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MULTI-CARRIER CDMA IN INDOOR WIRELESS RADIO NETWORKS

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Abstract

This paper presents a novel digital modulation technique called Multi-Carrier Code Division Multiple Access (MC-CDMA) in which symbols are transmitted at multiple subcarriers where each subcarrier is encoded with a phase offset of $\pi$ or $\pi$ based on a pseudo-random (pn) sequence. Analytical results are presented on the performance of this modulation scheme in an indoor wireless multipath radio channel.

Introduction

Recently, there has been a growing interest in indoor wireless radio communication [1]. This paper examines the performance of a new spread spectrum transmission method called "MC-CDMA" in an indoor wireless radio environment. MC-CDMA may be a suitable modulation technique in the indoor environment since the small rms delay spreads characteristic of indoor environments, typically in the range of 1-56 ns [2], allow for the exploitation of this technique.

Narrowband communications [3] has the desirable property of being relatively immune to intersymbol interference (ISI), but has the undesirable property of being susceptible to flat fading. To avoid deep fades over the entire signal bandwidth, spread spectrum transmission can be used.

Two parameters that are often used to characterize multipath channels are the delay spread and the coherence bandwidth [4]. The delay spread $T_d$ is a measure of the length of the impulse response of the channel, while the coherence bandwidth, which is found to be

$$ Bc = \frac{1}{2\pi T_d}, $$

is a measure of the correlation of the fading between frequencies. If two frequencies lie within the coherence bandwidth, they are likely to experience correlated fading. As in the case of narrowband communications, i.e., if the symbol duration is much larger than the delay spread ($T_d \ll T_b$), then the signal experiences negligible overlapping intersymbol interference (ISI) between adjacent transmitted symbols. However, the condition $T_d \ll T_b$ also implies that the signal bandwidth is much smaller than the coherence bandwidth. Depending on the location of the receiver antenna, the entire narrowband signal bandwidth may be located in a deep fade in which case the signal experiences significant attenuation.

With conventional Code Division Multiple Access-Spread Spectrum (CDMA-SS) systems [5], resistance to fading is achieved by spreading the signal energy over a larger bandwidth than necessary to contain the user signal. The larger bandwidth is achieved by multiplying each user symbol with a fast binary m-sequences where the chip duration is $T_c/N$. If the spreading factor $N$ is sufficiently large, the signal bandwidth can be much larger than the coherence bandwidth. This signal experiences frequency selective fading, so it is unlikely that all of the signal bandwidth is located in a deep fade. However, in the process of providing resistance to deep fading, the signal is affected by delay spreads to a greater extent, and the signal experiences considerable inter-chip interference.

MC-CDMA addresses the issue of how to spread the signal bandwidth without increasing the adverse effect of the delay spread. With MC-CDMA, the data symbol is transmitted over $N$ narrowband subcarriers with each subcarrier being encoded with the $\pi$ or $\pi$ phase offset. The resulting signal has a spread structure in the frequency domain. If the number of and spacing between subcarriers is appropriately chosen, it is unlikely that all of the subcarriers will be located in a deep fade and consequently frequency diversity is achieved. As a MC-CDMA signal is composed of $N$ narrowband subcarrier signals each of which has a symbol duration much larger than the delay spread, a MC-CDMA signal will not suffer from inter-chip interference and ISI. In addition, multiple access may be possible by having different users transmit at the same subcarriers but with a different pn-code that is orthogonal to the codes of other users.

Basic Principles of MC-CDMA

The generation of an MC-CDMA signal can be described as follows. A single data symbol is replicated into $N$ parallel copies. Each branch of the parallel stream is multiplied by a chip of a pseudo-random (pn) or some other code of length $N$ and then BPSK modulated to a different subcarrier spaced apart from its neighboring subcarriers by $F/T_b$ where $F$ is an integer number. The transmitted signal consists of the sum of the outputs of these branches. This process yields a multi-carrier signal with the subcarriers containing the pn-coded data symbol. An example of a MC-CDMA signal with $F=1$ is shown in Fig. 1. Demodulation of this signal would involve collapsing the chip structure in frequency and demodulating the resulting multi-carrier signal.
Transmitter Model

Shown in Fig. 2 is the model of the transmitter. The input data symbols, \( a_{\text{in}}(k) \), are assumed to be binary antipodal where \( k \) denotes the \( k \)th bit interval and \( m \) denotes the \( m \)th user. It is assumed in the analysis that \( a_{\text{in}}(k) \) takes on values of \(-1\) and \(1\) with equal probability. As illustrated in Fig. 2, the transmitted signal corresponding to the \( k \)th data bit of the \( m \)th user is

\[
s_n(t) = \sum_{i=0}^{N-1} c_{m,i}(0) a_{\text{in}}(k) \cos \left(2\pi f_d t + \frac{2\pi}{T_d} t\right) p_{\text{a}}(t - kT_d)
\]

where \( c_{m,i}(0) \), \( i = 1, \ldots, N \), represents the pn-code of the \( m \)th user and \( p_a(t) \) is defined to be an unit amplitude pulse that is non-zero in the interval of 0 to \( T_d \).

Since the MC-CDMA signal is best described in the frequency domain, it is natural to define the transfer function of the channel also in the frequency domain. The transfer function of the continuous-time fading channel assumed for the \( m \)th user in the indoor environment can be represented as

\[
H_m\left(f - \frac{f_c}{T_d}\right) = \rho_{m,i} e^{j\theta_{m,i}}
\]

where \( \rho_{m,i} \) and \( \theta_{m,i} \) are the random amplitude and phase effects of the channel of the \( m \)th user at frequency \( f_c + (m/T_d) \).

- Uplink

For transmissions from terminals to the base station (uplink), the base station receives each signal from different users through different channels depending on the location of the terminal. For the \( m \)th user, the random amplitudes, \( \rho_{m,i} \) for \( i = 0, 1, \ldots, N - 1 \), are assumed to be iid Rayleigh random variables. In an indoor environment, a Ricean amplitude distribution would be more appropriate since there is often a dominant component in the form of the line-of-sight (LOS) path [7]. Rayleigh fading would correspond to the case of having an object or person blocking the direct path from the transmitter to the receiver. However, by analyzing the performance of MC-CDMA with Rayleigh fading, a conservative estimate of the performance is obtained since radio communications tend to have better performance when a LOS component is present. For the \( m \)th user, the random phases, \( \theta_{m,i} \) for \( i = 0, 1, \ldots, N - 1 \), are assumed to be iid uniform random variables on the interval 0 to \( 2\pi \). \( \rho_{m,i} \) and \( \theta_{m,i} \) are also assumed to be independent of each other and between users. This assumption is appropriate for channels in which \( F \gg BW, T_d \). By assuming that the amplitude and phase effects of fading at each subcarrier are independent of the fading at other subcarriers, it is believed that a conservative analysis is performed since having uncorrelated subcarriers removes the benefits of the orthogonal code.

- Downlink

For transmissions from the base station to the terminals (downlink), the terminal receives interfering signals designated for other users (\( m = 1, 2, \ldots, M - 1 \)) through the same channel as the wanted signal (\( m = 0 \)). The user index may be represented as follows

\[
\rho_{m,i} = \rho_{1,i} \quad \theta_{m,i} = \theta_{1,i}
\]

for \( m = 0, 1, \ldots, M - 1 \).

Channel Model: Dispersive Rayleigh Fading

In this paper, we address a frequency-selective channel with \( 1/T_d \ll BW \ll 1/T_0 \). This model implies that each modulated subcarrier with transmission bandwidth of \( 1/T_0 \) does not experience significant dispersion \( (T_d > T_0) \). It is also assumed that the amplitude and phase remain constant over a symbol duration, \( T_d \) (i.e., Doppler shifts due to the motion of terminals is negligible.) This agrees with measurements revealing that Doppler shifts are very small and typically in the range of 0.5-6.1 Hz [6].
\[ t(t) = \sum_{m=1}^{M-1} \sum_{n=0}^{N-1} p_{m,n} c_m(i) s_m[k] \]
\[ \times \cos \left( 2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,n} \right) + n(t) \]

where \( n(t) \) is additive white Gaussian noise (AWGN). The local-mean power at the \( i \)th subcarrier of the \( m \)th user is defined to be
\[
\overline{p_{m,n}} = E \left[ p_{m,n} \cos \left( 2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,n} \right) \right]^2 = \frac{1}{2} E \overline{p_{m,n}}. \]

The local-mean powers of the subcarriers are assumed to be equal. Thus, the total local-mean power of the \( m \)th user is defined to be \( P_m = N \overline{p_{m,n}} \). Shown in Fig. 3 is the model of the receiver. To simplify the analysis, it is assumed that exact synchronization with the desired user \( (m = 0) \) is possible. The first step in obtaining the decision variable involves demodulating each of subcarriers of the received signal, which includes applying a phase correction, \( \theta_{0,0} \), and multiplying the \( i \)th subcarrier signal by a gain correction, \( g_{i0} \). In the analysis, it is assumed that perfect phase correction can be obtained, i.e., \( \theta_{0,0} = \theta_{0,0} \). After adding the subcarrier signals together, the combined signal is then integrated and sampled to yield the decision variable, \( v_b \). For the \( k \)th bit, the decision variable is
\[
v_b = \sum_{m=1}^{M-1} \sum_{i=0}^{N-1} p_{m,n} c_{n}(i) d_{n,m} [k] \frac{2}{T_b} \]
\[ \times \int \cos \left( 2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,n} \right) \]
\[ \times \cos \left( 2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,n} \right) dt + \eta \]

where the corresponding AWGN term, \( \eta \), is given as
\[
\eta = \sum_{i=0}^{N-1} \int_{T_b} n(t) \frac{2}{T_b} d_{n,m} \cos \left( 2\pi f_c t + 2\pi i \frac{F}{T_b} t + \theta_{m,n} \right) dt. \]

The value of the gain correction depends on the particular diversity method chosen. We will consider two methods: Equal Gain Combining (EGC) and Maximum Ratio Combining (MRC).

- Equal Gain Combining

With Equal Gain Combining, the gain correction removes the code structure but does not try to equalize the effects of the amplitude scaling introduced by different channel attenuations at different subcarrier frequencies. The gain factor at the \( i \)th subcarrier is given as
\[
d_{n,m} = c_{n}(i). \]

This method yields the following decision variable
\[
u_b = a_{i} [k] \sum_{i=0}^{N-1} p_{0,i} + \beta_{int} + \eta \]

where the interference term, \( \beta_{int} \), is given as
\[
\beta_{int} = \sum_{m=1}^{M-1} \sum_{i=0}^{N-1} a_{m}[k] c_{m}(i) \frac{2}{T_b} \overline{p_{m,n}} \cos \left( \theta_{0,0} - \theta_{m,n} \right). \]

As the phase difference \( \overline{\theta_{0,0}} = \theta_{0,0} - \theta_{m,n} \) is a sum of independent random variables, carrying out the convolution of the pdfs results in \( \overline{\theta_{0,0}} \), still being uniformly distributed on 0 to 2\( \pi \). Since the \( 0 \)-phase component, \( p_{0,n} \cos \theta_{0,0} \), relative to the phase of the received wanted signal is Gaussian and \( a_{m}[k] c_{m}(i) \), \( i \in \{0, 1, \ldots, 1\} \), the interference term, \( \beta_{int} \), has a Gaussian distribution. The mean and variance of the interference term and the variance of the noise component are
\[
E \beta_{int} = 0, \quad \sigma_{\beta_{int}}^2 = (M - 1) \overline{p_{0,n}}, \quad \sigma_{\eta}^2 = N_0 \frac{2}{T_b} \]

respectively.

- Maximum Ratio Combining

The motivation behind Maximum Ratio Combining is that the components of the received signal with large amplitudes are likely to contain relatively less noise. Thus, their effect on the decision process is increased by squaring their amplitudes. For MRC, the gain factor at the \( i \)th subcarrier is
\[
d_{n,m} = p_{0,n} c_{m}(i). \]

The corresponding decision variable for MRC is
\[
u_b = a_{i} [k] \sum_{i=0}^{N-1} p_{0,i} + \beta_{int} + \eta \]

where the interference term is given as
\[
\beta_{int} = \sum_{m=1}^{M-1} \sum_{i=0}^{N-1} a_{m}[k] p_{0,n} c_{m}(i) \frac{2}{T_b} \overline{p_{m,n}} \cos \theta_{0,0} \]

As the terms of the summation in parenthesis are assumed to be iid with respect to "\( m \)" and "\( t \)" and the inner summation can be approximated by a Gaussian distribution for large \( N \). Using this approximation, the interference term, \( \beta_{int} \), is a sum with respect to "\( n \)" of \( M-1 \) independent Gaussian random variables. Thus, \( \beta_{int} \) itself is Gaussian. The mean and variance of \( \beta_{int} \) and the variance of the noise component are
\[
E \beta_{int} = 0, \quad \sigma_{\beta_{int}}^2 = 2 \left( \frac{M - 1}{N} \right) \overline{p_{0,n}}, \quad \sigma_{\eta}^2 = N_0 \frac{2}{T_b} \]

respectively.

Uplink Bit Error Rate (BER)

The probability of making a decision error conditioned on the amplitudes of the desired signal, \( \{ p_{0,i}, i = 0, 1, \ldots, N-1 \} \), and the local-mean interference power, \( \sigma_{p_{0,i}}^2 \), is
\[
Pr(\text{error} | \{ p_{0,i}, i = 0, 1, \ldots, N-1 \}, \sigma_{\eta}^2) = \frac{\text{Pr} \left( \sum_{i=0}^{N-1} d_{n,m} c_{m}(i) p_{0,i} + \beta_{int} + \eta \right)}{\text{Pr} \left( \sum_{i=0}^{N-1} d_{n,m} c_{m}(i) p_{0,i} + \beta_{int} + \eta \right)}. \]

As the interference term and the noise term are independent and Gaussian, the sum to the right of the inequality has a zero-mean Gaussian distribution with a variance equal to the sum of their variances. Thus, the probability of a decision error can be represented as
\[
\Pr\left(\text{error} \mid \rho_{0,i} \right) \sim \frac{1}{2} \text{erfc}\left(\frac{\pi}{\sqrt{2N} \sigma_0^2} \rho_{0,i} \frac{T_b}{(M-1) P_m T_b + NN_0}\right)
\]

where \( b = \frac{f_{m}}{N}\left(\frac{2N}{11}\right)^{1/2} \).

3. A third possible approximation can be obtained by applying the Central Limit Theorem (CLT) for the limiting case of large \( N \). Using the CLT results in a BER of

\[
\Pr\left(\text{error} \mid p_m, p_{m'}\right) \approx \frac{1}{\sqrt{2\pi \sigma_{p_m}^2}} e^{-\frac{(p_m - p_{m'})^2}{2\sigma_{p_m}^2}}
\]

where \( \mu_{p_m} = \frac{N}{2} \bar{N} p_0 \) and \( \sigma_{p_m}^2 = \left(\frac{2 - \pi}{\pi}\right) p_0 \).

\* MRC

The conditional instantaneous BER for MRC can be determined to be

\[
\Pr\left(\text{error} \mid \rho_{0,i}, \rho_{0,i} \right) \sim \frac{1}{2} \text{erfc}\left(\frac{\pi}{\sqrt{2N} \sigma_0^2} \rho_{0,i} \frac{T_b}{(M-1) P_m T_b + 2N_0}\right)
\]

The exact description for the distribution of the sum $\sum_{i=1}^{N-1} \rho_{0,i}^2$ is the gamma distribution $[3]$

\[
f(t \mid \bar{N}) = \frac{1}{\bar{N}^{N-1}} \frac{t^{N-1}}{(N-1)!} e^{-t / \bar{N}}.
\]

Using Eq.(26) and Eq.(27), the BER for MRC is

\[
\Pr\left(\text{error} \mid p_m, p_m\right) = \int_0^{\infty} f(t \mid \bar{N}) \left(1 - \frac{t T_b}{N P_m T_b + N_0}\right) dt.
\]

An approximation for the BER, obtained by applying the LLN, is

\[
\Pr\left(\text{error} \mid p_m, p_m\right) \approx \left(\frac{p_m T_b}{N P_m T_b + N_0}\right)
\]

Downlink Bit Error Rate

With the downlink, all signals experience the same channel. The received signal is the same as Eq.(5) with $\rho_{m,i} = \rho_{m,i}$ and $\theta_{m,i} = \theta_{m,i}$. Note that when phase correction is applied, all signals have the correct phase alignment. Note that for the codes to be orthogonal, this requires that half the chip products, $c_{m}(i) c_{0}(i)$, be positive and the other half be negative. Using these observations, the BER for EGC and MRC in the downlink can be calculated in a similar manner as above.
• EGC

The decision variable is now given as

\[ u_n = a_n |k| \sum_{l=0}^{N-1} \sum_{m=1}^{M-1} \sum_{p=0}^{P-1} \sum_{q=0}^{Q-1} c_{l,m} \epsilon_{l,m} (l) p_{o,n} + n. \]  

(30)

Because of the orthogonality of the codes, the interference has a variance of \( \sigma_n^2 = 2 (M-1) \frac{\pi}{2} \frac{1}{4} \). The corresponding BER is

\[ \Pr(\text{error} | p_0) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\pi}{6} \frac{P_0 T_0}{(M-1) N} \frac{1}{4} \left( \frac{\pi}{4} \frac{1}{4} \frac{P_0 T_0}{(M-1) N} + N_0 \right)} \right). \]  

(31)

• MRC

For MRC in the downlink, the BER can be determined to be

\[ \Pr(\text{error} | p_0) = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\pi}{6} \frac{P_0 T_0}{(M-1) N} \frac{1}{4} \left( \frac{\pi}{4} \frac{1}{4} \frac{P_0 T_0}{(M-1) N} + N_0 \right)} \right) \]

where the variance of the interference is

\[ \sigma_n^2 = (M-1) N \left[ \frac{E^2_P}{2} - \left( \frac{E^2_P}{2} \right)^2 \right] = \frac{(M-1) N}{N} \sigma_p^2. \]  

(32)

Numerical Results

Uplink

Plots of the bit error rates versus the number of co-channel interferers are given in Fig. 4. To calculate the BERs, it was assumed that each interfering signal has a local mean power equal to the local mean power of the wanted signal. For the sake of comparison, the BER for MRC using the LLN approximation is included. As it can be seen in Fig. 4, the LLN approximation, the small parameter approximation, and the CLT approximation for EGC produce relatively close curves. According to all approximation, MRC outperforms EGC for any number of interfering signals. Comparing Eq. (23) and Eq. (28), the improvement in performance between MRC and EGC can be quantified by the ratio

\[ \frac{E^2_P}{E^2_P - \frac{1}{2} \frac{\pi}{4}} - 1 \frac{\pi}{4} = -1.05 \text{dB}. \]  

(34)

If the acceptable BER is \( 10^{-3} \) as in some video applications, using MRC over EGC translates to an increase in CDMA user capacity of 50%.

![Fig. 4 BER for EGC using the small argument approx. (1), CLT (2), and LLN (3) and for MRC exact (4) and approx. using LLN (5) in the uplink versus the number of interferers in 0, 1, ..., 17. The SNR is 10 dB and N = 128.](image)

Downlink

Plots of the BER for downlink transmissions are given in Fig. 5. For both diversity methods, the bit error rates were approximated by using the LLN. Examining the curves in Fig. 5, it can be seen that for a small number of users (i.e., in a noise limited channel) MRC outperforms EGC. However, for a large number of users, EGC has a superior performance.

![Fig. 5 BER for EGC (__) and MRC (---) in the downlink versus the number of interferers in 0, 1, ..., 127. The SNR is 10 dB and N = 128.](image)

Conclusion

In this paper, a new spread spectrum technique was introduced and its bit error rate for a Rayleigh fading dispersive channel was analyzed. For the two diversity techniques considered, MRC had a better performance than EGC in the uplink but appeared less effective in combating interference in the downlink. While being a better combatter against noise, MRC appears to distort the orthogonality of the codes and consequently performed worse for large number of users in the downlink. Comparing the performance of EGC in the uplink to the downlink at a bit error rate of \( 10^{-3} \), there is an increase in capacity from 8 users to 20 users in the downlink. This improvement is due to the greater degree of phase control in the downlink that allows for some of the benefits from the orthogonality of the codes to be utilized.

References


Dear MC-CDMA friend,

I am very pleased to see that MC-CDMA is appearing regularly in the scientific literature as an interesting solution for wireless communication over multipath channels. Regularly I get requests for a reprint of the pioneering paper of 1993 in which this transmission concept was baptized MC-CDMA, and I certainly appreciate the citation to this paper. While there is no original electronic file available, I have scanned the original conference text. Apologies that the file is 2.8 Mbytes.

Kind Regards; Success with your research

Jean-Paul Linnartz
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[02-C3] Stefano Tomasin, Alexei Gorokhov, Haibing Yang, Jean-Paul Linnartz, “Reduced Complexity Doppler Compensation for Mobile DVB-T”, PIMRC 2002, Lisbon, paper 1584