Direct-Sequence Spread-Spectrum

Basic Principle

- Generate DS signal by multiplying a (narrowband) signal with a fast chip sequence $c(t)$.

- In the receiver, the signal is again multiplied by $c(t)$.
- $c(t) = +1$ or -1, so $c(t)c(t) = 1$
- Often: Time Average $c(t)c(t - T) \approx 0$ if T not 0
- Multi-user: Time Average $c_{\text{user 1}}(t)\ c_{\text{user 2}}(t - T) \approx 0$

Effect of Interchip Interference

- Any signal bandlimited to B_T can resolve the channel with resolution $1/B_T$.
- DS-CDMA with nyquist pulse shapes, $B_T = 1/T_c$.
- The multipath channel is seen as a discrete set of reflections

- Interarrival time is (integer multiple of) chip time.
- Number of relevant reflections, L, is called the number of resolvable paths.

Approximate model

$$L = \left\lceil \frac{T_c}{T_D} \right\rceil \quad \textbf{(1)}$$

with

T_c the chip duration

T_D the total duration of the delay spread

but ...... some papers say T_D is rms delay spread

Channel Model

Impulse response

$$h(t) = \sum_{k=0}^{L} h_k \, \delta(t - kT_c)$$

where

h_k is the amplitude of the k-th path

- Expected power in each resolvable path is found from delay profile, and

$$\bar{p} = \sum_{k=0}^{L} \mathrm{E}\ h_k h_k^*$$

- IID (independent, identically distributed model):

  h_1, h_1, ... h_L are independent and

$$\mathrm{E}\ h_k h_k^* = \frac{\bar{p}}{L} \qquad \forall_k$$

Link Model

- Transmitter waveform $m_0(t)$ for a digital "0" and $m_1(t)$ for a digital "1", with $m_0(t)$ = - $m_1(t)$.

- Received complex low-pass equivalent signal is

$$r_i(t) = \sum_{k=0}^{\infty} h_k \; m_i(t - kT_c) + z(t)$$

where

z(t) describes AWGN.

i is "0" or "1"

h_k is the amplitude of th *k*-th path

- Intersymbol interference is ignored.

  Reasonable if delay spread << 0.1 bit duration.

Optimum receiver

- According to Matched Filter theory: Multiply *r(t)* with expected waveforms if "0" or "1" was sent.
- Define reference (expected receive) waveforms

$$y_i(t) = \sum_{k=0}^{\infty} h_k(t)\ m_i(t-kT_c)$$

.

Schemetic diagram of optimum Matched Filter receiver

- The optimal receiver uses knowledge on
  - message waveforms and
  - channel impulse response.

Matched filter receiver for DS

For BPSK, the decision variable becomes

$$
\begin{aligned}
\nu &= \mathrm{Re}\left\{\int_0^{T_b} r(t)\; y_1^*(t)\; dt\right\} \\
&= \mathrm{Re}\sum_{k=0}^{L}\int_0^{T_b} r(t)\; h_k^*\; m_i^*(t-kT_c)\; dt \qquad \textbf{(7)} \\
&= \mathrm{Re}\, h_k^* \sum_{k=0}^{L}\int_0^{T_b} r(t)\; m_i^*(t-kT_c)\; dt
\end{aligned}
$$

- the optimum receiver can be implemented by
  1) *L* correlators each the received signal *r(t)* with a delayed version of the TRANSMIT waveform
  2) Weighing the outputs of the correlators according to the path amplitude

- Similar to Matched Filter and Maximum Ratio Combining: weigh each path in proportion to its envelope

RAKE Receiver

- Rake receiver is optimum if
  - no interference (AWGN)
  - no ISI
  - for any waveform (even if not orthogonal)
- The difference between an *Equalizer* and a *Rake* is in "the way you set the taps".

Bit Error Rates

- Insert *r(t)* in the decision variable ν.

In the event "1", this becomes

$$\nu = \mathrm{Re}\sum_{k=0}^{L} h_k^* \sum_{m=0}^{L} h_n R_m(n-k) \qquad \text{(signal)}$$

$$+\mathrm{Re}\sum_{k=0}^{L} h_k^* \int_0^{T_b} z(t) m_1^*(t-kT_c)dt \qquad \text{(noise)}$$

where

z(t) is AWG noise

R() is the autocorrelation function

$$R(n-k) \triangleq \int m_1(t-nT_c)\ m_1^*(t-kT_c)\ dt$$

For ideally orthogonal waveforms, the autocorrelation is

$$R_m(n) \triangleq \begin{cases} 0, & n=0 \\ T_b, & n\neq 0 \end{cases}$$

For ideally orthogonal waveforms

$$\nu = \mathrm{Re}\sum_{k=0}^{L} h_k^* h_k T_b + \mathrm{Re}\sum_{k=0}^{L} h_k^* \int_0^{T_b} z(t) m_1^*(t-kT_c)\,dt$$

where

first term represents the wanted signal and

second term is additive noise.

The signal-to-noise ratio is at the Rake output

$$\gamma = \frac{\left[\sum_{k=1}^{L} |h_k|^2\right]^2 T_b^2 \overline{p}}{\sum_{k=1}^{L} |h_k|^2 N_0 T_b} = \sum_{k=1}^{L} \gamma_k$$

where

γ_k is signal-to-noise ratio in k-th resolvable path.

Inherent Diversity

- Rake performance is identical to MRC for L branches
  - (if waveforms were truely orthogonal)

For a channel with known impulse response, the BER is

$$P_b(e \mid \{h_k\}) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\sum_{k=1}^{L} \gamma_k}$$

Wideband Rayleigh-fading channel

- h_1, h_2, ... h_L are independent complex Gaussian
- ensemble average BER is

$$\overline{P}_b(e) = \frac{1}{2}\sum_{k=1}^{L} \prod_{\substack{i=1 \\ i \neq k}}^{L} \frac{\overline{\gamma}_k}{\overline{\gamma}_k - \overline{\gamma}_i}\left[1 - \sqrt{\frac{\overline{\gamma}_k}{1+\overline{\gamma}_k}}\right]$$

Large signal-to-noise ratios

$$\overline{P}_b(e) \rightarrow \binom{2L-1}{L} \prod_{i=1}^{L} \frac{1}{4\overline{\gamma}_k}$$

Autocorrelation and crosscorrelation of code sequences

- Optimum diversity gain: good autocorrelation
- Optimum suppression of interference:
  good crosscorrelation

1) Maximal Length PN-sequences

- Generated by shift register of length m
- Contains 2^m - 1 chips
- Different codes are time-shifted versions of each other
- Two sequences of odd length cannot be truely orthogonal
- Good autocorrelation

- Often used, particularly for peer-to-peer communication

2) **Walsh Hadamard codes**

- Length 2^m

- Truely orthogonal

$$C_0 = [1] \qquad C_{n+1} = \begin{bmatrix} C_n & C_n \\ C_n & -C_n \end{bmatrix} \quad \textbf{(16)}$$

- Useful for synchronous downlink transmission

- Poor spectral characteristics

  Code 0: {1, 1, 1,....., 1}: no bandspreading

- Can be multiplied by PN-sequence to improve sprectral spreading

Spectrum Efficiency

Can a synchronous downlink "Code Division Multiplexing" link with spread factor N handle N simultaneous users?

- In a perfect LTI nondispersive AWGN channel: "Yes"

- In a dispersive multipath channel: "May be"
  - Walsh-Hadamard coding ensures orthogonality of direct-line-of-sight signals.
  - Delay spread erodes orthogonality, but offers diversity.

How does the capacity depend on the delay profile?

Additional fingers introduce more interference, many existing studies assumpted i.i.d. paths and random codes.

In an indoor system, can we enhance capacity by adding another base station?

Sometimes NOT!

Channel model:

- Exponential delay profile with Line-of-Sight
- Self interference adds 'coherently':
  BER = Q(√Coherent/Incoherent Noise)
  - Self interference (SI) cannot be modelled as 'noise'
  - SI adds to the sigma of Rician wanted compenent
  - SI is constant during a packet time
- Interference from all other users (MUI) arrives over the same channel
- Synchronous Multi-User Interference (MUI) components cancel
- Optimum Receiver: Cancel Delayed MUI Interference

DS-CDMA for Indoor High-Speed Downlink

- A fully loaded DS-CDMA system may work if LOS is strong enough and the delay spread small enough.

- Adding a second base station in the same room may reduce performance.