Orthogonal Frequency Division Multiplexing and Multi-Carrier CDMA

Jean-Paul Linnartz[1,2] & Nathan Yee[1]

[1]Dept. of Electrical Engineering and Computer Science

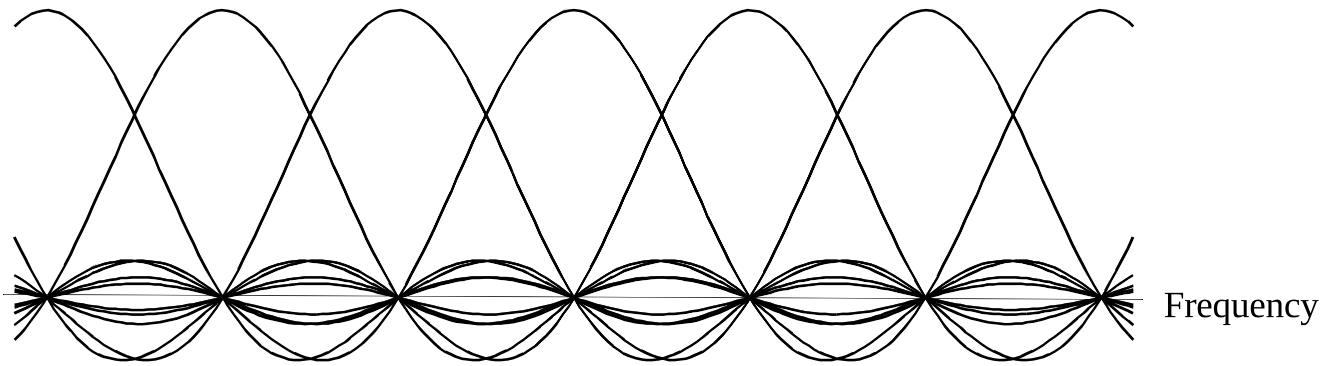
University of California, Berkeley CA 94720-1770 USA

[2]Natuurkundig Laboratorium, Philips Research,

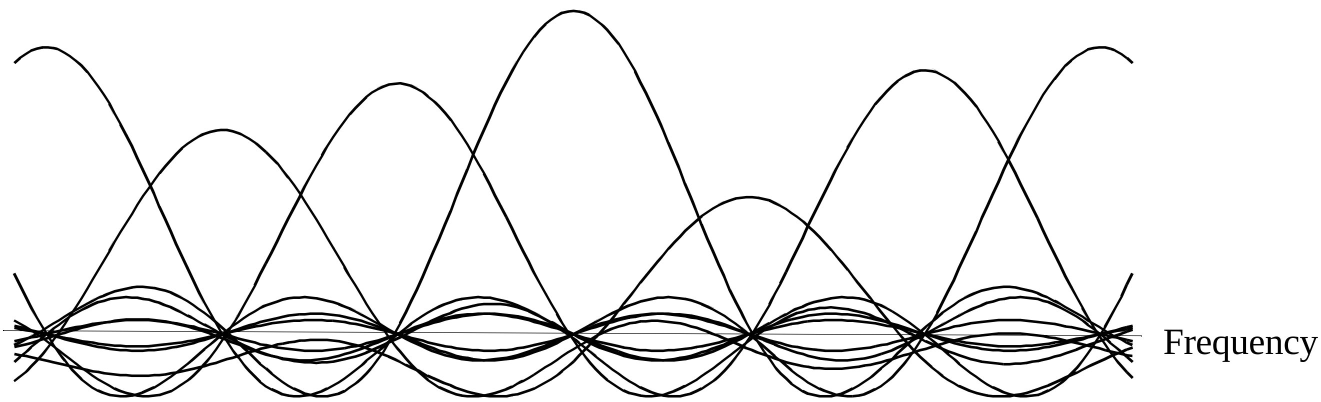
5656 AA Eindhoven, The Netherlands

Multi-Carrier Modulation Signal Spectra

• MCM Transmit Signal Spectrum



• MCM Received Signal Spectrum



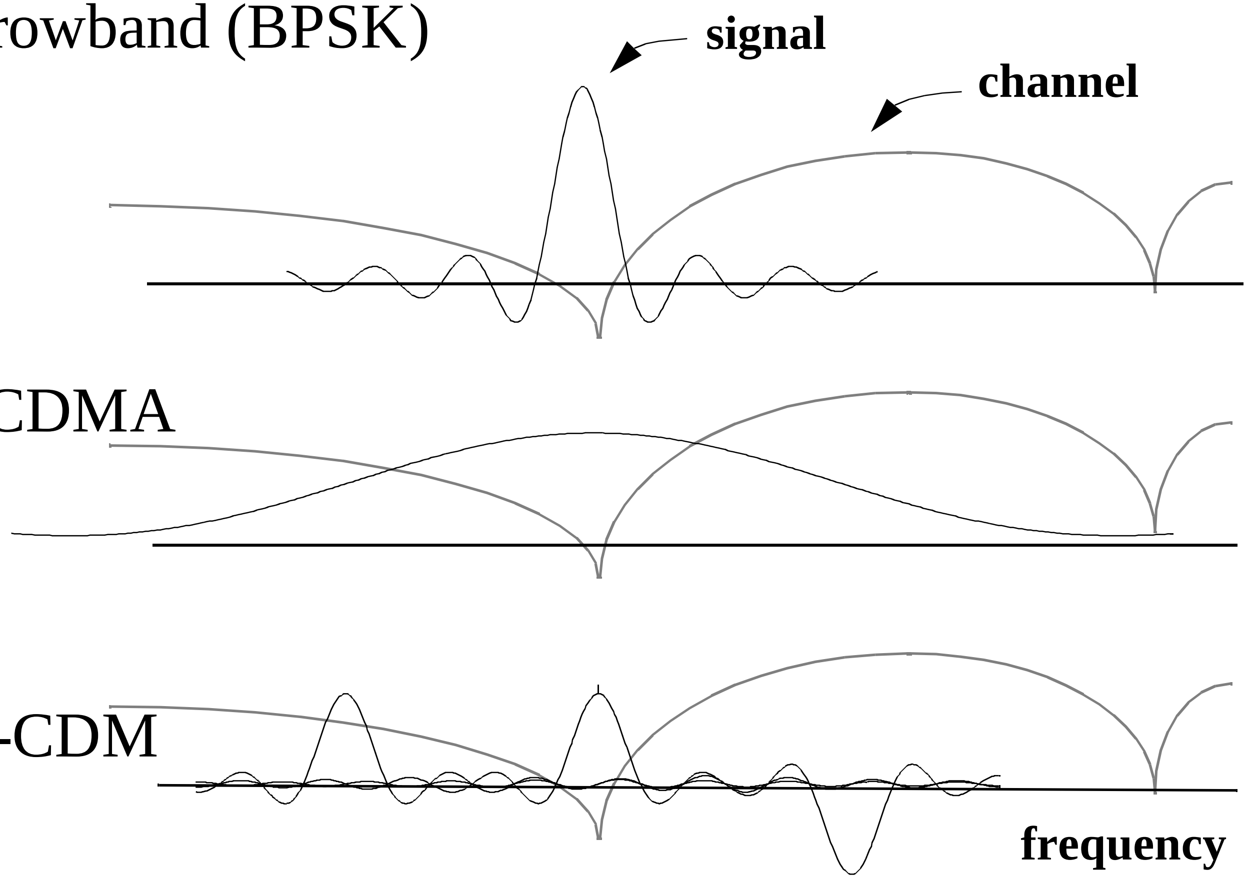
• Subcarrier orthogonality is not eroded by dispersive multi-path propagation if $T_b \gg T_{rms}$ and $f_D \ll 1/T_b$

What is MC-CDMA?

- A digital modulation / multiple access technique
- A combination of OFDM and DS-SS
- A CDMA method using the FFT of DS-SS signals
- A spread spectrum modulation technique in which each bit is modulated on multiple subcarriers with relative phase polarity according to a spreading code

Comparison of Modulation Technique Spectrums

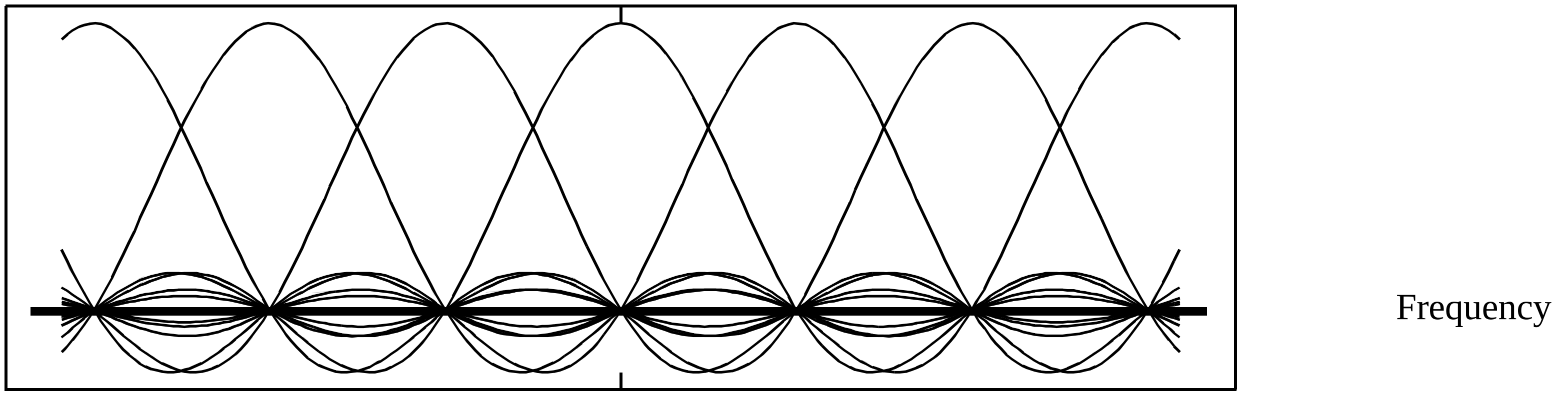
- Narrowband (BPSK)
- SS-CDMA
- MC-CDM



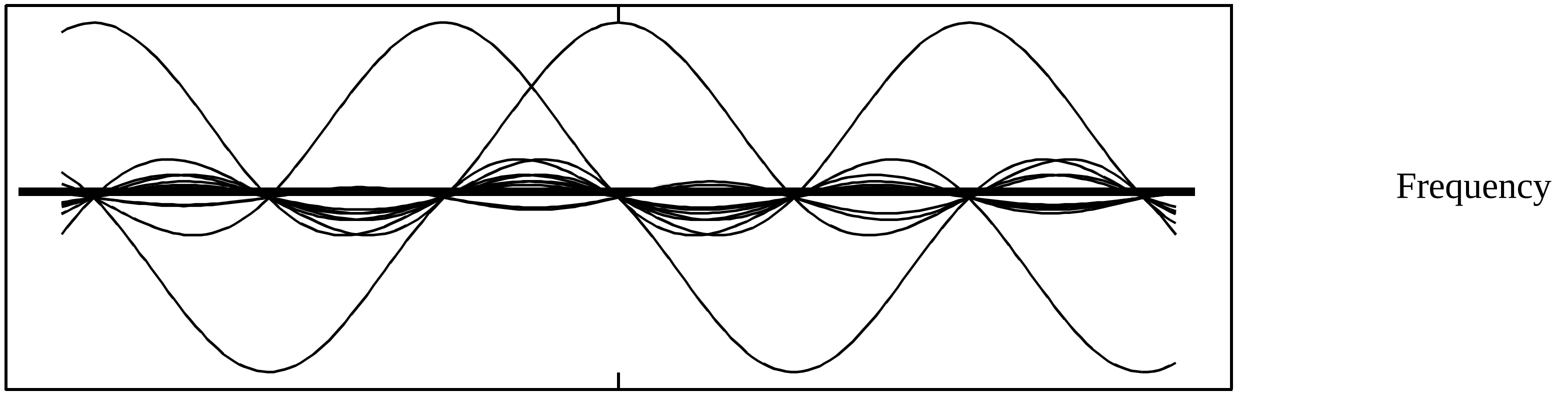
Motivation

- Narrowband susceptible to flat fading in frequency selective channels
- CDMA-SS spreads signal energy over large bandwidth for frequency diversity
- However, wideband signals more sensitive to delay spreads because of inter-chip interference
- Use multi-carrier modulation with narrowband subcarriers
- Lower spreading factor required

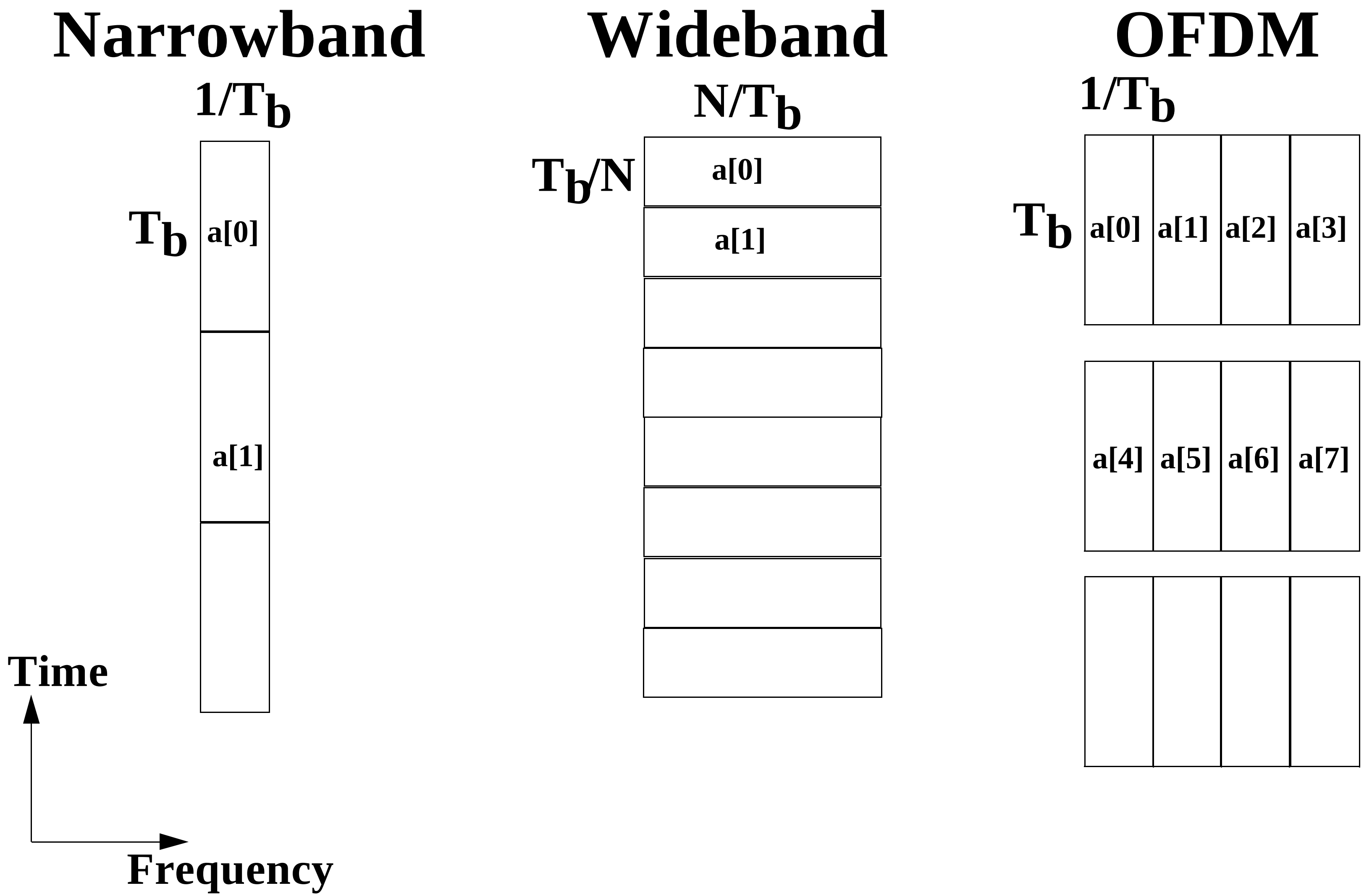
OFDM signal



MC-CDMA signal

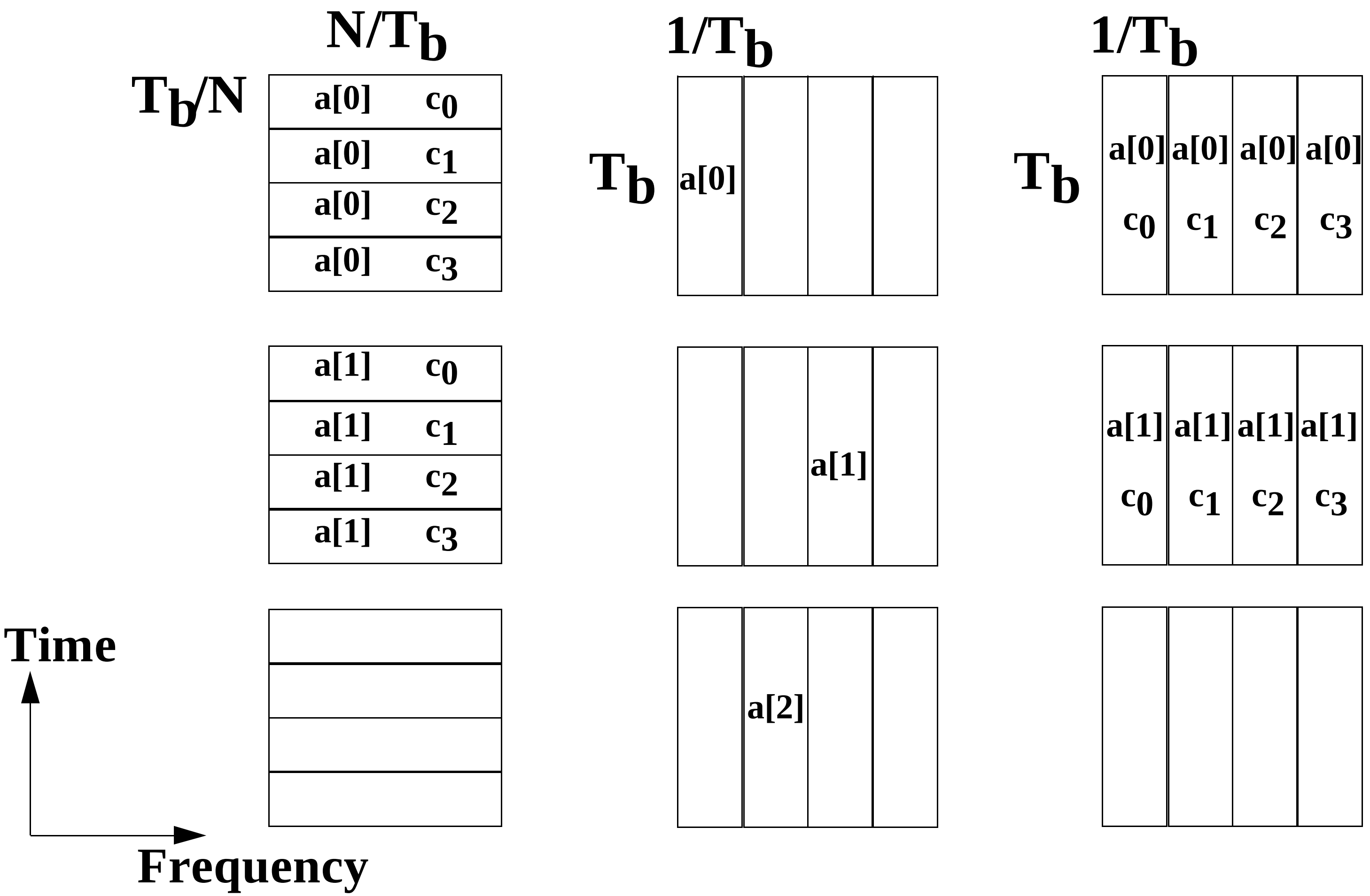


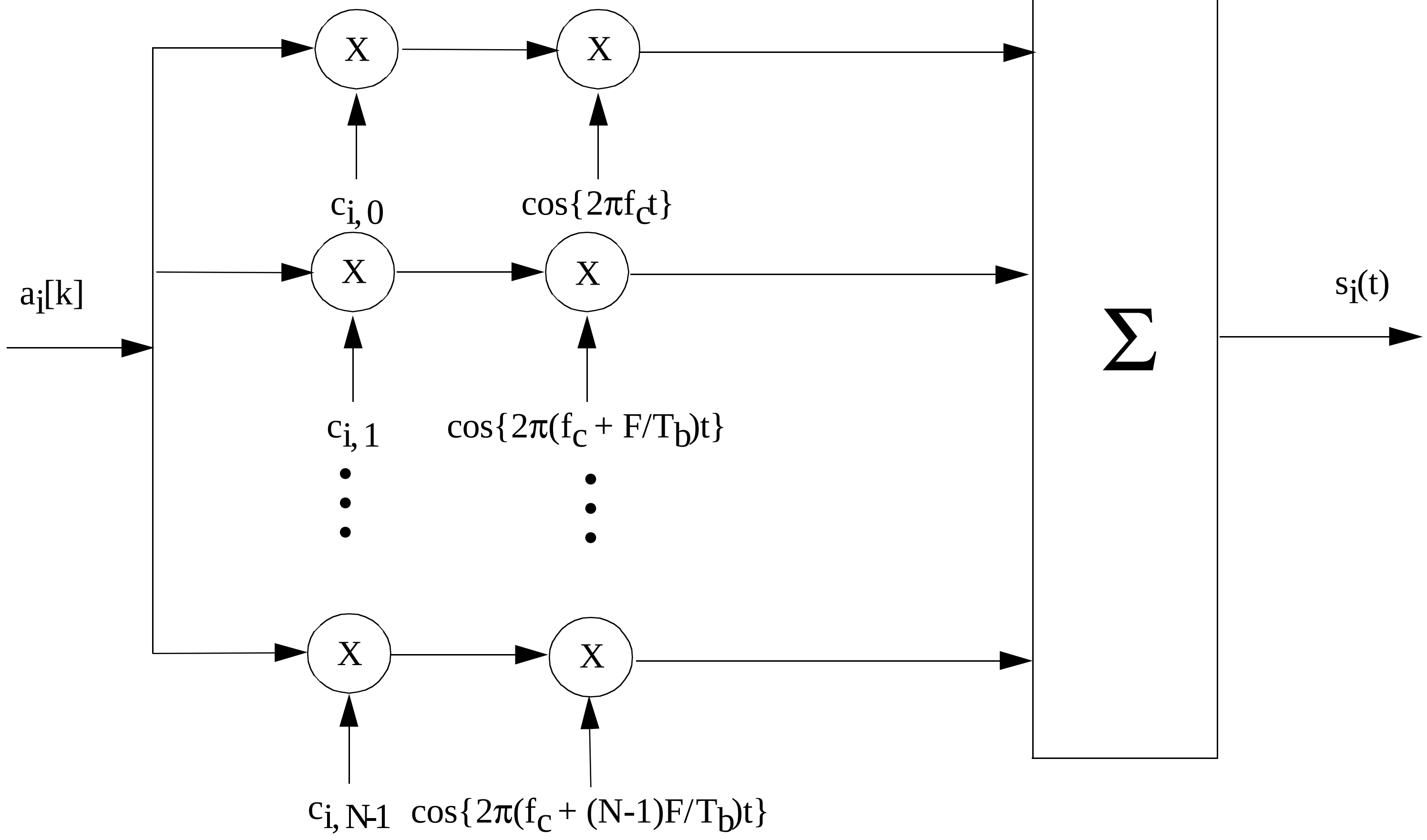
Time - Frequency Diagram



Time - Frequency (cont.)

DS-CDMA FH-SS MC-CDMA





Transmitter Model

Implementation Aspects

- MC-CDMA requires no chip synchronization. It requires only carrier and bit synchronization.
- MC-CDMA may use FFT devices
- Charged-capacitor domain detector circuit?
- MC-CDMA receiver might be simpler than DS-SS receiver with full MRC for all resolvable paths. MC-CDMA does not require multiple despreaders
- Subcarrier amplitude estimation can be simple for the downlink

Synchronization & Estimation in Downlink

Frequency

N	-N	N	-N	$\sum a_m c_{m,1}$
N	-N	N	-N	$\sum a_m c_{m,2}$
N	-N	N	-N	
N	-N	N	-N	
N	-N	N	-N	
N	-N	N	-N	$\sum a_m c_{m,N}$

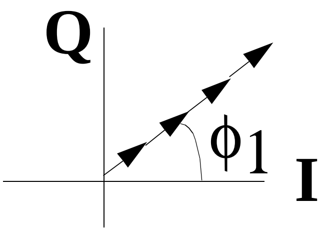
Time

syncword identical for all users **MC-CDMA**

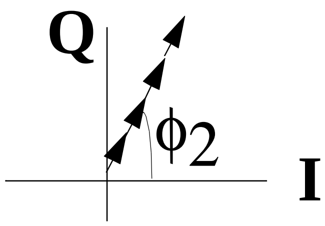
- "Interfering" signals to other users help
- Rapid acquisition of synchronization and channel estimation
- Improvement: Use PN sequence in Frequency Domain

Joint Signal in Downlink

- Interference helps during acquisition



Subcarrier 1



Subcarrier 2

Simplified Channel Model

- Frequency selective channel with

$$\frac{1}{T_b} \ll BW_c \ll \frac{F}{T_b}$$

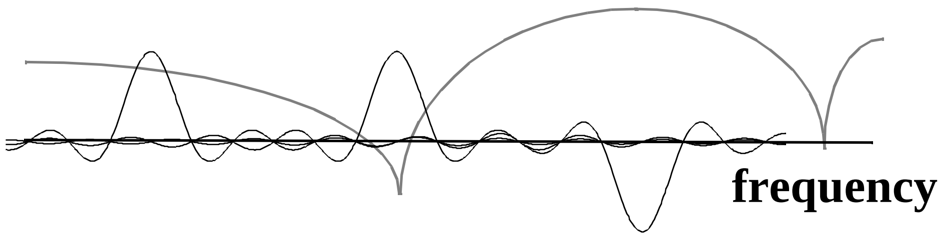
- Narrowband behavior of subcarrier

- $H_m\left(f_c + \frac{i}{T_b}F\right) = \rho_{m,i} e^{j\theta_{m,i}}$

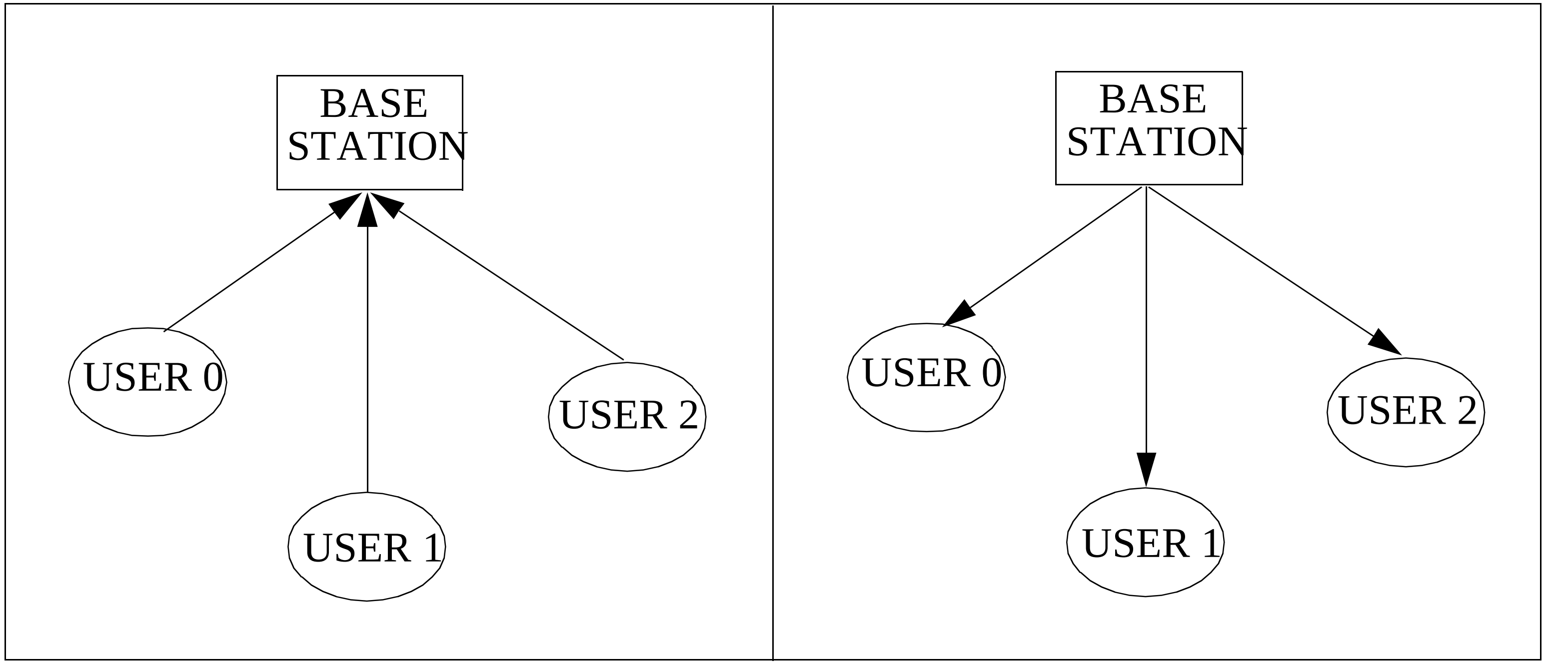
- Rayleigh / Rician amplitude and uniform phase
- IID fading at the subcarriers

Channel

Signal

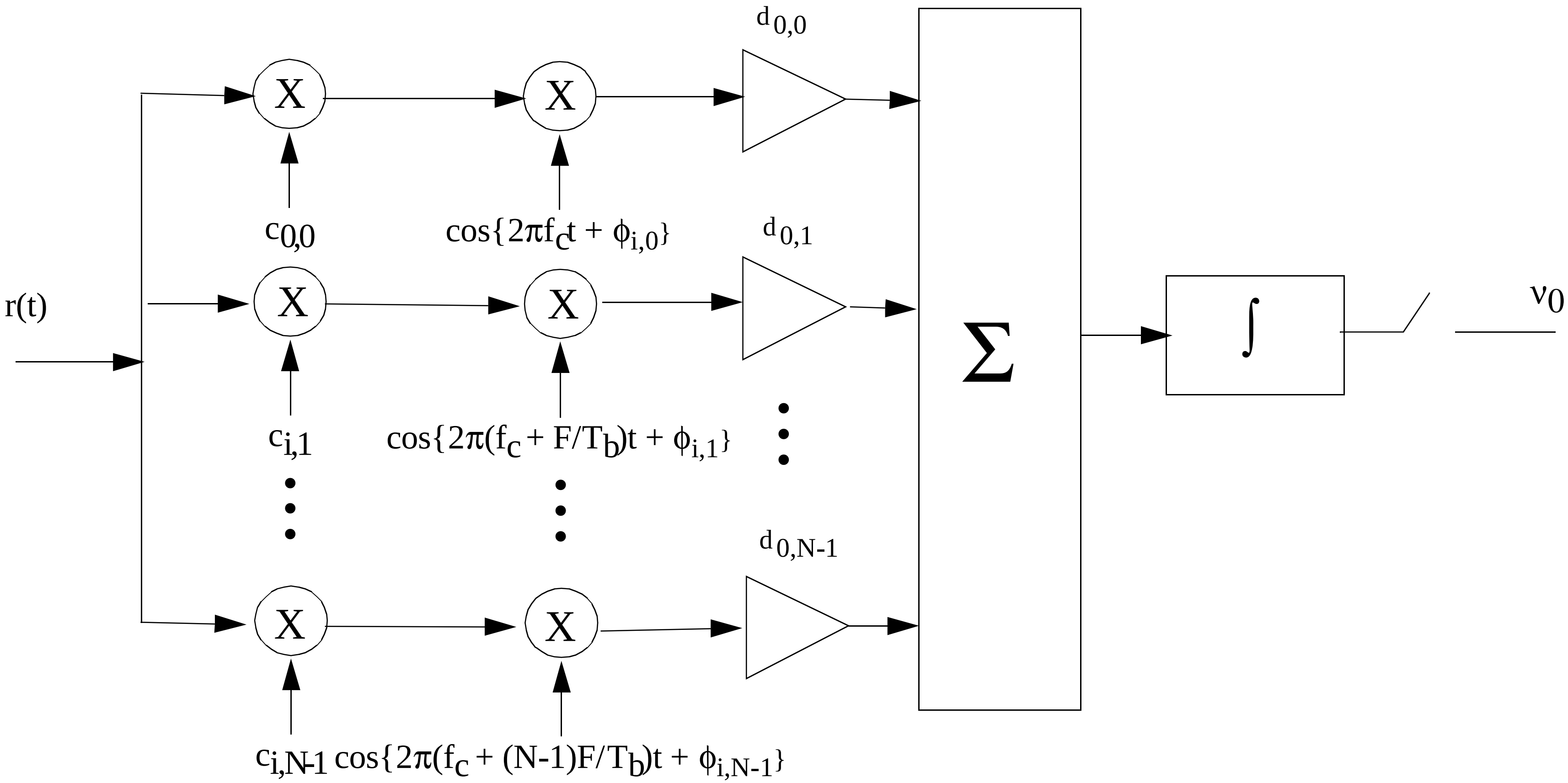

frequency

Uplink vs. Downlink



$$\{\rho_{m,i}, \theta_{m,i}\}_{m=0}^{M-1}$$

$$\rho_{m,i} = \rho_{0,i} \qquad \theta_{m,i} = \theta_{0,i} \qquad \forall m$$



Receiver Model

ASSUMPTIONS

- LARGE NUMBER OF SUBCARRIERS
- FREQUENCY NON-SELECTIVE FADING FOR SUBCARRIER BANDWIDTH (F/T_b)
- IID RAYLEIGH FADING AT SUBCARRIERS OR RANDOM CODE SEQUENCES
- PERFECT PHASE CORRECTION AT RECEIVER FOR WANTED SIGNAL

Analysis of Performance

- Transmitted Signal of the mth user

$$s_m(t) = \sum_{i=0}^{N-1} c_{m,i} a_m[k] \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t\right) p_{T_b}(t - kT_b)$$

$$c_{m,i} \{-1, 1\} \qquad \sum_{i=0}^{N-1} c_{l,i} c_{m,i} = N\delta_{l,m}$$

- Received Signal

$$r(t) = \sum_{m=0}^{M-1} \sum_{i=0}^{N-1} \rho_{m,i} c_{m,i} a_m[k] \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,i}\right) + n(t)$$

- Decision Variable

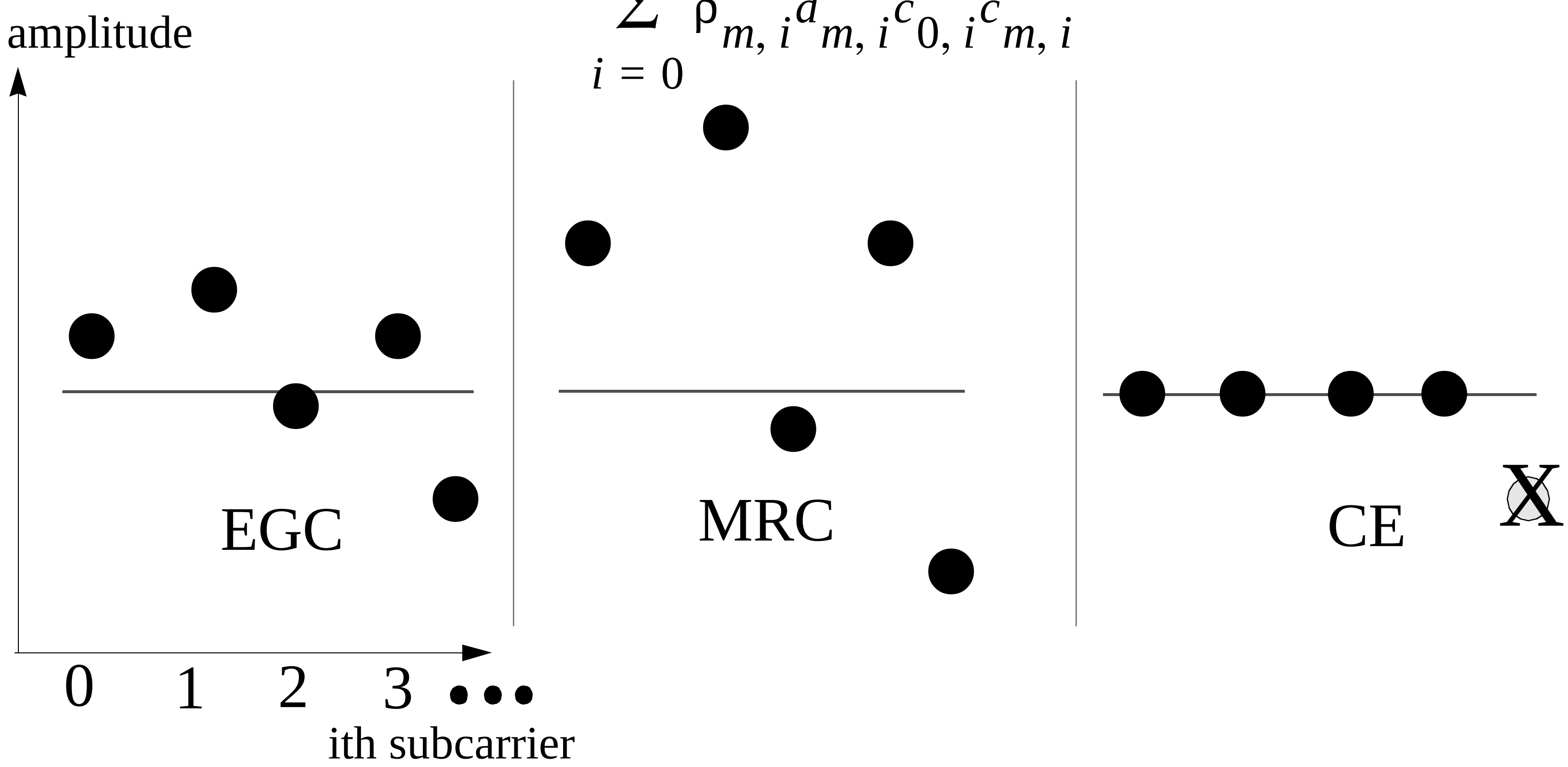
$$\nu_0 = a_0[k] \sum_{i=0}^{N-1} \rho_{0,i} d_{0,i} + \sum_{m=1}^{M-1} a_m[k]$$

$$\times \sum_{i=0}^{N-1} \rho_{m,i} d_{0,i} \cos(\theta_{0,i} - \theta_{m,i}) c_{m,i} c_{0,i} + \eta$$

Receiver Equalization Techniques

- Contribution to decision variable by mth interferer in downlink is proportional to

$$\sum_{i=0}^{N-1} \rho_{m,i} d_{m,i} c_{0,i} c_{m,i}$$



Receiver Equalization Techniques

- Equal Gain Combining (simplicity)

$$d_{0,i} = 1$$

- Maximum Ratio Combining (combats noise)

$$d_{0,i} = \rho_{0,i}$$

- Controlled Equalization (restores orthogonality)

$$d_{0,i} = \frac{1}{\rho_{0,i}} u(\rho_{0,i} - \rho_{THRESH})$$

- Optimum Filtering

$$d_{0,i} = \frac{\rho_{0,i}}{\rho_{0,i}^2 + c}$$

EGC and MRC in Uplink

- Decision Variable for Uplink:

$$\nu_0 = a_0[k] \sum_{i=0}^{N-1} \rho_{0,i} d_{0,i} + \sum_{m=1}^{M-1} a_m[k]$$

$$\times \sum_{i=0}^{N-1} \rho_{m,i} d_{0,i} \cos(\theta_{0,i} - \theta_{m,i})\, c_{m,i} c_{0,i} + \eta$$

- Conditional BER for EGC:

$$Pr\left(error \mid \{\rho_{0,i}\}_{i=0}^{N-1},\ (M-1)\overline{p_m}\right) =$$

$$= \frac{1}{2} erfc\left(\sqrt{\frac{\frac{1}{2}\left(\sum_{i=0}^{N-1} \rho_{0,i}\right)^2 T_b}{(M-1)\overline{p_m} T_b + N N_0}}\right)$$

- Distribution of amplitudes

$$\sum_{i=0}^{N-1} \rho_{0,i} \sim \underset{small\ \ argument}{\overset{NE\rho_{0,i}}{N(\mu, \sigma^2)}}$$

EGC and MRC in Uplink (cont.)

- Average BER for EGC (LLN)

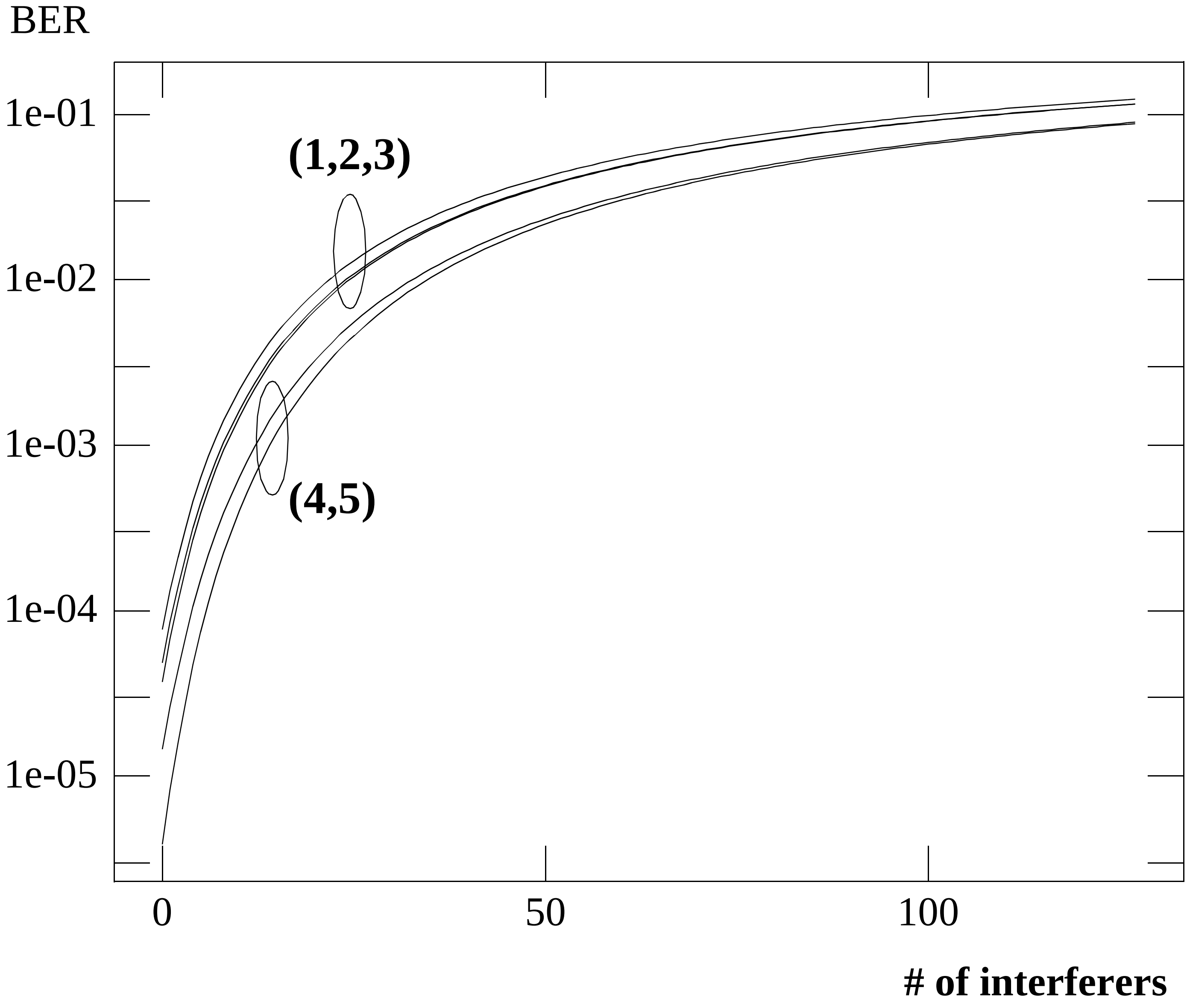
$$BER \cong \frac{1}{2} erfc\left(\sqrt{\gamma \frac{\overline{p_0} T_b}{\frac{(M-1)}{N} \overline{p_m} T_b + N_0}}\right)$$

$$\gamma = \frac{\pi}{4}\left(\frac{e^{-K}}{K+1}\right)\left[(1+K) I_0\left(\frac{K}{2}\right) + K \times I_1\left(\frac{K}{2}\right)\right]^2$$

- Average BER for MRC (LLN)

$$BER \cong \frac{1}{2} erfc\left(\sqrt{\frac{\overline{p_0} T_b}{\frac{(M-1)}{N} \overline{p_m} T_b + N_0}}\right)$$

Uplink BER for EGC & MRC vs. # of Interferers (Rayleigh, SNR=10dB)



BER vs. the # of interferers for EGC using small argument approx. (1), CLT (2), and LLN (3) and MRC exact (4) and LLN (5). N = 128.

EGC and MRC in Downlink

- Decision Variable:

$$\nu_0 = a_0[k]\sum_{i=0}^{N-1}\rho_i d_i + \sum_{m=1}^{M-1} a_m[k]\sum_{i=0}^{N-1}\rho_i d_i c_{m,i} c_{0,i} + \eta$$

$$= a_0[k]\sum_{i=0}^{N-1}\rho_i d_i + \sum_{m=1}^{M-1} a_m[k]\sum_{j=0}^{N/2}\left(\rho_{a_j} d_{a_j} - \rho_{b_j} d_{b_j}\right) + \eta$$

- Average BER for EGC (LLN)

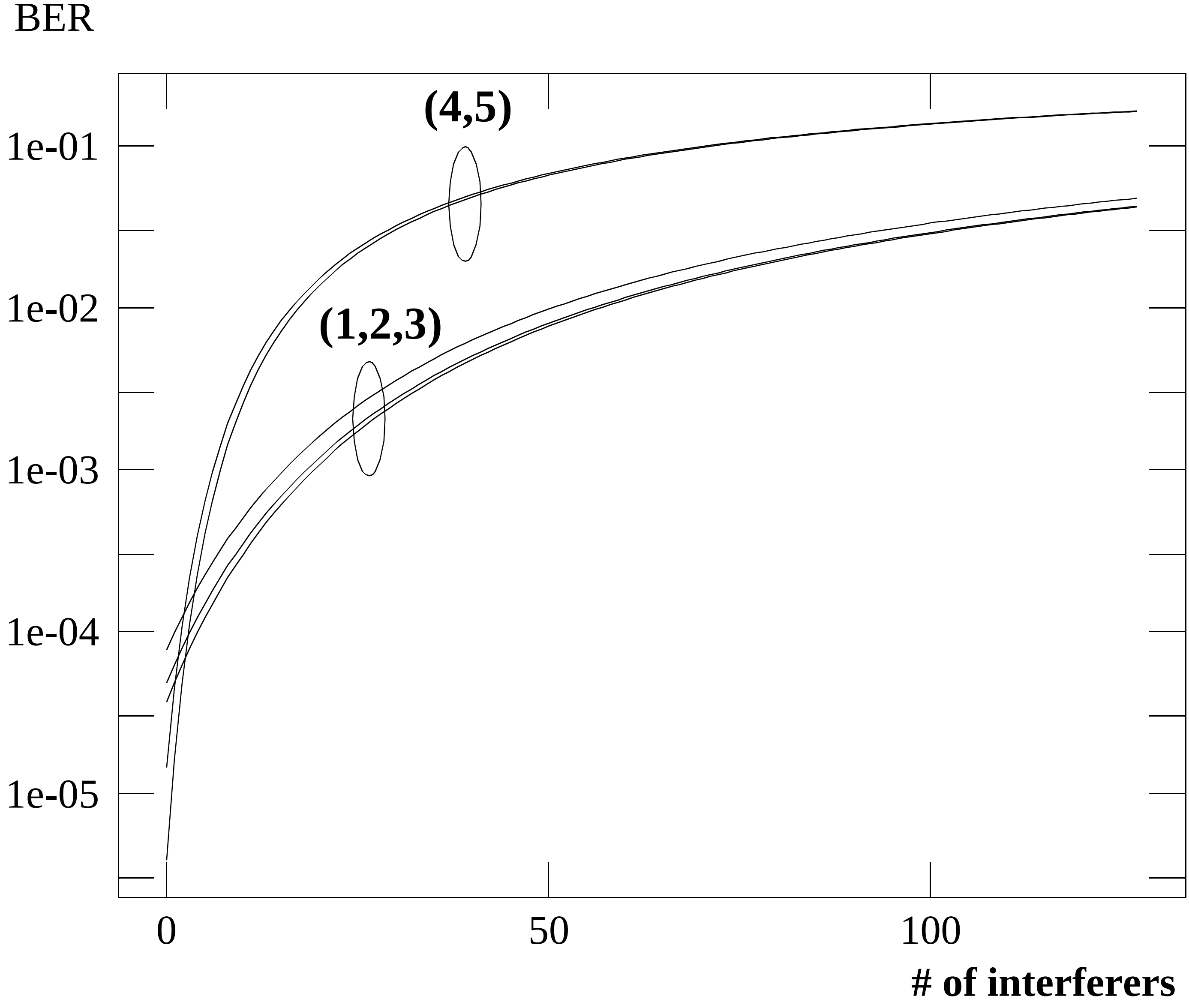
$$BER \cong \frac{1}{2} erfc\left(\sqrt{\frac{\gamma \bar{p} T_b}{2\left(\frac{M-1}{N}\right)[1-\gamma]\bar{p} T_b + N_0}}\right)$$

$$\gamma = \frac{\pi}{4}\left(\frac{e^{-K}}{K+1}\right)\left[(1+K) I_0\left(\frac{K}{2}\right) + K \times I_1\left(\frac{K}{2}\right)\right]^2$$

- Average BER for MRC (LLN)

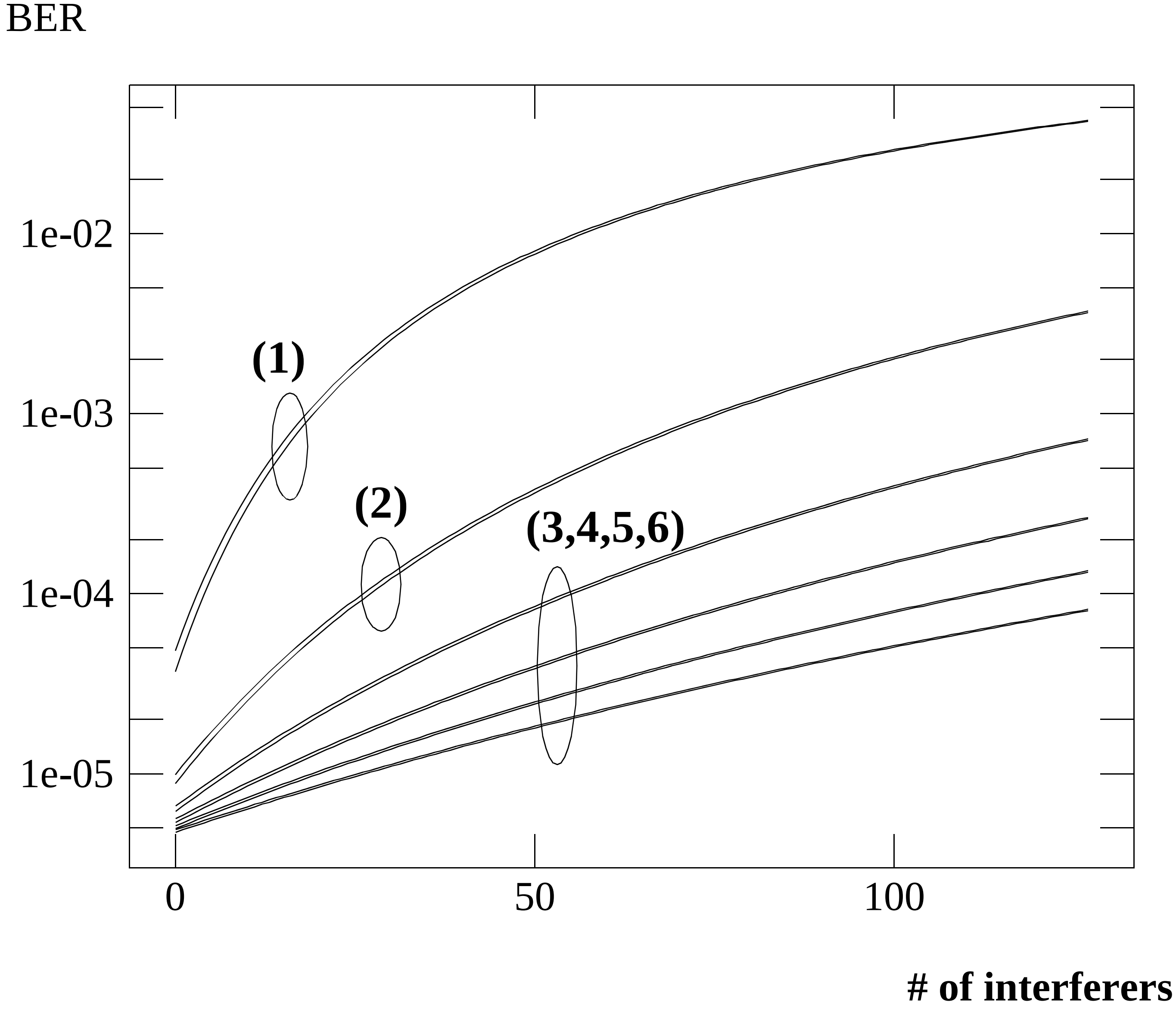
$$BER \cong \frac{1}{2} erfc\left(\sqrt{\frac{\bar{p} T_b}{\left(\frac{M-1}{N}\right)\left[\frac{4K+2}{(K+1)^2}\right]\bar{p} T_b + N_0}}\right)$$

Downlink BER for EGC & MRC vs. # of Interferers (Rayleigh, SNR=10dB)



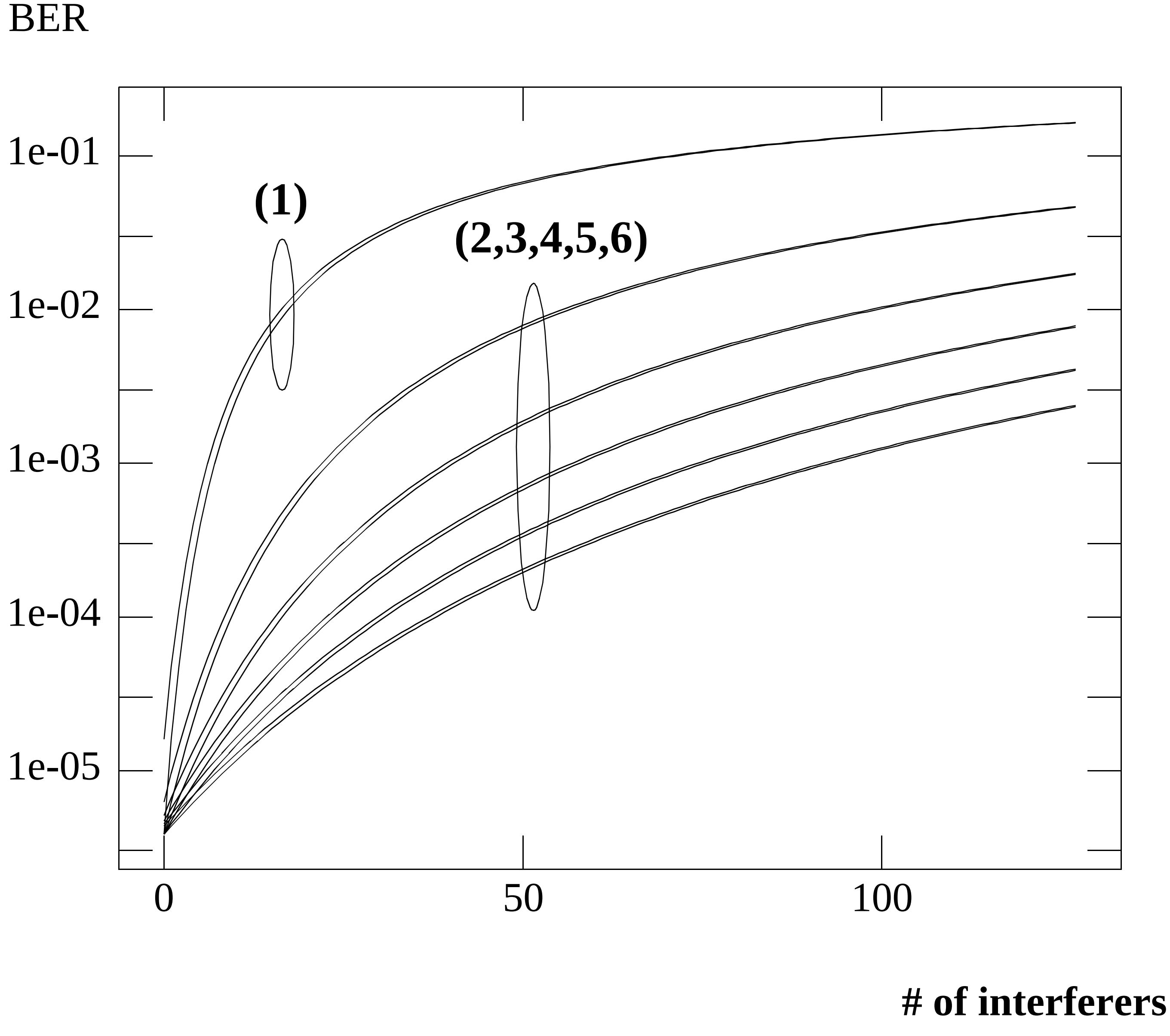
BER vs. the # of interferers for EGC using small argument approx. (1), CLT (2), and LLN (3) and MRC exact (4) and LLN (5). N = 128.

Downlink BER for EGC vs. # of Interferers
(Rician Fading, SNR=10dB)



BER vs. the # of interferers for EGC with Rician Fading using CLT and LLN approximations for (1) K=0, (2) K=5, (3) K=10, (4) K=15, (5) K=20, (6) K=25. N = 128.

Downlink BER for MRC vs. # of Interferers
(Rician Fading, SNR=10dB)



BER vs. the # of interferers for MRC with Rician Fading using CLT and LLN approximations for (1) K=0, (2) K=5, (3) K=10, (4) K=15, (5) K=20, (6) K=25. N = 128.

Controlled Equalization (CE)

$$d_{0,i} = \frac{1}{\rho_{0,i}} u(\rho_{0,i} - \rho_{THRESH})$$

- Reduce interference by normalizing and restoring orthogonality
- Greatest effect in downlink where phase correction of interferers may be performed
- Reduce amplification of noise by discarding subcarriers below threshold

Analysis of CE in Downlink

- Decision variable given n_0 subcarriers are above threshold

$$\nu_0 \big| n_0 = a_0[k]\, n_0 + \sum_{m=0}^{M-1} a_m[k] \sum_{j=0}^{n_0-1} c_{m,j} c_{0,j} + \eta$$

- Interference, n_i:

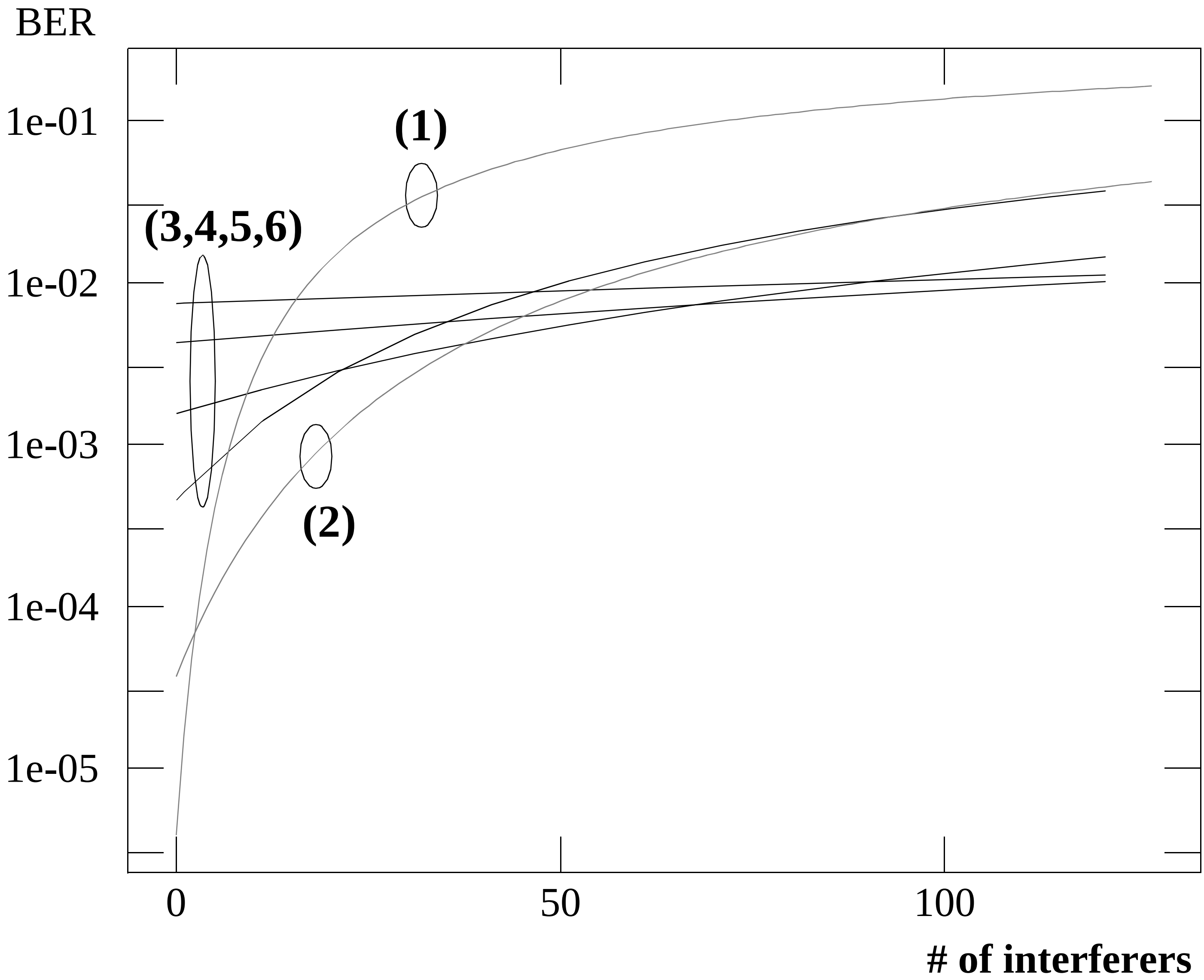
$$p_{\Sigma_m|n_0}\left(\sigma_m \big| n_0\right) = \frac{\binom{N/2}{(n_0+\sigma_m)/2}\binom{N/2}{(n_0-\sigma_m)/2}}{\binom{N}{n_0}}$$

$$p_{n_i|n_0}\left(n_i \big| n_0\right) = \otimes\, p_{\Sigma_m|n_0}\left(\sigma_m \big| n_0\right)$$

- BER

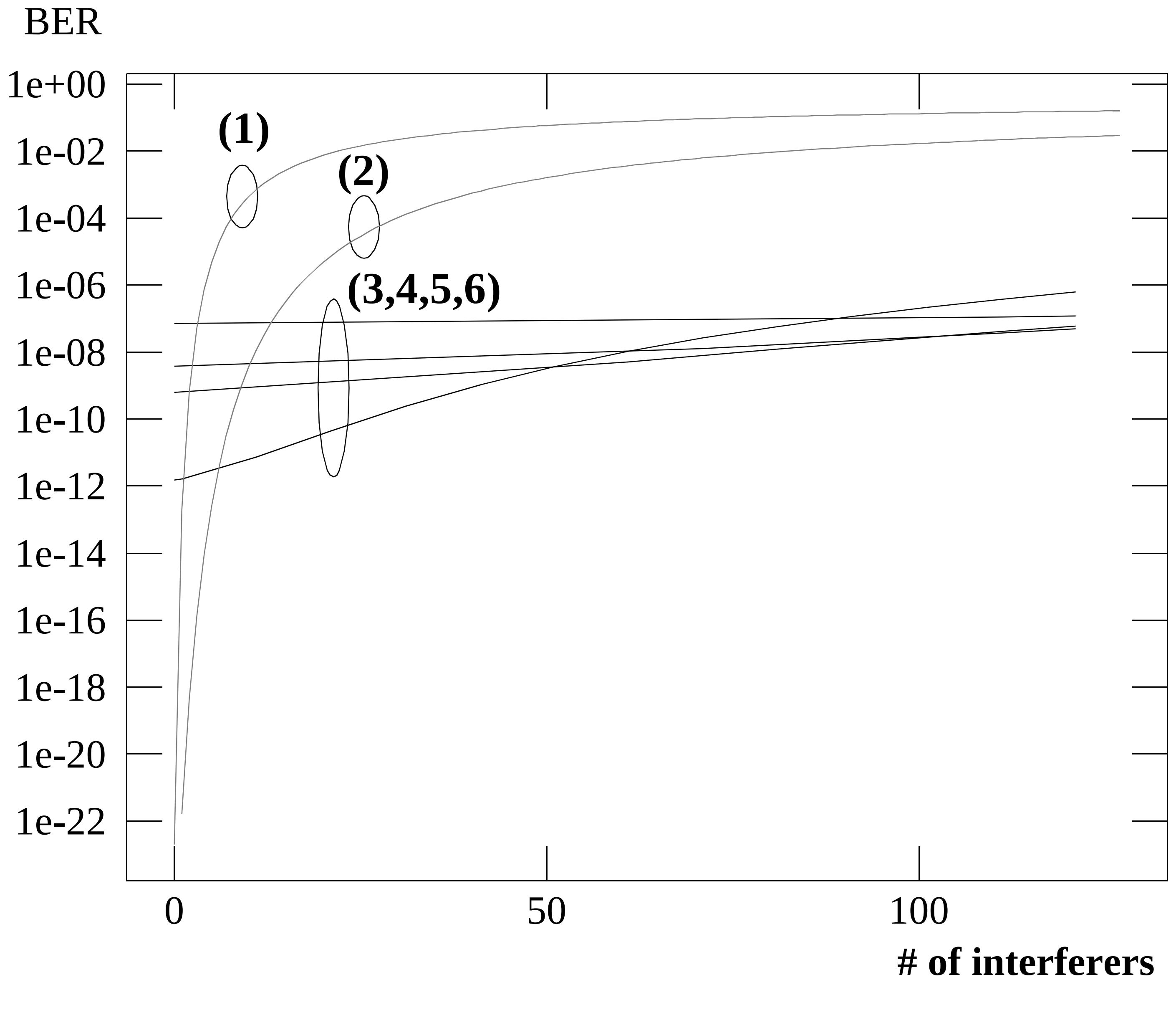
$$P_b(e) = \sum_{n_0=0}^{N} \binom{N}{n_0} p_a^{n_0} (1-p_a)^{N-n_0} \sum_{n_i} p_{n_i|n_0}\left(n_i \big| n_0\right)$$

Downlink BER for CE vs. # of Interferers (Rayleigh, SNR=10dB)



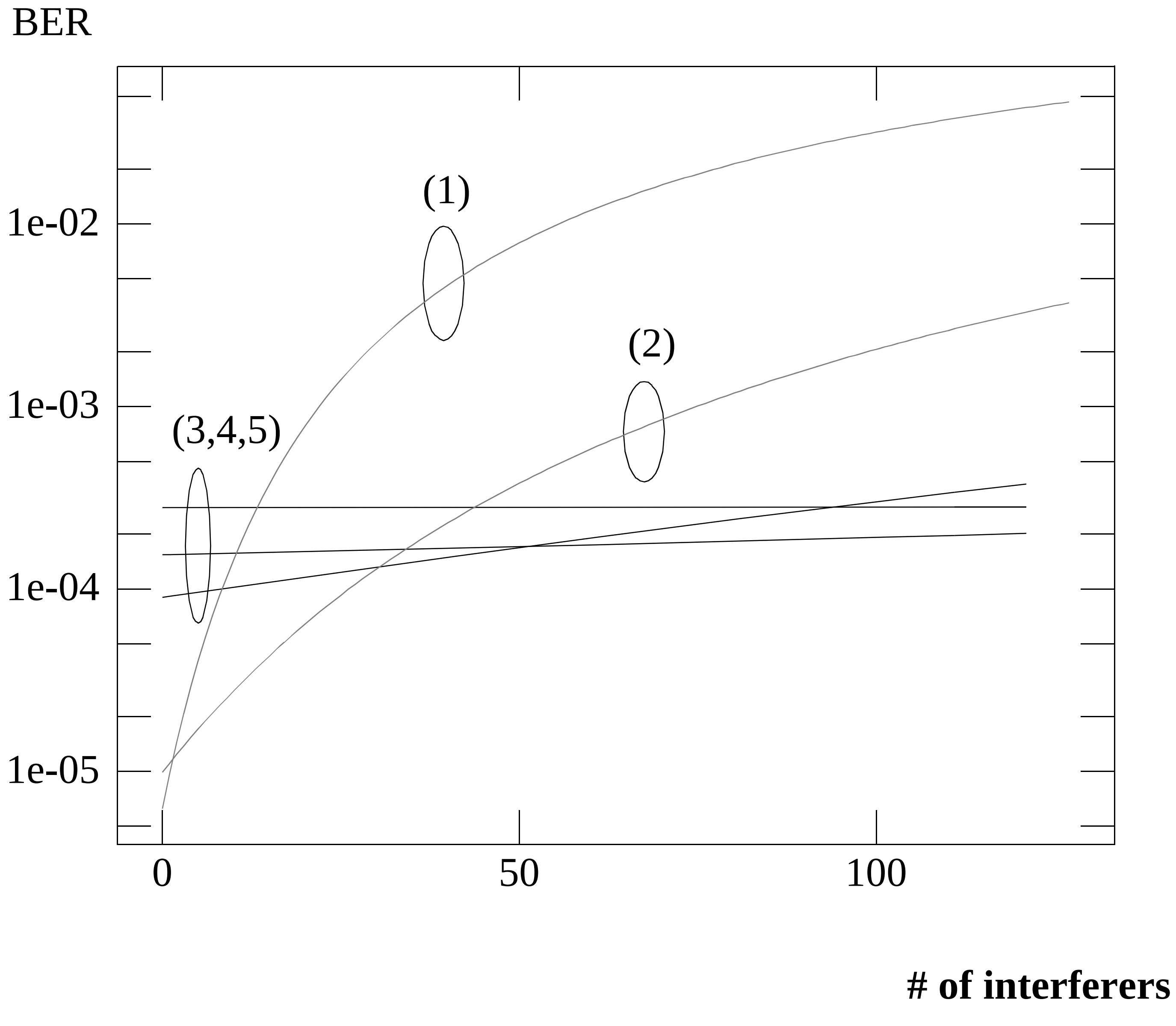
BER vs. the # of interferers for MRC (1), EGC (2), and CE for $p_{min} = 0.006$ (3), $p_{min} = 0.008$ (4), $p_{min} = 0.012$ (5), and $p_{min} = 0.018$. The SNR is 10dB, $\bar{p}_0 = 0.1$, and N = 128.

Downlink BER for CE vs. # of Interferers (Rayleigh, SNR=20dB)



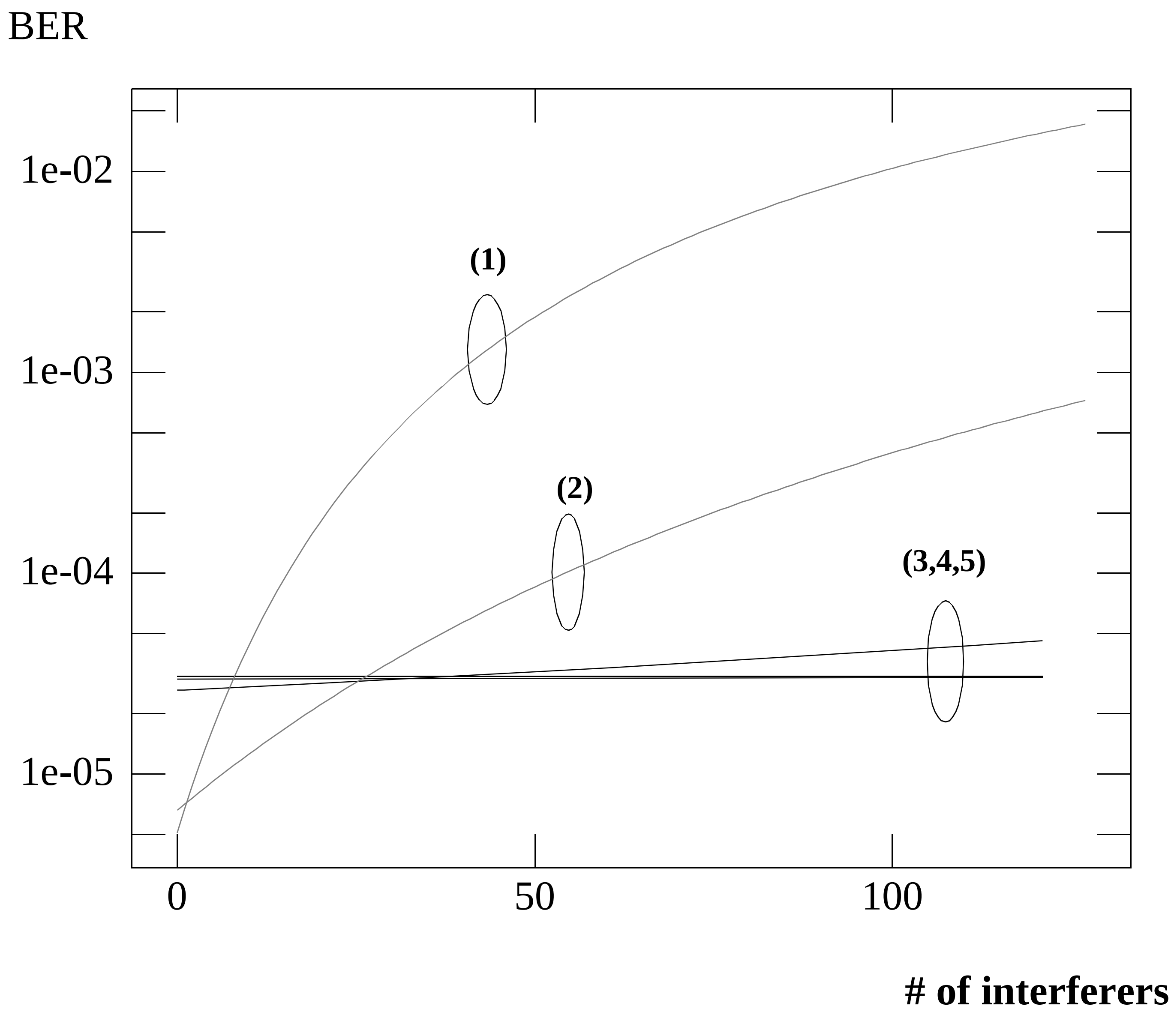
BER vs. the # of interferers for MRC (1), EGC (2), and CE for p_{min} = 0.0008 (3), p_{min} = 0.0015 (4), p_{min} = 0.002 (5), and p_{min} = 0.004 (6). The SNR is 20dB, $\bar{p}_0$ = 0.1, and N = 128.

Downlink BER for CE vs. # of Interferers
(Rician: K=5, SNR=10dB)



BER vs. the # of interferers for MRC (1), EGC (2), and CE for p_{min} = 0.002 (3), p_{min} = 0.008 (4), and p_{min} = 0.014 (5). The SNR is 10dB, $\bar{p}_0$ = 0.1, and N = 128.

Downlink BER for CE vs. # of Interferers
(Rician: K=10, SNR=10dB)



BER vs. the # of interferers for MRC (1), EGC (2), and CE for p_{min} = 0.016 (3), p_{min} = 0.002 (4), and p_{min} = 0.008 (5). The SNR is 10dB, $\bar{p}_0$ = 0.1, and N = 128.

Wiener Filtering

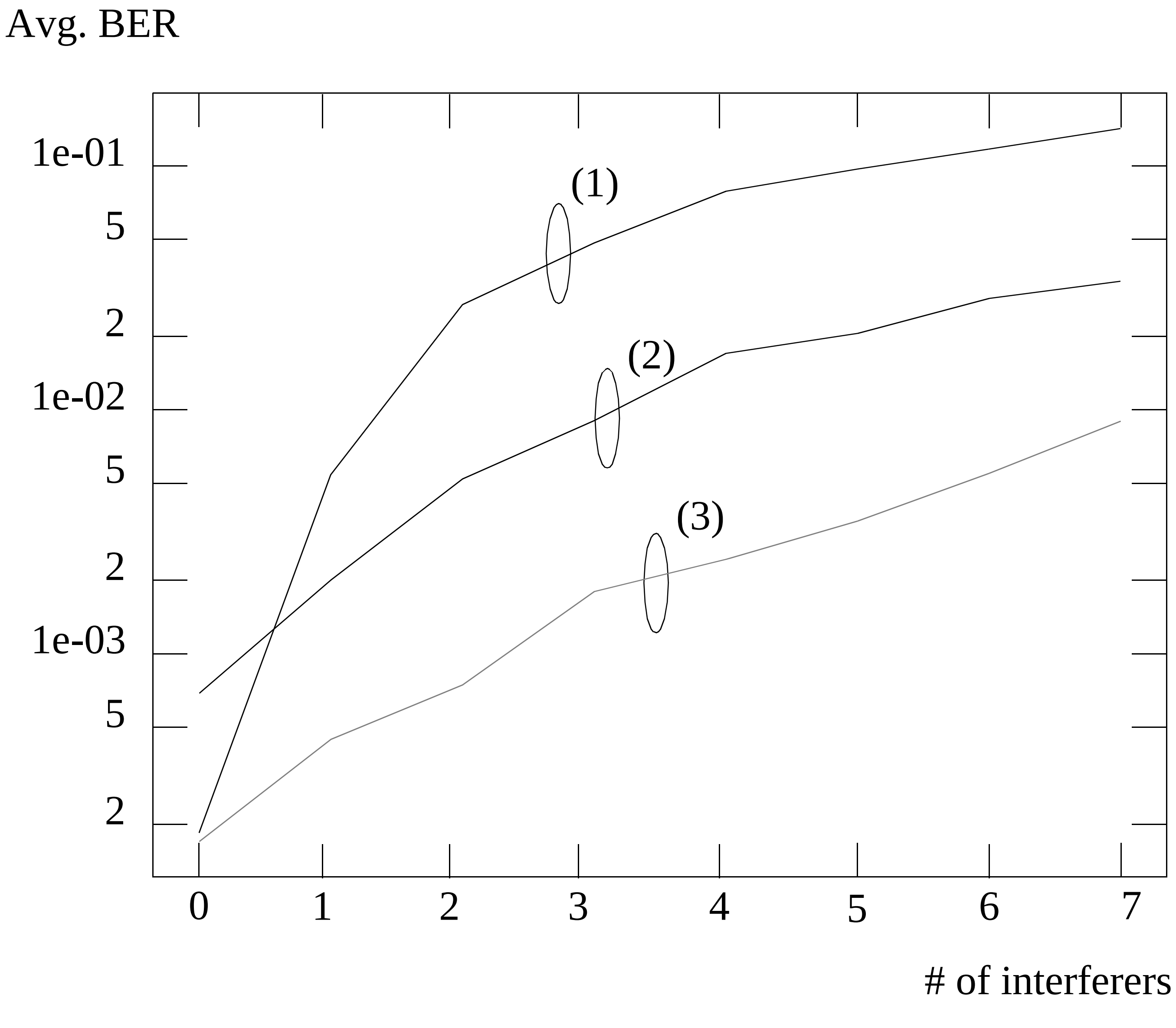
- Optimum weighting vector in a mean-squared error sense
- Weighting vector

$$D = R_Y^{-1} R_{a_0 Y}$$

- Decision variable

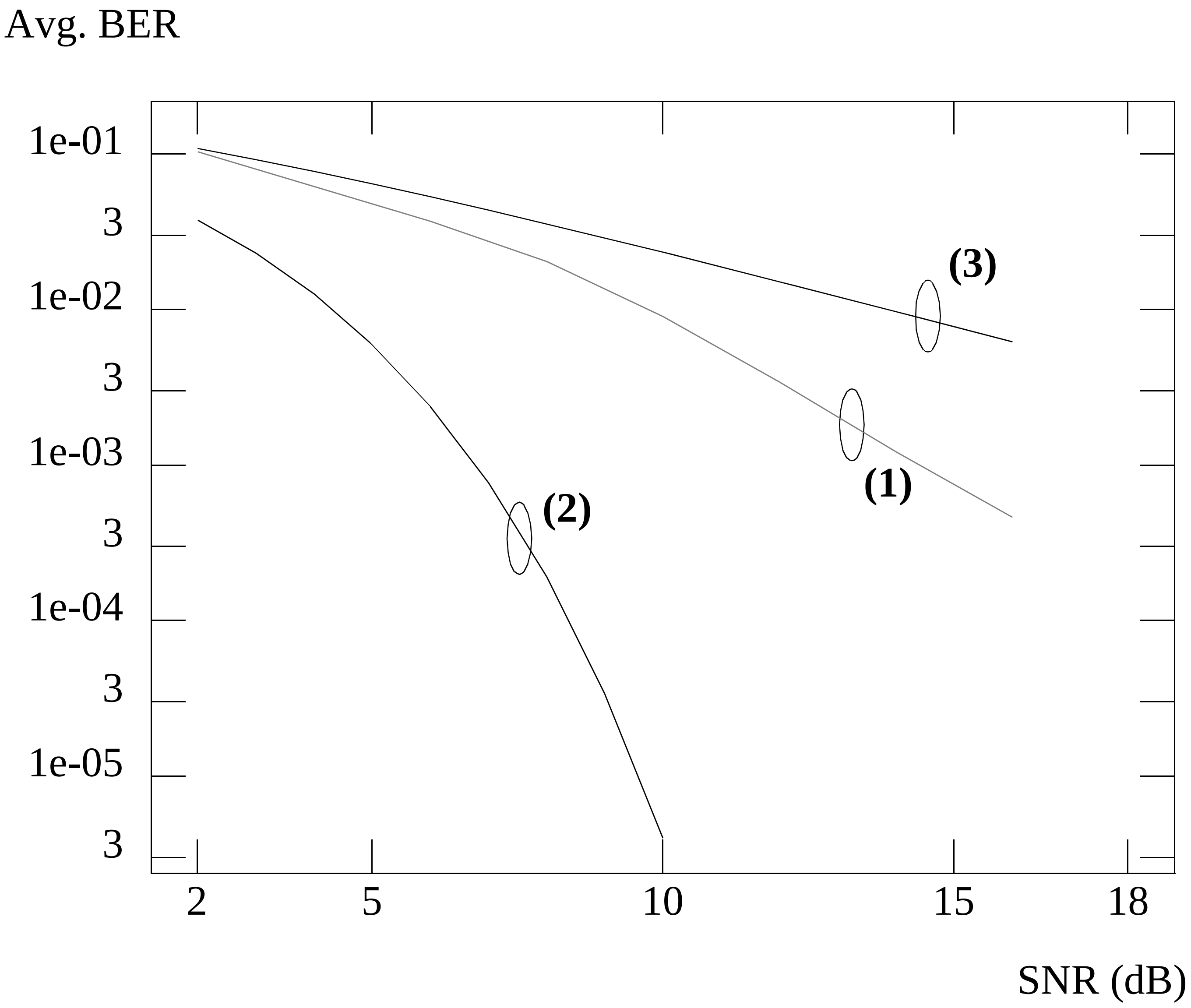
$$\nu_0 = D \bullet Y$$

Downlink BER for Wiener vs. # of Interferers (Rayleigh, SNR=10dB, N=8)



Simulation results for the average BER vs. the # of interferers for (1) MRC, (2) EGC, and (3) Wiener filtering.

Downlink BER for Wiener vs. SNR (Rayleigh, Full Load, N=8)



Simulation results for the average BER vs. the SNR for (1) Wiener filtering with a, full load, (2) single subcarrier with noise only, and (3) single subcarrier with noise and Rayleigh fading.

Conclusions

- MC-CDMA is a new, promising CDMA method
- Most effective in downlink
- Can operate well with large delay spreads
- Controlled equalization better than MRC and EGC for combating interference
- If F>>1, exploit frequency diversity without excessive spreading