Digital Modulation for Wireless Communication

Requirements for modulation method

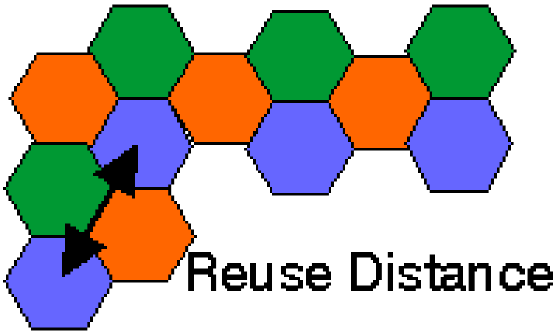
- Compact spectrum (many bits/sec/Hz)
- Immune to interference

  1.Cochannel interference

  2.Adjacent channel interference
- Robust against fading (frequency dispersion) and multipath (time dispersion)
- Simple implementation of transmitter and receiver
- Low-power consumption of transmit power amplifiers

  For instance: "constant envelope signals" allow the use of nonlinear Class C amplifiers

Spectral Efficiency of Cellular Network



Two aspects: Reuse distance R_u determines

- propagation distances of interfering signals, thus the C/I ratio, and
- the cluster size, thus the number of channels needed

C/I ratio γ is proportional to R_u^β where β path loss exponent

For a certain required protection ratio z, one must take

$$C/I \propto R_u^\beta > z \text{ so } R_u > z^{1/\beta}$$

Number of channels C : proportional to R_u^2

Hence, the number of channels needed is proportional to $z^{2/\beta}$

Minimize Bandwidth of Cellular System

The system designer can choose the modulation method.

- This affects the transmit bandwidth B and
- the required reuse distance R_u

Typically, one can trade transmit bandwidth B for good immunity to interference and fading.

Optimum spectrum efficiency:

- Minimize $B\, z^{2/\beta}$

Extreme choices:

- DS-CDMA: large B, small z, very dense reuse
- Analog AMPS: small B, but large reuse distance

Detection of Digital Signals

Theory of digital communication is most detailed for channels

- with Additive White Gaussian Noise (AWGN)
- no interference
- Linear Time-Invariance
- Non-dispersive

One of the most important results is
the “matched filter receiver”.

However, wireless communication suffers

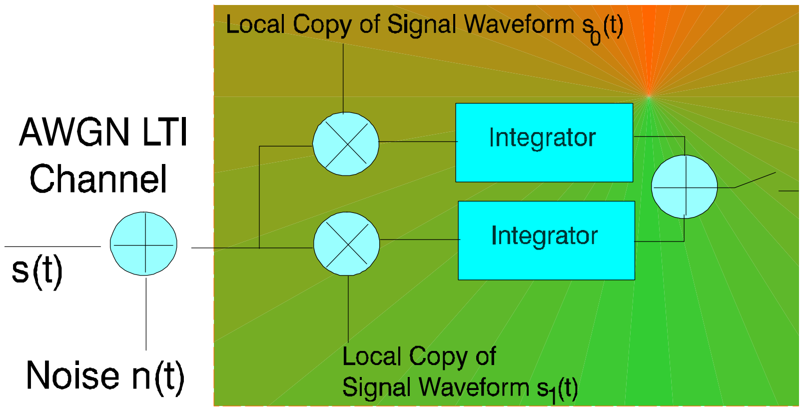
- Time varying channels
- Frequency disperisive channels
- Interference from other users

Matched Filter Receiver

- Matched Filter:

  Multiply received signal by reference signalwaveform

- This is equivalent to filtering with time-inverse impulse response
- Decision Variable is "Sufficient Statistic"
- Interpretation:
  - weigh strong components heavily
  - ignore weak components



Matched Filter is optimum if

- Noise is AWGN (presence of interference may require a more complex receiver structure)
- Channel is non-selective, Linear Time-Invariant (LTI)
- Exact time reference is available

One Shot Transmission overDispersive Channel

- How to apply Matched Filter concept for LTI AGWN channel with dispersion?

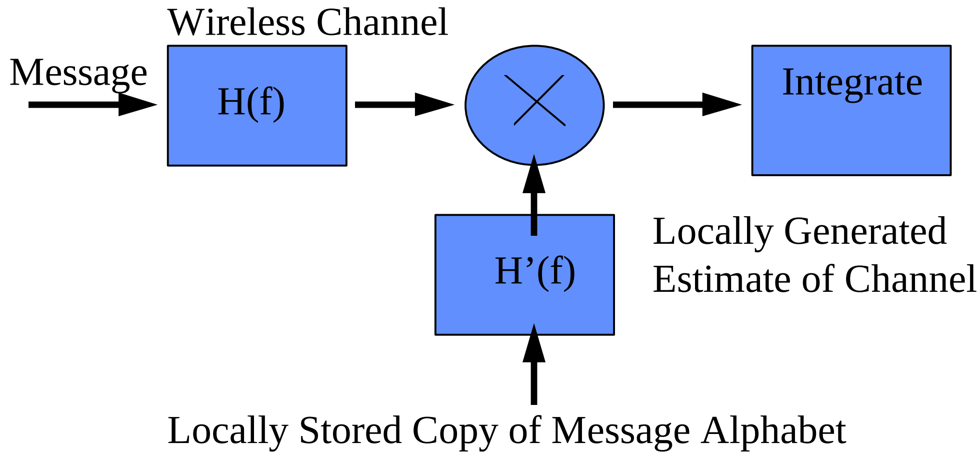


Figure: Optimum One-Shot Receiver

- Match the filter to expected *receive* waveform
- Hence, receiver needs to know channel behavior
- For CDMA this leads to the rake receiver
- Transmission of multiple signals:Intersymbol Interference occurs.

  further investigation is needed, e.g. treat total bit stream as one composite symbol, correlate received signal with all possible sequences

Transmission over slowly fading, non-dispersive channel

Assumptions

- No ISI
- Perfect Synchronization
- Fading affects only received energy per bit

    Amplitude is Rician distributed, phase is known

Definitions

- Instantaneous BER:

    BER experienced at a particular location

- Local-mean BER:

    BER averaged over multipath fading fading

- Fast fading and slow fading:

    fast/slow compared to symbol duration

    fast/slow compared to block duration

Effect of Fading on BER

For Very Slow Fading

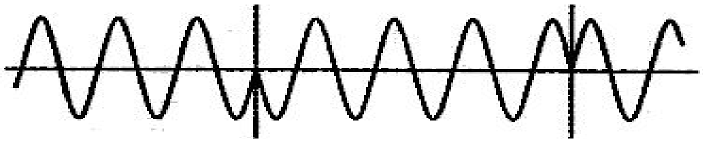
- Fading increases the BER: with some probability E_b is small, even for large average E_b.
- Waterfall behavior vanishes:
  only a slow decrease of BER with improving C/N
- Most bit errors occur during deep fades, in bursts
- The burst length depends on the speed of antenna
- Stationary users do not experience burst errors,
  but receive constant power.
  Then the BER depends highly on location
- Coding and interleaving may help
- Local-mean BER does not say so much in practice

Other Effects

- Rapid (Doppler) channel changes causes bit errors
- Doppler affects accuracy of phase reference
- Channel amplitude changes may also cause errors in QAM
- ISI due to delay spread

COHERENT BPSK

- Binary Phase Shift Keying



For 0 and 1: carrier has opposite phase

Instantaneous Bit Error Rate

- For coherent detection
- Non-selective Linear Time-Invariant (LTI) channels
- Additive White Gaussian Noise (AWGN)

$$P_b(error) = \tfrac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}\ .$$

where

- E_b is the energy per bit

  $E_b = p_0 T_b$ with T_b the bit duration, and
- N_0 is the one-sided spectral power density of the AWGN.

  E_b / N_0 is the signal-to-noise ratio γ.
- erfc: complementary error function: $\mathrm{erfc}(k) = 2Q(\sqrt{2}\,k)$.

COHERENT BPSK

Local-mean BER

- Slowly Rayleigh-fading channel
- Exponentially distributed power
- Local-mean: Take average over the PDF of received signal power. For Rayleigh fading:

$$\overline{P}_b = \int_0^{\infty} \frac{1}{\overline{p}} \exp\left\{-\frac{p}{\overline{p}}\right\} \frac{1}{2} erfc\left\{-\frac{p T_b}{N_0}\right\} dp$$

so

$$\overline{P}_b = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p} T_b}{\overline{p} T_b + N_0}}$$

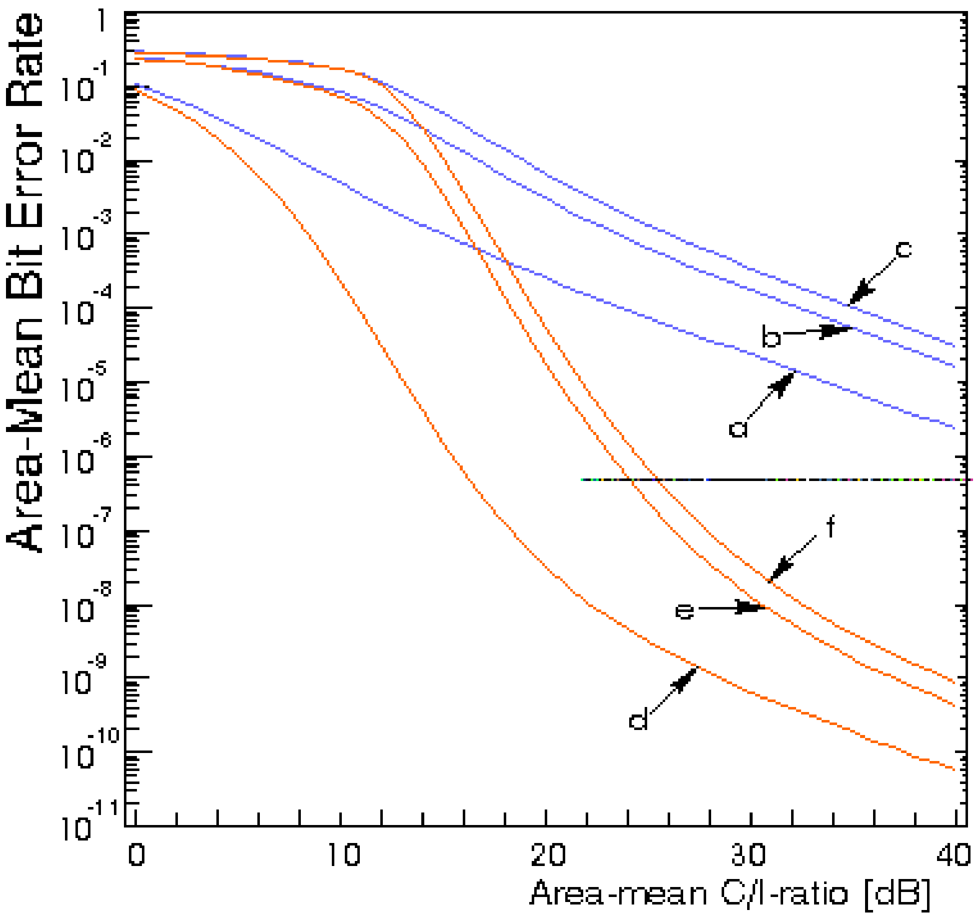
- Asymptotically

$$\overline{P}_b \rightarrow \frac{N_0}{4\overline{p}T_b} = \frac{1}{4\overline{g}}$$

Inversely proportional to signal-to-noise ratio, i.e., only a slow improvement in BER with increasing C/N

Area-mean BER

Bit error rate averaged over amplitude distribution of due to fading and shadowing



Curve c-f: 12 dB shadowing
Curve b-e: 6 dB shadowing
Curve a-d: no shadowing

Wanted signal: Rician fading, K=4 (a,b,c) K=16 (d,e,f)
Interference: Rayleigh fading

Average BER for fading channel

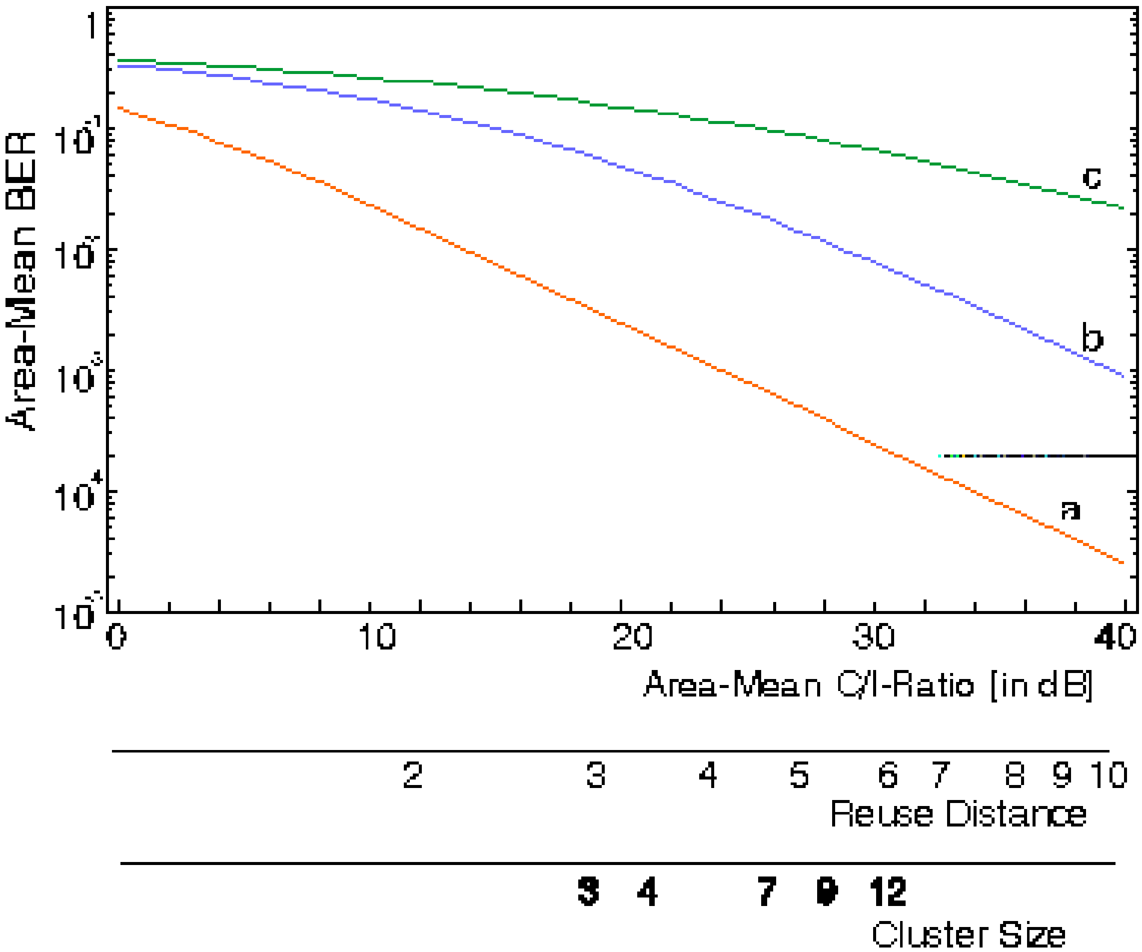


Figure: Area-mean BER versus C/I ratio.

C/I ratio can be converted into the required reuse distance (using a 40 log *d* path loss model) and into a cluster size.

Curve

a Rayleigh fading, no shadowing

b Rayleigh fading, 6 dB shadowing

c Rayleigh fading, 12 dB shadowing

Fast Fading

There are different definitions of fast fading.

1) Packet Duration >> Coherence Time of Channel

2) Bit Duration >> Coherence Time of Channel

We use definition 1 and assume that

- Bit Duration << Coherence time of Channel
- Amplitude and phase statistically independent from bit to bit
- Receiver stays in lock

Packet Success Probability

$$\Pr(s_i) = \sum_{m=0}^{M} \binom{L}{m} \left[1 - \overline{P_b}\right]^{L-m} \left[\overline{P_b}\right]^{m}$$

where

L packet length in bits

M bit error correcting code

Slow Fading

Amplitude and phase constant for duration of a packet.

⇔ During one packet time,

motion of the mobile << wave length

Probability of not more than M bit errors in block of L bits

$$\Pr\left(s_i \mid p_0\right) = \sum_{m=0}^{M} \binom{L}{m} \left[1 - \frac{1}{2}\operatorname{erfc}\left\{\sqrt{\frac{p_0 T_b}{N_0}}\right\}\right]^{L-m} \left[\frac{1}{2}\operatorname{erfc}\left\{\sqrt{\frac{p_0 T_b}{N_0}}\right\}\right]^{m}$$

Slow Fading:

- First compute conditional Packet success probability;
- Then average over all possible channels

Fast Fading:

- First find *average* BER;
- Then compute Packet Success Probability

Performance comparison

For Slow Fading

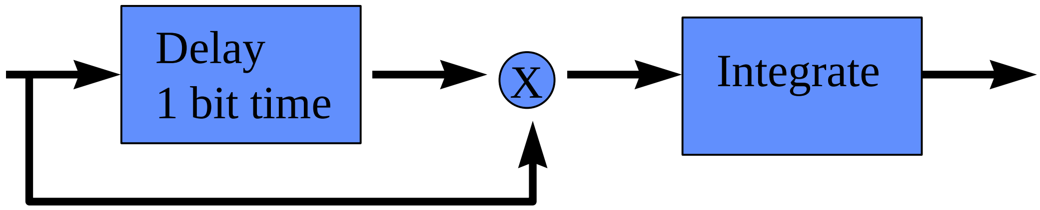
- the number of bit errors is not Binomial
- errors are highly correlated (bursts)
- error correction is less effective

Performance

- if no coding is used: slow fading performs better
- if coding is used: BER better for fast fading if C/N is sufficiently large (cellular telephony)
- with fast fading (independent bit errors), coding can change to slope of the BER vs. C/N curve. Increasing C/N has more effect for improving BER (Consider coding as a kind of diversity)
- better for slow fading if C/N is very small (random access, collision channels)

BER of Binary Phase Shift Keying (continued)

Noncoherent Detection for BPSK



- No carrier phase reference needed:
  - simplicity of receiver
  - Advantageous in fading channel
- Typically 3 dB more sensitive to noise
- Previous bit is used as reference:

  Differential encoding

  0: no phase reversal

  1: phase reversal
- BER analysis is complicated due to inaccurate phase of reference bit

  BER analysis simple for binary DPSK

Differential BPSK

For LTI AWGN, the **instantaneous BER** is

$$P_b = \tfrac{1}{2} \exp\left\{ - \frac{p_0 T_b}{N_0} \right\}$$

Local-mean BER

For a flat, slow Rayleigh-fading channel

$$\overline{P} = \frac{N_0}{2\overline{p}T_b + 2N_0}$$

- Asymptotically 3 dB more vulnerable to AWGN than coherent BPSK

Example:
Noncoherent detection of D-BPSK in a slow Rayleigh fading channel

Transmit digital words of L bits.

Find the probability of success for a code that can correct up to M bit errors.

The conditional probability of successful reception for slow fading is

$$P(S|p) = \sum_{m=0}^{M} \binom{M}{m} \left(\frac{1}{2} \exp\left\{ - \frac{p T_b}{N_0} \right\} \right)^{m} \left(1 - \frac{1}{2} \exp\left\{ - \frac{p T_b}{N_0} \right\} \right)^{L-m}$$

Averaging over PDF of received power gives the local-mean success rate.

Closed-form solutions only exist for special cases.

Special Case

- no error correction is used ($M = 0$)
- A slow fading channel

The L-th order binomial can be rewritten as a sum of terms.

$$P(S) = \int_0^\infty \frac{1}{\bar{p}} \exp\left\{-\frac{p}{\bar{p}}\right\} \sum_{i=0}^{L} \binom{L}{i} \left(-\frac{1}{2}\right)^i \exp\left\{-\frac{ipT_b}{N_0}\right\} dp$$

After interchanging sum and integral, this gives

$$\mathrm{P}(S) = \sum_{i=0}^{L} \binom{L}{i} (-2)^{-i} \frac{N_0}{N_0 + iT_b\bar{p}}$$

$$\rightarrow 1 - \frac{N_0}{\bar{p}T_b} \sum_{i=1}^{L} \binom{L}{i} \frac{1}{i} \left(-\frac{1}{2}\right)^i$$

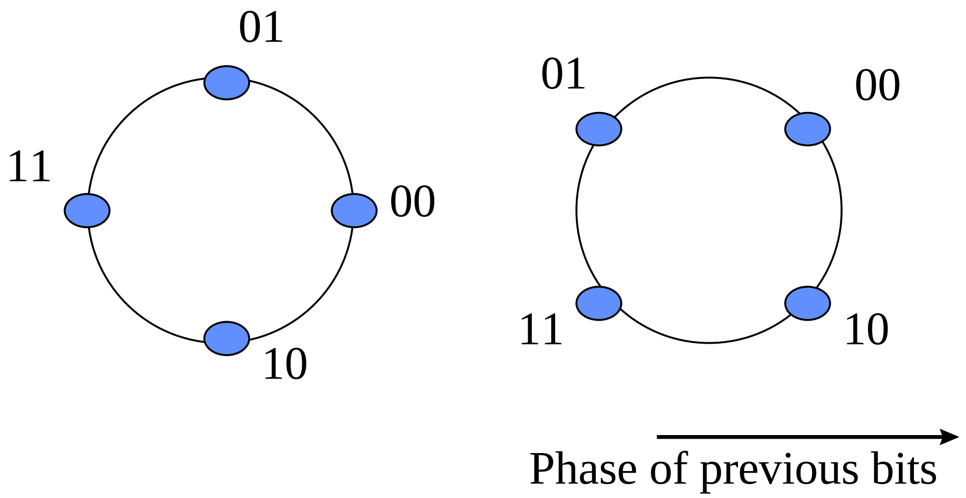
- Probability of error is inversely proportional to C/N

For slow fading, coding does not change the slope of the BER curve.

DQPSK:

Differential Quadrature Phase Shift Keying

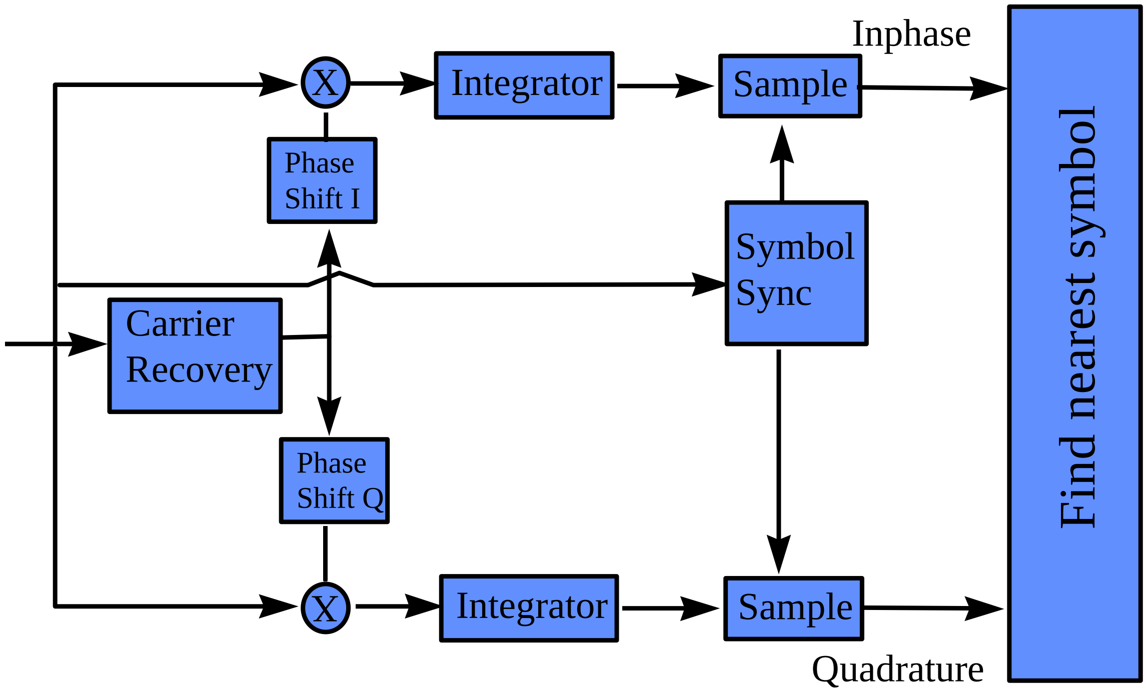
- Used in U.S. and Japanese standards for digital telephony, because of
  - simplicity of implementation,
  - good spectrum efficiency.



Initial phase: $\lambda = 0$ $\lambda = \pi/4$

Figure: Signal constellations
Relative phase shift between
successive bit intervals for D - Q PSK

QPSK Receivers



Coherent detection of QPSK signals
Phase shift I: 0, Phase shift Q: $\pi/2$

For noncoherent Detection:

- Carrier recovery is replaced by a delay line

Phase shifts

	$\lambda = 0$	$\lambda = \pi/4$
Phase Shift I	0	$\pi/4$
Phase Shift Q	$\pi/2$	$-\pi/4$

Differential Quadrature Phase Shift Keying

Instantaneous BER

Pierce (1962): BER for DQPSK,

given instantaneous amplitude ρ of wanted signal

$$P_b(error| r\ ) \;=\; Q(a,b) - \tfrac{1}{2} I_0(ab)\ \exp\left(-\frac{a^2 + b^2}{2} \right)$$

where

- $Q(a,b)$ is the Marcum Q-function, i.e.,
  - cumulative distribution of Rician pdf
  - Prob ($x > b$)

$$Q(a,b) \;=\; \int_b^{\infty} x \exp\left(-\frac{a^2 + x^2}{2} \right) I_0(ax)\, dx$$

with

- I_k is the k-th order modified Bessel function, and
- a and b are defined as

$$a = \sqrt{g\ [2 - \sqrt{2}\ ]} \quad \text{and} \quad b = \sqrt{g\ [2 + \sqrt{2}\ ]}$$

DIFFERENTIAL QPSK

The local-mean BER forRayleigh fading,

$$\overline{P_b}(error) = \frac{1}{2\sqrt{1+4\bar{g}+2\bar{g}^2}} \frac{\bar{g}\sqrt{2} + [\sqrt{2} - 1][1 + 2\bar{g} - \sqrt{1+4\bar{g}+2\bar{g}^2}\,]}{\bar{g}\sqrt{2} - [\sqrt{2} - 1][1 + 2\bar{g} - \sqrt{1+4\bar{g}+2\bar{g}^2}\,]}$$

COMPARISON BPSK, QPSK and FSK

COHERENT BPSK

$$P_b(error|r_0) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\frac{1}{2}r_0^2 T_b}{\sum_{i=1}^{N}\overline{p}_i T_b + N_0}}$$

COHERENT FSK

$$P_b(error|r_0) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\frac{1}{2}r_0^2 T_b}{\sum_{i=1}^{N}\overline{p}_i T_b + 2N_0}}$$

COHERENT QPSK

$$P_b(error|r_0) = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{\frac{1}{2}r_0^2 T_b}{\sum_{i=1}^{N}\overline{p}_i T_b + 2N_0}}$$

Discussion

- Interference affects BER in a different way than AWGN.

- Increasing T_b reduces effect of noise
  but has no effect on interference

- FSK is fairly robust

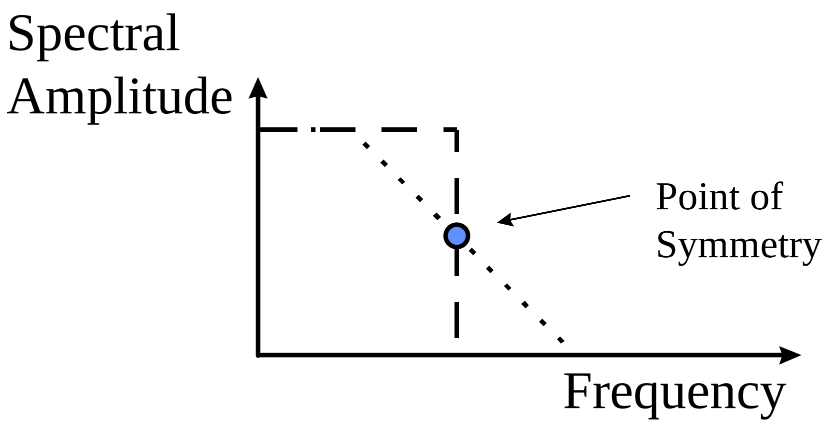
- QPSK is more vulnerable to interference than expected

  Intuition: take BPSK; double T_b; insert Q-phase;
  reduce amplitude by $\sqrt{2}$

  - interference amplitude stays constant
  - amplitude of wanted signal drops 3 dB

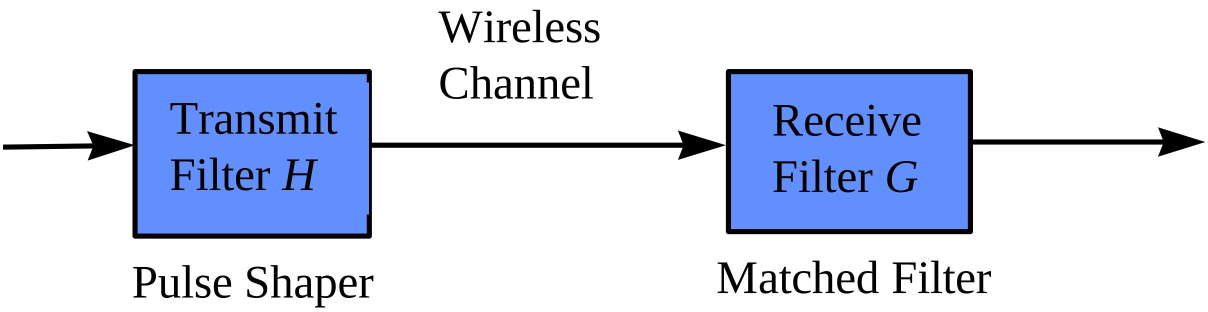
Pulse Shaping

- Pulse shaping is needed make transmit spectrum bandlimited



- Rectangular spectrum requires SINC-shaped pulses. Disadvantage: the addition of many randomly polarized, time-shifted SINC may give large signal peaks.
- Smoother behavior is guaranteed by raised-cosine type spectra.
- Symmetric spectral roll-off ensures absence of ISI

Square Root Nyquist



Two requirements for pulse shape

- Matched Filter

  Receive Filter Characteristic = Transmitwaveform

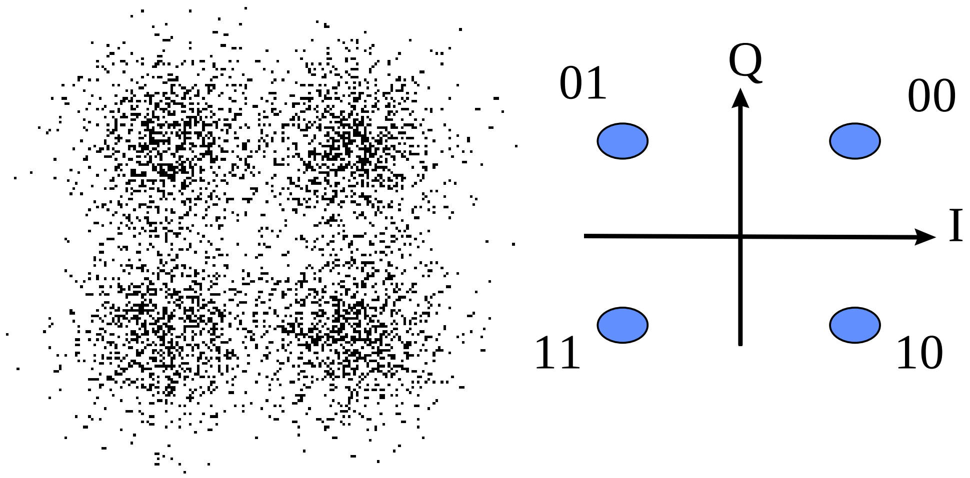
- No Intersymbol Interference

  Transfer function of all filters should be symmetric.

Take the transfer function such that

- *H G* is symmetric
- Frequency transfer G = H

Quadrature Amplitude Modulation

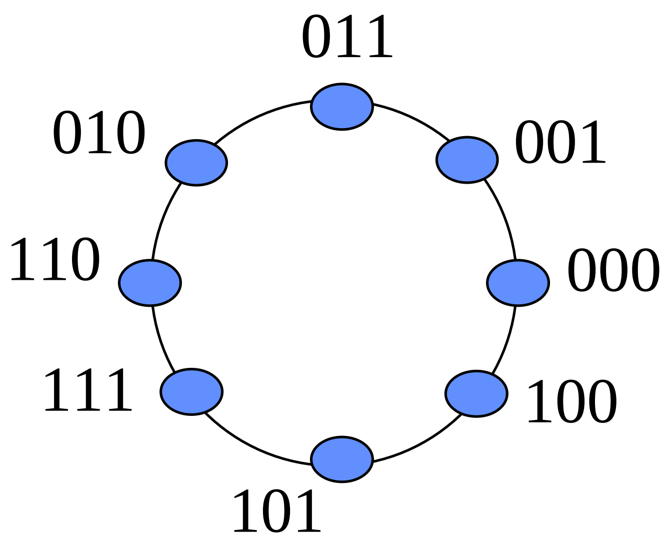


I and Q diagram of 4-QAM Signal with noise

- multi-level modulation packs more bits/sec per Hz
- But: QAM is more vulnerable to interference

  Larger reuse distance needed: Less Spectrum-Efficient?
- But: QAM is more vulnerable to AWG noise
- Application:
  - Radio relay links
  - Cable TV including "Wireless Cable" Microwave Multi-Point Video Distribution

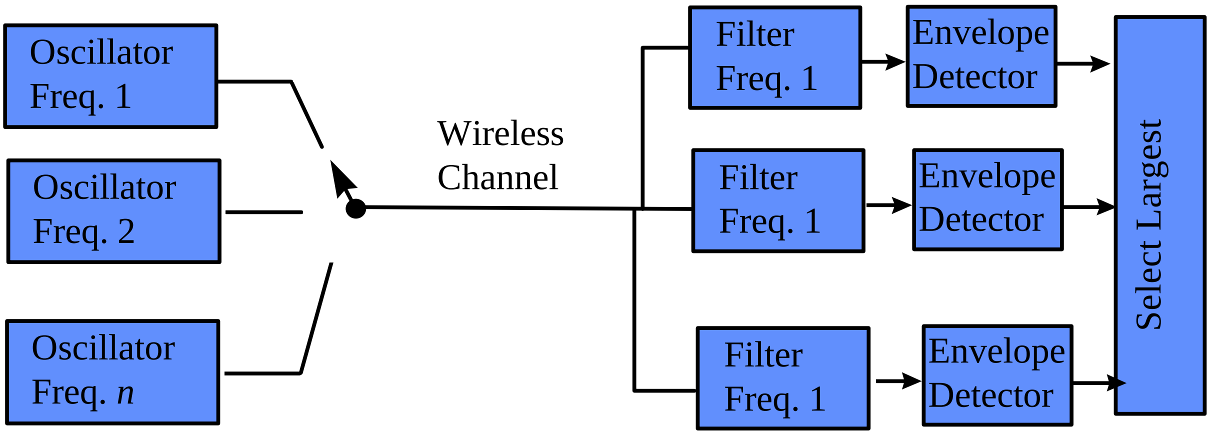
M-ary Phase Shift Keying (PSK)



M-PSK Constellation, M = 8

- For large M: Theoretically, BER worse than QAM
- Constant amplitude: simple power amplifiers
- Use Gray code
- QPSK is particularly popular ($M = 4$)
- Special case Continuous Phase- Phase Shift Keying

Frequency Shift Keying (FSK)



FSK communication system

FSK:

- Theoretically sub-optimum, compared to BPSK
- Noncoherent detection
    - Simple to implement, robust
    - Noncoherent FSK works well on fading channel
- Special case: Continous Phase FSK

Coherent Frequency Shift Keying (FSK)

Instantaneous BER

For LTI AWGN, the BER is

$$P_b(error) = \tfrac{1}{2} \operatorname{erfc} \sqrt{\frac{E_b}{2N_0}} .$$

Local-mean BER

$$\overline{P}_b(error) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + 2N_0}}$$

- Coherent FSK is 3 dB less immune to noise than BPSK (FSK is orthogonal, BPSK is antipodal)

Non-coherent FSK

Instantaneous BER

For LTI AWGN, the BER is approximately

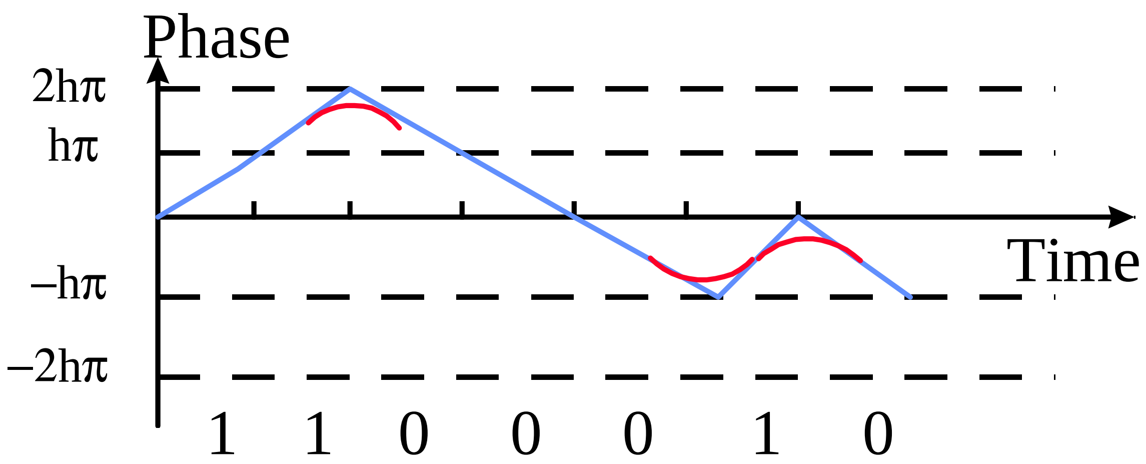
$$P_b(e) = \tfrac{1}{2}\,\exp\left\{-\frac{p_0 T_b}{2 N_0}\right\}$$

Local-mean BER

$$\overline{P_b}(error) = \frac{N_0}{\overline{p}_0 T_b + 2 N_0}$$

Continuous Phase FSK (CP FSK)

- CP reduces spectral sidelobes
- Data is embedded in the "phase trajectory"
- Can be interpreted as Phase Shift Keying or as Frequency Shift Keying



Phase trajectory for

- Binary Continuous Phase Frequency Shift Keying (blue)
- Filtered CP FSK to reduce spectral sidelobes, MSK (Red)

Minimum Shift Keying (MSK)

- Special case of CP FSK:
  phase trajectory is 'filtered'
- Special case of phase modulation
  MSK can be interpreted as a form of two time-shifted
  binary PSK signals, each with a sinusoidal envelope
- Very compact spectrum
  h is chosen as small as possible, but large enough to
  ensure orthogonal signal waveforms, $h = 0.5$
- Robust against interference
- Anti-podal signals: robust against noise
- Constant envelope
- Used in GSM, Hiperlan, ...

MSK, QPSK and Offset QPSK

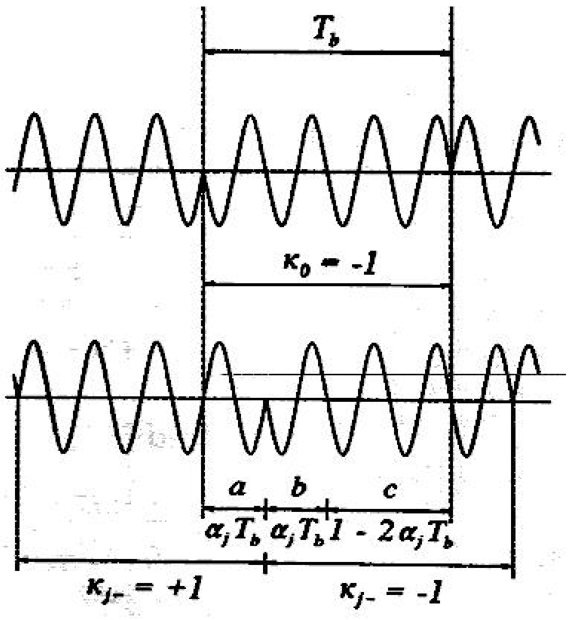
During the transmission of a single bit the MSK signal smoothly increases or reduces the signal phase by 90°.

In normal QPSK, abrupt phase changes of 90° or 180° can occur at the end of each symbol transmission (2 bits)

In Offset QPSK, abrupt phase changes of 90° occur but twice as frequent as in normal QPSK.

INTERFERING BPSK SIGNALS

The effect of interference differs significantly from the effect of AWGN



Wanted BPSK signal with a single interferer.

Effect of interfering BPSK Signal depends on

- Relative amplitude
- Phase offset
- Bit timing offset

INTERFERING FADING CARRIERS

Received signal

$$r(t) = a_0\, r_0 \cos(w_c t + f) + \sum_{i=1}^{N} r_i \cos\left(2p(f_c + f_i)t + f_i\right) + n_I$$

with bit $a_0 = \underline{+}1$. After matched filtering, the decision variable becomes

$$v = a_0\, r_0 T_b + \sum_{i=1}^{N} x_i \int_0^{T_b} \cos(w_i t)dt$$

$$+ \sum_{i=1}^{N} z_i \int_0^{T_b} \sin(w_i t)dt + n_I$$

where

ξ_i and ζ_i are inphase and quadrature component of i-th interfering signal with

$\xi_i = \rho_i \cos \theta_i,$ (zero-mean Gaussian)

$\zeta_i = \rho_i \sin \theta_i$ (zero-mean Gaussian)

and $E\xi_i^2 = E\zeta_i^2 = {}_i$.

Variance of the interference term is $\Sigma_i T_b^2 \operatorname{sinc} f_i T_b$.

Effect of BPSK Interference

- Bit synchronisation offset α_i , α_i uniformly distributed between 0 and 1.

Decision variable for a receiver in perfect lock to the wanted signal:

$$n = r_0 k_0 + \sum_{i=1}^{n} z_i \left\{ k_{i\leftarrow} a_i + k_{i\rightarrow}(1-a_i) \right\} + n_I \ .$$

This contains components from previous and next bit interfering signal

Remarks:

- Interference is NOTGaussian
- Interference cancellation is easier if bit timing is synchronous
- Interference affects synchronization; it does not "average out" in the receiver PLL

Average BER for synchronization offset

- Bit-timing offset α_i is uniformly distributed for $0 < \alpha_i < 1$

If phase reversal occurs at the instant $(k + \alpha_i)T_b$,

the variance of the interference sample is $p_t(1 - 2\alpha_i)^2$.

- If there is a timing offset, interference components are a little weaker

Assumption

- "0"'s and "1"'s are equiprobable and independent,
- the interference is Rayleigh fading, and
- the receiver locks perfectly to the wanted signal

the bit error probability is

$$\overline{P}_b = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + \overline{p}_1 T_b + N_0}}$$

$$- \frac{1}{4}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + \overline{p}_1 T_b (1 - 2a_1)^2 + N_0}} .$$

- We only modeled the effect on the decision variable, not on the timing recovery

- In practice the effect on receiver synchronization may be more significant

Average BER for Sync. Random Offset

For a Rayleigh-fading wanted signal, the average BER is

$$\overline{P}_b = \frac{1}{2} - \frac{1}{4}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + \overline{p}_1 T_b + N_0}}$$

$$+ \tfrac{1}{4}\sqrt{\frac{\overline{p}_0}{\overline{p}_1}} \ln\left(\frac{\sqrt{\overline{p}_0 T_b + \overline{p}_1 T_b + N_0} - \sqrt{\overline{p}_1 T_b}}{\sqrt{\overline{p}_0 T_b + N_0}}\right)$$

- Non-synchronous interference is effectively 1.8 dB weaker than synchronous interference

Many interfering signals

- Relevant case in Spread Spectrum systems
- Many weak signals may not have such a dramatic effect on receiver synchronization
- CLT: The sum of many weak signals is a Gaussian distributed random variable.

Variance of interference

- If an interfering signal makes a phase reversal, the variance of the interference sample is

$$E[z_i^2(1-2a_i)^2] = \frac{\overline{p_1}}{3}$$

- If no phase reversal occurs, the variance is

$$E[z_i^2] = \overline{p_1}$$

Many interfering signals

Local-mean BER

$$\overline{P}_b = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + \frac{2}{3}\overline{p}_t T_b + N_0}}$$

For Spread Spectrum with Random Codes

$$\overline{P}_b = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\overline{p}_0 T_b}{\overline{p}_0 T_b + \frac{2}{3C}\overline{p}_t T_b + N_0}}$$

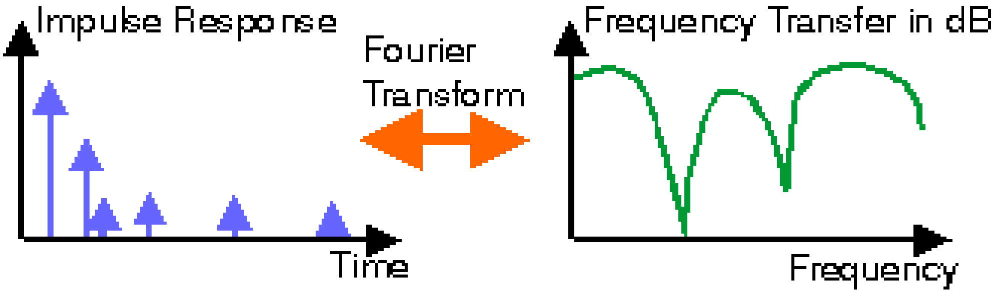
where

- C is the spread factor
- Chips of different users have random timing offset

Effect of Intersymbol Interference

The impulse response of a discrete-time multipath channel.

$$h(t) = \sum_{k=0}^{\infty} h_k d(t + k T_b)$$



Example of (discrete time) impulse response multipath channel and frequency transfer function.

Effect of Intersymbol Interference

Irreducable BER

- Excessively delayed reflections cause intersymbol interference (ISI).
- If unequalized, this results in a residual BER, even if no noise is present.
- If perfect synchronization to the first (resolvable) path, the decision variable becomes

$$n_n = a_n h_0 + \sum_{k=1}^{\infty} h_k a_{n-k}$$

For a known signal power p_0 in the first resolvable path, the BER becomes

$$P_b(error|p_0) = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{p_0 T_b}{\sum_{k=1}^{\infty} p_k T_b + N_0}}\,.$$

Effect of Intersymbol Interference

2. Noise penalty due to channel equalization

`zero forcing equalizer'

- eliminates the effect ofintersymbol interference altogether.
- Disadvantage: it excessively enhances the noise.

Signal power density spectrum $S(f)$

AWGN channel with frequency transfer function $H(f)$

Zero Forcing filter $1/H(f)$

Received power spectrum

$$R(f) = S(f) + \frac{N_0}{H(f)H^*(f)}$$

Thus signal to noise ratio

$$\frac{S}{N} = \frac{E_b T_b}{N_0}\left(\int_{B_T} \frac{1}{|H(f)|^2}\, df\right)^{-1}$$

Equalizer Approaches

- Best equalization minimizes the signal-to-interference-plus-noise, but this is not necessarily the best receiver design:

- **Maximum likelihood (ML) detector**
  Find the bit sequence that was most likely

- **Minimize the Mean Square Error** (MSE) caused by ISI and noise

  LMS: Least mean square algorithm recursively adjusts tap weights