Random Access

- Many terminals communicate to a single base station
- Fixed multiple access methods (TDMA, FDMA, CDMA) become inefficient when the traffic is bursty.
- Random Access works better for
  - many users, where ..
  - each user only occasionally sends a message

Suitable Protocols

- ALOHA
- Carrier Sense
- Inhibit Sense
- Collision Resolution
  - Stack Algorithm
  - Tree Algorithm
- Reservation methods
  - Reservation ALOHA
  - Packet Reservation Multiple Access
ALOHA Protocol

- Developed early 70s at University of Hawaii
- First realization used radio links to connect terminals on islands with main computer
- Basic idea is very simple but many modifications exist (to optimize retransmission policy)
- Any terminal is allowed to transmit without considering whether channel is idle or busy
- If packet is received correctly, the base station transmits an acknowledgement.
- If no acknowledgement is received by the mobile,  
  1) it assumes the packet to be lost  
  2) it retransmits the packet after waiting a random time
- Critical performance issue: "How to choose the retransmission parameter?" 
  - Too long: leads to excessive delay 
  - Too short: stirs instability
- Unslotted ALOHA: transmission may start anytime
  Slotted ALOHA: packets are transmitted in time slots
ALOHA Algorithm: Terminal Behavior

ALOHA

Idle

packet

transmit

collision

Yes

Random Delay

No

idle
Carrier Sense Multiple Access: CSMA

- "Listen before talk"
- No new packet transmission is initiated when the channel is busy
- Reduces collisions
- Performance is very sensitive to delays in Carrier Sense mechanism
- CSMA is useful if channel sensing is much faster than packet transmission time
  - satellite channel with long roundtrip delay: just use ALOHA
- Hidden Terminal Problem:
  mobile terminal may not be aware of a transmission by another (remote) terminal.
  Solution: Inhibit Sense Multiple Access (ISMA)
- Decision Problem: how to distinguish noise and weak transmission?
  Solution: Inhibit Sense Multiple Access (ISMA)
Inhibit Sense Multiple Access: ISMA

Busy Tone Multiple Access: BTMA

- If busy, base station transmits a "busy" signal to inhibit all other mobile terminals from transmitting

- Collisions still occur, because of Signalling delay
  - New packet transmissions can start during a delay in the broadcasting of the inhibit signal,
  - Persistent terminals
  - after the termination of transmission, packets from persistent terminals, awaiting the channel to become idle, can collide.
Transmission Attempt Persistency in CSMA

Non-persistent
- Random waiting time after sensing the channel busy
- High throughput, but long delays

1-Persistent
- Waiting terminal may start transmitting as soon as previous transmission is terminated
- Short delays, but more severe stability problems

$p$-Persistent
- The channel has mini-slots, much shorter than packet duration
- Transmission attempt takes place with probability $p$

NB: One may combine a very persistent channel sensing method with a more sophisticated Collision Resolution method
1 and $p$-Persistent CSMA Algorithm:
Terminal Behavior

- **p-Persistent CSMA**
  - **Idle**
  - **Packet**
  - **Busy**
    - $x := \text{random}$
    - $x < p$?
      - No: **Wait till next slot**
      - Yes: **Transmit**
    - **Busy**
      - No: **Wait till next slot**
      - Yes: **Collision**
    - **Random Delay**
    - **Idle**

- **1-pers CSMA**
  - **Idle**
  - **Packet**
  - **Busy**
    - Yes: **Transmit**
    - No: **idle**
  - **Collision**
    - Yes: **Wait till next slot**
    - No: **Random Delay**
Parameter Definitions and Notation

\[ G, \ G \] Total offered traffic
Expected number of transmitted packets per slot

\[ G(r) \] Offered traffic per unit area at distance \( r \)

\[ S, \ S \] Total throughput
Expected number of successfully received packets packet per slot

Approaches for Performance Analysis

- Input traffic \( \lambda \) (packets per second), \( T \) is packet duration
- Realistic model
  \[ S = \lambda T \]
  All unsuccessful traffic is retransmitted
- Simple Model
  \[ G = \lambda T \]
  Attempted Traffic is fixed, as e.g. in telemetry.
Throughput - Offered Traffic \((S-G)\) Relation

The ideal wired (LAN) channel:

- No packet is lost, unless a collision occurs (no fading, no ISI, no noise)
- All packets involved in a collision are lost
- Perfect feedback
Common Performance Analysis Assumptions

- All packets are of uniform duration, unit of time = packet duration + guard time
- Acknowledgements are never lost
- Steady-state operation (stability)
- Poisson distributed attempts

Steady-state operation:

- Random waiting times need to be long enough to ensure uncorrelated interference during the initial and successive transmission attempts.
- This is an approximation: dynamic retransmission control is needed in practice
- N.B. ALOHA without capture, with infinite population is always unstable
Throughput Curves

Unslotted ALOHA

\[ S_t = G_t \exp(-2G_t) \]

Slotted ALOHA

\[ S_t = G_t \exp(-G_t) \]

Carrier Sense Multiple Access

- Non-persistent

\[ S_t = \frac{G_t}{1 + G_t} \]

- 1-Persistent

\[ S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)} \]
Probability of successful reception

.. depends on

- Receiver capture performance
- Distance from the central receiver, path loss
- Channel fading and dispersion
- Shadowing
- Contending packet traffic (from same cell)
- Interference from co-channel cells
- Channel noise
- Modulation method
- Type of coding
- Signal processing at the receiver (diversity, equalization, ...)
- Initial Access protocol:
  slotted ALOHA, Carrier Sense (CSMA) or Inhibit Sense Multiple Access (ISMA)
- Retransmission policy
Useful probabilities of successful reception:

\[ Q(r) \]  probability of successful reception of a particular test packet

- Packet is generated at a distance \( r \)
- Taking account of the probability of permission to transmit
- Averaged over the number of interfering packets
- Averaged over the unknown positions of the interfering terminals.
- Sometimes called "near-far effect"
- Determines fairness of system

\[ q_n(r) \]  probability of correct reception of a particular test packet

- Transmitted from a distance \( r \)
- Given the number of interfering packets \( n \),
- Averaged over the unknown positions of interfering signals.
- Can usually be calculated
$C_{n+1}$  **Expected number of successful packets**

- given that $n+1$ packets collide.
- If receiver capture is mutually exclusive (no multi-signal detection), $C_{n+1} < 1$.
- Often equals $n + 1$ time probability of success for one particular packet.
- Typically decreases with $n + 1$
- Behavior is critical for stability

$S_t$  **Total throughput**

- expected number of successful packets per unit of time

$$S_t = \int_0^\infty 2\pi r Q(r) G(r) \, dr$$
Total Throughput versus Offered Traffic

- ALOHA (orange), 1-persistent CSMA (green), non-persistent CSMA (blue)
- $d$: Carrier Sensing Delay, relative to packet time
- Unit of throughput: packets per slot time
- Mobile slowly Rayleigh-fading channel
- Plane-earth path loss
- Quasi-uniform distribution of terminals in circular area
- Capture threshold $z = 4$ (6 dB C/I ratio needed)
Probability of Successful Transmission

- ALOHA (orange), 1-persistent CSMA (green), non-persistent CSMA (blue)
- $d$: Carrier Sensing Delay, relative to packet time
- Offered Traffic: average of 1 packet per slot time
- Mobile slowly Rayleigh-fading channel
- Plane-earth path loss
- Uniform distribution of terminals in circular area
- Capture threshold $z = 4$ (6 dB C/I ratio needed)
- For non-persistent CSMA, some attempts do not lead to transmission: P(success) is not unity for terminal near base station
Packet Success Probability in Slotted ALOHA

- Fundamental property of independent (Poisson) arrivals:

  Probability of a total of \( n \) packets =

  Probability of \( n \) packets interfering with test packet (total \( n+1 \) packets)

  Poisson probability \( P_n(n) \) of \( n \) contending signals in same slot is

  \[
P_n(n) = \frac{G_t^n}{n!} \exp(-G_t).
  \]

  The probability \( Q(r) \) of a successful transmission is

  \[
  Q(r) = \sum_{n=0}^{\infty} P_n(n) q_n(r).
  \]

  The total packet throughput is

  \[
  S_t = G_t \sum_{n=0}^{\infty} P_n(n) q_n = \sum_{i=1}^{\infty} P_n(i) C_i
  \]

  where

  \( q_n \) is the probability that one test packet captures,
  while \( C_i \) is the probability that one out of \( i \) captures
Throughput of Slotted ALOHA

If no capture

\[ q_n(r) = q_n = 0 \text{ if } n = 1, 2, \ldots. \]
\[ q_0 = 1 \]

The probability \( Q(r) \) of a successful transmission is

\[ Q(r) = P_n(0) = \exp(-G_r) \]

The total packet throughput is

\[ S_t = G_t P_n(0) = P_n(1) \]

Both methods give the classical expression

\[ S_t = G_t \exp(-G_t) \]
INHIBIT SENSE MULTIPLE ACCESS

Outbound signalling channel:

- receiver status: *busy* or *idle*.
- acknowledgements

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A **Busy period** contains an

- Inhibited period
  
  = (period in which the base station sends busy signal)

plus a

- Vulnerable period
  
  = (packet has arrived but no busy tone yet)

  ■ Duration: signalling delay $d$.  

---

Time-Space diagram for ISMA
Length of busy period

- Assumption: same delay for all terminals

Add:
+ one packet time (unity duration)
+ busy-tone turn-off delay ($d$)
+ additional duration because of colliding packets
  ($d$ - length of period with no arrivals)

The busy period has average duration

$$E \ B = 1 + 2d - \frac{1}{G_t} [1 - \exp(-dG_t)].$$
Idle period

- Idle period is the time interval from end of busy tone till arrival of new packet
- For Poisson arrivals and no propagation delays:
  - Memoryless property of Poisson arrivals:
  - Expected duration $I$ of idle period
    $= \text{the average time until a new packet arrival occurs}$,
- Thus, $E I = G_t^{-1}$

Cycle

one cycle = idle period + busy period

Renewal Reward Theorem

Throughput per unit of time =

\[
\frac{\text{Expected throughput per cycle}}{\text{Expected length per cycle}}
\]
Non-p. ISMA without Delay without Capture

If a packet arrives when the base station transmits a "busy" signal

- The attempt fails.
- The packet is rescheduled for later transmission.
- It contributes to $G$, but not to $S$
- Retransmissions also contribute to $G$

If a packet arrives in the idle period

- The transmission is successful
  - No interference can occur ($d = 0$)
  - Channel is assumed perfect
- This occurs with probability $EI/(EI + EB)$

Using the renewal reward theorem, the throughput becomes

$$S_t = \frac{EB}{EI + EB} = \frac{1}{G_t + 1}$$
Non-persistent ISMA in Mobile Channel

Probability of successful transmission $Q(r)$

• Take account of the three possible events
  · Arrival in idle period
  · Arrival in vulnerable period
  · Arrival in busy period

If a packet arrives when the base station transmits a "busy" signal

• The attempt fails.

If a packet arrives in the idle period

• This occurs with probability $E_I/(E_I + E_B)$
• We call this packet an "initiating packet"
• A collision occurs if other terminals start transmitting during delay $d$ of the inhibit signal.
• Probability of $n$ interfering transmissions is Poissonian, with

$$\frac{(dG_i)^n}{n!} \exp(-dG_i).$$
Non-persistent ISMA: Probability of success

If a packet arrives in the vulnerable period

- Channel is "busy" but seems "idle"
- It occurs with probability \( d/(EB + EI) \).
- Packet is NOT inhibited
- It always interferes with the initiating packet
- This packet experiences interference from at least one other packet
- Additional \( n - 1 \) contending signals are Poisson distributed. Conditional probability of \( n \) interferers is

\[
\frac{(dG)^{n-1}}{(n-1)!} \exp(-dG)
\]

with \( n = 1, 2, \ldots \).
Total Throughput of non-persistent ISMA

Use the following results:

- Average cycle length $EI + EB$
- Initiating packet plus Poisson arrivals during period $d$

So

$$S_t = \frac{\exp(-dG_t)}{EB + EI} \sum_{n=0}^{\infty} \frac{d^n G_t^n}{n!} C_{n+1}$$

**Special case**: instantaneous inhibit signalling ($d \to 0$)

- collisions can never occur in non-persistent ISMA.
- $S_t \to G_t (1 + G_t)^{-1}$.
- $S_t \to 1$ for $G_t \to \infty$. 
1-Persistent unslotted ISMA

- Waiting terminal may start transmitting as soon as previous transmission is terminated
- Busy period can consist of a number of packet transmissions in succession
- We consider no signalling delay ($d = 0$).
- For large offered traffic ($G \to \infty$), throughput rapidly decreases (with $\exp\{-G\}$)

Transmission cycle in 1-p ISMA
Throughput of 1-Persistent unslotted ISMA

Cycle-initiating packet

- If a packet arrives during idle period
- Probability of correct reception is \( q_0(r) \).

During transmission of (initiating) packet

- A random number of \( k \) terminals sense the channel busy
- \( k \) is Poissonian with probability \( P_n(k) \).
- When the channel goes idle, \( k \) terminals start transmitting

Probability that busy period terminates

- Probability that no terminals starts transmitting, \( k = 0 \) is \( \exp(-G_t) \)
- Probability \( P_m(m) \) of transmissions during \( m \) units of time, concatenated to initiating packet is

\[
P_m(m) = \exp(-G_t) [1 - \exp(-G_t)]^m.
\]

- Average duration of busy period

\[
E B = E[1 + mP_m(m)] = e^{G_t}.
\]
Probability of a successful transmission $Q(r)$

Successful packet arrive in idle or vulnerability period

Capture probability:

$$Q(r) = \frac{EI}{EB+EI} q_0(r)$$

$$+ \frac{EB}{EB+EI} \sum_{n=0}^{\infty} \frac{G_t^n}{n!} e^{-G_t} q_n(r).$$

Inserting $EB$ and $EI=G_t^{-1}$ and capture probabilities gives

$$Q(r) = \frac{q_0(r) + G_t \sum_{n=0}^{\infty} \frac{G_t^n}{n!} q_n(r)}{1 + G_t \exp(G_t)}$$
Throughput of 1-Persistent unslotted ISMA

Total channel throughput $S_t$

$$S_t = G_t \frac{C_1 + \sum_{i=1}^{\infty} \frac{G_t^i}{i!} C_t}{1 + G_t \exp(G_t)}$$

where

$C_1$ is probability of success if no interference is present

$C_i$ is probability of success when $i$ packets collide

Special case

- 1-persistent CSMA on wired channels
  
  $(q_0=1$ and $q_n=0$ for $n = 1, 2, \ldots)$

$$S_t = \frac{G_t + G_t^2}{1 + G_t \exp(G_t)}$$
A Capture Model: C/I Ratio Threshold

- Successful reception if C/I is above threshold $z$
- The probability of capture $q_n(r)$
  - given location of test packet
  - given $n$ interferers

$$q_n(r) \triangleq \Pr\left(\frac{p_0}{p_t} > z \mid p_0, n\right)$$

$$= \int_{0}^{\infty} f_{p_t}(x) \int_{x}^{\infty} f_{p_0}(y) \ dy \ dx$$

$$= \int_{0}^{\infty} \exp\left(-\frac{yz}{p_0}\right) f_{p_t}(y) \ dy$$

- Recognize that this is a Laplace Transform

- This can also be written as

$$q_n(r) = \left[\frac{1}{G} \int_{0}^{\infty} \frac{x^4}{x^4 + zr^4} 2\pi x G(x) dx\right]^n.$$  

- Capture Probability, directly expressed in terms of traffic intensity $G(r)$
Probability of successful transmission

Slotted ALOHA:

\[ Q(r) = \exp\{-G_t(1-q_1(r))\} \]

\[ = \exp\{- \int_{\text{area}} \frac{x^4}{x^4 + z^4} G(x) dx\} \]

Non-persistent ISMA:

\[ Q(r) = \frac{\exp\{-dG_t(1-q_1(r))\} (1+q_1(r)dG_t)}{G_t(1+2d) + e^{-dG_t}} \]

1-persistent ISMA with zero signalling delay

\[ Q(r) = \frac{1 + G_t \exp\{G_t q_1(r)\}}{1 + G_t \exp(G_t)} \]
Discussion of results

- Performance of access protocols depends on the channel.
- In typical mobile networks, data packet arriving without interference experiences an outage probability of a few percent.
- In radio systems, capture occurs.
- For best performance, keep packets short. C/I ratios are small, particularly during collisions.
- Models for packet error rates produce largely different estimates of the probability capture.
- Slotted ALOHA results in the most significant near-far unfairness.
- Non-persistent ISMA without delay ($d=0$) gives a uniform probability of access.
- Signalling delay degrades average network performance.
- Nonetheless, nearby users benefit from a small signalling delay.
- For low offered traffic loads ($G_i < 1$ ppt), slotted ALOHA and non-persistent ISMA (or $p < 0.1$) have almost equal performance.
• For exceptionally high traffic loads, the total channel throughput approaches an identical non-zero limit for 1-persistent ISMA and slotted ALOHA.

• For reasonably high traffic loads ($3 < G < 10\ ppt$), non-persistent ISMA outperforms slotted ALOHA and 1-persistent ISMA.
Capture Probability

- Finite population of $N$ terminals with known positions
- Transmissions are independent from slot to slot
- Conditional on local-mean power of test packet

Capture probability for test packet $j$ is

$$\Pr(capt_j | p_j) = \prod_{k=1}^{N} \mathcal{Q}\left\{ f_{p_k}, \frac{z}{p_j} \right\}$$

where

- $f_{p_k}$ is (unconditional) PDF of interference power, considering
  - probability $P(k_{OFF})$ that terminal is idle ($p_k = 0$ if $k_{OFF}$)
  - path loss variations, shadowing and multipath fading.
Multiple interfering signals

- Incoherent cumulation
  Interference power = sum of powers for interferers
- PDF of interference power is $n$-fold convolution of PDF of power of single signal
- Laplace image is $n$-th power

For $n$ interferers, the capture probability $q_n(r_j)$ is

$$q_n(r_j) = \left[ \frac{1}{G_t} \int_0^\infty \frac{r^\beta}{r^\beta + zr_j^\beta} 2\pi r G(r) dr \right]^n$$

- Note: $q_n(r) = q_j^n(r)$
- Note: This is an integral transform of the offered traffic $G(r)$
Poisson field of interferers

Poisson distributed number of interfering packets,

Capture probability is

\[
Q(r) = \sum_{n=0}^{\infty} \frac{G_t^n}{n!} \exp(-G_t)q_n(r)
\]

\[
= \exp(-G_t) \exp(+G_tq_1(r))
\]

This can also be written as

\[
Q(r) = \exp\left\{ -\int_{\text{area}} W(r,x)G(x)\,dx \right\}
\]

Interpretation:

- Interfering traffic intensity \( G(r) \) is multiplied by a weight factor
- This factor is determined by propagation attenuation and receiver capture ratio \( z \)
- Interference from remote areas \( (r_i >> r_j) \) is weak: \( W(r_j,0) \to 0 \)
- Nearby interference causes destructive collisions: weigh by unity \( (W(r_j,0) = 1) \).
Vulnerability Weight Function

Interpretation:

a test signal from distance \( r_j \) is vulnerable to interference \( k \) from distance \( r_k \) to an extent quantified by \( W(r_j, r_k) \)

Factor \( W(1, r_k) \) to weigh the vulnerability of a test packet from unity distance \( (r_j = 1) \) to an interfering signal from \( r_k \). Receiver threshold \( z = 1 \) (0 dB).
Vulnerability circle

• Test Packet is lost if and only if interference occurs within vulnerability circle
• Proposed by Abramson (1977)
• Weight function is replaced by a step function: Interference is harmful, only iff transmitted from within vulnerability circle
Ring distribution of offered traffic

- All signals have the same local-mean power thus, no near-far effect and shadowing
- Realistic model for (slow) adaptive power control
- Insert spatial distribution

\[ G(r) = \frac{G_t}{2\pi r} \delta(r-1) \]

This gives

\[ q_n = q_n(1) = \frac{1}{(z+1)^n} \]

Total throughput is [Verhulst et al.], [Arnbak et al.]

\[ S_t = G_t \exp\left\{-\frac{z}{z+1}G_t\right\}. \]
Ring distribution of offered traffic

Total throughput

\[ S_t = G_t \exp\left\{ -\frac{z}{z+1} G_t \right\}. \]

Throughput \( S \) versus offered traffic \( G \),

- Rayleigh fading channel
- Receiver threshold \( z = 1, 4 \) and infinity (no capture)
Uniform distribution with infinite extension

- offered traffic $G(r) \equiv G_0$ everywhere ($0 < r < \infty$)
- Note: total offered traffic $G_t$ is unbounded.
- Example: uncontrolled burst transmissions in ISM bands;

Probability of a successful transmission from distance $r$ is

$$Q(r) = \exp \left\{ -\frac{2\pi^2 G_0}{\beta} \frac{2}{\beta \sin \frac{2\pi}{\beta}} r^2 \right\}.$$ 

Special case: plane earth loss ($\beta = 4$)

$$Q(r) = \exp \left\{ -\frac{\pi^2}{2} G_0 \sqrt{\frac{2}{\pi}} r^2 \right\}.$$
Effect of traffic load for uniform $G(r)$

Probability of successful access

$$Q(r) = \exp\left\{-\frac{\pi^2}{2} G_0 \sqrt{z} r^2\right\}.$$ 

Probability of successful reception versus location of terminal

- Rayleigh-fading channel, Plane earth loss, Receiver capture threshold $z = 4$, Attempted traffic $G_0 = 0.1, 1$ and 10 packets per slot per unit of area, uniform offered traffic
- Example:
  Intelligent Vehicle Highway System: Collecting travel times from probe vehicles
Maximum traffic capacity

- minimum success rate \( Q(r) \geq Q_{MIN} \) for any \( 0 \leq r \leq 1 \)

Packet traffic per unit of area offered to the network must be bounded by

\[
G_0 < -\frac{2}{\pi^2 \sqrt{z}} \ln Q_{MIN} .
\]

- Slow Rayleigh fading
- PEL path loss \((40 \log d)\)
- Infinitely large Poisson field of interferers
Total Throughput

- Special case: plane earth loss $\beta = 4$

The total throughput $S_t$ is

$$S_t = \frac{2}{\pi \sqrt{z}} \approx \frac{0.64}{\sqrt{z}}$$

Typically, $z = 4$. Then $S_t = 0.32$ packets per slot

**Effect of noise:**

- reduces throughput at cell fringe
- reduces total throughput
- leads to more attempts
The total throughput is

\[ S_t = \frac{\beta^2}{2 \pi} \sin \left( \frac{2\pi}{\beta} \right). \]

- Throughput as a function of path loss exponent \( \beta \)
- Slowly Rayleigh-fading channel
- Infinitely large Poisson field of interferers
- Various receiver thresholds \( z \)
For free-space propagation ($\beta = 2$),

The throughput degrades to zero.

- Why? The amount of interference accumulates to infinity
- Why does it get dark at night?
  Why does the amount of light from all the stars remain finite?
Uniform distribution in circular band

- Offered traffic uniformly distributed with intensity $G_0$ between radius $r_1$ and $r_2$, i.e.,

\[
G(r) = \begin{cases} 
  G_0 = \frac{G_t}{\pi (r_2^2 - r_1^2)}, & r_1 < r < r_2 \\
  0, & \text{elsewhere}
\end{cases}
\]

The probability of a successful transmission becomes

\[
Q(r) = \exp \left\{ -\pi G_0 \int_{\lambda=r_1}^{r_2} \frac{zr^\beta}{\lambda^\beta + zr^\beta} d\lambda^2 \right\}.
\]

For plane-earth loss $\beta = 4$,

\[
Q(r) = \exp \left\{ -\sqrt{z} \pi r^2 G_0 \arctan \left( \frac{\sqrt{z} r^2 (r_2^2 - r_1^2)}{z r^4 + r_1^2 r_2^2} \right) \right\}.
\]
Throughput for uniform distribution in circular band

- Slow Rayleigh fading, PEL path loss
- Blue: no capture, Orange: receiver threshold $z = 4$
- $r_1 = 0$:
  Terminals can come arbitrarily close to base station
  Throughput $S_t \to 2 / (\pi \sqrt{z})$
- $r_1 > 0$:
  Throughput decreases to zero for large offered traffic
Uniform distribution within unit cell

\[
G(r) = \begin{cases} 
G_0 = \frac{G_t}{\pi}, & 0 < r < 1 \\
0, & \text{elsewhere}
\end{cases}
\]

The throughput is

\[
Q(r) = \exp\left\{-\sqrt{z}r^2 G_t \arctan\left(\frac{1}{\sqrt{z}r^2}\right)\right\}.
\]

Minimum success rate \(Q_{\text{MIN}}\) at cell boundary

- Packet traffic must be bounded by

\[
G_0 < \frac{-\ln Q_{\text{MIN}}}{\pi \sqrt{z} \arctan\left(\frac{1}{\sqrt{z}}\right)}.
\]

- For near-perfect capture \((1 < z < 4)\), \(\arctan(1) = \frac{1}{4}\sqrt{\pi}\).
- Thus: Maximum offered traffic is (slightly less than) twice the traffic for uniform spatial distribution with infinite extension \((r_2 \to \infty)\)

What does this mean for frequency reuse?

- Optimum reuse factor is \(C = 1\)
Limit for high offered traffic ($G_t \to \infty$)

- Uniform offered traffic gives non-zero limits
- only nearby packets contribute to the throughput

Slotted ALOHA and $p$-persistent ISMA ($p > 0$) without signalling delay:
Throughput $S_t \to 2 / (\pi \sqrt{z})$

ISMA with a propagation delay

$$\lim_{G_t \to \infty} S_t = \frac{2}{\sqrt{z} \pi (1 + 2d)}$$

- Delay reduces the throughput
- Throughput ISMA (with $d$) is less than for slotted ALOHA
- Theoretical limit is approached very slowly.
- For reasonably high traffic loads ($3 < G_t < 10$) and small delay, non-persistent ISMA outperforms slotted ALOHA.
Uniform throughput

- Remote terminal:
  capture probability is less
  more retransmissions needed
- Attempted traffic increases with distance

Define $S(r)$

Expected number of packets per slot per unit area
transmitted from distance $r$.

Take uniform throughput within a cell

$S(r) = S_0$ for $0 < r < 1$

Mathematical Solution: Find $G(r)$ such that

$$S_0 = G(r) \exp\left\{-\int_0^\infty 2\pi \frac{zr^\beta}{\lambda^\beta + zr^\beta} G(\lambda) \lambda d\lambda\right\}.$$ 

Do recursive estimation of $G(r)$
**Uniform throughput**

- Uniform throughput
- Total throughput $S_t = 0.4$ packet per slot
- Noise mainly affects remote users

Cluster size $C = 4$: orange
Cluster size $C = 9$: blue
Cluster size $C$ infinite (no co-ch. interference): green
Cellular Reuse for ALOHA system

A simple case study
- Consider two cells
- Arrival rate per second per cell is $\lambda$
- Bandwidth is such that we can transmit one packet in $T$ seconds

Case I: Each cells has its own channel
- Each cell has only half the bandwith
- Packet transmission time is $2T$
- Success probability (no capture) is $\exp\{-\lambda 2T\}$
- Delay proportional to $2T \exp\{-2\lambda T\}$

Case II: The cells share the same channel
- Each cell has the full bandwith
- Packet transmission time is $T$
- Success probability (no capture) is $\exp\{-2\lambda T\}$
- Delay proportional to $T \exp\{-2\lambda T\}$

Conclusion:
- Contiguous Frequency Reuse gives best performance
DS-CDMA ALOHA Network

- Under ideal signal separation conditions, DS-CDMA can enhance the capacity
- Make a fair comparison!
  Spreading by $N$ in the same transmit bandwidth implies slots that are $N$ times longer. The arrival rate per slot is $N$ times larger
- Assumption for simple analysis:
  All packets in a slot are successful iff the number of packets in that slot does not exceed the spreading gain.
- Probability of success = $\text{Prob}(n \leq N) =
  \[ P(\text{capt}) = \sum_{n=1}^{N} \frac{(NG)^n}{n!} \exp(-NG) \]
• Spread Factors 1, 2, 5 and 10
• Perfect Capture; perfect signal separation
• Throughput seems to increase with spread factor
Intuition

• Compare the ALOHA system with an embarkment quay
• People arrive with Poisson arrival rate $\lambda < 1$ person per unit of time
• Boats of seat capacity $N$ at regular intervals of duration $N$
• Thus: total seat capacity is 1 person per unit of time
• The boat sinks and the passengers drown if the number of people exceeds $N$

Case I: $N = 1$ (ALOHA without spreading)
• Boats arrive very frequently
• Probability of survival is $\exp\{-\lambda\}$

Case II: Large $N$
• Fewer but larger boats arrive
• Average waiting time is $N$ times larger
• Probability of survival is larger, because of the law of large numbers
DS-CDMA ALOHA Delay

- Spread Factors $N = 1, 2, 5$ and $10$
- Perfect Capture; perfect signal separation
- Small load: small $N$ preferable
- Large load: high $N$ preferable
Direct sequence spread spectrum with imperfect signal separation

- Spread factor $N$
- CDMA codes typically attenuate interference by factor $N$
- Receiver threshold: success if $C/N > z/N$
- Fixed system bandwidth, thus transmission time increases by factor $N$
  offered traffic per slot increases by factor $N$

Capture probability

\[ Q(r) = \exp \left\{ -2\pi NG_0 \int_0^\infty \frac{z\lambda^\beta}{z\lambda^\beta + Nr^\beta} \, \lambda \, d\lambda \right\}. \]

- Capture probability decreases with increasing $N$
- DS-spreading is harmful to performance
- This is at odds with previous conclusion that CDMA improves performance
ISM Applications: Assumptions

- ISM band $B_N = 2400$-$2483.5$ MHz
- Required C/N after despreading 6 dB ($z = 4$)
- $\eta_r = 1$ bit/s/Hz or 1 chip/s/Hz
- $40 \log d$: $\beta = 4$
- Range $r = 5$ meters
- offered traffic:
  - two devices per $10$ m\(^2\) room
  - peak rate 10 Mbit/s, average activity 5%
    Average data rate $q = 0.1$ Mbit per $1$ m\(^2\)
    (cf. AT&T $q = 6$ kbit/s/m\(^2\))

- Offered load $G = N q / (\eta B_N)$ packets per packet time

Capture probabability

$$Q(r) = \exp \left\{ - \frac{Ng}{\eta r B_N} - \frac{\pi r^2}{\beta \sin \frac{2\pi}{\beta}} \left( \frac{z}{\beta} \right)^2 \right\}.$$
Slow Frequency Hopping

- Narrowband (unspread) transmission of each packet
- Receiver threshold remains unchanged
- $N$ parallel channels, each with rate $1/N$
- Traffic load per slot remains unchanged

Average Capture probability remains

$$Q(r) = \exp\left\{-\frac{\pi^2}{2} G_0 \sqrt{\frac{2}{r^2}} \right\}.$$  

Advantages:
- Frequency diversity:
  fading on different carrier uncorrelated
  capture probabilities independent
  improved performance
- Less Intersymbol Interference

Disadvantages
- longer delay