Orthogonal Multi-Carrier CDMA

Abstract

Orthogonal MC-CDMA is a novel modulation methods that combines the advantages of Multi-Carrier Modulation and Direct Sequence CDMA. It is similar to Direct Sequence-CDMA, except that we multiply the user signal by the Fourier Transform of a spreading code, rather than by the (binary) spreading code itself. The result can beinterpreted as if each data symbol is simultaneously transmitted at N binary phase shift-keying (BPSK) narrowband subcarriers, each separated by $1/T_b$ with the T_b bit time. Each of the N subcarrier waveforms is modulated by a single, time constant chip belonging to a spreading code of length N. Different users transmit at the same set of subcarriers but with a different spreading code in the frequency domain. In a downlink scenario, a fully loaded system, i.e., one with N simultaneous users can operate with good BER, even if the channel is highly dispersive.

Introduction and Basic Principle

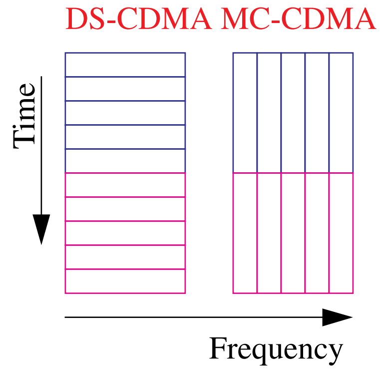

Figure1 . Time-Bandwidth occupation for Direct Sequence CDMA and Multi-Carrier CDMA. First bit: blue. Second bit: magenta.

As the time-bandwidth diagram (Figure 1) illustrates, with orthogonal MC-CDMA, each data symbol is simultaneously transmitted at N binary phase shift-keying (BPSK) narrowband subcarriers, each separated by F/T_b Hz where F is an integer. As shown in the transmitter model of Fig. 2a, each of the N subcarrier waveforms is modulated (multiplied) by a single (time constant) chip belonging to a spreading code of length N. Different users transmit at the same set of subcarriers but with a different spreading code in the frequency domain. In contrast to Direct-Sequence CDMA (or other wideband) signals, MC-CDMA signals are not significantly affected by delay spreads due to their narrowband composition. This modulation technique should not be confused with transmitting multiple DS-CDMA signals at different frequencies as done so (non-orthogonal) Multi-Carrier DS-CDMA. Note that the signal structure of MC-CDMA is similar to that of Orthogonal Frequency Division Multiplexing (OFDM) but the manner in which the signals are used is very different.

Due to its special signal structure, MC-CDMA signals will not experience significant linear distortion in fading channels where the symbol duration, T_b, is much larger than the delay spread, T_d. However, the flat fading that each subcarrier with result in an amplitude scaling.

Figure 3 and 4 describe possible transmitter and receiver implementations. Here, the receiver is taken linear, that is, the decision variable is a linear combination of the subcarrier components. In the receiver, the choice of the subcarrier weight factor d_i is critical.

If only a single MC-CDMA signal is present, the scheme is similar to a frequency diversity system. The conventional diversity combining methods, such as equal gain combining (EGC) and maximal ratio combining (MRC) can be used. With EGC, the weight factor is constant, identical for all subcarriers, and independent of the received signal power in the various subcarriers. With MRC, the weight factor is proportional to the amplitude of the signal in that subcarrier. In such case, strong subcarriers are prominent in the detection of user bits. this agrees with the intuition that weak subcarriers contain relatively the most noise. In an additive white Gaussian noise (AWGN) Linear Time-Invariant channel, Maximum Ratio Combining is known to be optimum.

If, on the other hand, the MC-CDMA signal is composed of signal from different users with orthogonal codes, unequal scaling a different subcarriers distorts the orthogonality between users. MRC or EGC detection methods do neither directly restore orthogonality nor does it provide for interference cancellation. Consequently, these detection methods do not perform as well in interference limited channels. To optimally restore the orthogonality of the signals, one could make the weight factor inversely proportional to the signal amplitude of that subcarrier. This is exactly the opposite strategy of MRC. Not surprisingly, this 'equalization' excessively boosts the noise in subcarriers where the signal is in a fade.

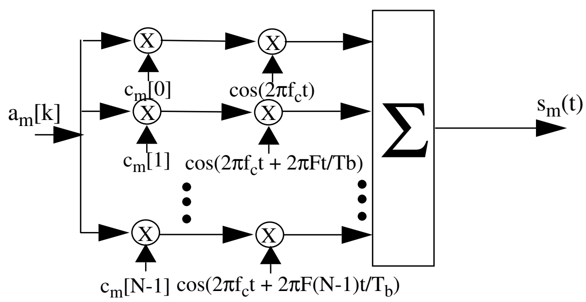

Figure2a: Transmitter Model of the *mth* user signal. Otheruser signalsare generated similarly and added.

One can apply the theory of Wiener filtering to find appropriate receivers for the detection of MC-CDMA signals. Wiener filtering is optimal in a mean-squared error sense with respect to both the noise and the interference. In most fading channels, the determination of the Wiener coefficients needs to be performed adaptively. We will show that Wiener filtering suggests a method that is a compromise between MRC and equalization. Weak subcarriers are attenuated according to MRC, because the signal to noise ratio is too low to considers these fully in the decision variable. The strong subcarriers are equalized, to give them equal signal amplitudes. This restores orthogonality among different user signals and reduces multi-user interference. Table 1 summarizes these receiver strategies.

TABLE 1. Comparison of various subcarrier handling methods for various types of channel conditions.

	AWGN channel	**noise-free, interference-limited channel**	**noise plus interference**
Equal Gain Combining	good	not so good	poor performance
Maximum Ratio Combining	optimum	poor performance	poor performance
Equalization	Poor: exececssively boosts noise	optimum	Poor: excesivly boosts noise
Wiener Filtering	optimum	optimum	optimum
	converts to MRC)	(converts to equalization)	

System Analysis: Channel Model

Let's consider a downlink system in which a basetransmits to N different user terminals. The base station can easily guarante that all user signal are synchronous. In MC-CDMA, intersymbol interference can be avoided by guard times, in a manner very similar to common practice in OFDM multi-carrier systems. Therefor we restrict our analysis to a single, say the k-th, bit interval. We d enote the vector of the binary data symbols to all N users as

$$A = \left[a_0[k] \;\; a_1[k] \;\; \dots \;\; a_{N-1}[k]\right]^T \tag{1}$$

where $a_m[k] \in \{-1, 1\}$ corresponds to the data symbol of the *mth* user.

The multiple access features are provided through the use of spreading codes, which we denote by the code matrix C

$$C = \begin{bmatrix} c_0[0] & c_1[0] & \dots & c_{N-1}[0] \\ c_0[1] & c_1[1] & \dots & c_{N-1}[1] \\ \dots & \dots & \dots & \dots \\ c_0[N-1] & c_1[N-1] & \dots & c_{N-1}[N-1] \end{bmatrix}$$

Here the *mth* column vector of *C* corresponds to the spreading code, $\{c_m[i] \mid i = 0, 1, \ldots, N-1\}$, of the *mth* user. Using this vector and matrix notation, an equivalent discrete vector representation of the subcarrier components of the transmitted signal is

$$S = \left[s_0[k] \;\; s_1[k] \;\; \dots \;\; s_{N-1}[k]\right]^T = CA \tag{2}$$

where $s_i[k]$ represents the signal components of all users at the *ith* subcarrier for the *kth* symbol interval. The actual continuous-time transmitted signal corresponding the *kth* data symbol of the *mth* user, $a_m[k]$, is

$$s_m(t) = a_m[k] \sum_{i=0}^{N-1} c_m[i] \cos\left\{2\pi\left(f_c + \frac{F}{T_b} i\right)t\right\} \times p_{T_b}(t - kT_b) \tag{3}$$

where f_c is the carrier frequency and $p_T(t)$ is a unit pulse that is non-zero for $t \in [0, T)$ and zero otherwise. Two important interpretations of this expression are

- The user bits $a_m[k]$ are put on *N* different subcarriers, similar to frequency diversity transmission. The phase of each subcarrier is determined by a time-constant factor $c_m[i]$.
- The user bits $a_m[k]$ are spread, i.e., multiplied, by a spreading sequence of the form.

$$\sum_{i=0}^{N-1} c_m[i] \cos\left\{2\pi\left(f_c + \frac{F}{T_b} i\right)t\right\}$$

This happens to be equal to a Discrete Cosine or Fourier-like transform of the sequence $c_m[0]$ $c_m[1]$.. $c_m[N]$

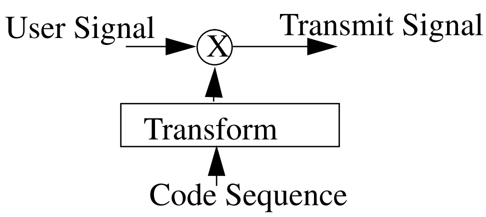

Figure 2b: MC-CDMA transmitter for a single user signal.

- A third interpretation is seen after rewriting
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That is, use signals are code multiplexed and the resulting signal is transformed before transmission.

$$s_m(t) = \sum_{i=0}^{N-1} a_m[k]\, c_m[i] \cos\left\{2\pi\left(f_c + \frac{F}{T_b} i\right)t\right\}$$
$$\times p_{T_b}(t - kT_b)$$

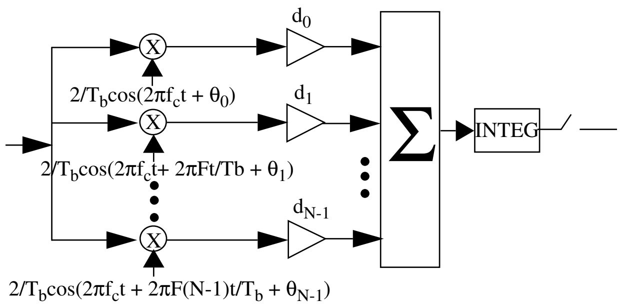


Fig. 3 Receiver Model

To see the multuple accessproperty of this signal, we note that in a distortionless channel, that data symbol of the *mth* user can be recovered using the orthogonality of the codes as indicated by the following operation

$$a_m[k] = \frac{1}{N} C_m \bullet S \qquad \textbf{(4)}$$

where $\bullet$ denotes the inner product between two vectors and

$$C_m = \left[c_m[0]\ \ c_m[1]\ \ \ldots\ \ c_m[N-1]\right]^T \qquad \textbf{(5)}$$

is a vector containing the spreading code of the *mth* user.

In downlink transmissions, i.e., from the base station to the mobiles, the wanted signal and the interference signals all arrive over the same propagation channel. Assuming that the delay spread is much smaller than the symbol duration, the effect of the channel at the *ith* subcarrier may be approximated by a constant amplitude scaling, $p_i[k]$, and a constant phase offset, $\theta_i[k]$, over the symbol duration. Using this assumption, the index k will be suppressed throughout the rest of this text. Applying the received signal to the receiver model shown in Fig. 3, the equivalent discrete representation of the received signal is

$$Y = \left[y_0\ y_1\ \ldots\ y_{N-1}\right]^T = HCA + N_1 \qquad \textbf{(6)}$$

where y_i represents the component of the received signal at the *ith* subcarrier, the channel matrix H is defined to be

$$H = \begin{bmatrix} \rho_0 & \ldots & 0 & 0 \\ \ldots & \rho_1 & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \rho_{N-1} \end{bmatrix}, \qquad \textbf{(7)}$$

and $N_1 = \left[\eta_0\ \eta_1\ \dots\ \eta_{N-1}\right]^T$ is a vector containing the corresponding AWGN terms with η_i representing the noise term at the *ith* subcarrier with power N_0/T_b. Obviously, the discrete representation of the demodulated signal given in Eq.(7) is an approximation. However, under the condition $T_b \gg T_d$, each subcarrier faces a relatively flat channel and Eq.(7) provides a reasonable representation.

Optimum Filtering

The vector channel is illustrated below. User bits *A* are Code Division Multiplexed by Matrix *C* and transmitted on different subcarriers *S*. Each subcarriers see a different channel attenuation and phase shift, represented by diagonal matrix *H*. The receiver sees vector *Y*, which also contains noise *N*. The

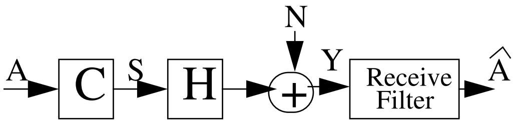


Figure: Vector representation of MC-CDMA system.

receiver filters this *Y* to generate an estimate of *A*. Next, we will study which (linear) filter can best be used to estimate *A*.
The application of Wiener filtering to this received signal involves linearly combining the different subcarrier diversity components to yield the decision variable

$$\nu_0 = D \bullet Y \tag{8}$$

where the vector $D = \left[d_0\ d_1\ \dots\ d_{N-1}\right]^T$ represents the optimal weighting coefficients. These coefficients can also be viewed as an amplitude equalization to compensate for the fading at the subcarriers. Arbitrarily choosing *m = 0* as the desired signal, the optimal choice of the equalization vector *D* in the mean-squared sense can be determined to be

$$D = R_{Y|H}^{-1} R_{a_0 Y|H} \tag{9}$$

where

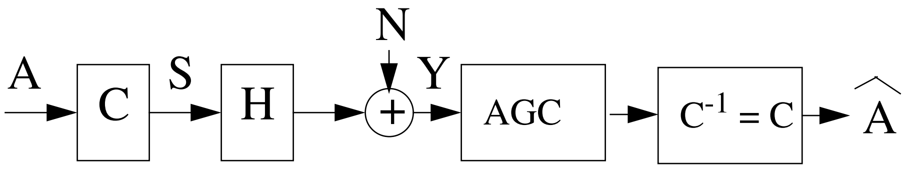
$$R_{Y|H} = E\{YY^T\} = HCE(AA^T)C^T H + \frac{N_0}{T_b} I \tag{10}$$

represents the autocorrelation matrix of the received vector *Y* and $R_{a_0 Y|H}$ represents the cross-correlation vector between the desired symbol, a_0, and the received signal vector, *Y*. In the formulation of this method, it is assumed that estimates of the channel amplitudes, $\{\rho_i | i = 0, 1, \dots, N-1\}$, are available and are consequently treated as deterministic constants.

A closed form solution of Eq.(10) is difficult to obtain except for the case of a full load, i.e., when all *N* users are active. The corresponding equalization coefficient at the *ith* subcarrier for a full load is

$$d_i = \frac{\rho_i}{N\rho_i^2 + N_0/T_b} c_0[i] \,. \tag{11}$$

As expected, Eq.(12) indicates that the linear combination of the subcarrier components should include the inner product of the desired user's spreading code vector with the received signal vector. For small ρ_i, the equalization coefficient should be small to avoid excessive amplification of the noise. For large ρ_i, the correction factor should be proportional to taking the inverse of the channel, $1/\rho_i$, in order to restore the orthogonality between users.



The expression can be interpreted as an automatic gain control per subcarrier, combined by an inverse code matrix operation. The automatic gain control amplifies the subcarrier signal with by a factor

$$d_i = \frac{\rho_i}{N\rho_i^2 + N_0/T_b}$$

Bit Errror Rate

For sufficiently large values of N, the average bit error rate (BER) for the case of a full load may be approximated using Eq.(9) and the Central Limit Theorem by

$$BER \cong \frac{1}{2} erfc \sqrt{\frac{1}{2} \frac{(NEw_i)^2}{N^2\sigma_{w_i}^2 + N\frac{N_0}{T_b}Ev_i^2}} \tag{12}$$

where

$$w_i = \frac{\rho_i^2}{N\rho_i^2 + N_0/T_t}$$

and

$$v_i = \frac{\rho_i}{N\rho_i^2 + N_0/T_b}.$$

In obtaining this expression, independent and identically distributed (IID) fading at the subcarriers was assumed.

Correlated Fading on Adjacent Subcarriers

To study the effect of correlated fading through simulations, the following algorithm for generating correlated Rayleigh r.v.s was used. For Rayleigh fading, the set of channel amplitudes is zero-mean jointly complex Gaussian, so their statistical behavior is fully characterized by the covariance matrix. The correlation between two frequencies separated by $\Delta w = 2\pi F(n-l)/T_b$ radians in a complex gaussian fading channel is

$$R_{\rho}(\Delta w) = \frac{1 + j\dfrac{2\pi F(n-l)}{T_b}T_d}{1 + \left(\dfrac{2\pi F(n-l)}{T_b}\right)^2 T_d^2}\frac{2\overline{p_m}}{N}. \tag{13}$$

where $j = \sqrt{-1}$ and $\overline{p_m}$ is the local-mean power of the *mth* user. Thus, the MC-CDMA transmission can be simulated by generating a set of N correlated jointly Gaussian channel amplitudes. In fact, the vector $\overline{H}$, of dimension N, whose elements, h_i for $i = 0, 1, ..., N-1$, are the complex correlated gaussian fading variables at the different subcarriers, can be generated by taking linear combinations of IID unit normal gaussian r.v.s:

$$\overline{H} = AG \tag{14}$$

where G is a vector of dimension N containing IID unit normal r.v.s.

The autocorrelation matrix of $\overline{H}$, $R_{\overline{H}} = E\overline{H}\overline{H}^*$ A possible choice of the $N \times N$ transformation matrix A is

$$A = \frac{1}{\sqrt{2}}E\Lambda^{1/2} \tag{15}$$

where E is an $N \times N$ matrix with the eigenvectors of $R_{\overline{H}}$ as its column vectors and $\Lambda \in \Re^{N \times N}$ is a diagonal matrix with the eigenvalues of $R_{\overline{H}}$ as its entries. The relationship between the fading variables of Eq.(8) and $\overline{H}$ is

$$\rho_i = |h_i|. \tag{16}$$

Simulation and Numerical Results

Due to the lengthy time consumption of the simulations, results for only spreading factors of $N = 8$ and $N = 64$ were obtained. Shown in Fig. 3 is a plot of the average BER versus the signal-to-noise ratio (SNR) in a Rayleigh fading channel for a full load with $N = 8$. IID fading at the subcarriers is assumed. It is also assumed that all users are received with equal power. Note that the analytical results for Wiener filtering differ significantly from the simulation results for large SNR. This discrepancy is due to the application of the Central Limit Theorem to small spreading factors. Shown in Fig. 4 are the results for $N = 64$. Examining Fig. 4 and 5, it can be seen that Eq.(13) is more accurate for large N. Also included in Fig. 4 are the curves for a single narrowband subcarrier with AWGN, and with and without fading. Note that the performance of MC-CDMA is better than a single narrowband transmission due to frequency diversity even with a fully loaded system.

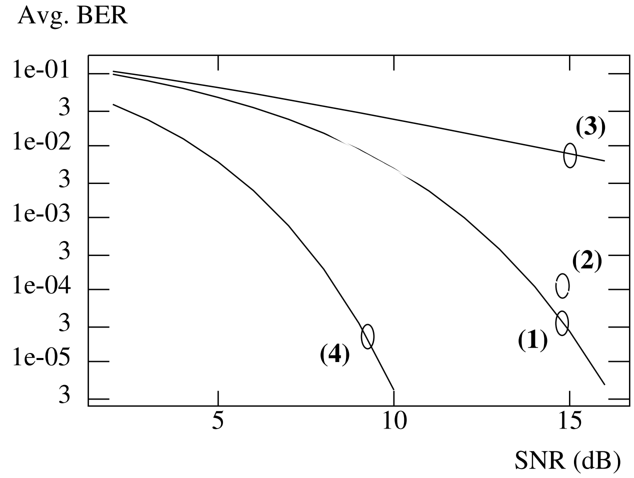


Figure 5.Average BER vs SNR with Rayleigh fading and a full load for Wiener Filtering: (1) analytical, N -> INF, (2) simulation, N = 64, (3) analytical, N = 1 (single narrowband), and (4) analytical, N = 1 with linear time invariant AWGN channel.

The average BER versus the SNR for correlated fading was also examined for the case of $N = 8$ (see Fig. 6). The degree of correlation was measured in terms of

$$\alpha = \frac{T_d}{T_b}. \tag{17}$$

Small α's correspond to highly correlated fading at the subcarriers while large α's correspond to IID fading. Ideally, the larger the value of α, the higher the degree of frequency diversity. In a DS-CDMA system, the number of resolvable paths is equal to

$$L = \lfloor N\alpha \rfloor + 1\,. \tag{18}$$

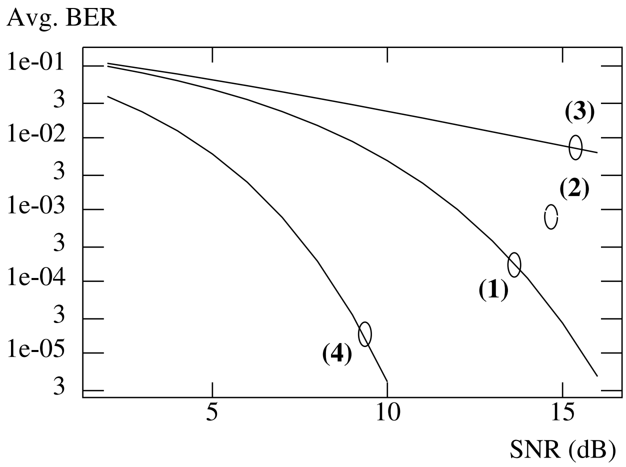


Figure 4. Average BER vs SNR with Rayleigh fading and full load for Wiener Filtering: (1) analytical, N -> INF, (2) simulation, N = 8, (3) analytical, N = 1 (single narrowband), and (4) analytical, N = 1 with linear time invariant AWGN channel.

Note that for α = *0.125 (L = 2)* most of the benefits in performance from frequency diversity have already been achieved.

Shown in Fig. 6 is a plot of the simulation results for the average BER versus the number of interferes. The SNR was chosen to be 10dB. Because of the time consuming nature of the simulator, a relatively small spreading factor of $N = 8$ was used. Besides the curves for Wiener filtering, the simulation results of conventional EGC and MRC detection are also included [1]. As expected, Wiener filtering outperforms EGC and MRC substantially in combating interference.

IV. Conclusion

Wiener filtering can be applied to the detection process of MC-CDMA. In contrast to conventional diversity combining techniques, this detection technique directly addresses the effects of the interference on the BER. It was shown that even with a full load, MC-CDMA with Wiener filtering outperforms single narrowband transmissions in a Rayleigh fading channel. It was also shown that Wiener filtering outperforms conventional detection methods substantially. Analytical approximations for $N \to \infty$ appeared reasonably accurate for SNRs up to 12dB and for $N = 64$ subcarriers. As the number of subcarriers is increased, the accuracy of the analytical results improves. Most of the benefits of frequency diversity can be achieved even in a channel with a significant degree of correlated fading at the subcarriers.

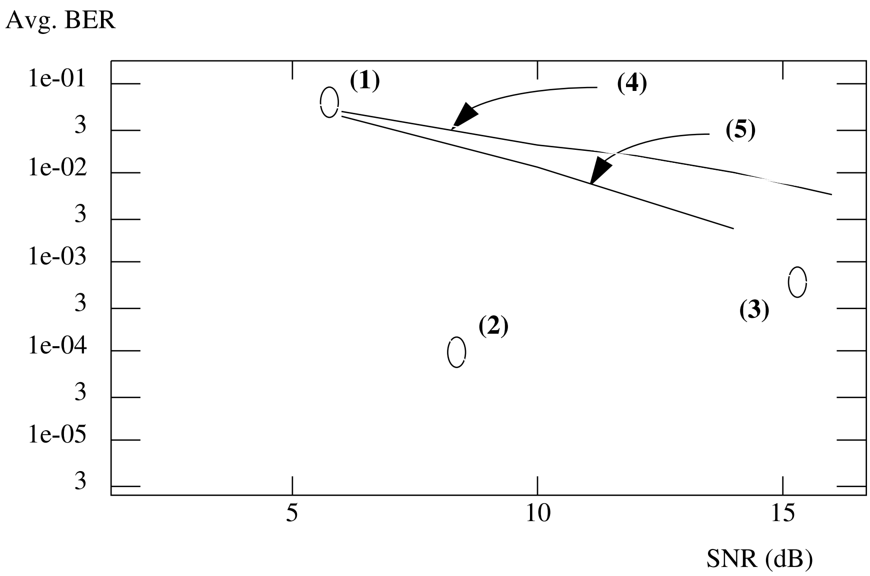


Figure 6. Average BER vs SNR with Rayleigh fading and a full load for Wiener Filtering: (1) analytical, N = 1, (single subcarrier), (2) analytical, N = 1 with linear time invariant AWGN channel, (3) simulation, N = 8, with IID fading (α = INF), (4) simulation, N = 8, correlated fading with α = 0.001 (L = 1) and (5) α = 0.125 (L = 2).