Outage Performance of Cellular Networks for Wireless Communications

Abstract Cellular frequency reuse is known to be an efficient method to allow many wireless telephone subscribers to share the same frequency band. However, frequency reuse laeds to mutual interference among co-channel cells. We review the technique for computing and modelling link performance in cellular networks.

1 Introduction

A crucial aspect in the evaluation and planning of radio networks is the computation of the effect of co-channel interference in radio links. The amount of interference that can be tolerated determines the required separation distance between co-channel cells and therefore also the efficiency of the network. The link performance of cellular telephone networks was first studied around 1980 by Gosling [4], French [5] and Cox [6]. Initial analyses were limited to outage probabilities in continuous wave (CW) voice communication, taking into account path loss and flat (frequency non-selective) Rayleigh fading. In the 1980's the technique for computing outage probabilities was refined step by step, see e.g. [1 - 13], considering among other things shadowing [3], multiple interfering signals cumulating coherently [5] or, more realistically, incoherently [6], the modulation technique and error correction method [14], and more recently the presence of a dominant line-of-sight propagation path, as it occurs in micro-cellular networks [8, 9, 14]. The stochastic occupation of nearby co-channel cells according to the traffic laws by Erlang was included in some studies.

This text analyses generic systems, without making specific assumptions about the frequency bands or propagation environments considered. The assumption of a constant channel transfer function during the transmission of a packet of data is appears realistic for the parameters chosen in most modern VHF or UHF wireless networks. A second assumption, considering a flat transfer function over the transmit bandwidth, is reasonable as long a the symbol time is longer than, say, ten times the rms delay spread. In systems with large cells and high bit rates as in GSM, this assumption may not be satisfied strictly, but the model used here may be sufficiently realistic for the purpose of the paper.

We review the technique for computing and modelling link performance in cellular networks, which is used later on as a tool to compute the network performance. Section 2 starts with a statistical description of the multipath radio channel. Section 3 shows that Laplace Transforms facilitate some of the analyses. Accurate numerical evaluations can nonetheless be lengthy, but section 4 proposes series expansions that can be used in computationally intensive tasks, such as the planning of practical nets from topographical data bases.

2 Radio Channel Characterization

A typical radio channel exhibits multipath reception, which causes fading. We address narrowband systems, that is, we assume that the channel transfer function is virtually constant over the signal bandwidth. This corresponds to the assumption that Intersymbol Interference does not play a major role in the performance of the radio links. A method to include the effect of channel dispersion on outage probabilities was proposed in [19].

The signal amplitude ρ_i and phase received from user i varies randomly with antenna location and carrier frequency. Several statistical models have been proposed to model the stochastic behavior of the signal amplitude and power. Most commonly accepted is Rician fading, which assumes a dominant line-of-sight component and a large set of reflected waves. The instantaneous power p_i received from the i-th user, with $p_i = {}^1/2\rho_i^2$, has the probability density function (pdf)

$$f_{p_i}(p_i \mid \bar{p}_i) = \frac{(1+K)e^{-K}}{\bar{p}_i} \exp\left\{-\frac{1+K}{\bar{p}_i} p_i\right\} I_0\left(\sqrt{4K(1+K)\frac{p_i}{\bar{p}_i}}\right). \quad \textbf{(1)}$$

where the Rician K-factor is defined as the ratio of the power in the dominant component and the scattered (multipath) power, $\bar{p}_i$ is the total local-mean power in the dominant and scattered waves, and $I_0(.)$ denotes the modified Bessel function of the first kind and order zero. In the special case that the dominant component is zero ($K = 0$), Rayleigh fading occurs, with an exponentially distributed power with mean $\bar{p}_i$.

Another experimentally verified model for multipath reception is Nakagami fading [15, 16]. In this case the instantaneous power has the gamma pdf

$$f_{p_i}(p_i \mid \bar{p}_i) = \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{p}_i}\right)^m p_i^{m-1} \exp\left\{-\frac{m p_i}{\bar{p}_i}\right\}. \quad \textbf{(2)}$$

where $\Gamma(m)$ is the gamma function, with $\Gamma(m + 1) = m!$ for integer m. The parameter m is called the 'shape factor' of the distribution. In the special case that $m = 1$, Rayleigh fading is recovered, while for larger m the fluctuations of the signal strength are less, compared to Rayleigh fading. For ease of analysis, the Nakagami model is sometimes used to approximate the pdf of the power of a Rician fading signal [16]. Matching the first and second moments of the Rician and Nakagami pdfs gives

$$m = \frac{K^2+2K+1}{2K+1} \quad \textbf{(3)}$$

which tends to $m = K/2$ for large K. However section 4 shows that in contrast to common belief, this approximation is less suitable to model deep signal fades of the wanted signal. The Nakagami model is nonetheless relevant as it models the received signal after m-branch diversity with Maximum Ratio Combining. In a Rayleigh-fading channel, this signal becomes m-Nakagami [1, 13].

A second propagation effect is shadowing, resulting in a slow variation of the local-mean power as the antenna moves over distances larger than a few meters. Measurements indicate that the received local-mean power converted into logarithmic values, such as dB or neper, has a normal distribution. The local-mean power $\bar{p}_i$ in absolute units (e.g. watts) thus has the log-normal pdf

$$f_{\bar{p}_i}(\bar{p}_i) = \frac{1}{\sqrt{2\pi}\,\sigma_s \bar{p}_i} \exp\left\{-\frac{1}{2\sigma_s^2}\ln^2\left(\frac{p_i}{\bar{\bar{p}}_i}\right)\right\}, \quad \mathbf{(4)}$$

where σ_s is the logarithmic standard deviation of the shadowing and $\bar{\bar{p}}_i$ is the logarithmic area-mean power of the i-th user. A typical mobile channel suffers from both shadowing and either Rician or Nakagami fading.

The area-mean power $\bar{\bar{p}}_i$ can be estimated from the distance r_i between the i-th terminal and the base station. The most simple path loss model used for analysis of generic radio systems is

$$\bar{\bar{p}}_i = r_i^{-\beta} \quad \mathbf{(5)}$$

where β is on the order of 2 to 5. Harley [17] suggested

$$\bar{\bar{p}}_i = r_i^{-2}\left(1 + \frac{r_i}{r_g}\right)^{-2} \quad \mathbf{(6)}$$

to improve the accuracy for short-range micro-cellular propagation, where r_g is a turnover distance, often in the range 100 m to 500m.

3 Method for Link Evaluation

In certain situations [1 - 13], it is a sufficiently good approximation to assume that a message is received successfully if and only if the signal-to-interference-plus-noise ratio exceeds a certain threshold z. Typically z is on the order of 2 to 10 (3 to 10 dB). Assuming constant received power during a packet transmission time, the probability that the wanted signal power p_0 sufficiently exceeds the joint interference plus noise power p_t is

$$\Pr\left(p_0 > z p_t\right) = \int_{0-}^{\infty} f_{p_t}(x) \int_{zx}^{\infty} f_{p_0}(y)\, dy\ dx \quad \textbf{(7)}$$

where we insert the appropriate pdfs of received signal power as presented in section 2. The joint interference signal p_t is the incoherent sum of multiple individual signals. For independent fading, the pdf of the joint interference power is the convolution of the pdf of individual interference powers.

In the special case of a Rayleigh-fading wanted signal, its pdf of signal power is an exponential one. Hence, for probabilities conditional on the local-mean $\bar{p}_0$, the integral over y can be solved analytically. An elegant mathematical framework has been developed by interpreting the result as a Laplace transform of the pdf of joint interference power. For a wanted signal subject to Rayleigh fading, this probability can be expressed in the form [1, 10, 11, 18]

$$\Pr\left(p_0 > z p_t \middle| \bar{p}_0\right) = \mathfrak{L}\left\{f_{p_t}; s\ \right\}\Big|_{s=\frac{z}{\bar{p}_0}} \quad \textbf{(8)}$$

where $\mathfrak{L}\{f,s\}$ denotes the one-sided Laplace transform of the function f at the point s. We will now show that this approach can be applied to a Rician-fading wanted signal, using the series expansion

$$I_0(z) = \sum_{n=0}^{\infty} \frac{1}{(n!)^2}\left(\tfrac{1}{4}z^2\right)^n \quad \textbf{(9)}$$

for the modified Bessel function I_0. This gives

$$\begin{aligned}\Pr(p_0 > z p_t | \bar{p}_0) &= \sum_{n=0}^{\infty}\sum_{k=0}^{n}\int_{0-}^{\infty} \exp\left\{-K - zx\frac{K+1}{\bar{p}}\right\}\frac{K^n}{n!}\left(zx\frac{K+1}{\bar{p}}\right)^k f_{p_t}(x)dx \\ &= \sum_{n=0}^{\infty}\frac{K^n}{n!}e^{-K}\sum_{k=0}^{n}\frac{s^k}{k!}\int_{0-}^{\infty} x^k s^{-sx} f_{p_t}(x)dx\end{aligned} \quad \textbf{(10)}$$

Using the properties of the Laplace Transform, this can be written as the series

$$\Pr(p_0 > z p_t | \bar{p}_0) = e^{-K}\sum_{n=0}^{\infty}\sum_{k=0}^{n}(-1)^k\frac{K^n}{n!}\ \frac{s^k}{k!}\ \frac{d^k}{ds^k}\mathfrak{L}\left\{f_{p_t}(x), s\right\} \quad \textbf{(11)}$$

For a Nakagami-fading wanted signal, a similar method has been proposed by [12, 13]. Inserting the gamma pdf, one obtains

$$\begin{aligned}\Pr(p_0 > zp_t|\bar{p}_0) &= \sum_{n=0}^{m-1} \frac{s^i}{i!}\int_{0-}^{\infty} x^i e^{-sx} f_{p_t}(x)dx \\ &= \sum_{n=0}^{m-1} \frac{s^i}{i!}\frac{d^i}{ds^i}\mathfrak{L}\left\{f_{p_t}(x);s\right\}\end{aligned} \quad \textbf{(12)}$$

We conclude that in both cases, this probability can be expressed in the generalized form

$$\Pr\left(p_0 > zp_t\middle|\bar{p}_0\right) = \sum_{i=0}^{\infty} a_i s_0^i \frac{(-1)^i}{i!} \frac{d^i}{ds^i}\mathfrak{L}\left\{f_{p_t};s\right\}\Big|_{s=s_0} \quad \textbf{(13)}$$

where Table 1 gives the appropriate coefficients a_i and argument s_0.

Table 1: coefficients a_i and argument s_0 for link success probability ?

Channel Fading	a_i	s_0
Rayleigh	$\begin{cases} a_0 = 1 \\ a_i = 0 \text{ for } i = 1,2,\ldots \end{cases}$	$\frac{z}{\bar{p}_0}$
Rician	$\begin{cases} a_0 = 1 \\ a_i = 1 - e^{-K}\sum_{n=0}^{i-1}\frac{K^n}{n!} \end{cases}$	$\frac{z(K+1)}{\bar{p}_0}$
m-Nakagami	$\begin{cases} a_i = 1 \text{ for } i = 0,1,\ldots,m-1 \\ a_i = 0 \text{ for } i = m, m+1,.. \end{cases}$	$\frac{mz}{\bar{p}_0}$

Typically, terminals with bursty traffic transmit discontinuously, say with probability $\Pr(i_{ON}) = P_{on}$, to minimize interference to other users. Taking this activity factor into account, the Laplace image of the pdf of interference power becomes after some straightforward mathematical manipulations [1],

$$\mathfrak{L}\left\{f_{p_0};s\right\} = 1 - \Pr(i_{ON})\left[1 - \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-x^2)dx}{1 + s\bar{\bar{p}}_i\exp\left(\sqrt{2}x\sigma_s\right)}\right] \quad \textbf{(14)}$$

The Laplace Transform of the pdf of joint interference power, as needed in (13), is the product of the Laplace transforms of individual interfering pdfs, each of the form of (14). The effect of (man-made) noise can also be modelled as a Laplace image [19].

If the wanted signal is subject to both multipath fading and shadowing, area-mean probabilities are obtained by averaging the above conditional probabilities over the lognormal local-mean power of the wanted signal (4). The probability of successful transmission, given the area-mean power, or equivalently, given the propagation distance r_0, is

$$Q(r_0) = \Pr(p_0 > z p_t | r_0) = \int_0^{\infty} \Pr\left(p_0 > z p_t | \bar{p}_0\right) f_{\bar{p}_0}(\bar{p}_0 | r_0) d\bar{p}_0 \quad (15)$$

where the probability in the integrand is of the form of (3). The outage probability equals 1 - *Q(r)*.

4 Practical expressions for outage probability in telephone nets

The above method to find the probability of a signal outage can be significantly faster than brute-force averaging over all pdfs of multipath fading and shadowing, of the wanted and all interfering signals. In hexagonal networks, this may require a 14 fold integration. To the author's best knowledge, no paper endeavors such numerical evaluation. Authors either use approximate methods, particularly the one by Schwartz and Yeh [20], or, as is increasingly often performed in recently published work, use Laplace Techniques for Nakagami or Rayleigh channels, e.g. [10 - 13]. Nonetheless the Laplace method can be time-consuming and a faster method can be developed for the planning practical networks.

The probability of a signal outage at large signal-to-interference ratios ($\bar{p}_0 >> \mathrm{E}p_t$) is found from the behavior of the Laplace expression at small values for s. Expanding the Laplace transform into a McLaurin series gives

$$\mathcal{L}\{f_{p_t}, s\} = 1 - s\mathrm{E}p_t + \frac{s^2}{2!}\mathrm{E}p_t^2 - \cdots . \quad \textbf{(16)}$$

Inserting this in expression ? gives the outage probability

$$\begin{aligned} \Pr(\mathrm{out}) &= 1 - \Pr\left(p_0 > z p_t \middle| \bar{p}_0\right) \\ &= \frac{z}{m!}\frac{\mathrm{E}p_t^m}{\bar{p}_0^m} + 0\left(\frac{\mathrm{E}\ p_t^{m+1}}{\bar{p}_0^{m+1}}\right) \end{aligned} \quad \textbf{(17)}$$

for Nakagami fading with integer m. For Rician fading, we find

$$\Pr(\mathrm{out}) = s e^{-K}\mathrm{E}p_t + (1-K)\frac{s^2}{2}e^{-K}\mathrm{E}p_t^2 + 0\left(\frac{\mathrm{E}\ p_t^3}{\bar{p}_0^3}\right) \quad \textbf{(18)}$$

with s given in Table 1. The results are strikingly different for m larger than one. As the approximation in (3) was based on the first and second moments, it is likely to be most accurate for values close to the mean. Outage probabilities however highly depend on the tail of the pdf for small power of the wanted signal. We conclude that approximating the pdf of a Rician-fading *wanted* signal by a Nakagami pdf is highly inaccurate: Results differ even in first-order. This is due to fact that Rician-fading signals exhibit relatively deep fades with a non-zero probability density for small received power. This is in sharp contrast to vanishing probability density at zero power for a Nakagami-fading signal if m = 2, 3, .. .

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