequence spread spectrum with imperfect signal separation

- Spread factor *N*
- CDMA codes typically attenuate interference by factor *N*
- Receiver threshold: sucess if C/N > *z / N*
- Fixed system bandwidth, thus
  transmission time increases by factor *N*
  offered traffic per slot increaes by factor *N*

Capture probability

$$Q(r) = \exp\left\{ -2\pi N G_0 \int_0^\infty \frac{z\lambda^\beta}{z\lambda^\beta + N r^\beta} \lambda d\lambda \right\} .$$

- Capture probability decreases with increasing *N*
- DS-spreading is harmful to performance
- This is at odds with previous conclusion that CDMA improves performance

ISM Applications: Assumptions

- ISM band B_N = 2400-2483.5 MHz
- Required C/N after despreading 6 dB (z = 4)
- η_r = 1 bit/s/Hz or 1 chip/s/Hz
- 40 log d: $\beta = 4$
- Range r = 5 meters
- offered traffic:

  two devices per 10 m^2 room

  peak rate 10 Mbit/s, average activity 5%

  Average data rate q = 0.1 Mbit per 1 m^2

  (cf. AT&T q = 6 kbit/s/m^2)

- Offered load $G = N\, q\, /\, (\eta\, B_N)$ packets per packet time

Capture probabability

$$Q(r) = \exp\left\{ -\frac{Ng}{\eta_r B_N} \frac{\pi r^2}{\beta \sin \frac{2\pi}{\beta}} \left(\frac{z}{N} \right)^{\frac{2}{\beta}} \right\} .$$

Slow Frequency Hopping

- Narrowband (unspread) transmission of each packet
- Receiver threshold remains unchanged
- N parallel channels, each with rate $1/N$
- Traffic load per slot remains unchanged

Average Capture probability remains

$$Q(r) = \exp\left\{ -\frac{\pi^2}{2} G_0 \sqrt{z}\, r^2 \right\} \ .$$

Advantages:

- Frequency diversity:
  fading on different carrier uncorrelated
  capture probabilities independent
  improved performance
- Less Intersymbol Interference

Disadvantages

- longer delay