ANALOGUE TRANSMISSION OVER FADING CHANNELS

Amplitude modulation

Various methods exist to transmit a baseband message $m(t)$ using an RF carrier signal $c(t) = A_c \cos(\omega_c t + \theta)$. In linear modulation, such as Amplitude Modulation (AM) and Single Side band (SSB) the amplitude A_c is made a function a function of the message $m(t)$. In broadcast AM with carrier component, the transmit signal has the form

$$s(t) = A_c(1 + \mu\, m(t)) \cos(\omega_c t + \phi) \tag{1}$$

where μ is the modulation index. The transmission bandwidth B_T of this signal is twice the bandwidth of the message ($B_T = 2W$). For the special case of modulation by a sinusoidal message with frequency f_m, Figure 1 gives the inphase and quadrature phase components of the signal. It is seen that three frequency components occur: the carrier an upper side band and a lower side band. The quadrature phase components of these two sidebands cancel, such that the resulting signal has an inphase component only.

Exercise

> A message $m(t)$ is a zero-mean wide-sense stationary (WSS) random process with autocorrelation $R_m(\tau)$ and corresponding power spectral density $G_m(f)$ The carrier $c(t) = A_c \cos(\omega_c t + \theta)$ is a randomly phase sinusoid with uniform pdf $f_\theta(\theta) = 1/2\pi$. Show that the autocorrelation function of the carrier is $R_c(\tau) = A_c^2 \cos(\omega_c \tau)/2$. Show that the autocorrelation function of the AM signal $s(t)$ is $R_s(\tau) = R_c(\tau) + \mu^2 R_c(\tau)\, R_m(\tau)$ with power spectral density

$$G_s(f) = \frac{A_c^2}{4}\left[\delta(f-f_c) + \delta(f+f_c) + \mu^2 G_m(f+f_c) + \mu^2 G_m(f-f_c)\right] \tag{2}$$

Exercise

> In the previous exercise, $m(t)$ is a Gaussian random process with variance $R_m(0)$. The AM transmitter is overmodulated if $\mu|m(t)| > 1$. Find the total transmit power if the probability of overmodulation should be less than 1%.

For amplitude modulation with modulation index μ, the signal-to-noise ratio experienced γ_d by the user is

$$\gamma_d = 2\,\frac{\mu^2 S_m}{1 + \mu^2 S_m}\,\frac{p}{N_0 B_T} \tag{3}$$

where $S_m = R_m(0)$ is the average power in the modulating (voice) signal, p the received RF signal power and N_0 the one-sided spectral noise power density at the receiver input. A comparison of postdetection signal-to-noise ratios is given in Figure 4.

AM has the advantage that the detector circuit can be very simple. This allows inexpensive production of mediumwave broadcast receivers. The transmit power amplifier, however, needs to be highly linear and therefor expensive and power consuming.

If the message has no DC component, the short-term average received signal power is $A_c^2 \rho^2(t)\ [\ 1 + \mu^2 <m^2(t)>]/2$ where <x> denotes the short-term average and $\rho(t)$ describes the random fading of the channel. For mobile reception of AM audio signals above 100 MHz, the spectrum of fluctuations in $\rho(t)$ and in $m(t)$ overlap. Hence the Automatic Gain Control in the receiver IF stages can not distinguish the message and channel fading. AGC will thus distort $m(t)$.

AM is only rarely used for mobile communication, although it is still used for radio broadcasting.

Figure 1: Phasor diagram for (a) broadcast AM and (b) SSB with tone modulation.

Single Side Band

In the frequency power spectrum (2) of AM signals we recognize an upper side band and a lower side sideband, with frequency components above and below the carrier at f_c. In Single Side Band transmission, the carrier and one of these side bands are removed. In time domain, we can denote Upper and Lower Single Sideband signals as

$$s(t) = A_c m(t) \cos(\omega_c t + \phi) \pm A_c \hat{m}(t) \sin(\omega_c t + \phi) \qquad \textbf{(4)}$$

where $\hat{m}(t)$ is the Hilbert transform of the message, i.e., $\hat{m}(t) = 1/(\pi t) * m(t)$. In the frequency domain, a Hilbert Transform Filter gives a $\pi/2$ phase shift over the entire frequency band, thus $H(f) = -j\ \mathrm{sgn}(f)$. In Equation (4) the +-sign holds for Upper Side Band (USSB) and the --sign holds for Lower Sideband LSSB.

The message can be recovered by multiplying the received signal by $\cos(\omega_c t + \phi_R)$. If the local oscillator has a phase offset $(\phi_R - \phi \neq 0)$, the detected signal is a linear combination of $m(t)$ and $\hat{m}(t)$. The human ear is not very sensitive to phase distortion; therefor $\hat{m}(t)$ sounds almost identical to $m(t)$.

Exercise

Find the frequency transfer function from transmitter input to receiver output as a function of the phase offset $\phi_R - \phi$ of the local oscillator.

The effect of a frequency error Δf in the local oscillator is more dramatic. It can best be understood from the frequency domain description of the SSB signals that this results in a frequency shift of all baseband tones in frequency by Δf. In this case, the harmonic relation between audio tones is lost and the signal sounds very artificial.

The signal-to-noise ratio at the detector output is $\gamma_d \approx p/N_0W$ where p is the received RF signal power. The transmission bandwidth B_T of SSB is $B_T = W$. This suggests high bandwidth efficiency if used for mobile radio. However, SSB is relatively sensitive to interference, which requires large frequency reuse spacings and reduces the spatial spectrum efficiency. AGC to reduce the effect of amplitude fades substantially affects the message signal. Furthermore, SSB requires very sharp filters, which are mostly sensitive to physical damage, temperature and humidity changes. This makes SSB not very attractive for mobile communication.

PHASE MODULATION

In phase modulation, the transmit signal has the constant-amplitude form $s(t) = A_c \cos(\omega_c t + \phi_\Delta m(t))$ where ϕ_Δ is the called the *phase deviation*.

Exercise

Show that for Narrowband Phase Modulation (NBPM) with $\phi_\Delta << 1$, Phase modulation can be approximated by the linear expression $s(t) = A_c \cos(\omega_c t) + \phi_\Delta m(t) \cos(\omega_c t)$. Compare NBPM with AM by drawing the phasor diagrams, computing the transmit power in the carrier and each sideband, the signal-to-noise ratio for coherent detection. Explain why in mobile communications NBPM has advantages over AM. Consider transmitter power amplifier implentation aspects and the effect of fading on the received signal.

FREQUENCY MODULATION

A radio link using Frequency Modulation (FM) is depicted in Figure 2. For frequency deviation f_Δ, the transmit signal is of the form

$$s(t) = A_c \cos\left(\omega_c t + 2\pi f_\Delta \int_{-\infty}^{t} m(\lambda)\, d\lambda \right) \qquad (5)$$

For a message bandwidth W, the transmit bandwidth B_T can be approximated by the Carson bandwidth

$$B_T = 2(f_\Delta + W) \tag{6}$$

In the event of $2W < f_\Delta < 10\,W$, a better approximation is

$$B_T = 2(f_\Delta + 2W) \tag{7}$$

Exercise

Find B_T for FM broadcasting with f_Δ = 75 kHz and W = 15 kHz. Cellular telephone nets with W = 3000 Hz typically transmit over B_T = 12.5 or 25 kHz Find f_Δ.

Figure 2: Schematic diagram of an FM radio link

After frequency-nonselective multipath propagation, the received signal is

$$r(t) = \rho(t) A_c \cos\left(\omega_c t + 2\pi f_\Delta \int_{-\infty}^{t} m(\lambda)\, d\lambda + \theta(t) \right) + n_I(t)\cos(\omega_c t) - n_Q(t)\sin(\omega_c t) \tag{8}$$

where ρ and θ are the random amplitude and phase caused by multipath reception and $n_I(t)$ and $n_Q(t)$ are the inphase and quadrature phase components of the noise, respectively. The joint received signal can be expressed in terms of the amplitude $y(t)$ and phase $\phi_y(t)$, where

$$r(t) = y(t) \cos\left(\omega_c t + \phi_y(t) \right) \tag{9}$$

The signal $d(t)$ at the FM detector output is the derivative of the phase $\phi_y(t)$ with respect to time. The measure $f(t) = f_c + (2\pi)^{-1} d\phi_y(t)/dt$ is called the *instantaneous frequency* of the received signal.

For ease of analysis, we assume that the noise is bandpass filtered by the IF stages of the receiver with a Gaussian frequency transfer function, with negligible phase shifts. We write

$$H_{IF}(f) = \exp\left\{ -\pi \frac{(f-f_c)^2}{B^2} \right\} + \exp\left\{ -\pi \frac{(f+f_c)^2}{B^2} \right\} \tag{10}$$

It is assumed that the wanted signal is not affected by this filter.

For AWGN with one-sided spectral power density N_0 experienced at the receiver front-end, the double-sided power density spectrum of the noise at the output of the last IF stage is $N(f) = N_0/2\ H_{IF}(f)$. Hence, the total predetector noise is $2BN_0$ and the instantaneous signal-to-noise ratio is $\gamma = A_c^2\rho^2(t)\ /\ 2BN_0$.

A general expression for $d\phi_y(t)/dt$ is complicated to use in further analysis. Approximate results can be given for the special cases of large and small signal-to-noise ratios. Initially we address the non-fading case, i.e., ρ and θ are assumed to be constant.

Reception above the FM-Threshold

If the signal-to-noise ratio is sufficiently large, $r(t)$ is dominated by the wanted signal. This minimum signal-to-noise ratio for which this assumption is reasonable is called the FM capture threshold. Typically, $\gamma_T \approx 10$ (10 dB).

Figure 3: Inphase / Quadrature phase diagram for FM signal in the presence of noise, received over a fading channel.

Figure 3 illustrates that for large signal-to-noise ratios, the phase can be closely approximated by the addition of the integrated wanted signal multiplied by $2\pi f_\Delta$, the "quadrature" noise component, and some Doppler phase modulation θ(t). So $\phi_y(t) \approx \phi_L(t)$ where

$$\phi_L(t) = 2\pi f_\Delta \int_{-\infty}^{t} m(\lambda)\, d\lambda + \theta(t) + \frac{n_Q(t)}{A_c\rho(t)} \qquad \textbf{(11)}$$

Here, $n_Q(t)$ is the noise component orthogonal to the dominant wanted signal. For time being we ignore the noise caused by the random frequency modulation $\theta(t)$. Also, we assume $\rho(t)$ to change relatively slowly. This quasi-static approach resembles the treatment of noise in FM reception over non-fading channels in many textbooks.

Rice showed that in this case, the detected signal $d(t)$ can be approximated as

$$d(t) \approx \frac{d\phi_L(t)}{dt}\,[\ 1 - \exp(-\gamma)\ ] \qquad \textbf{(12)}$$

that is, the detected signal is almost undistorted but attenuated by a small amount depending on the instantaneous signal-to-noise ratio γ. Hence, the power spectral density of $d(t)$ is

$$G_d(f) = S_d(f) + N_D(f) = [\ 1 - \exp(-\gamma)\ ]^2 S_{\dot{\phi}_L}(f) = (2\pi f)^2 [\ 1 - \exp(-\gamma)\ ]^2 S_{\phi_L}(f) \quad (13)$$

where $S_{\phi_L}(f)$ is the spectral power density function of the phase ϕ_L. Since the signal component in ϕ'_L is $2\pi f_\Delta m(t)$, the power spectral density of the detected signal component is

$$S_d(f) = \left(2\pi f_\Delta\right)^2 [\ 1 - \exp(-\gamma)\ ]^2 M(f) \quad (14)$$

where $M(f)$ is the power spectral density of the message $m(t)$. The total wanted-signal power at the detector output is

$$S_d = \int_{-W}^{W} S_d(f) df = \left(2\pi f_\Delta\right)^2 [\ 1 - \exp(-\gamma)\ ]^2 \mathrm{E}m^2(t) \quad (15)$$

where $\mathrm{E}m^2(t) = S_m$.

The power spectral density of the noise term in ϕ_L (see equation (11)) is $N_Q(f)\ /\ A_c^2\rho^2(t)$ where the quadrature phase component of the noise $N_Q(f)$ is subject to bandpass filtering with $N_Q(f) = N(f - f_c) + N(f + f_c) = N_0 \exp\{-\pi f^2/B^2\}$. After detection, the noise power spectral density is

$$N_d(f) = \frac{(2\pi f)^2}{2B_T\gamma} [\ 1 - \exp(-\gamma)\ ]^2 \exp\left(-\pi \frac{f^2}{B^2}\right) \quad (16)$$

where we used $N_0\ /\ A_c^2\rho^2(t) = 1/\ 2B\gamma$.

Noise Power for signals below the FM threshold

Below the FM capture threshold, the wanted signal no longer dominates the received signal. In such case one can not distinguish signal and noise the detector output. In fact the recovered signal is distorted and attenuated by $\exp\{-\gamma\}$. Rice studied the noise, which appeared to be best characterised by *clicks* or *spikes*. Davis [] gave approximate results for the click noise power, with

$$N_{clicks}(f) = \frac{4\pi B \exp(-\gamma)}{\sqrt{2}\ (\gamma + 2.35)} \quad (17)$$

Noise in the nonfading case

The total noise power can be found by integrating the spectral densities of the noise (for signal above and below the FM threshold) over the frequency pass band of the postdetection filter. So

$$N_{stat} = \int_{-W}^{W} N_d(f) + N_{clicks}(f)\ df \tag{18}$$

where we insert (16) and (17). The result is a function of the instantaneous signal-to-noise ratio γ. Hence the total noise power for the non-fading case can be written as

$$N_{stat}(\gamma) = a\frac{(1-e^{-\gamma})^2}{\gamma} + \frac{8\pi BW e^{-\gamma}}{\sqrt{2(\gamma+2.35)}} \tag{19}$$

where the constant a is defined as

$$a \triangleq \frac{(2\pi)^2}{B}\int_{0}^{W} f^2 \exp\left\{-2\pi\frac{f^2}{B^2}\right\} df \tag{20}$$

The integral is an incomplete gamma function. A MacLaurin expansion is found by rewriting the exponential factor as a series a integrating term by term. This gives

$$a = \frac{4\pi W^3}{3B}\left[1 - \frac{6\pi}{10}\left(\frac{W}{B}\right)^2 + \cdots\right] \tag{21}$$

For wideband FM ($B >> W$) this can be closely approximated by the first term only. Higher order terms depend on the particular shape of the predetection IF filters

For large γ, the noise is approaches $N \to 4\pi W^3 / (3B\gamma)$ and the destination signal-to-noise ratio becomes

$$\left(\frac{S}{N}\right)_D \approx \frac{3}{4\pi}\ \frac{B}{W^3} f_{\Delta}^2\ S_m\ \gamma \tag{22}$$

Exercise

> Compare this signal-to-noise ratio with baseband transmission over a channel with bandwidth W and received message power $A_c^2\rho^2/2$. Show that FM increases the signal-to-noise ratio by a factor $3/4\pi(f_{\Delta}/W)2\ S_m$, provided that $\gamma >> 10$.

Figure 4 compares the perceived signal-to-noise ratio for various modulation techniques for $S_m = 0.5$.

In non-linear modulation, such as phase modulation (PM) or frequency modulation (FM), the post-detection signal-to-noise ratio can be greatly enhanced as compared to baseband transmission or compared to linear modulation. This enhancement occurs as long as the received pre-detection signal-to-noise ratio is above the threshold. Below the threshold the signal-to-noise ratio deteriorates rapidly. This is often perceived if the signal-to-noise ratio increases slowly: a sudden transition from poor to good reception occurs. The signal appears to "capture" the receiver at certain point. A typical threshold value is $\gamma_r \approx 10$ (10 dB).

Effects of Rayleigh fading on FM reception

In a rapidly fading channel, the events of crossing the FM capture threshold may occur to frequency to be distinguished individually. The performance degradation is perceived as an average degradation of the channel. We will see next that the capture effect and the FM threshold vanish in such situations.

Initially we compute the average noise power by integrating (19) over the expentially distributed γ. This gives []

$$\mathrm{E}N_{stat} = \frac{a}{\bar{\gamma}} \ln\left(\frac{(1+\bar{\gamma})^2}{1+2\bar{\gamma}} \right) + 8BW \sqrt{\frac{\pi}{2\bar{\gamma}(1+\bar{\gamma})}} \, e^{2.35\frac{1+\bar{\gamma}}{\bar{\gamma}}} \operatorname{erfc} \sqrt{2.35\frac{1+\bar{\gamma}}{\bar{\gamma}}} \qquad \textbf{(23)}$$

However, two more mechanisms contribute to the postdetection noise: random signal suppression and random FM.

Effect of Amplitude variations

Fluctuations of the signal-to-noise ratio γ cause fluctuations of received noise power (18) and fluctuations of the amplitude of the detected wanted signal (12). In this section we assume that the difference between the detected signal and the expected signal is perceived as a noise type of disturbance. It is called the *signal-suppression* 'noise', even though disturbances that are highly correlated with the signal are mostly perceived as 'distortion' rather than as noise.

We consider the amplitude fading as a random process, and the message $m(t)$ as a known (deterministic) signal. The expected signal at the detector output is found by taking the expectation over γ, so

$$\mathbf{E}d(t) = \mathbf{E}[m(t)\ (1 - \exp(-\gamma)\ )] = m(t)\frac{\overline{\gamma}}{(1+\overline{\gamma})} \qquad \textbf{(24)}$$

This signal has the same form as the message $m(t)$ but is attenuated by a factor ($\overline{\gamma}$ / 1 + $\overline{\gamma}$). The expected signal power is $\mathrm{E}S_d = (\overline{\gamma} / 1 + \overline{\gamma})^2 S_m$.

The signal-suppression noise is defined as

$$n_{SSN}(t) \triangleq d(t) - \mathbf{E}d(t) = m(t)\left[1 - \frac{\overline{\gamma}}{1+\overline{\gamma}} - e^{-\gamma}\right] \qquad \textbf{(25)}$$

The local-mean signal-suppression noise power is thus $N_{\mathrm{SSN}} = S_m\ \mathrm{E}[\ (1 / 1 + \overline{\gamma}) - e^{-\gamma}]^2$ or

$$N_{SSN} = S_m\left[\frac{1}{1+2\overline{\gamma}} - \frac{1}{(1+\overline{\gamma})^2}\right] \qquad \textbf{(26)}$$

Exercise

Show that for AM transmission, the expected signal is $\mathrm{E}d(t) = [1 + \mu m(t)]\ \sqrt{(\pi b_0/2)}$ with b_0 the local-mean carrier power ($b_0 = \mathrm{E}A_c\ \rho^2(t)$). Show that the average ratio between the signal and the signal-suppression noise is $S_m/ (1+ S_m)\ \pi/(2(2 - \pi/2))$.

Effect of Random FM

For voice communication with audio passband 300 - 3000 Hz, the noise contribution due to random FM is

$$N_{RFM} \approx \int_{300\text{ Hz}}^{3000\text{ Hz}} \frac{2\pi^2 f_D^2}{f}\, df = 2\pi^2 f_D^2 \ln 10 \quad (27)$$

For large local-mean signal-to-noise ratios ($\bar{\gamma} \rightarrow \infty$), this is the only remaining term. So the postdetection signal-to-noise ratio tends to

$$\gamma_d \rightarrow \frac{S_d}{N_{RFM}} \approx \frac{2}{\ln 10}\left(\frac{f_\Delta}{f_D}\right)^2 S_m \quad (28)$$

which does not depend on additive predection noise. Wideband transmission (large f_Δ) is thus significantly less sensitive to random FM than narrowband FM.

Exercise:

An national regulatory agency meets to decide upon transmission standards for a cellular telephone network. Which parties should be consulted in this process? Compare different modulation techniques from the point of view of the network operator, the terminal manufacturing industry, the network manufacturing industry, the customer, the international regulatory agency allocating frequency bands, the computer industry interested in developing additional data transmission schemes.

Exercise:

Review techniques for computing outage probability. Show that the optimum spectrum efficiency is achieve by minimizing $z^\beta B_T^2$ where z is the receiver threshold and β is the path loss law. Optimize the FM frequency deviation for a given β.